doi:10.20944/preprints201901.0024.v3

Nikfar Domination Versus Others: Restriction, Extension Theorems and Monstrous Examples

Mohammadesmail Nikfar

Independent Scholar

nikfarmath1@gmail.com

Abstract

The aim of this expository article is to present recent developments in the centuries-old discussion on the interrelations between several types of domination in graphs. However, the novelty even more prominent in the newly discovered simplified presentations of several older results. Domination can be seen as arising from real-world application and extracting classical results as first described by this article.The main part of this article, concerning a new domination and older one, is presented in a narrative that answers two classical questions: (i) To what extend must closing set be dominating? (ii) How strong is the assumption of domination of a closing set? In a addition, we give an overview of the results concerning domination. The problem asks how small can a subset of vertices be and contain no edges or, more generally how can small a subset of vertices be and contain other ones. Our work was as elegant as it was unexpected being a departure from the tried and true methods of this theory that had dominated the field for one fifth a century. This expository article covers all previous definitions. The inability of previous definitions in solving even one case of real-world problems due to the lack of simultaneous attentions to the worthy both of vertices and edges causing us to make the new one. The concept of domination in a variety of graphs models such as crisp, weighted and fuzzy, has been in a spotlight. We turn our attention to sets of vertices in a fuzzy graph that are so close to all vertices, in a variety of ways, and study minimum such sets and their cardinality. A natural way to introduce and motivate our subject is to view it as a real-world problem. In its most elementary form, we consider the problem of reducing waste of time in transport planning. Our goal here is to first describe the previous definitions and the results, and then to provide an overview of the flows ideas in their articles. The final outcome of this article is twofold: (i) Solving the problem of reducing waste of time in transport planning at static state; (ii) Solving and having a gentle discussions on problem of reducing waste of time in transport planning at dynamic state. Finally, we discuss the results concerning holding domination that are independent of fuzzy graphs. We close with a list of currently open problems related to this subject. Most of our exposition assumes only familiarity with basic linear algebra, polynomials, fuzzy graph theory and graph theory.

Keywords: fuzzy graph, fuzzy bridge, α-strong edge, nikfar domination, dynamic networks.

AMS Subject Classification: 05C72, 05C69, 03E72, 94D05

 doi:10.20944/preprints201901.0024.v3

Contents

1 Introduction and Overview

Domination are among the most fundamental concepts of graph theory. Also, domination can behave in many strange ways. For instance, besides the classical definitions of domination, there are many characterization of this concept. One of this characterization due to A. Somasundaram and S. Somasundaram (Ref. [31]), see also Refs. [8, 13–15, 17, 22–24, 30, 32] for further generalizations. One the contrary and quite surprisingly, there are nowhere these definitions Solving the problem of reducing waste of time in transport planning and also (separately) all others real-world problems see 6. Somehow, a key direction of study of domination deals with trying to provide a clear structure of what the dominating set of vertices looks like. The leading theme of this expository article is to discuss the following two questions concerning fuzzy graphs

Q1: How much closing does dominating imply?

Q2: How much dominating does closing imply?

They will be addressed in sections 2 and 6, respectively. The main narrative presented in these sections is independent of any results from graph theory and/or calculus. The purpose of this expository article is to provide an overview of the authors' recent series of work (Refs. [8, 13–15, 17, 22–24, 30–32]), in which a positive answer to the problem of reducing waste of time in transport planning for the our new definition is given.

Consider a set of cities connected by communication paths, Which cities is connected to others by roads? We face with a graph model of this situation. But the cities are not same and they have different privileges in low traffic levels and this events also occur for the roads in low-cost levels. So we face with the weighted graph model, at first. These privileges are not crisp but they are vague in nature. So we don't have a weighted graph model. In other words, we face with a fuzzy graph model, which must study the concept of domination on it.

Next we turn our attention to sets of vertices in a fuzzy graph G that are close to all vertices of G, in a variety of ways, and study minimum such sets and their cardinality.

In 1998, the concept of effective domination in fuzzy graphs was introduced by A. Somasundaram and S. Somasundaram (Ref. [31]) as the classical problems of covering chess board with minimum number of chess pieces. In 2010, the concept of 2-strong(weak) domination in fuzzy graphs was introduced by C. Natarajan and S.K. Ayyaswamy (Ref. [23]) as the extension of strong (weak) domination in crisp graphs. In 2014, the concept of 1-strong domination in fuzzy graphs was introduced by O.T. Manjusha and M.S. Sunitha (Ref. [14]) as the extension of domination in fuzzy graphs with strong edges. In 2015, the concept of 2-domination in fuzzy graphs was introduced by A. Nagoor Gani and K. Prasanna Devi (Ref. [22]) as the extension of 2-domination in crisp graphs. In 2015, the concept of strong domination in fuzzy graphs was introduced by O.T. Manjusha and M.S. Sunitha (Ref. [13]) as reduction of the value of old domination number and extraction of classic results. In 2016, the concept of $(1,2)-$domination in fuzzy graphs was introduced by N. Sarala and T. Kavitha (Ref. [30]) as the extension of $(1,2)-$domination in crisp graphs. A few researchers studied other domination variations which are based on above definitions. So we only compare our new definition with the fundamental dominations.

A *fuzzy set* on a given set V, is a map assigning to every its elements a real number from unit interval $[0,1]$; this number is called *value* of element in V.

The *number* of a fuzzy set is a summation on values of all its elements.

A *fuzzy graph* G is an ordered pair (V,E) consisting of a fuzzy set V of *vertices* and a fuzzy set E, disjoint from V, of *edges*, together with an *incidence function* ϕ_G that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G. If e is an edge and u and v are vertices such that $\phi_G(e)=\{u,v\}$, then e is said to *join* u and v, and the vertices u and v are called the *ends* of e. In a fuzzy graph, value of every vertices are at least equal to value of their ends. We denote the numbers of vertices and edges in G by $n(V)$ and $n(E)$; these two basic parameters are called the *order* and *size* of G, respectively. In section **??**, some examples should serve to clarify the definition. For notational simplicity, we write uv for the unordered pair $\{u,v\}$.

We don't speak about a graph. So when we write vertices or edges, we talk about a fuzzy graph.

A *path* is a sequence of vertices $v_0v_1\cdots v_n$ such that $v_{i-1}v_i$ is an edge for any $1\leq i\leq n$. The least value between edges in a path is called its value. In other words, if e has the least value in a path P, we would call $E(e)$ value of path and it is denoted by the same notation $E(P)$. In this case, we have $E(e)=E(P)$. The greatest value between all paths from the vertices x to y in a fuzzy graph $G=(V,E)$ is called *value between x and y* and is denoted by $E(x,y)$.

A fuzzy graph $G=(V,E)$ is *connected* if for every x,y in V, $E_G(x,y)>0$.

Note that $E_{G-xy}(x,y)$ is the strength of connectedness between x and y in the fuzzy graph obtained from G by deleting the edge xy. An edge xy in G is *α-strong* if $E(xy)>E_{G-xy}(x,y)$. An edge xy in G is *β-strong* if $E(xy)=E_{G-xy}(x,y)$. An edge xy is a *strong* edge if it is either $\alpha-$strong or $\beta-$strong. An edge uv of a fuzzy graph is called an *M-strong* edge, In order to avoid confusion with the notion of strong edges, we shall call strong in the sense of Mordeson as M-strong, if $E(uv)=V(u)\wedge V(v)$. If $E(uv)>0$, then u and v are called the *neighbors*. The set of all neighbors of u is denoted by $N(u)$. Also v is called the *α-strong neighbor* of u, if the edge uv is α-strong. The set of all α-strong neighbors of u is denoted by $N_s(u)$. The degree of a vertex v is defined as $d(v)=V_{u\neq v}E(uv)$. The *α-strong degree* of a vertex $v\in V$ is defined as the sum of membership values of all α-strong edges incident at v; It is denoted by $d_s(v)$; That is $d_s(v)=\Sigma_{u\in N_s(v)}E(uv)$. v is called the *effective neighbor* of u, if the edge uv is M-strong. The set of all M-strong neighbors of u is denoted by $N_e(u)$. The *M-strong degree* of a vertex $v\in V$ is defined as the sum of membership values of all M-strong edges incident at v; It is denoted by $d_e(v)$; That is $d_e(v)=\Sigma_{u\in N_e(v)}E(uv)$.

