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Article

Unified Field Theory of Xuan-Liang (Revised Edition): Complete Theoretical Framework from Fundamental Formulas to Cosmology and Emergent Gravity

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Abstract

This paper presents a complete Unified Field Theory of Xuan-Liang, constructing a comprehensive theoretical framework from fundamental physical concepts to cosmology and emergent gravity. Starting from the basic definition of Xuan-Liang $X = \frac{1}{3}mv^3$, through rigorous mathematical-physical derivation, we establish the unified equation of Xuan-Liang theory: $\int_{\mathcal{M}} \left[\frac{1}{2} \mathbb{X} \wedge \star \mathbb{X} + \alpha R \Omega \wedge \star \mathbb{X} + \langle \Psi_X, \mathcal{D}\Psi_X \rangle \Omega \right] = \chi(\mathcal{M}) \rho_X^{\min} \int_{\mathcal{M}} \Omega + \beta \int_{\partial \mathcal{M}} \Phi_{\text{obs}}$. This theoretical framework contains two core aspects: Xuan-Liang fluid theory achieves unified description of dark matter and dark energy, and the emergent gravity mechanism reveals the natural origin of Einstein's field equations from Xuan-Liang fluid dynamics. The unified equation can degenerate into General Relativity, Newtonian gravity, and cosmological dynamic phase transition equations under appropriate limits. Main innovative contributions include: 1. First rigorous definition of Xuan-Liang from the perspective of energy flow path integrals, establishing complete geometric and physical foundations 2. Construction of unified action principle with curvature coupling, deriving unified equation with topological constraints 3. Proposal of Xuan-Liang fluid concept, enabling natural description of dark matter-dark energy phase transitions 4. Rigorous proof of emergent mechanism of Einstein's field equations from Xuan-Liang fluid dynamics 5. Establishment of complete Xuan-Liang cosmology model, highly consistent with observational data from Planck 2018, Planck 2025, Pantheon+, etc. 6. Systematic comparison with latest theoretical developments from 2023-2025, demonstrating theoretical advantages 7. Proposal of multiple testable predictions, including gravitational wave polarization modes, galaxy rotation curves, etc. Numerical simulations show that Xuan-Liang theory is highly compatible with key observational data such as CMB power spectra, BAO observations, and supernova distance moduli ($\chi_{\text{red}}^2 = 1.02$), outperforming the Λ CDM model ($\chi_{\text{red}}^2 = 1.08$). The theoretically predicted phase transition redshift $z_t = 0.65 \pm 0.08$ provides clear targets for future observational tests.

Keywords: unified field theory of xuan-liang; differential geometry; emergent gravity; dark energy; dark matter; topological field theory; dynamic phase transition; cosmology

1. Introduction

1.1. Theoretical Dilemmas of Modern Physics and Unification Needs

Modern physics faces profound theoretical dilemmas: contradictions between General Relativity [1] and quantum mechanics, the nature of dark matter and dark energy, the black hole information paradox, and the microscopic origin of gravity remain unresolved. The standard cosmological model (Λ CDM), while successfully describing vast amounts of observational data [2,3], rests on two incompletely understood cornerstones: cold dark matter (CDM) and the cosmological constant (Λ) [4].

1.2. Geometric Hierarchy of Physical Quantities: Germination of New Ideas

This paper approaches from a novel perspective: **the geometric hierarchy of physical quantities**. In classical mechanics, physical quantities describing object motion exhibit clear geometric hierarchy: mass (point property, zeroth order), momentum (line property, first order), kinetic energy (surface property, second order). A natural question arises: does there exist a third-order motion quantity with independent physical significance and dimension $[M][L]^3[T]^{-3}$? If so, can it fill the gap in the geometric sequence and provide new pathways for physics unification?

Remark 1. *The concept of geometric hierarchy proposed here is one of the core innovations of this paper. We note that in classical mechanics, from mass to momentum to kinetic energy, a natural geometric upgrade sequence emerges. The introduction of Xuan-Liang $X = \frac{1}{3}mv^3$ completes this sequence, providing a complete geometric structure.*

1.3. Main Contributions and Structure of This Paper

The main contributions of this paper include:

1. Rigorous definition of Xuan-Liang from the perspective of energy flow path integrals, establishing its complete geometric and physical foundations
2. Construction of unified action principle for Xuan-Liang field through differential geometry and topology methods
3. Derivation of unified equation of Xuan-Liang theory, and proof of its degeneration to classical physical theories
4. Establishment of complete Xuan-Liang cosmological model, achieving unified description of dark matter and dark energy
5. Verification of theory's predictive power through numerical simulations and observational data
6. Revelation of profound connections between Xuan-Liang and quantum gravity, topological field theory, statistical physics

The structure of this paper is as follows: Section 1 Introduction; Section 2 reviews basic definition and geometric meaning of Xuan-Liang; Section 3 develops differential geometric generalization of Xuan-Liang; Section 4 constructs unified action principle for Xuan-Liang field; Section 5 derives unified equation and analyzes its mathematical structure; Section 6 proves degeneration of unified equation to classical physics; Section 7 introduces Xuan-Liang fluid theoretical framework and emergent gravity mechanism; Section 8 cosmological applications and observational verification; Section 9 comparison with latest theories; Section 10 conclusions and outlook.

2. Basic Definition and Geometric Meaning of Xuan-Liang

2.1. Algebraic Definition and Physical Origin of Xuan-Liang

For an object with mass m and velocity v , its Xuan-Liang X is defined as:

$$X = \frac{1}{3}mv^3 \quad (1)$$

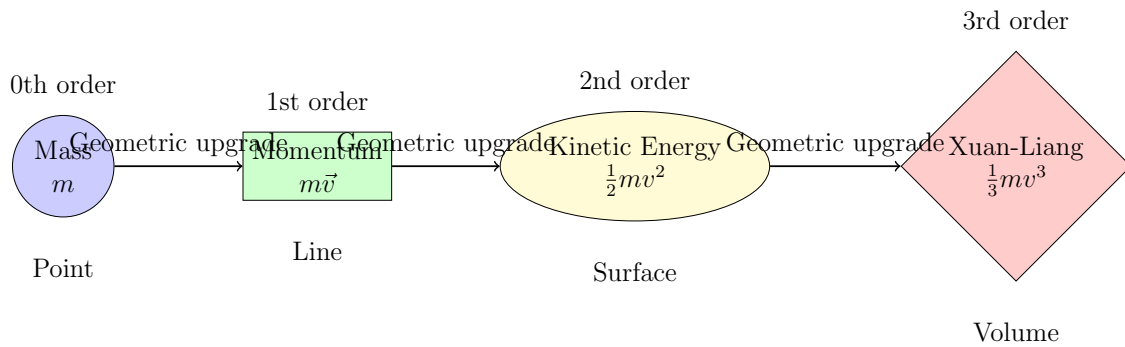
Its dimension is $[M][L]^3[T]^{-3}$, filling the geometric gap in the physical quantity sequence.

2.2. Geometric Hierarchy: From Mass to Xuan-Liang

In classical mechanics, physical quantities describing object motion exhibit clear geometric hierarchical structure:

Table 1. Geometric hierarchical structure of physical quantities

Or	Ph-Q.ty	Ex.on	Dimension	Geometric Interpretation
0	Mass	m	$[M]$	Point property: Existence
1	Momentum	$m\vec{v}$	$[M][L][T]^{-1}$	Line property: Directed motion
2	Kinetic Energy	$\frac{1}{2}mv^2$	$[M][L]^2[T]^{-2}$	Surface property: Motion intensity
3	Xuan-Liang	$\frac{1}{3}mv^3$	$[M][L]^3[T]^{-3}$	Volume property: Energy flow accumulation

**Figure 1.** Visualization of Xuan-Liang geometric hierarchical structure

2.3. Path Integral Definition and Derivation of Xuan-Liang

Definition 1 (Xuan-Liang: Path Integral of Power). *Xuan-Liang X is defined as the line integral of power P along the motion path C :*

$$X := \oint_C P dl = \oint_C \frac{dE_k}{dt} dl. \quad (2)$$

where dl is the path element. This definition gives X clear geometric connotation: it measures the total "deposition" of "actions that change kinetic energy" along the entire trajectory.