A fuzzy graph $G=(V,E)$ is said *complete* in **Ref.**([20], Definition 2.3, p.28) if $E(uv)=V(x)\wedge V(y)$ for all $u,v\in V$.

The *complement* of a fuzzy graph $G=(V,E)$ denoted by $\bar{G}$, is defined to $\bar{G}=(V,\bar{E})$, where $\bar{E}(xy)=V(x)\wedge V(y)-E(xy)$ for all $x,y\in V$.

A fuzzy graph $G=(V,E)$ is said *bipartite* if the vertex set V can be partitioned into two nonempty sets V_1 and V_2 such that $E(v_1v_2)=0$ if $v_1,v_2\in V_1$ or $v_1,v_2\in V_2$. Moreover, if $E(uv)=V(u)\wedge V(v)$ for all $u\in V_1$ and $v\in V_2$ then G is called a *complete* *bipartite fuzzy graph* and is denoted by K_{V_1,V_2} , where V_1 and V_2 are respectively the restrictions of V to V_1 and V_2. In this case, If either $|V_1| = 1$ or $|V_2| = 1$ then the complete bipartite fuzzy graph is said a *star fuzzy graph* which is denoted by $K_{1,V}$.

A vertex u is said *isolated* if $E(uv) = 0$ for all $v \neq u$.

Now, we will define some special operations on fuzzy graphs. The pages of references will show the proof of validity of them.

The *cartesian product* in **Ref.**([19], Proposition 2.1, pp.160,161) $G = G_1 \times G_2$ of two fuzzy graphs $G_i = (V_i, E_i)$ on $V_i, i = 1, 2$ is defined as a fuzzy graph $G = (V_1 \times V_2, E_1 \times E_2)$ on $V \times V$ where $E = \{\{uu_2, uv_2\}|u \in V_1, u_2v_2 \in E_2\} \cup \{\{u_1w, v_1w\}|u_1v_1 \in E_1, w \in V_2\}$. Fuzzy sets $V_1 \times V_2$ and $E_1 \times E_2$ are defined as $(V_1 \times V_2)(u_1, u_2) = V_1(u_1) \wedge V_2(u_2)$ and $\forall u \in V_1, \forall u_2v_2 \in E_2, (E_1 \times E_2)(\{uu_2, uv_2\}) = V_1(u) \wedge E_2(u_2v_2)$ and $\forall u_1v_1 \in E_1, \forall w \in V_2, (E_1 \times E_2)(\{u_1w, v_w\}) = E_1(u_1v_1) \wedge V_2(w)$.

The *union* $G = G_1 \cup G_2$ in **Ref.**([19], Proposition 3.1, pp.166,167) of two fuzzy graphs $G_i = (V_i, E_i)$ on $V_i, i = 1, 2$ is defined as a fuzzy graph $G = (V_1 \cup V_2, E_1 \cup E_2)$ on $V_1 \cup V_2$ where $E = E_1 \cup E_2$. Fuzzy sets $V_1 \cup V_2$ and $E_1 \cup E_2$ are defined as $(V_1 \cup V_2)(u) = V_1(u)$ if $u \in V_1 - V_2$, $(V_1 \cup V_2)(u) = V_2(u)$ if $u \in V_2 - V_1$, and $(V_1 \cup V_2)(u) = V_1(u) \vee V_2(u)$ if $u \in V_1 \cap V_2$. Also $(E_1 \cup E_2)(uv) = E_1(uv)$ if $uv \in E_1 - E_2$ and $(E_1 \cup E_2)(uv) = E_2(uv)$ if $uv \in E_2 - E_1$, and $(E_1 \cup E_2)(uv) = E_1(uv) \vee E_2(uv)$ if $uv \in E_1 \cap E_2$.

Let $G = G_1 + G_2$ denote the *join* in **Ref.**([19], Proposition 3.3, p.168) of two fuzzy graphs $G_i = (V_i, E_i)$ on $V_i, i = 1, 2$ is defined as a fuzzy graph $G = (V_1 + V_2, E_1 + E_2)$ on $V_1 \cup V_2$ where $E = E_1 \cup E_2 \cup E^{'}$ and $E^{'}$ is the set of all edges joining vertices of V_1 with the vertices of V_2, and we assume that $V_1 \cap V_2 = \emptyset$. Fuzzy sets $V_1 + V_2$ and $E_1 + E_2$ are defined as $(V_1 + V_2)(u) = (V_1 \cup V_2)(u)$ and $\forall u \in V_1 \cup V_2; (E_1 + E_2)(uv) = (E_1 \cup E_2)(uv)$ if $uv \in E_1 \cup E_2$ and $(E_1 + E_2)(uv) = V_1(u) \wedge V_2(v)$ if $uv \in E^{'}$.

Definition 1.1. Let $G = (V, E)$ be a fuzzy graph. Then

(i) (**Ref.** [31], Definition 2.9, p.3). $D \subseteq V$ is said to be *effective dominating set*, if for every $v \in V - D$, there exists u in D such that $E(uv) = V(u) \wedge V(v)$. Let S be the set of all effective dominating sets in G. The *effective domination number* of G is defined by $\gamma(G) = \min_{D \in S}(\Sigma_{u \in D}V(u))$.

(ii) (**Ref.** [23],p.1035). $D \subseteq V$ is said to be *2-strong(weak) dominating set*, if for every $v \in V - D$, there exists u in D such that $E(uv) = V(u) \wedge V(v)$ and $d_e(u) \geq d_e(v)$. Let S be the set of all 2-strong(weak) dominating sets in G. The *2-strong(weak) domination number* of G is defined by $\gamma_{sf}(G)(\gamma_{wf}(G)) = \min_{D \in S}(\Sigma_{u \in D}V(u))$.

(iii) (**Ref.** [14], Definition 4.1(c), p.3208). $D \subseteq V$ is said to be *1-strong dominating set*, if for every $v \in V - D$, there exists u in D such that $E(uv) \geq E_{G-xy}(u, v)$. Let S be the set of all 1-strong dominating sets in G. The *1-strong domination number* of G is defined by $\gamma_{Sn}(G) = \min_{D \in S}(\Sigma_{u \in D}V(u))$.

(iv) (**Ref.** [22], Definition 3.1, p.120). $D \subseteq V$ is said to be *2-dominating set*, if for every $v \in V - D$, there exists two vertices like u in D such that $E(uv) = E_{G-xy}(u, v)$. Let S be the set of all 2-dominating sets in the fuzzy graph G. Then The *2-domination number* of G is defined by $\gamma_2(G) = \min_{D \in S}(\Sigma_{u \in D}V(u))$.

(v) (**Ref.** [13], Definition 3.1, p.372). $D \subseteq V$ is said to be *strong dominating set*, if for every $v \in V - D$, there exists u in D such that $E(uv) \geq E_{G-xy}(u, v)$. Let S be the set of all strong dominating sets in G. The *strong domination number* of G is defined by $\gamma_s(G) = \min_{D \in S}(\Sigma_{u \in D}t(u, v))$ where $t(u, v)$ is the minimum of the membership values (weights) of the edge uv such that $E(uv) \geq E_{G-xy}(u, v)$.

(vi) (**Ref.** [30], Definition 3.1, p.16502). $D \subseteq V$ is said to be $(1, 2)-$*dominating set*, if for every $v \in V - D$, there exists at least one vertex in D at distance 1 from v and a second vertex in D at distance almost 2 from v. Let S be the set of all $(1, 2)-$dominating sets in G. The $(1, 2)-$*domination number* of G is defined by

$\gamma_{(1,2)}(G) = \min_{D\in S}(\Sigma_{u\in D}V(u))$.

Remark 1.2. For the sake of simplicity, we do sometimes saying $V(x)$ and $E(xy)$ with different literatures, e.g. value, weight, membership value and etc.

Definition 1.3. Let $G = (V, E)$ be a fuzzy graph. Then

(i) (**Ref.** [31], Definition 2.9, p.3). $D \subseteq V$ is said to be *effective dominating set*, if for every $v \in V - D$, there exists u in D such that $E(uv) = E(u) \wedge E(v)$. Let S be the set of all effective dominating sets in G. The *effective domination number* of G is defined by $\gamma(G) = \min_{D\in S}(\Sigma_{u\in D}V(u))$.

(ii) (**Ref.** [23],p.1035). $D \subseteq V$ is said to be *2-strong(weak) dominating set*, if for every $v \in V - D$, there exists u in D such that $E(uv) = E(u) \wedge E(v)$ and $d_e(u) \geq d_e(v)$. Let S be the set of all 2-strong(weak) dominating sets in G. The *2-strong(weak) domination number* of G is defined by $\gamma_{sf}(G)(\gamma_{wf}(G)) = \min_{D\in S}(\Sigma_{u\in D}V(u))$.

(iii) (**Ref.** [14], Definition 4.1(c), p.3208). $D \subseteq V$ is said to be *1-strong dominating set*, if for every $v \in V - D$, there exists u in D such that $E(uv) \geq E_{G-xy}(u, v)$. Let S be the set of all 1-strong dominating sets in G. The *1-strong domination number* of G is defined by $\gamma_{Sn}(G) = \min_{D\in S}(\Sigma_{u\in D}V(u))$.

(iv) (**Ref.** [22], Definition 3.1, p.120). $D \subseteq V$ is said to be *2-dominating set*, if for every $v \in V - D$, there exists two vertices like u in D such that $E(uv) = E_{G-xy}(u, v)$. Let S be the set of all 2-dominating sets in the fuzzy graph G. Then The *2-domination number* of G is defined by $\gamma_2(G) = \min_{D\in S}(\Sigma_{u\in D}V(u))$.

(v) (**Ref.** [13], Definition 3.1, p.372). $D \subseteq V$ is said to be *strong dominating set*, if for every $v \in V - D$, there exists u in D such that $E(uv) \geq E_{G-xy}(u, v)$. Let S be the set of all strong dominating sets in G. The *strong domination number* of G is defined by $\gamma_s(G) = \min_{D\in S}(\Sigma_{u\in D}t(u, v))$ where $t(u, v)$ is the minimum of the membership values (weights) of the edge uv such that $E(uv) \geq E_{G-xy}(u, v)$.

(vi) (**Ref.** [30], Definition 3.1, p.16502). $D \subseteq V$ is said to be $(1,2)-$*dominating set*, if for every $v \in V - D$, there exists at least one vertex in D at distance 1 from v and a second vertex in D at distance almost 2 from v. Let S be the set of all $(1,2)-$dominating sets in G. The $(1,2)-$*domination number* of G is defined by $\gamma_{(1,2)}(G) = \min_{D\in S}(\Sigma_{u\in D}V(u))$.

Remark 1.4. For the sake of simplicity, we do sometimes saying $V(x)$ and $E(xy)$ with different literatures, e.g. value, weight, membership value and etc.

2 New definition versus other ones: Restrictions, Extension Theorem and Monstrous examples

Consider a set of cities connected by communication paths. Which cities have these properties? Having low traffic levels and other cities associating with at least ones by low-cost roads. We call this question as problem of reducing wast of time in transport planning. As outlined in Section 6, the previous definitions didn't consider values of vertices and edges, simultaneously. These parameters are simultaneously affected on any decision and analysis in transport planning. So those can't provide the appropriate solution to the problem. Therefore we decided to provide a new definition for the domination in fuzzy models.

To describe its generalization to fuzzy graph, it is helpful to reformulate this structure in three successive steps. The nikfar domination number of a fuzzy graph is defined in a classic way, (Definitions 2.1, 2.3 and 2.4) as reducing waste of time in transportation planning.

Definition 2.1. Let $G = (V, E)$ be a fuzzy graph and $x, y \in V$. We say that x *dominates* y in G as α-*strong*, if the edge xy is α-strong.

Example 2.2. Let $G = (V, E)$ be a fuzzy graph as Figure 1. Then the edges $\{v_2v_5, v_2v_4, v_3v_4, v_1v_3\}$ are α-strong and the edges $\{v_1v_4, v_1v_2, v_4v_5\}$ are not α-strong.

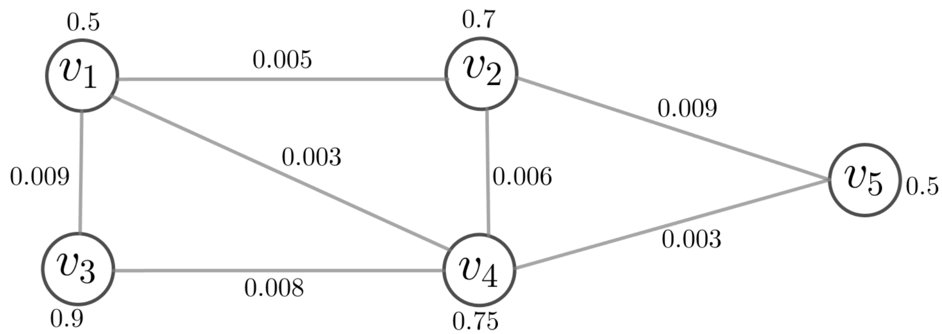

Figure 1. nikfar domination

Definition 2.3. Let $G = (V, E)$ be a fuzzy graph. A subset S of V is called a *α-strong dominating set* in G, if for every $v \in V - S$, there is $u \in S$ such that u dominates v as α-strong.

Definition 2.4. Let $G = (V, E)$ be a fuzzy graph. For every $u \in V$, the *nikfar weight* of u is defined by $w_v(u) = V(u) + \frac{d_s(u)}{d(u)}$. If $d(u) = 0$, for some $u \in V$. Then we consider $\frac{d_s(u)}{d(u)}$ equal with 0. For any $S \subseteq V$, natural extension of this concept to a set, is as follows. We also say the *nikfar weight* of S, it is defined by $w_v(S) = \Sigma_{u\in S}(w_v(u))$. Now, let U be the set of all α-strong dominating sets in G. The *nikfar domination number* of G is defined as $\gamma_v(G) = \min_{D\in U}(w_v(D))$. The α-strong dominating set that is correspond to $\gamma_v(G)$ is called by *nikfar dominating set*.

In what follows, we will work under this generality.

Example 2.5. Let $G = (V, E)$ be a fuzzy graph as Figure 1. The set $S = \{v_2, v_3\}$ is an α-strong dominating set. This set is also nikfar dominating set in fuzzy graph G. Hence $\gamma_v(G) = 1.75 + 0.9 + 0.7 = 3.35$. So $\gamma_v(G) = 3.35$.

Example 2.6. The following is a table consist of a brief fundamental comparison between types of domination in fuzzy graphs. There are two different types of the complete bipartite fuzzy graphs as Figures 2 and 3, which compare types of domination in fuzzy graphs.