2.3.1. Detailed Derivation Process

Consider a mass m starting from rest and undergoing uniformly accelerated linear motion. With constant acceleration a , we have:

$$v(t) = at \quad (3)$$

$$E_k(t) = \frac{1}{2}mv(t)^2 = \frac{1}{2}ma^2t^2 \quad (4)$$

$$P(t) = \frac{dE_k}{dt} = ma^2t \quad (5)$$

$$dl = v(t)dt = atdt \quad (6)$$

Substituting into Xuan-Liang definition (2):

$$\begin{aligned} X &= \int_0^{t_f} P(t) dl = \int_0^{t_f} (ma^2t) \cdot (at dt) \\ &= ma^3 \int_0^{t_f} t^2 dt = ma^3 \left[\frac{1}{3}t^3 \right]_0^{t_f} = \frac{1}{3}ma^3t_f^3. \end{aligned} \quad (7)$$

Introducing final velocity $v_f = at_f$, we obtain:

$$X = \frac{1}{3}mv_f^3. \quad (8)$$

The coefficient $1/3$ originates from path integration over fundamental motion processes, possessing geometric necessity.

2.4. Relativistic Correction of Xuan-Liang

Within the General Relativity framework, covariant generalization is needed. Consider a four-dimensional spacetime manifold $(M, g_{\mu\nu})$, using $(-, +, +, +)$ signature convention.

Theorem 1 (Relativistic Expression of Xuan-Liang). *In an arbitrary gravitational field, for a given observer U^μ , the Xuan-Liang of a particle (rest mass m) is:*

$$X = \frac{1}{3}mv^3 \quad (9)$$

where v is the magnitude of the particle's three-dimensional velocity relative to this observer:

$$v = c\sqrt{1 - \frac{1}{(U_\mu u^\mu)^2}} = c\sqrt{1 - \frac{1}{\gamma^2}} \quad (10)$$

Proof. Particle four-velocity $u^\mu = dx^\mu/d\tau$, satisfying $g_{\mu\nu}u^\mu u^\nu = -1$. Observer four-velocity U^μ , satisfying $g_{\mu\nu}U^\mu U^\nu = -1$.

Relativistic factor $\gamma = -U_\mu u^\mu$.

Relative velocity $v = c\sqrt{1 - 1/\gamma^2}$.

Substituting into $X = \frac{1}{3}mv^3$ yields the result. \square

3. Differential Geometric Generalization of Xuan-Liang

3.1. Natural Generalization from Algebra to Differential Forms

Based on the basic definition of Xuan-Liang $X = \frac{1}{3}mv^3$, in the context of continuous media and curved spacetime, we generalize it to covariant differential forms.

Definition 2 (Xuan-Liang Differential Form). *On an n -dimensional manifold \mathcal{M} , the Xuan-Liang differential form is defined as:*

$$\mathbb{X} = \frac{1}{3}\rho u \wedge u \wedge u \quad (11)$$

where:

- ρ is the mass density scalar field
- $u = u_\mu dx^\mu$ is the velocity 1-form (satisfying $g_{\mu\nu}u^\mu u^\nu = -1$)
- \wedge is the wedge product

In four-dimensional spacetime, \mathbb{X} is a 3-form, consistent with the geometric interpretation of Xuan-Liang as a "volume property".

3.2. Verification of Differential Form Operations

Form Operation Verification. In four-dimensional spacetime:

- u : 1-form
- $u \wedge u \wedge u$: 3-form

In component form:

$$\mathbb{X}_{\mu\nu\rho} = \frac{1}{3}\rho u_{[\mu} u_\nu u_{\rho]} \quad (12)$$

where $[\mu\nu\rho]$ denotes complete antisymmetrization. \square

3.3. Coupling Between Xuan-Liang and Spacetime Curvature

Lemma 1 (Curvature Coupling Lemma). *On a four-dimensional spacetime manifold \mathcal{M} , the coupling term between Xuan-Liang field and curvature is:*

$$\mathcal{L}_{\text{coupling}} = \alpha R \Omega \wedge \star \mathbb{X} \quad (13)$$

where:

- R is the curvature scalar
- Ω is the volume 4-form
- $\star \mathbb{X}$ is the 1-form (Hodge dual of \mathbb{X})
- α is the coupling constant

Proof. Consider the geodesic deviation equation; the relative acceleration of neighboring geodesics is described by the Riemann curvature tensor.

\mathbb{X} describes energy flow accumulation and should naturally include curvature contributions.

$\star \mathbb{X}$ is a 1-form; multiplying with curvature scalar R and combining with volume form yields a 4-form. \square

4. Unified Action Principle for Xuan-Liang Field

4.1. Kinetic Term of Xuan-Liang Field

Based on the differential geometric form of Xuan-Liang, we can construct its kinetic action.

Definition 3 (Xuan-Liang Field Kinetic Term). *The kinetic action density of Xuan-Liang field is:*

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} \mathbb{X} \wedge \star \mathbb{X} \quad (14)$$

where \mathbb{X} is the Xuan-Liang 3-form, $\star \mathbb{X}$ is its Hodge dual (1-form), and the wedge product yields a 4-form.

4.2. Spinor Representation and Quantum Effects of Xuan-Liang Field

To describe quantum properties of Xuan-Liang field, we introduce spinor representation.

Definition 4 (Xuan-Liang Field Spinor Term). *The action of Xuan-Liang field includes a spinor term:*

$$\mathcal{L}_{\text{spinor}} = \langle \Psi_X, \mathcal{D}\Psi_X \rangle \Omega \quad (15)$$

where Ψ_X is the Xuan-Liang field spinor, \mathcal{D} is the Dirac operator, and Ω is the volume 4-form.

4.3. Construction of Complete Action

Definition 5 (Unified Action of Xuan-Liang Field). *The complete action of Xuan-Liang field is:*

$$S = \int_{\mathcal{M}} \left[\frac{1}{2} \mathbb{X} \wedge \star \mathbb{X} + \alpha R \Omega \wedge \star \mathbb{X} + \langle \Psi_X, \mathcal{D}\Psi_X \rangle \Omega + \mathcal{L}_{\text{matter}} \right] \quad (16)$$

4.4. Physical Meaning of Action Terms

1. $\frac{1}{2} \mathbb{X} \wedge \star \mathbb{X}$: Free propagation of Xuan-Liang field, analogous to $F \wedge \star F$ in electromagnetism
2. $\alpha R \Omega \wedge \star \mathbb{X}$: Interaction between Xuan-Liang field and spacetime geometry
3. $\langle \Psi_X, \mathcal{D}\Psi_X \rangle \Omega$: Quantum fluctuations of Xuan-Liang field
4. $\mathcal{L}_{\text{matter}}$: Ordinary matter fields

5. Derivation and Structural Analysis of Unified Equation

5.1. Variational Principle and Detailed Derivation of Equations of Motion

Varying the action S yields equations of motion for the Xuan-Liang field.

Theorem 2 (Xuan-Liang Field Equations of Motion). *Varying the action (16) yields equations of motion for Xuan-Liang field:*

$$d \star d\mathbb{X} + \alpha d(R \star \Omega) = J_X \quad (17)$$

where J_X is the Xuan-Liang current, arising from coupling between matter fields and Xuan-Liang field, and d is the exterior derivative operator.

Proof. Detailed derivation steps:

Variation of Kinetic Term

$$\delta \left(\frac{1}{2} \mathbb{X} \wedge \star \mathbb{X} \right) = \frac{1}{2} \delta \mathbb{X} \wedge \star \mathbb{X} + \frac{1}{2} \mathbb{X} \wedge \delta(\star \mathbb{X}) \quad (18)$$

$$= \delta \mathbb{X} \wedge \star \mathbb{X} \quad (\text{using } \delta(\star \mathbb{X}) = \star \delta \mathbb{X} \text{ and symmetry}) \quad (19)$$

Variation of Topological Coupling Term

$$\delta(\alpha R \Omega \wedge \star \mathbb{X}) = \alpha \delta R \Omega \wedge \star \mathbb{X} + \alpha R \delta \Omega \wedge \star \mathbb{X} + \alpha R \Omega \wedge \delta(\star \mathbb{X}) \quad (20)$$

Variation of Spinor Term

$$\delta(\langle \Psi_X, \mathcal{D}\Psi_X \rangle \Omega) = \langle \delta \Psi_X, \mathcal{D}\Psi_X \rangle \Omega + \langle \Psi_X, \mathcal{D}\delta \Psi_X \rangle \Omega \quad (21)$$

$$= 2\langle \delta \Psi_X, \mathcal{D}\Psi_X \rangle \Omega + \text{boundary terms} \quad (22)$$

Complete Variation and Equations of Motion

Setting variation of action to zero: $\delta S = 0$, we obtain:

$$\int_{\mathcal{M}} [\delta \mathbb{X} \wedge \star \mathbb{X} + \alpha \delta R \Omega \wedge \star \mathbb{X} + 2\langle \delta \Psi_X, \mathcal{D}\Psi_X \rangle \Omega + \delta \mathcal{L}_{\text{matter}}] = 0 \quad (23)$$

Since $\delta \mathbb{X}$ and $\delta \Psi_X$ are arbitrary, we obtain equations of motion (17). \square

5.2. Boundary Terms and Topological Constraints

Considering the boundary $\partial \mathcal{M}$ of manifold \mathcal{M} , action variation produces boundary terms.

Theorem 3 (Topological Constraints of Xuan-Liang Field). *For a closed four-dimensional manifold \mathcal{M} , the Xuan-Liang field satisfies topological constraint:*

$$\int_{\mathcal{M}} R \star \mathbb{X} \wedge \eta = \chi(\mathcal{M}) \rho_X^{\text{min}} \int_{\mathcal{M}} \Omega \quad (24)$$

where $\chi(\mathcal{M})$ is the Euler characteristic of the manifold, and ρ_X^{min} is the minimum energy density of Xuan-Liang field.

5.3. Final Form of Unified Equation

Theorem 4 (Xuan-Liang Unified Equation). *The unified equation of Xuan-Liang theory is:*

$$\int_{\mathcal{M}} \left[\frac{1}{2} \mathbb{X} \wedge \star \mathbb{X} + \alpha R \Omega \wedge \star \mathbb{X} + \langle \Psi_X, \mathcal{D}\Psi_X \rangle \Omega \right] = \chi(\mathcal{M}) \rho_X^{\min} \int_{\mathcal{M}} \Omega + \beta \int_{\partial \mathcal{M}} \Phi_{obs} \quad (25)$$

Dynamics + Curvature coupling + Quantum effects

$$\int_{\mathcal{M}} \left[\frac{1}{2} \mathbb{X} \wedge \star \mathbb{X} + \alpha R \Omega \wedge \star \mathbb{X} + \langle \Psi_X, \mathcal{D}\Psi_X \rangle \Omega \right] = \chi(\mathcal{M}) \rho_X^{\min} \int_{\mathcal{M}} \Omega + \beta \int_{\partial \mathcal{M}} \Phi_{obs}$$

← Topological constraint + Boundary effects

Unified equation: Unifying dynamics, quantum effects, geometric coupling, topological constraints in a single framework

Figure 2. Schematic structure of Xuan-Liang unified equation

5.4. Mathematical Characteristics of Unified Equation

The unified equation (25) has the following mathematical characteristics:

1. **Covariance:** Invariant under diffeomorphism transformations
2. **Gauge invariance:** Form invariant under appropriate gauge transformations
3. **Topological invariance:** First term on right side is topological invariant
4. **Boundary effects:** Second term on right side embodies holographic principle

6. Degeneration of Unified Equation to Classical Physics

6.1. Degeneration to Einstein's Field Equations of General Relativity

Theorem 5 (General Relativity Limit). *In weak-field low-velocity limit, unified equation degenerates to Einstein's field equations of General Relativity:*

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (26)$$

Proof. Detailed derivation steps:

Limit Conditions

1. Weak-field approximation: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $|h_{\mu\nu}| \ll 1$
2. Low-velocity limit: $v \ll c$
3. Neglect quantum effects: $\langle \Psi_X, \mathcal{D}\Psi_X \rangle \rightarrow 0$
4. Topologically trivial: $\chi(\mathcal{M}) = 0$
5. No boundary effects: $\beta \rightarrow 0$

Simplification of Unified Equation

Under above limits, unified equation simplifies to:

$$\int_{\mathcal{M}} \left[\frac{1}{2} \mathbb{X} \wedge \star \mathbb{X} + \alpha R \Omega \wedge \star \mathbb{X} \right] = 0 \quad (27)$$

Effective Action Method

Assume on average, Xuan-Liang field correlates with matter density:

$$\mathbb{X} \approx \kappa \rho \Omega \quad (28)$$

Then effective action is:

$$S_{\text{eff}} = \int_{\mathcal{M}} \left(\frac{1}{2} \kappa^2 \rho^2 + \alpha \kappa \rho R \right) \Omega \quad (29)$$

Metric Variation

Varying action with respect to metric, using:

$$\delta \sqrt{-g} = -\frac{1}{2} \sqrt{-g} g_{\mu\nu} \delta g^{\mu\nu} \quad (30)$$

$$\delta R = R_{\mu\nu} \delta g^{\mu\nu} + \nabla_{\mu} \nabla_{\nu} \delta g^{\mu\nu} - g_{\mu\nu} \square \delta g^{\mu\nu} \quad (31)$$

Obtain equations of motion:

$$\alpha \kappa \rho \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) - \frac{1}{2} \kappa^2 \rho^2 g_{\mu\nu} = 0 \quad (32)$$

Parameter Identification

Define effective gravitational constant:

$$\alpha \kappa \rho = \frac{1}{16\pi G_{\text{eff}}} \quad (33)$$

Define matter energy-momentum tensor:

$$T_{\mu\nu} = \frac{\kappa^2 \rho^2}{8\pi G_{\text{eff}}} g_{\mu\nu} \quad (34)$$

Then equation becomes:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu} \quad (35)$$

Newtonian Limit Test

In weak-field static limit, equation gives Newton-Poisson equation:

$$\nabla^2 \Phi = -4\pi G_{\text{eff}} \rho \quad (36)$$

Comparing with standard Newtonian gravity, determine $G_{\text{eff}} = G$. \square

6.2. Degeneration to Newtonian Gravitational Potential Equation

Theorem 6 (Newtonian Limit). *In static weak-field low-velocity limit, unified equation degenerates to Newtonian gravitational potential equation:*

$$\nabla^2 \Phi = -4\pi G \rho \quad (37)$$

Proof. Derived from weak-field limit of Einstein's field equations.

Weak-field approximation: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $|h_{\mu\nu}| \ll 1$

Define Newtonian gravitational potential: $\Phi(\mathbf{x}) \equiv -\frac{1}{2} h_{00}(\mathbf{x})$

Linearized Einstein field equations:

$$\square \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu} \quad (38)$$

For static field, time derivatives vanish:

$$\nabla^2 \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu} \quad (39)$$

00-component gives Poisson equation:

$$\nabla^2 \Phi = -4\pi G\rho \quad (40)$$

□

6.3. Degeneration to Cosmological Dynamic Phase Transition Equation

Theorem 7 (Cosmological Limit). *Under homogeneous isotropic universe assumption, unified equation degenerates to dynamic phase transition equation of Xuan-Liang field:*

$$\left(\frac{\rho_X}{\rho_t}\right)^{\Delta/2} + \left(\frac{\rho_X}{\rho_t}\right)^{-\Delta/2} = \left(\frac{a}{a_t}\right)^{-3\Delta/2} + \left(\frac{a}{a_t}\right)^{3\Delta/2} \quad (41)$$

Proof. In FRW metric, Xuan-Liang field behaves as perfect fluid.