The types of edges	Types of Numbers	Figure 2	Figure 3
$M-$strong	Scalar cardinality	$\gamma(G) = 1$	$\gamma(G) = 0.9$
$M-$strong and $d_e(u) \geq d_e(v)$	Scalar cardinality	$\gamma_{sf}(G) = 1.2$	$\gamma_{sf}(G) = 1.1$
$M-$strong and $d_e(u) \geq d_e(v)$	Scalar cardinality	$\gamma_{wf}(G) = 1$	$\gamma_{wf}(G) = 1.1$
strong	Scalar cardinality	$\gamma_{Sn}(G) = 0.9$	$\gamma_{Sn}(G) = 1.6$
$\beta-$strong	Scalar cardinality	$\gamma_2(G) = 2.2$	$\gamma_2(G) = 1.6$
strong	$\Sigma_{u\in D}t(u, v)$	$\gamma_s(G) = 0.4$	$\gamma_s(G) = 0.2$
Distance	Scalar cardinality	$\gamma_{(1,2)}(G) = 0.9$	$\gamma_{(1,2)}(G) = 0.9$
$\alpha-$strong	nikfar weight	$\gamma_v(G) = 2.2$	$\gamma_v(G) = 2.1$

3 Nice properties of domination

It is well known and generally accepted that the problem of determining the domination number of an arbitrary graph is a difficult one. Because of this, researchers have turned

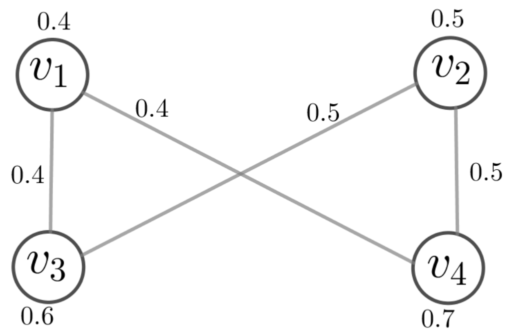

Figure 2. Comparison of dominations

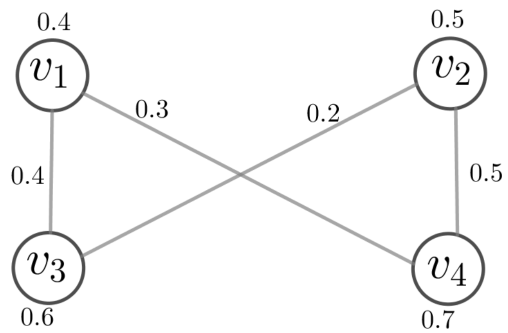

Figure 3. Comparison of dominations with different values

their attention to the study of classes of graphs for which the domination problem can be solved in polynomial time.

We determine nikfar domination number for several classes of fuzzy graphs consists of complete fuzzy graph, (Proposition 3.1), empty fuzzy graph, (Proposition 3.2), and complete bipartite fuzzy graph, (Proposition 3.4).

Proposition 3.1 (Complete fuzzy graph)**.** *Let $G = (V, E)$ be a complete fuzzy graph such that there is exactly one path with strength of $E(u, v)$. Then* $\gamma_v(G) = \min_{u\in V}(V(u)) + 1$.

Proof. Let G be a complete fuzzy graph. The strength of path P from u to v is of the form $V(u) \wedge \cdots \wedge V(v) \leq V(u) \wedge V(v) = E(uv)$. So $E(u, v) \leq E(uv)$. uv is a path from u to v such that $E(uv) = V(u) \wedge V(v)$. Therefore $E(u, v) \geq E(uv)$. Hence $E(u, v) = E(uv)$. Then $E(uv) > E^{'\infty}(u, v)$. It means that the edge uv is α-strong. All edges are α-strong and each vertex is adjacent to all other vertices. So $D = \{u\}$ is a α-strong dominating set and $d_s(u) = d(u)$ for each $u \in V$. The result follows. □

Proposition 3.2 (Empty fuzzy graph)**.** *Let $G = (V, E)$ be a edgeless fuzzy graph. Then $\gamma_v(G) = p$ where p denotes the order of G.*

Proof. G is edgeless. Hence V is only α-strong dominating set in G and there is no α-strong edge. So by Definition 2.4, we have $\gamma_v(G) = \min_{D\in S}[\Sigma_{u\in D}V(u)] = \Sigma_{u\in v}V(u) = p$. Therefore $\gamma_v(\bar{K_n}) = p$ by our notations. □

It is interesting to note the converse of Proposition 3.2, that does not hold.

 doi:10.20944/preprints201901.0024.v3

Example 3.3. We show that the converse of Proposition 3.2 does not hold. For this purpose, Let $V = \{v_1, v_2, v_3, v_4, v_5\}$. We define the fuzzy set V by

$$V(v_1) = 0.5, V(v_2) = 0.7, V(v_3) = 0.9, V(v_4) = 0.75, V(v_5) = 0.5$$

Now, the fuzzy set E is defined by $E(v_1v_2) = 0.005$,

$$E(v_1v_4) = 0.003, E(v_1v_3) = 0.009, E(v_2v_4) = 0.006, E(v_2v_5) = 0.009,$$

$E(v_3v_4) = 0.008, E(v_4v_5) = 0.003$ such that $\forall u, v \in V, E(uv) \leq V(u) \wedge V(v)$. The edges $\{v_2v_5, v_2v_4, v_3v_4, v_1v_3\}$ are α-strong and the edges $\{v_1v_4, v_1v_2, v_4v_5\}$ are not α-strong. So the set $\{v_2, v_3\}$ is the α-strong dominating set. This set is also nikfar dominating set in fuzzy graph G. Hence $\gamma_v(G) = 1.75 + 0.9 + 0.7 = 3.35 = \Sigma_{u\in v} V(u) = p$. Therefore $G \neq \bar{K}_5$ but $\gamma_v(G) = p$.

Proposition 3.4 (Complete bipartite fuzzy graph)**.** *Let $G = (V, E)$ be the complete bipartite fuzzy graph such that there is exactly one path with strength of $E(u, v)$. Then $\gamma_v(G)$ is either $V(u) + 1$, $u \in V$ or $\min_{u\in V_1, v\in V_2}(V(u) + V(v)) + 2$.*

Proof. Let $G = (V, E)$ be the complete bipartite fuzzy graph such that there is exactly one path with strength of $E(u, v)$. By analogues to the proof of Theorem 3.1, all the edges are α-strong.

If G be the star fuzzy graph with $V = \{u, v_1, v_2, \cdots, v_n\}$ such that u and v_i are the center and the leaves of G, for $1 \leq i \leq n$, respectively. Then $\{u\}$ is the nikfar dominating set of G. Hence $\gamma_v(G) = V(u) + 1$.

Otherwise, both of V_1 and V_2 include more than one vertex. Every vertex in V_1 is dominated by every vertices in V_2, as α-strong and conversely. Hence in K_{V_1,V_2}, the α-strong dominating sets are V_1 and V_2 and any set containing 2 vertices, one in V_1 and other in V_2. So $\gamma_v(K_{V_1,V_2}) = \min_{u\in V_1, v\in V_2}(V(u) + V(v)) + 2$. The result follows. □

Definition 3.5. (**Ref.** [20], Section 2.1, p.21) Let $G = (V, E)$ be a fuzzy graph and xy be an edge. Then xy is called a *bridge* if $E^{'}(u, v) < E(uv)$ for some $u, v \in V$, where $E^{'}(xy) = 0$ and $E^{'} = E$ otherwise.