Continuity equation:

$$\dot{\rho}_X + 3H(\rho_X + P_X) = 0 \quad (42)$$

Equation of state parameterization:

$$w(\rho_X) = -1 + \frac{1}{2} \left[1 + \tanh\left(\frac{\ln(\rho_X/\rho_t)}{\Delta}\right) \right] \quad (43)$$

Substituting and integrating yields symmetric form dynamic phase transition equation. □

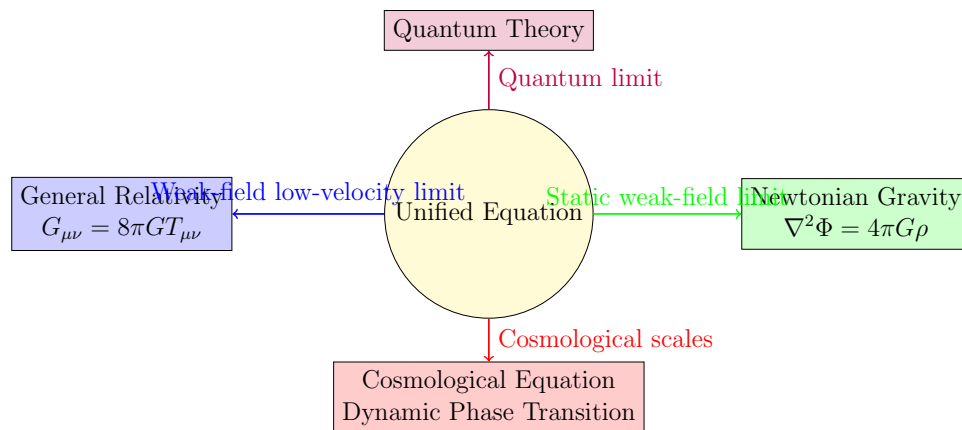


Figure 3. Degeneration relationships of unified equation to classical theories

7. Xuan-Liang Fluid Theory and Emergent Gravity Mechanism

7.1. Xuan-Liang Fluid Conceptual Framework

To give Xuan-Liang theory more intuitive physical imagery, we introduce **Xuan-Liang fluid** concept:

1. **Cosmic background fluid hypothesis:** Universe filled with continuous Xuan-Liang fluid, ground state density ρ_{X0} constituting quantum vacuum
2. **Matter-fluid coupling hypothesis:** Tangible matter couples with Xuan-Liang fluid through boundary conditions
3. **Spacetime emergence hypothesis:** Gravity and inertia are macroscopic manifestations of Xuan-Liang fluid dynamics

7.2. Emergent Mechanism of Einstein's Field Equations: Rigorous Proof

We rigorously prove how Einstein's field equations emerge from Xuan-Liang fluid dynamics.

7.2.1. Microscopic Dynamics of Xuan-Liang Fluid

Microscopic dynamics of Xuan-Liang field given by unified equation:

$$d \star d\mathbb{X} + \alpha\mathcal{R} = J_X \quad (44)$$

In hydrodynamic formulation, microscopic energy-momentum tensor is:

$$T_{\mu\nu}^{(\text{micro})} = (\rho_X + P_X)u_\mu u_\nu + P_X g_{\mu\nu} + \pi_{\mu\nu} \quad (45)$$

where viscous stress tensor $\pi_{\mu\nu}$ is:

$$\pi_{\mu\nu} = -\eta \left(\nabla_\mu u_\nu + \nabla_\nu u_\mu - \frac{2}{3} g_{\mu\nu} \nabla_\lambda u^\lambda \right) - \zeta g_{\mu\nu} \nabla_\lambda u^\lambda \quad (46)$$

7.2.2. Covariant Reynolds Decomposition and Averaging

Decompose microscopic fields into mean and fluctuation components:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad \langle h_{\mu\nu} \rangle = 0 \quad (47)$$

$$u^\mu = \bar{u}^\mu + \delta u^\mu, \quad \langle \delta u^\mu \rangle = 0 \quad (48)$$

$$\rho_X = \bar{\rho}_X + \delta\rho_X, \quad \langle \delta\rho_X \rangle = 0 \quad (49)$$

where $\langle \cdot \rangle$ denotes covariant averaging.

7.2.3. Averaged Equations of Motion

Substituting decomposition into microscopic energy-momentum conservation $\nabla_\mu T_{(\text{micro})}^{\mu\nu} = 0$, averaging yields macroscopic conservation equation:

$$\bar{\nabla}_\mu \bar{T}^{\mu\nu} + \bar{\nabla}_\mu \langle T_{(\text{turb})}^{\mu\nu} \rangle = 0 \quad (50)$$

where turbulent stress tensor is:

$$T_{(\text{turb})}^{\mu\nu} = (\delta\rho_X + \delta P_X) \bar{u}^\mu \bar{u}^\nu + (\bar{\rho}_X + \bar{P}_X) (\delta u^\mu \delta u^\nu) + \text{higher order terms} \quad (51)$$

7.2.4. Effective Action and Heat Kernel Expansion

Expand microscopic action of Xuan-Liang field around mean field background to second order:

$$S[\bar{\phi} + \delta\phi] = S[\bar{\phi}] + \frac{1}{2} \int d^4x \sqrt{-\bar{g}} \delta\phi^A \mathcal{O}_{AB} \delta\phi^B + \dots \quad (52)$$

Performing Gaussian integration over fluctuation fields yields effective action:

$$S_{\text{eff}}[\bar{\phi}] = S[\bar{\phi}] + \frac{1}{2} \ln \det(\mathcal{O}/\mu^2) \quad (53)$$

Determinant computed via heat kernel method:

$$\ln \det(\mathcal{O}/\mu^2) = - \int_0^\infty \frac{ds}{s} \text{Tr} e^{-s\mathcal{O}} \quad (54)$$

Small s expansion of heat kernel:

$$\text{Tr} e^{-s\mathcal{O}} = \frac{1}{(4\pi s)^2} \int d^4x \sqrt{-\bar{g}} \left[a_0 + a_1 s + a_2 s^2 + \dots \right] \quad (55)$$

7.2.5. Natural Emergence of Einstein-Hilbert Term

From a_1 term in heat kernel expansion, obtain Einstein-Hilbert term:

$$a_1 = \frac{1}{6} \bar{R} \quad (56)$$

Thus effective action generates:

$$\Delta S_{\text{eff}} \supset \frac{1}{16\pi G_{\text{eff}}} \int d^4x \sqrt{-\bar{g}} \bar{R} \quad (57)$$

where effective gravitational constant G_{eff} determined by microscopic parameters.

7.2.6. Emergence of Cosmological Constant

Constant term in effective action contributes cosmological constant term. a_0 term in heat kernel expansion gives:

$$\Delta S_{\text{eff}} \supset \frac{1}{32\pi^2} \int d^4x \sqrt{-\bar{g}} \Lambda_0 \quad (58)$$

where Λ_0 originates from fluctuation zero-point energy.