Theorem 3.6. *(**Ref.** [20], Theorem 2.4, pp.21,22) Let $G = (V, E)$ be a fuzzy graph and $xy \in E$. Let $E^{'}$ be the fuzzy subset of E such that $E^{'}(xy) = 0$ and $E^{'} = E$ otherwise. Then* $(3) \Leftrightarrow (2) \Leftrightarrow (1)$:

(1) *xy is a bridge;*

(2) $E^{'}(x, y) < E(xy)$;

(3) *xy is not the weakest edge of any cycle.*

Corollary 3.7. *Let $G = (V, E)$ be a fuzzy graph and $xy \in E$. xy is an α-strong edge if and only if xy is a bridge.*

Proof. By Theorem 3.6, the result is obviously hold. □

Definition 3.8. (**Ref.** [20], Section 2.1, pp.22,23) A (crisp) graph that has no cycles is called acyclic or a forest. A connected forest is called a tree. A fuzzy graph is called a forest if the graph consisting of its nonzero edge is a forest and a tree if this graph is also connected. We call the fuzzy graph $G = (V, E)$ a *fuzzy forest* if it has a partial fuzzy spanning subgraph which is a forest, where for all edges xy not in $F[E(xy) = 0]$, we have $E(xy) < E(x, y)$. In other words, if xy is in G, but not F, there is a path in F between x and y whose strength is greater than $E(xy)$. It is clear that a forest is a fuzzy forest. If G is connected, then so is F since any edge of a path in G is either in F, or can be diverted through F. In this case, we call G a *fuzzy tree.*

Theorem 3.9. ***(Ref.*** *[20], Proposition 2.7, p.24) Let $G = (V, E)$ be a fuzzy forest. Then the edges of $F = (\tau, \nu)$ are just the bridges of G.*

Corollary 3.10. *Let $G = (V, E)$ be a fuzzy forest. Then the edges of $F = (\tau, \nu)$ are just the α-strong edges of G.*

Proof. By Theorem 3.9 and Corollary 3.7, the result follows. □

Proposition 3.11. *Let $T = (V, E)$ be a fuzzy tree. Then $D(T) = D(F) \cup D(S)$, where $D(T), D(F)$ and $D(S)$ are nikfar dominating sets of T, F and S, respectively. S is a set of edges which has no edges with connection to F.*

Proof. By Corollary 3.10, the edges of $F = (\tau, \nu)$ are just the α-strong edges of G. So by using Definition 2.4, the result follows. □

4 Some related results independent of classes of fuzzy graphs

We give an upper bound for the nikfar domination number of fuzzy graphs, Proposition 4.1.

Proposition 4.1. *For any fuzzy graph $G = (V, E)$ on V, we have $\gamma_v \leq p$.*

Proof. By Proposition 3.2, $\gamma_v(\bar{K_n}) = p$. So the result follows. □

The classical paper in Ref. [26] of Nordhaus and Gaddum established the inequalities for the chromatic numbers of a graph $G = (V, E)$ and its complement $\bar{G}$. We are concerned with analogous inequalities involving domination parameters in graphs. For any fuzzy graph the Nordhaus-Gaddum(NG)'s result holds, (Theorem 4.2).

Theorem 4.2. *For any fuzzy graph $G = (V, E)$, the Nordhaus-Gaddum result holds. In other words, we have $\gamma_v + \bar{\gamma_v} \leq 2p$.*

Proof. Let G be a fuzzy graph. So $\bar{G}$ is also fuzzy graph. We implement Theorem 4.1, on G and $\bar{G}$. Then $\gamma_v \leq p$ and $\bar{\gamma_v} \leq p$. Hence $\gamma_v + \bar{\gamma_v} \leq 2p$. □

Definition 4.3. An α-strong dominating set D is called a *minimal* α-strong dominating set if no proper subset of D is a α-strong dominating set.

Theorem 4.4. *Let $G = (V, E)$ be a fuzzy graph without isolated vertices. If D is a minimal α-strong dominating set then $V - D$ is a α-strong dominating set.*

Proof. By attentions to all edges between two sets, which are only α-strong, the result follows. □

A domatic partition is a partition of the vertices of a graph into disjoint dominating sets. The maximum number of disjoint dominating sets in a domatic partition of a graph is called its domatic number.

Finding a domatic partition of size 1 is trivial and finding a domatic partition of size 2 (or establishing that none exists) is easy but finding a maximum-size domatic partition (i.e., the domatic number), is computationally hard. Finding domatic partition of size two in fuzzy graph G of order $n \geq 2$ is easy by the following.

Theorem 4.5. *Every connected fuzzy graph $G = (V, E)$ of order $n \geq 2$ has an α-strong dominating set D such that whose complement $V - D$ is also an α-strong dominating set.*

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 14 May 2019 doi:10.20944/preprints201901.0024.v3

Proof. For every connected fuzzy graph, V is an α-strong dominating set. By analogous to proof of Theorem 4.4, we can obtain the result. □

We improve the upper bound for the nikfar domination number of fuzzy graphs without isolated vertices, (Theorem 4.6).

Theorem 4.6. *For any fuzzy graph $G = (V, E)$ without isolated vertices, we have $\gamma_v \leq \frac{p}{2}$.*

Proof. Let D be a minimal dominating set of G. By Theorem 4.5, V-D is an α-strong dominating set of G. Hence $\gamma_v(G) \leq w_v(D)$ and $\gamma_v(G) \leq w_v(V-D)$.

Therefore $2\gamma_v(G) \leq w_v(D) + w_v(V-D) \leq p$ which implies $\gamma_v \leq \frac{p}{2}$. Hence the proof is completed. □

We also improve Nordhaus-Gaddum (NG)'s result for fuzzy graphs without isolated vertices, (Corollary 4.7).

Corollary 4.7. *Let $G = (V, E)$ be a fuzzy graph such that both of G and $\bar{G}$ have no isolated vertices. Then $\gamma_v + \bar{\gamma_v} \leq p$, where $\bar{\gamma_v}$ is the nikfar domination number of $\bar{G}$. Moreover, the equality holds if and only if $\gamma_v = \bar{\gamma_v} = \frac{p}{2}$.*

Proof. By the Implement of Theorem 4.6, on G and $\bar{G}$, we have $\gamma_v(G) = \gamma_v \leq \frac{p}{2}$, and $\gamma_v(\bar{G}) = \bar{\gamma_v}(G) = \bar{\gamma_v} \leq \frac{p}{2}$. So $\gamma_v + \bar{\gamma_v} \leq \frac{p}{2} + \frac{p}{2} = p$. Hence $\gamma_v + \bar{\gamma_v} \leq p$.

Suppose $\gamma_v = \bar{\gamma_v} = \frac{p}{2}$. Then obviously, $\gamma_v + \bar{\gamma_v} = p$. Conversely, suppose $\gamma_v + \bar{\gamma_v} \leq p$. Then we have $\gamma_v \leq \frac{p}{2}$ and $\bar{\gamma_v} \leq \frac{p}{2}$. If either $\gamma_v < \frac{p}{2}$ or $\bar{\gamma_v} < \frac{p}{2}$, then $\gamma_v + \bar{\gamma_v} < p$, which is a contradiction. Hence the only possible case is $\gamma_v = \bar{\gamma_v} = \frac{p}{2}$. □

Proposition 4.8. *Let $G = (V, E)$ be a fuzzy graph. If all edges have equal value, then G has no α-strong edge.*

Proof. By using Definition of α-strong edge, the result is hold. □

The following example illustrates this concept.

Example 4.9. In Figure 4, all edges have the same value but there is no α-strong edges in this fuzzy graph.

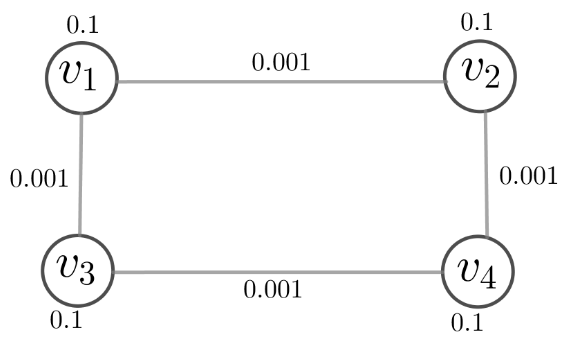

Figure 4. Identical edges and α-strong edges

We give the relationship between M-strong edges and α-strong edges, (Corollary 4.10).

Corollary 4.10. *Let $G = (V, E)$ be a fuzzy graph. If all edges are M-strong, then G has no α-strong edge.*

Proof. By Proposition 4.8, the result follows. □

We give a necessary and sufficient condition for nikfar domination number which is half of order under the conditions. In fact, the fuzzy graphs which whose nikfar domination number is half of order, are characterized under the conditions, (Theorem 4.11).

Theorem 4.11. *In any fuzzy graph $G = (V, E)$ such that values of vertices are equal and all edges have same value, i.e. $\forall u_i, u_j \in V$ and $\forall e_i, e_j \in E$, we have $V(u_i) = V(u_j)$ and $E(e_i) = E(e_j)$. $\gamma_v = \frac{p}{2}$ if and only if for any nikfar dominating set D in G, we have $|D| = \frac{n}{2}$.*

Proof. Suppose D has the conditions. By Proposition 4.8, $d_s(D) = 0$. So by using Definition 2.4, $\gamma_v(G) = V_{u\in D}V(u)$. Since values of vertices are equal and $|D| = \frac{n}{2}$, we have $\gamma_v(G) = \Sigma_{u\in D}V(u) = \frac{n}{2}V(u) = \frac{1}{2}(nV(u)) = \frac{1}{2}(\Sigma_{u\in V}V(u)) = \frac{1}{2}(p) = \frac{p}{2}$. Hence the result is hold in this case.

Conversely, suppose $\gamma_v = \frac{p}{2}$. Let $D = \{u_1, u_2, \cdots, u_n\}$ be a nikfar dominating set. By Proposition 4.8, $d_s(D) = 0$. So by using Definition 2.4, $\gamma_v(G) = \Sigma_{u\in D}V(u)$. Since $\gamma_v(G) = W_v(D)$, we have $\gamma_v = \frac{p}{2} = \frac{1}{2}(\Sigma_{u\in V}V(u)) = \Sigma_{u\in D}V(u)$. Suppose $n^{'} \neq \frac{n}{2}$. So $\Sigma_{i=1}^{n^{''}}V(v_i)) = 0$ which is a contradiction with $\forall u_i \in V, V(u_i) > 0$. Hence $n^{'} = \frac{n}{2}$, i.e. $|D| = n^{'} = \frac{n}{2}$. The result is hold in this case. □

5 dominating restrictions

The goal of this section is to prove some results concerning operations on a fuzzy graph and study some conjectures arising from it.

The nikfar domination of union of two fuzzy graphs is studied, (Proposition 5.1).

Proposition 5.1. *Let G_1 and G_2 be fuzzy graphs. The nikfar dominating set of $G_1 \cup G_2$ is $D = D_1 \cup D_2$ such that D_1 and D_2 are the nikfar dominating sets of G_1 and G_2, respectively. Moreover, $\gamma_v(G_1 \cup G_2) = \gamma_v(G_1) + \gamma_v(G_2)$.*

Proof. By using Definition of union of two fuzzy graphs, the result is obviously hold. □

Also the nikfar domination of union of fuzzy graphs family is discussed, (Corollary 5.2).

Corollary 5.2. *Let $G_1, G_2, \cdots, G_n$ be fuzzy graphs. The nikfar dominating set of $\cup_{i=1}^{n}G_i$ is $D = \cup_{i=1}^{n}D_i$ such that D_i is the nikfar dominating set of G_i. Moreover, $\gamma_v(\cup_{i=1}^{n}G_i) = \Sigma_{i=1}^{n}\gamma_v(G_i)$.*

Proof. By Proposition 5.1, the result is hold. □

The concepts of both monotone increasing fuzzy graph property, (Definition 5.3), and monotone decreasing fuzzy graph property, (Definition 5.5), are introduced.

Definition 5.3. We call a fuzzy graph property P monotone increasing if $G \in P$ implies $G + e \in P$, i.e., adding an edge e to a fuzzy graph G does not destroy the property.

Example 5.4. Connectivity and Hamiltonicity are monotone increasing properties. A monotone increasing property is nontrivial if the empty fuzzy graph $\bar{K}_V \notin P$ and the complete fuzzy graph $K_V \in P$.

Definition 5.5. A fuzzy graph property is monotone decreasing if $G \in P$ implies $G - e \in P$, i.e., removing an edge from a graph does not destroy the property.

Example 5.6. Properties of a fuzzy graph not being connected or being planar are examples of monotone decreasing fuzzy graph properties.

Remark 5.7. Obviously, a fuzzy graph property P is monotone increasing if and only if its complement is monotone decreasing. Clearly not all fuzzy graph properties are monotone. For example having at least half of the vertices having a given fixed degree d is not monotone.

Conjecture (Vizing **Ref.** [6]). For all graphs G and H, $\gamma(G)\gamma(H) \leq \gamma(G \times H)$.

By using α-strong edge and monotone decreasing fuzzy graph property, the result in relation with Vizing's conjecture is determined, (Theorem 5.8).

Theorem 5.8. *The Vizing's conjecture is monotone decreasing property in fuzzy graph G, if the edge e be α-strong and $\gamma_v(G-e) = \gamma_v(G)$.*

Proof. The fuzzy graph $(G-e) \times H$ is the spanning fuzzy subgraph of $G \times H$, for all fuzzy graph H. So $\gamma_v((G-e) \times H) \geq \gamma_v(G \times H) \geq \gamma_v(G)\gamma_v(H) = \gamma_v(G-e)\gamma_v(H)$. Hence Vizing's conjecture is also hold for $G-e$. Then the result follows. □

By α-strong edge and spanning fuzzy subgraph, some results in relation with Vizing's conjecture is studied, (Corollary 5.9).

Corollary 5.9. *Suppose the Vizing's conjecture is hold for G. Let K be the spanning fuzzy subgraph of G such that $\gamma_v(K) = \gamma_v(G)$. Then the Vizing's conjecture is hold for K.*

Proof. The fuzzy graph $K \times H$ is the spanning fuzzy subgraph of $G \times H$, for all fuzzy graph H. So $\gamma_v(K \times H) \geq \gamma_v(G \times H) \geq \gamma_v(G)\gamma_v(H) = \gamma_v(K)\gamma_v(H)$. Hence the Vizing's conjecture is also hold for K. So the result follows. □

The nikfar domination of join of two fuzzy graphs is studied, (Proposition 5.10).

Proposition 5.10. *Let G_1 and G_2 be fuzzy graphs. The nikfar dominating set of $G_1 + G_2$ is $D = D_1 \cup D_2$ such that D_1 and D_2 are the nikfar dominating set of G_1 and G_2, respectively. Moreover, $\gamma_v(G_1 + G_2) = \gamma_v(G_1) + \gamma_v(G_2)$.*

Proof. By using Definition of join of two fuzzy graphs in this case, M-strong edges between two fuzzy graphs is not α-strong which is a weak edge changing strength of connectedness of G. □

Also the nikfar domination of join of fuzzy graphs family is discussed, (Corollary 5.11).

Corollary 5.11. *Let $G_1, G_2, \cdots, G_n$ be fuzzy graphs. The nikfar dominating set of $+_{i=1}^{n} G_i$ is $D = +_{i=1}^{n} D_i$ such that D_i is the nikfar dominating set of G_i. Moreover, $\gamma_v(+_{i=1}^{n} G_i) = \Sigma_{i=1}^{n} \gamma_v(G_i)$.*

Proof. By Proposition 5.10, the result is hold. □

Conjecture (Gravier and Khelladi **Ref.** [34]). For all graphs G and H,

$$\gamma(G)\gamma(H) \leq 2\gamma(G + H).$$

By using α-strong edge and monotone decreasing fuzzy graph property, the result in relation with the Gravier and Khelladi's conjecture is determined, (Theorem 5.12).

Theorem 5.12. *The Gravier and Khelladi's conjecture is monotone decreasing property in fuzzy graph G, if the edge e be α-strong and $\gamma_v(G-e) = \gamma_v(G)$.*

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 14 May 2019 doi:10.20944/preprints201901.0024.v3

Proof. The fuzzy graph $(G - e) + H$ is the spanning fuzzy subgraph of $G + H$, for all fuzzy graph H. So $2\gamma_v((G - e) + H) \geq 2\gamma_v(G + H) \geq \gamma_v(G)\gamma_v(H) = \gamma_v(G - e)\gamma_v(H)$. Hence the Gravier and Khelladi's conjecture is also hold for $G - e$. Then the result follows. □

We conclude this section with some result in relation with the Gravier and Khelladi's conjecture, (Corollary 5.13).