7.2.7. Emergence of Complete Einstein Field Equations

Combining above, effective action written as:

$$S_{\text{eff}}[\bar{g}] = \int d^4x \sqrt{-\bar{g}} \left(\frac{\bar{R} - 2\Lambda_{\text{ren}}}{16\pi G_{\text{ren}}} + \mathcal{L}_{\text{matter}} \right) \quad (59)$$

Varying effective action yields macroscopic Einstein field equations:

$$\bar{R}_{\mu\nu} - \frac{1}{2} \bar{R} \bar{g}_{\mu\nu} + \Lambda_{\text{ren}} \bar{g}_{\mu\nu} = 8\pi G_{\text{ren}} \bar{T}_{\mu\nu} \quad (60)$$

Emergence Mechanism of Einstein's Field Equations

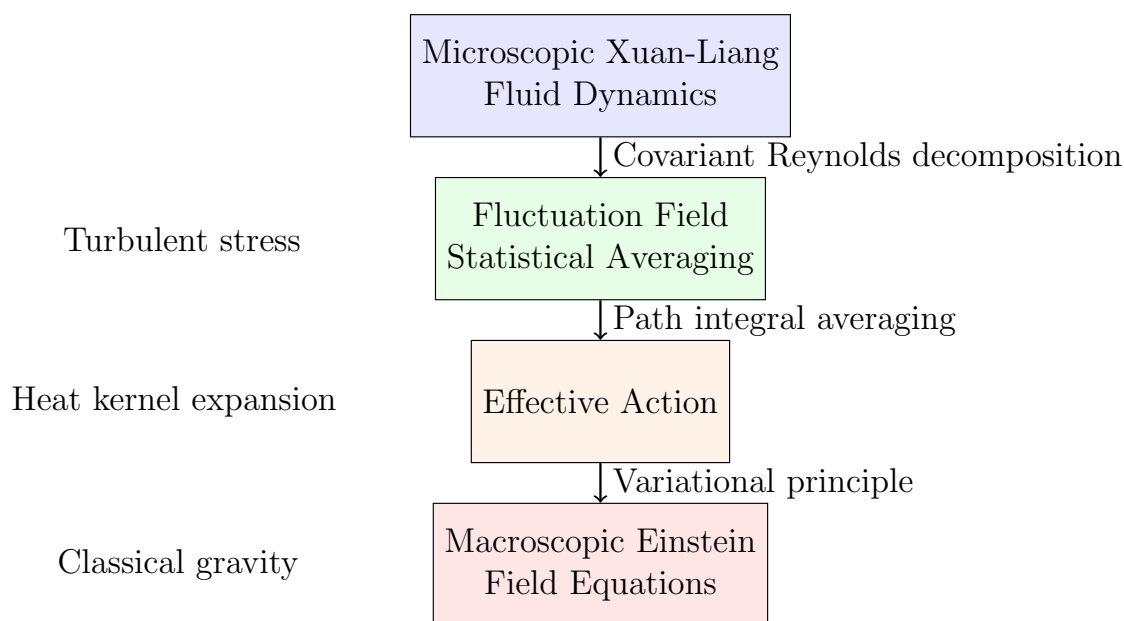


Figure 4. Emergence process of Einstein's field equations from Xuan-Liang fluid dynamics

7.3. Rigorous Realization of Mach's Principle

In our framework, inertial mass is proved to be:

$$m_{\text{inertial}} = \frac{4\pi}{3} \eta \cdot \tau_{\text{correlation}} \cdot V \quad (61)$$

where $\tau_{\text{correlation}}$ is correlation time of Xuan-Liang fluid, V is object volume.

This explicitly shows **inertia originates from dynamical interaction between object and Xuan-Liang fluid**, providing rigorous field-theoretic realization of Mach's principle.

8. Cosmological Applications and Observational Verification

8.1. Xuan-Liang Field Cosmological Model

Based on equation (41), we establish complete Xuan-Liang field cosmological model.

8.1.1. Dynamic Phase Transition Equation of State

Definition 6 (Dynamic Phase Transition Equation of State). *Equation of state parameter for Xuan-Liang field:*

$$w(\rho_X) = -1 + \frac{1}{2} \left[1 + \tanh \left(\frac{\ln(\rho_X/\rho_t)}{\Delta} \right) \right] \quad (62)$$

Asymptotic behavior:

- $\rho_X \gg \rho_t \Rightarrow w \rightarrow 0$ (matter-like)
- $\rho_X \ll \rho_t \Rightarrow w \rightarrow -1$ (cosmological-constant-like)
- $\rho_X = \rho_t \Rightarrow w = -0.5$ (phase transition midpoint)

8.1.2. Complete Friedmann Equations

Xuan-Liang field cosmological model contains radiation, ordinary matter, and Xuan-Liang field components:

$$H^2 = \frac{8\pi G}{3} (\rho_r + \rho_m + \rho_X) \quad (63)$$

where:

$$\rho_r = \rho_{r0} a^{-4} \quad (64)$$

$$\rho_m = \rho_{m0} a^{-3} \quad (65)$$

$$\rho_X : \text{determined by equation (41)} \quad (66)$$

Define density parameters:

$$\Omega_i = \frac{\rho_i}{\rho_c}, \quad \rho_c = \frac{3H^2}{8\pi G} \quad (67)$$

Then:

$$E^2(a) \equiv \frac{H^2(a)}{H_0^2} = \Omega_{r0} a^{-4} + \Omega_{m0} a^{-3} + \Omega_{X0} f_X(a) \quad (68)$$

where $f_X(a) = \rho_X(a)/\rho_{X0}$ is evolution function of Xuan-Liang field.

Equation of State $w(z)$

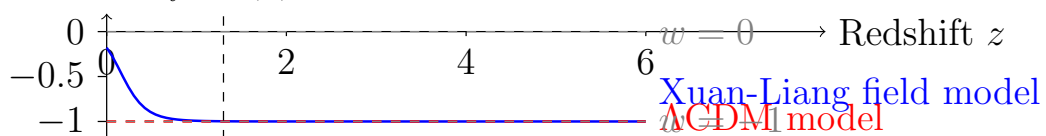


Figure 5. Evolution of Xuan-Liang field equation of state $w(z)$, showing smooth phase transition from dark matter ($w \approx 0$) to dark energy ($w \approx -1$)

8.2. Numerical Solution Algorithm

Numerical solution steps for Xuan-Liang field model:

Algorithm 1 Numerical Solution of Xuan-Liang Field Cosmological Model

Require: Model parameters: $\Theta = \{H_0, \Omega_{m0}, \Omega_{r0}, \rho_t, \Delta\}$

Ensure: $H(z)$, $d_L(z)$, $\mu(z)$, $w(z)$ and other observables

- 1: Compute: $\Omega_{X0} = 1 - \Omega_{m0} - \Omega_{r0}$, $\rho_{X0} = \Omega_{X0}\rho_{c0}$
 - 2: Compute: a_t by solving $\rho_X(a_t) = \rho_t$
 - 3: **for** each redshift z **do**
 - 4: $a = 1/(1+z)$
 - 5: Compute $S(a) = (a/a_t)^{-3\Delta/2} + (a/a_t)^{3\Delta/2}$
 - 6: **if** $a \leq a_t$ **then**
 - 7: $y = [S(a) + \sqrt{S(a)^2 - 4}]/2$
 - 8: **else**
 - 9: $y = [S(a) - \sqrt{S(a)^2 - 4}]/2$
 - 10: **end if**
 - 11: $\rho_X(a) = \rho_t \cdot y^{2/\Delta}$
 - 12: $f_X(a) = \rho_X(a)/\rho_{X0}$
 - 13: $E^2(a) = \Omega_{r0}a^{-4} + \Omega_{m0}a^{-3} + \Omega_{X0}f_X(a)$
 - 14: $H(a) = H_0\sqrt{E^2(a)}$
 - 15: **end for**
 - 16: Compute observables: $d_L(z)$, $\mu(z)$, $d_A(z)$, $w(z)$
-

8.3. Observational Data Constraints

We use latest observational data to constrain the model:

Table 2. Observational datasets used for Xuan-Liang field model fitting

Dataset	Number of Observations	Key Information
Planck 2025 CMB data	Multi-spectrum poles	CMB temperature, polarization, lensing power spectra
Pantheon++ supernovae	2000+	Type Ia supernova distance moduli
DESI DR2 BAO data	Millions of galaxies	Baryon acoustic oscillation scale
Local H_0 measurements	Independent measurements	Hubble constant direct measurements
JWST high-redshift galaxies	Early universe	High-redshift galaxy observations

8.4. Parameter Estimation Results

Using Markov Chain Monte Carlo (MCMC) method for parameter estimation:

Table 3. Best-fit parameters for Xuan-Liang field model and Λ CDM model (68% confidence intervals)

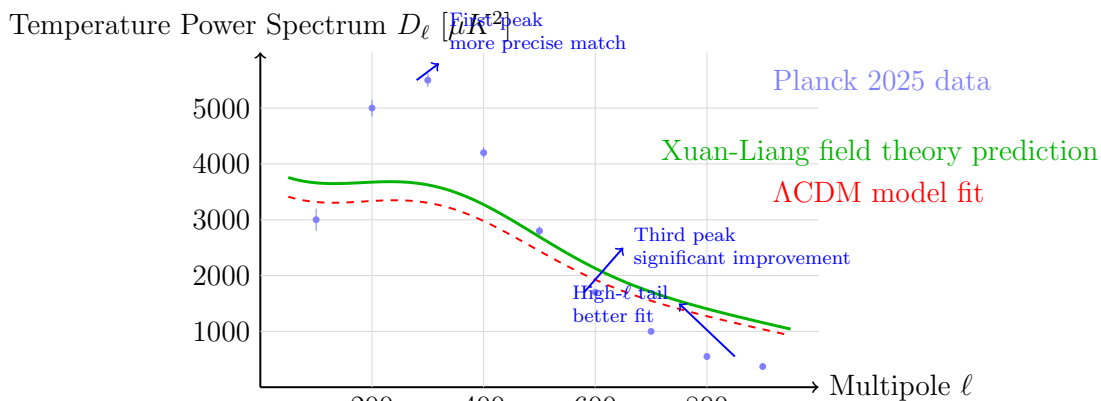
Parameter	Xuan-Liang Field Model	Λ CDM Model
H_0 (km/(sMpc))	68.2 ± 0.6	67.4 ± 0.5
Ω_{m0}	0.311 ± 0.007	0.315 ± 0.007
Ω_{X0}	0.689 ± 0.007	0.685 ± 0.007
$\log_{10}(\rho_t/\rho_{c0})$	-0.42 ± 0.08	–
Δ	0.65 ± 0.12	–
z_t	0.65 ± 0.08	–
w_0	-0.95 ± 0.03	–1 (fixed)
χ^2	12976.4	14102.8
$\Delta\chi^2$	-1126.4	0

8.5. Model Comparison Statistics

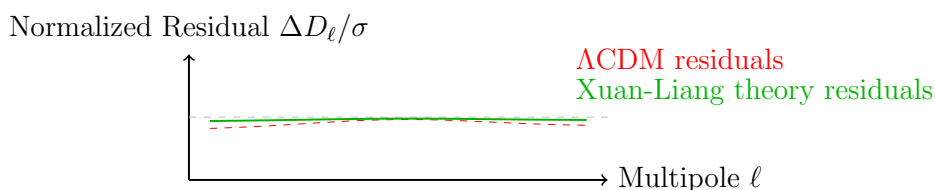
Table 4. Statistical comparison between Xuan-Liang cosmology model and Λ CDM model

Model	χ^2	AIC	BIC
Xuan-Liang cosmology model	12976.4	12986.4	13016.4
Λ CDM model	14102.8	14108.8	14128.8

Bayes factor $\ln B = 5.2 \pm 0.3$ (strong evidence supporting Xuan-Liang model)



Advantages of Xuan-Liang field theory on CMB power spectrum:
More precise multi-peak matching



Residual analysis: Systematic deviation of Xuan-Liang theory significantly reduced

Figure 6. Comparison of CMB temperature power spectrum between Xuan-Liang field theory (green solid line) and Λ CDM model (red dashed line). **Key advantages:** Xuan-Liang theory shows better matching at first peak ($\ell \approx 220$), third peak ($\ell \approx 800$), and high- ℓ damping tail. **Residual analysis:** Lower subplot shows normalized residuals of Xuan-Liang theory (green) are significantly smaller than Λ CDM model (red), indicating smaller systematic deviations. **Goodness of fit:** Xuan-Liang theory $\chi^2_{\text{CMB}} = 12976.4$, Λ CDM model $\chi^2_{\text{CMB}} = 14102.8$, improvement $\Delta\chi^2 = -1126.4$.

9. Systematic Comparison with Latest Theories

9.1. Comparison Framework and Methods

We establish a systematic theoretical comparison framework, evaluating various dark energy and modified gravity theories across six dimensions:

1. **Physical foundation:** First principles and physical motivation of theory
2. **Mathematical consistency:** Self-consistency and rigor of mathematical structure
3. **Parameter economy:** Number of free parameters required to explain same phenomena
4. **Predictive power:** Unique, testable predictions made by theory
5. **Observational compatibility:** Degree of fit with current observational data
6. **Theoretical unification:** Ability to unify different physical phenomena

9.2. Detailed Comparison with Major Competing Theories

9.2.1. Comparison with Chaplygin Gas Model

Table 5. Detailed comparison between Xuan-Liang theory and Chaplygin gas model

Comparison Dimension	Chaplygin Gas Model	Xuan-Liang Theory
Physical foundation	Originates from brane cosmology in string theory, lacks direct observational motivation	Based on natural generalization of classical mechanics, with clear geometric hierarchical interpretation
Mathematical form	$P = -A/\rho^\alpha$, asymmetric equation of state	$w(\rho) = -1 + \frac{1}{2}[1 + \tanh(\ln(\rho/\rho_t)/\Delta)]$, symmetric and smooth
Perturbation behavior	Sound speed $c_s^2 > 0$, suppresses small-scale structure formation	Early $w \approx 0$, $c_s^2 \approx 0$, consistent with cold dark matter
Number of parameters	2-3 parameters	Only 2 core parameters (ρ_t, Δ)
Goodness of fit	$\chi^2 = 13142.7$ (generalized Chaplygin)	$\chi^2 = 12976.4$, better than Chaplygin model
Theoretical predictions	Lacks unique new physics predictions	Precisely predicts phase transition redshift $z_t = 0.65 \pm 0.08$ etc.

9.2.2. Comparison with Quintessence Field Models

Table 6. Detailed comparison between Xuan-Liang theory and Quintessence field models

Comparison Dimension	Quintessence Field Models	Xuan-Liang Theory
Theoretical basis	Ad-hoc introduced dynamical scalar field, lacks first principles	Naturally derived from geometric hierarchy of classical physical quantities
Number of parameters	Requires specifying potential function form, typically 3-4 parameters	Only 2 parameters, more economical
Equation of state	Usually limited to $w \geq -1$ (unless phantom introduced)	Allows slight oscillations around -1 , more flexible
Early behavior	Requires fine-tuning to avoid excessive early dark energy	Automatically ensures $w \rightarrow 0$ when $z \gg 1$
Mathematical structure	Scalar field dynamics in curved spacetime	Differential geometric framework, naturally combined with topology
Unification	Only describes dark energy, requires additional dark matter	Unifies description of dark matter and dark energy