Corollary 5.13. *Suppose the Gravier and Khelladi's conjecture is hold for G. Let K be the spanning fuzzy subgraph of G such that $\gamma_v(K) = \gamma_v(G)$. Then the Gravier and Khelladi's conjecture is hold for K.*

Proof. The fuzzy graph $K + H$ is the spanning fuzzy subgraph of $G + H$, for all fuzzy graph H. So $2\gamma_v(K + H) \geq 2\gamma_v(G + H) \geq \gamma_v(G)\gamma_v(H) = \gamma_v(K)\gamma_v(H)$. Hence the Gravier and Khelladi's conjecture is also hold for K. The result follows. □

6 Dominating monster and other examples

In this section, we introduce one practical application in related to this concept. In the following, we will try to solve this problem by previous definitions. We show that these definitions are incapable of solving this problem and the new definition of this paper can give us a more realistic view of the situation and make it easier to understand the situation. In other words, this definition provides a solution to the problem that is consistent with reality. In the end, we will give a dynamic analysis of the status of this issue. In the dynamic state of this problem, we show that the previous definitions are even incapable of understanding the problem and we present dynamic and reality-based analysis by using the new definition.

Problem[reducing wast of time in transport planning] Consider a set of cities connected by communication paths. Which cities have these properties? Having low traffic levels and other cities associating with at least ones by low-cost roads.

The terms "low traffic" and "low-cost" are vague in nature. So we are faced with a fuzzy graph model. In other words, Let G be a graph which represents the roads between cities. Let the vertices denote the cities and the edges denote the roads connecting the cities. From the statistical data that represents the high traffic flow of cities and high-cost roads, the membership functions V and E on the vertex set and edge set of G can be constructed by using the standard techniques given in Bobrowicz et al. in **Ref.** [2], Reha Civanlar and Joel Trussel **Ref.** [28]. In this fuzzy graph, a dominating set S can be interpreted as a set of cities which have low traffic and every city not in S is connected to a member in S by a low-cost road. Suppose the Figure 5, the fuzzy graph model of the hypothetical condition of cities and the paths between them in a region. We now look at the answer to the problem raised by using the old and the new definitions. As you can see in this model, finding the desirable cities is more important than finding the domination number. Because the numbers given for the set and each situation are compared with each others in the context of the same definition, and this number is merely to compare the different sets of cities in the context of the same definition. Therefore, speaking of the magnitude of this number in other definitions is meaningless. The table below illustrates the solutions presented for this problem.

 doi:10.20944/preprints201901.0024.v3

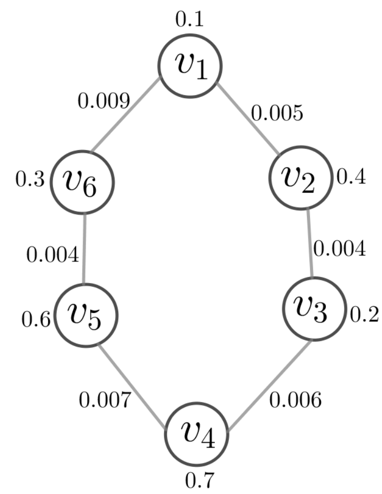

Figure 5. The exemplary scheme of road infrastructure

Definitions	Given desirable set
A. Somasundaram and S. Somasundaram (Ref. [31])	V
C. Natarajan and S.K. Ayyaswamy (Ref. [23])	V
O.T. Manjusha and M.S. Sunitha (Ref. [14])	$\{v_3, v_6\}$
A. Nagoor Gani and K. Prasanna Devi (Ref. [22])	V
O.T. Manjusha and M.S. Sunitha (Ref. [13])	$\{v_3, v_6\}$
N. Sarala and T. Kavitha (Ref. [30])	$\{v_3, v_6\}$
Our new definition	$\{v_1, v_4\}$

It is obvious from the above table and Figure 5 that the desirable cities given by previous definitions, are meaningless due to the lack of simultaneous attention to cities and roads.

7 Dynamic example on domination: Another monster

We are now presenting the dynamic status of the problem. The dynamic state is the situation in which the fuzzy graph model is found over time. Since over time, roads are becoming more affected and more precisely, they get worse, so the value of the roads increases, but cities do not change significantly over time, in their traffic. Because the traffic problem is an infrastructure problem. So in the Figure 6 presented by the dynamic fuzzy graph model, we present a situation in which, and over time, the value of the paths increases equally. In this situation, the answer given by the previous definitions reflects their inability to solve this problem, while the new definition adapts itself well to the new situation. The ineffectiveness and meaninglessness of previous definitions due to the lack of simultaneous attention to cities and roads.

Dynamic analysis of networks in the first row of Figure 6 are the following table.

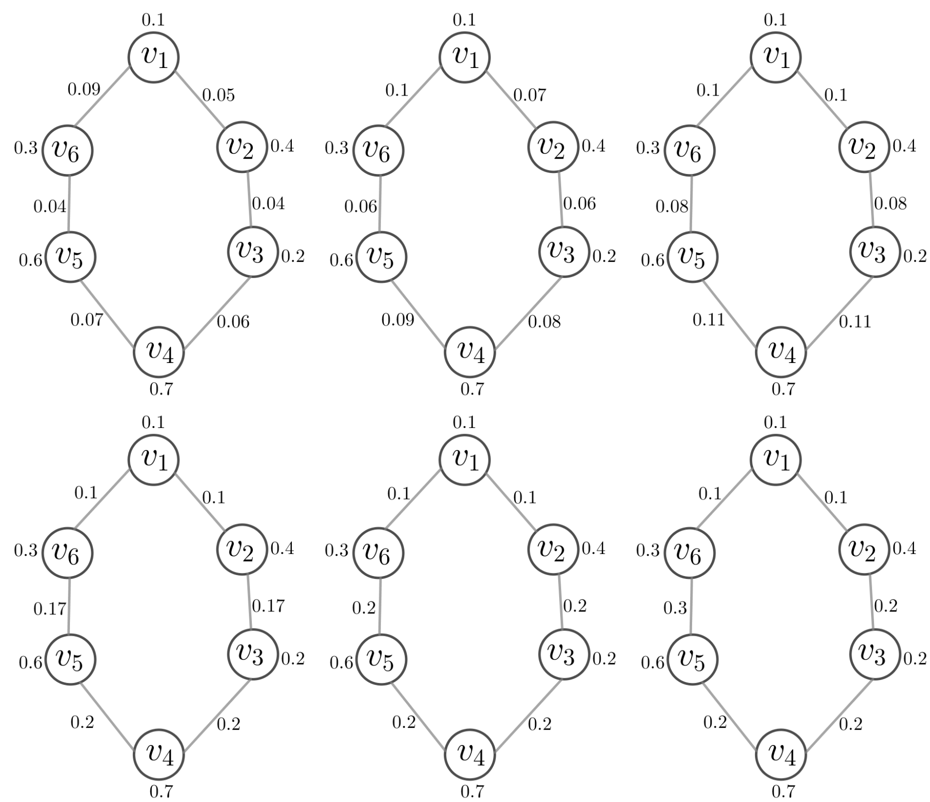

Figure 6. The dynamic scheme of road infrastructure

Definitions	Given desirable set
A. Somasundaram and S. Somasundaram (Ref. [31])	V, $V-\{v_6\}$, $V-\{v_2, v_6\}$
C. Natarajan and S.K. Ayyaswamy (Ref. [23])	V, $V-\{v_6\}$, $V-\{v_2, v_6\}$
O.T. Manjusha and M.S. Sunitha (Ref. [14])	$\{v_3, v_6\}$, $\{v_3, v_6\}$, $\{v_3, v_6\}$
A. Nagoor Gani and K. Prasanna Devi (Ref. [22])	V, V, V
O.T. Manjusha and M.S. Sunitha (Ref. [13])	$\{v_3, v_6\}$, $\{v_3, v_6\}$, $\{v_3, v_6\}$
N. Sarala and T. Kavitha (Ref. [30])	$\{v_3, v_6\}$, $\{v_3, v_6\}$, $\{v_3, v_6\}$
Our new definition	$\{v_1, v_4\}$, $\{v_1, v_4\}$, $\{v_1, v_4\}$

According to the upper and lower tables, the desirable set given over time by using of the previous definitions, either provided the same solutions such as O.T. Manjusha and M.S. Sunitha (Ref. [14]), O.T. Manjusha and M.S. Sunitha (Ref. [13]) and N. Sarala and T. Kavitha (Ref. [30]) or in spite of a tangible change in their solutions to different situations, the general solutions have given. Additionally, the solutions of these definitions to the problem is not consistent with reality.