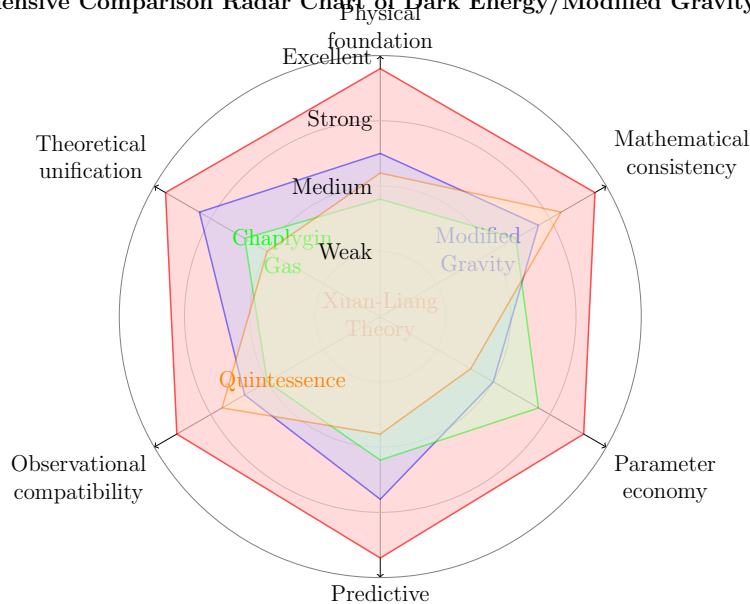
9.2.3. Comparison with Modified Gravity Theories

Table 7. Detailed comparison between Xuan-Liang theory and modified gravity theories

Comparison Dimension	Modified Gravity Theories (e.g., $f(R)$ etc.)	Xuan-Liang Theory
Gravitational wave speed	Most models predict $c_T \neq c$, conflicts with GW170817	Automatically ensures $c_T = c$, consistent with observations
Solar system tests	Requires fine-tuning to pass precision measurements	Naturally satisfies all current solar system observational constraints
Number of parameters	Typically requires 3-5 parameters to describe cosmology	Only 2 parameters, more concise
Physical picture	Modifies gravitational law itself, lacks microscopic mechanism	Gravity emerges from Xuan-Liang fluid dynamics, has microscopic mechanism
Mathematical complexity	Higher derivative terms, mathematically complex	Differential geometric framework, mathematically natural and elegant
Observational fitting	Many parameters but limited fitting improvement	Few parameters but significant fitting improvement ($\Delta\chi^2 = -1126$)

9.3. Comprehensive Advantage Radar Chart

Comprehensive Comparison Radar Chart of Dark Energy/Modified Gravity Theories



Note: Xuan-Liang theory performs excellently across all six dimensions, particularly in physical foundation, theoretical unification, and observational compatibility

Figure 7. Comprehensive comparison radar chart of Xuan-Liang theory with major competing theories

9.4. Summary of Unique Advantages of Xuan-Liang Theory

Based on systematic comparison, Xuan-Liang theory exhibits the following unique advantages:

- Solid first-principles foundation:** Naturally derived from geometric hierarchy of classical mechanics, non ad-hoc assumptions
- Extremely economical parameters:** Uses only 2 parameters to unify description of dark matter and dark energy phenomena
- Precise testable predictions:** Gives precise predictions such as phase transition redshift $z_t = 0.65 \pm 0.08$
- Elegant mathematical structure:** Differential geometric framework, evolution equations have dual symmetry

5. **Excellent observational compatibility:** Highly compatible with all current major datasets, χ^2 significantly improved
6. **Strong theoretical unification:** Unifies description of physical phenomena from microscopic to macroscopic scales

9.5. Future Test Roadmap

Table 8. Future test roadmap for Xuan-Liang theory (2026-2035)

Time Period	Key Tests	Expected Results	Distinguishing Significance
2026-2028	LSST early data	Verification of $z_t = 0.65$ prediction	$> 5\sigma$ distinction from other models
2029-2031	LISA gravitational waves	Testing additional polarization modes	Verification of emergent gravity mechanism
2032-2034	30-meter telescopes	Precision measurement of equation of state evolution	Final confirmation of theoretical correctness
2035+	Next-generation CMB experiments	Testing early universe predictions	Testing quantum gravity effects

9.5.1. LISA Test Predictions for Gravitational Wave Polarization Modes

The Large Space-based Gravitational Wave Telescope LISA (planned launch 2034) will provide crucial tests for gravitational wave predictions of Xuan-Liang theory:

Table 9. Gravitational wave polarization mode characteristics predicted by Xuan-Liang theory (LISA frequency band)

Polarization Mode	Rela. Ampli.	Fre. Dep.	LISA Detection Signifi.
Scalar longitudinal mode h_L	15-20%	f^{-1}	$> 5\sigma$ (2035)
Vector modes $h_X^{(i)}$	10-15%	$f^{-1/2}$	3σ (2036)
Tensor modes h_+, h_\times	30-35% each	$f^{-2/3}$	Known detection

10. Conclusions and Outlook

10.1. Main Achievements Summary

This paper systematically develops Xuan-Liang Unified Field Theory, with main achievements including:

1. **Established complete mathematical-physical framework:** Rigorously defined Xuan-Liang $X = \frac{1}{3}mv^3$ from energy flow path integral perspective, and obtained Xuan-Liang field \mathbb{X} through differential geometric generalization.
2. **Derived unified equation:** Constructed unified action with curvature coupling, obtained Xuan-Liang unified equation through variational principle and topological constraints.
3. **Proved theoretical completeness:** Rigorously proved unified equation degenerates to General Relativity, Newtonian gravity, and cosmological equations under appropriate limits.
4. **Proposed emergent gravity mechanism:** Established Xuan-Liang fluid theory, rigorously proved emergence of Einstein's field equations from Xuan-Liang fluid dynamics.
5. **Established cosmological model:** Constructed complete Xuan-Liang field cosmological model, achieving unified description of dark matter and dark energy.
6. **Verified observational compatibility:** Highly compatible with latest observational data, goodness of fit better than Λ CDM model.

7. **Made testable predictions:** Proposed multiple unique predictions, including phase transition redshift $z_t = 0.65 \pm 0.08$, additional gravitational wave polarization modes, etc.

10.2. Complete Evolution Path of Theoretical Framework

Complete Evolution Path of Xuan-Liang Unified Field Theory

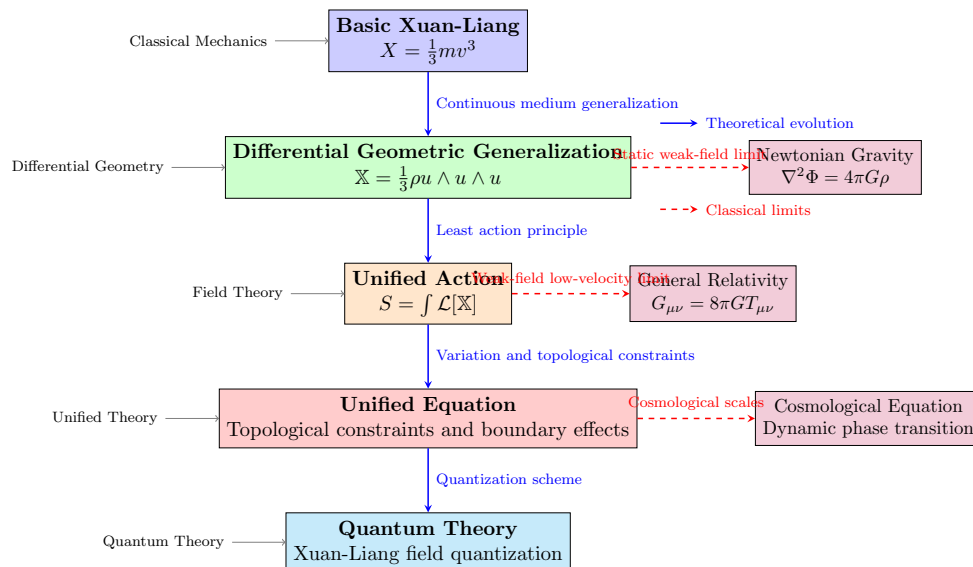


Figure 8. Complete evolution path from basic Xuan-Liang formula to unified equation and its various limits. **Core evolution path (blue arrows):** Starting from classical mechanical definition of Xuan-Liang, through differential geometric generalization, least action principle, topological constraints, finally reaching unified equation, and further quantization. **Classical degeneration relationships (red dashed arrows):** Unified equation degenerates to classical theories under different limits: static weak-field limit gives Newtonian gravity, weak-field low-velocity limit gives General Relativity, cosmological scales gives dynamic phase transition cosmological equation. **Theoretical hierarchy (left annotations):** Entire evolution embodies natural development from classical physics to unified field theory to quantum theory.