Dynamic analysis of networks in the second row of Figure 6 are the following table.

Definitions	Given desirable set
A. Somasundaram and S. Somasundaram (Ref. [31])	$\{v_1, v_4\}$, $\{v_1, v_3, v_6\}$, $\{v_3, v_6\}$
C. Natarajan and S.K. Ayyaswamy (Ref. [23])	$\{v_1, v_4\}$, $\{v_1, v_3, v_6\}$, $\{v_3, v_6\}$
O.T. Manjusha and M.S. Sunitha (Ref. [14])	$\{v_3, v_6\}$, $\{v_3, v_6\}$, $\{v_3, v_6\}$
A. Nagoor Gani and K. Prasanna Devi (Ref. [22])	$V-\{v_1\}$, $V-\{v_1\}$, $V-\{v_1\}$
O.T. Manjusha and M.S. Sunitha (Ref. [13])	$\{v_3, v_6\}$, $\{v_3, v_6\}$, $\{v_3, v_6\}$
N. Sarala and T. Kavitha (Ref. [30])	$\{v_3, v_6\}$, $\{v_3, v_6\}$, $\{v_3, v_6\}$
Our new definition	$\{v_3, v_6\}$, $\{v_3, v_6\}$, $\{v_3, v_6\}$

494

References

1. C. Berge, *The Theory of Graphs and its Applications*, Methuen, London, 1962.
2. O. Bobrowicz, C. Choulet, A. Haurat, F. Sandoz, M. Tebaa, *A method to build membership functions- Application to numerical/symbolic interface building*, in Proceedings of 3rd Internat. Conference on Information Processing and Management of Uncertainty in Knowledge Based System, Paris 1990.
3. B. Bollobás and E.J. Cockayne, unpublished.
4. Brešar, Boštjan; Dorbec, Paul; Goddard, Wayne; Hartnell, Bert L.; Henning, Michael A.; Klavžar, Sandi; Rall, Douglas F. (2012), *Vizing's conjecture: a survey and recent results*, Journal of Graph Theory 69 (1) 46-76.
5. G. Chartrand and L. Lesniak, *Graphs and Digraphs*, Third edition, CRC Press, 1996.
6. W.E. Clark and S. Suen, *An Inequality Related to Vizing's Conjecture*, Electronic J. Combinatorics 7 1 (2000) 1-3, 2000.
7. E.J. Cockayne and S.T. Hedetniemi, *Toward a theory of domination in graphs*, Networks 7 (1977) 247-261.
8. M. Ismayil and I. Mohideen, *Complementary nil domination in fuzzy graphs*, Annals of Fuzzy Mathematics and Informatics (2014) 1-8.
9. F. Jaeger and C. Payan, *Relations du type Nordhaus-Gaddum pour le nombre d'absorption d'un graphe simple*, C.R. Acad. Sci. Paris A 274 (1972) 728-730.
10. J.P. Joseph and S. Arumugam, *A note on domination in graphs*, lnternet. J. Management Systems, to appear.
11. S. Klavžar, B. Zmazek, *On a Vizing-like conjecture for direct product graphs*, Discrete Mathematics, 156 (1996) 243-246.
12. R. Laskar and K. Peters, *Vertex and edge domination parameters in graphs*, Congr. Numer. 48 (1985) 291-305.
13. O.T. Manjusha and M.S. Sunitha, *Strong Domination in Fuzzy Graphs*, Fuzzy Inf. Eng. 7 (2015) 369-377.
14. O.T. Manjusha and M.S. Sunitha, *Notes on domination in fuzzy graphs*, Journal of Intelligent and Fuzzy systems 27 (2014) 3205-3212.
15. O.T. Manjusha and M.S. Sunitha, *Connected domination in fuzzy graphs using strong edges*, Annals of Fuzzy Mathematics and Informatics 10 (6) (2015) 979-994.
16. O.T. Manjusha and M.S. Sunitha, *The Strong Domination Alteration Sets in Fuzzy Graphs*, International Journal of Mathematics and its Applications 4 (2-D) (2016) 109-123.
17. O.T. Manjusha and M.S. Sunitha, *Total Domination in Fuzzy Graphs Using Strong Edges*, Annals of Pure and Applied Mathematics 9 (1) (2015) 23-33.

18. S. Mathew and M.S. Sunitha, *Types of edges in a fuzzy graph*, Information Sciences 179 (2009) 1760-1768.

19. J.N. Mordeson and C.S. Peng, *Operations on fuzzy graphs*, Information Sciences 79 (1994) 159-170.

20. J.N. Mordeson and P.S. Nair, *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica-Verlag, 2000.

21. A. Nagoorgani and V.T. Chandrasekaran, *Domination in fuzzy graph*, Adv. in fuzzy sets and systems 1 (1) (2006) 17-26.

22. A. Nagoor Gani and K. Prasanna Devi, 2-*Domination in Fuzzy Graphs*, International Journal of Fuzzy Mathematical Archive 9 (1) (2015) 119-124.

23. C. Natarajan and S.K. Ayyaswamy, *on Strong (weak) domination in Fuzzy Graphs*, International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering 4 (7) (2010) 1035-1037.

24. A. Nagoorgani and P.Vadivel, *Relations between the parameters of Independent Domination and Irredundance in Fuzzy Graphs*, International Journal of Algorithms, Computing and Mathmatics 2 (1) (2009) 15-19.

25. H.T. Nguyen and E.A. Walker, *A First course in fuzzy logic*, CRC Press, 2006.

26. E.A. Nordhaus and J.W. Gaddum, *On complementary graphs*, Amer. Math. Monthly 63 (1956) 175-177.

27. O. Ore, *The Theory of Graphs*, Amer. Math. Soc. Colloq. Pub., providence, RI 38 1962.

28. M. Reha Civanlar, H. Joel Trussel, *Constructing membership functions using statistical data*, Fuzzy Sets Syst.,18 (1986) 1-13.

29. A. Rosenfeld, *Fuzzy Graphs*, In Fuzzy Sets and their Applications to Cognitive and Decision Processes, eds. L.A. Zadeh, K.S. Fu, K. Tanaka and M. Shimura, Academic Press, New York (1975) 77-95.

30. N. Sarala and T. Kavitha, $(1,2)$-*Vertex Domination in Fuzzy Graphs*, International Journal of Innovative Research in Science, Engineering and Technology 5 (7) (2016) 16501-16505.

31. A. Somasundaram and S. Somasundaram, *Domination in fuzzy graphs-I*, Pattern Recognition Letters 19 (1998) 787-791.

32. Dr.S. Vimala and J.S. Sathya, *Efficient Domination number and Chromatic number of a Fuzzy Graph*, International Journal of Innovative Research in Science, Engineering and Technology 3 (3) (2014)9965-9970.

33. R.T. Yeh, S.Y. Bang, *Fuzzy relations, fuzzy graphs and their applications to clustering analysis*, in: Fuzzy Sets and Their Applications to Cognitive and Decision Processes, eds. L.A. Zadeh, K.S. Fu, K. Tanaka and M. Shimura, Academic Press, (1975) 125-149.

34. L.A. Zadeh, *Fuzzy sets*, Information and Control 8 (1965) 338-353.