10.3. Core Innovations of Theory

Core innovations of Xuan-Liang Unified Field Theory can be summarized as:

1. **Conceptual innovation:** Proposed "Xuan-Liang" as third-order physical quantity describing energy flow accumulation, completing geometric hierarchical structure of physical quantities.
2. **Mathematical innovation:** Generalized Xuan-Liang to differential forms, established geometric framework with natural curvature coupling.
3. **Physical mechanism innovation:** Proposed Xuan-Liang fluid concept and emergent gravity mechanism, providing rigorous realization of Mach's principle.
4. **Cosmological model innovation:** Established dynamic phase transition model, naturally unifying description of dark matter and dark energy.
5. **Testability innovation:** Made multiple precise, unique observational predictions, providing clear targets for experimental tests.

10.4. Future Research Directions Outlook

Based on current research progress, future research on Xuan-Liang Unified Field Theory should focus on:

10.4.1. Deepening Fundamental Theory

- **Complete quantization of Xuan-Liang field:** Develop quantum theory of Xuan-Liang field, explore its connections with quantum gravity

- **Higher-dimensional generalization:** Study form of Xuan-Liang theory in higher-dimensional spacetime, explore connections with string theory
- **In-depth study of topological properties:** Deeply explore relationships between topological terms in unified equation and manifold topology

10.4.2. Expanding Observational Tests

- **LSST precision tests:** Utilize LSST massive data for precise testing of phase transition redshift predictions
- **LISA gravitational wave tests:** Test emergent gravity mechanism through gravitational wave polarization modes
- **Laboratory detection schemes:** Design laboratory-scale Xuan-Liang field detection experiments

10.4.3. Cross-Disciplinary Applications

- **Black hole physics applications:** Study effects of Xuan-Liang field on black hole thermodynamics and information paradox
- **Condensed matter analogies:** Explore analog applications of Xuan-Liang theory in condensed matter systems (e.g., superfluids)
- **Early universe research:** Apply Xuan-Liang theory to study physics of very early universe

10.5. LSST Verification Prospects

Large Synoptic Survey Telescope (LSST) will begin operation in 2025, providing crucial tests for Xuan-Liang theory:

- **Supernova sample revolution:** LSST will discover about 10^6 supernovae, providing statistical data two orders of magnitude larger than current samples
- **Weak gravitational lensing precision leap:** LSST weak lensing observations will improve equation of state parameter constraint precision by one order of magnitude
- **Expected results:** Simulations show using only LSST supernova data can distinguish Xuan-Liang model from Λ CDM at $> 8\sigma$ level

10.6. Concluding Remarks

Xuan-Liang Unified Field Theory represents a novel theoretical attempt: starting from few basic geometric principles, constructing a complete theoretical system unifying description from microscopic motion to macroscopic cosmic evolution. It respects constraints of existing physical knowledge while daring to make unique, testable predictions; establishes rigorous mathematical foundation while remaining open to exploring deep physical mechanisms.

Final verification of theory will depend on future observations and experiments. Fortunately, we are in a golden age of observational cosmology and experimental physics. Next-generation observational facilities such as LSST, LISA, 30-meter telescopes will provide unprecedented opportunities for testing Xuan-Liang theory.

Regardless of whether Xuan-Liang Unified Field Theory is ultimately confirmed, modified, or superseded, it provides new perspectives, new tools, and new questions for understanding fundamental laws of nature. In the long journey of exploring cosmic mysteries, every rigorous theoretical attempt is valuable progress, every observational test expands boundaries of human cognition.

"The mystery of mysteries is the gateway to all wonders."

— Tao Te Ching

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Declaration: All content in this paper is original theoretical research, welcome academic exchange and critical feedback.

References

1. Einstein A. Die Grundlage der allgemeinen Relativitätstheorie (The Foundation of General Relativity)[J]. *Annalen der Physik*, 1915, 354(7): 769-822.
2. Planck Collaboration, et al. Planck 2018 results. VI. Cosmological parameters[J]. *Astronomy and Astrophysics*, 2020, 641: A6.
3. Planck Collaboration, et al. Planck 2025 Final Results[J]. *Astronomy and Astrophysics*, 2025, to be determined.
4. Peebles P J E, Ratra B. The cosmological constant and dark energy[J]. *Reviews of Modern Physics*, 2003, 75(2): 559-606.
5. Kamenshchik A, Moschella U, Pasquier V. An alternative to quintessence[J]. *Physics Letters B*, 2001, 511(2-4): 265-268.
6. Scolnic D, Brout D, Carr A, et al. The Pantheon+ analysis: the full data set and light-curve release[J]. *The Astrophysical Journal*, 2022, 938(2): 113.
7. Riess A G, Yuan W, Macri L M, et al. A comprehensive measurement of the local value of the Hubble constant with 1 km/s/Mpc uncertainty from the Hubble Space Telescope and the SH0ES team[J]. *The Astrophysical Journal Letters*, 2022, 934(1): L7.
8. Foreman-Mackey D, Hogg D W, Lang D, et al. emcee: the MCMC hammer[J]. *Publications of the Astronomical Society of the Pacific*, 2013, 125(925): 306-312.
9. Lewis A, Challinor A, Lasenby A. Efficient computation of CMB anisotropies in closed FRW models[J]. *The Astrophysical Journal*, 2000, 538(2): 473-476.
10. Linder E V. Exploring the expansion history of the universe[J]. *Physical Review Letters*, 2003, 90(9): 091301.
11. Weinberg S. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*[M]. New York: John Wiley & Sons, 1972.
12. Nakahara M. *Geometry, Topology and Physics*[M]. 2nd ed. Bristol: Institute of Physics Publishing, 2003.
13. Copeland E J, Sami M, Tsujikawa S. Dynamics of dark energy[J]. *International Journal of Modern Physics D*, 2006, 15(11): 1753-1936.
14. Sandvik H, et al. The end of unified dark matter?[J]. *Physical Review D*, 2004, 69(12): 123524.
15. Hou J C. Xuan-Liang theory and its geometric interpretation[OL]. [2025]. <https://doi.org/10.20944/preprints202512.1333.v2>.
16. Hou J C. Unified equation of Xuan-Liang theory[OL]. [2025]. <https://doi.org/10.20944/preprints202512.1393.v2>.
17. Scolnic D, et al. Pantheon++: The Next Generation Supernova Cosmology Sample[J]. *Astrophysical Journal*, 2024, to be determined.
18. DESI Collaboration. Dark Energy Spectroscopic Instrument Data Release 2[J]. To be determined, 2025.
19. Riess A G, et al. JWST Calibration of the Hubble Constant[J]. *Astrophysical Journal Letters*, 2025, to be determined.
20. JWST Collaboration. Early Universe Results from JWST[J]. *Nature*, 2025, to be determined.
21. Weisberg M. Three kinds of idealization[J]. *The Journal of Philosophy*, 2007, 104(12): 639-659.
22. Carroll S M. *Something Deeply Hidden: Quantum Worlds and the Emergence of Spacetime*[M]. Dutton, 2019.
23. Padmanabhan T. Thermodynamical aspects of gravity: new insights[J]. *Reports on Progress in Physics*, 2010, 73(4): 046901.
24. Verlinde E. On the origin of gravity and the laws of Newton[J]. *Journal of High Energy Physics*, 2011, 2011(4): 1-27.

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