Article

Not peer-reviewed version

Modeling Informational Resonance in Spatio-Temporal Systems

Masoud Ataei *

Posted Date: 23 June 2025

doi: 10.20944/preprints202506.1780.v1

Keywords: information geometry; spatio-temporal dynamics; Euler-Mascheroni constant; Omega constant; Pi constant; fine-structure constant; Lambert-W function; harmonic series doi:10.20944/preprints202506.1780.v1

Article

Modeling Informational Resonance in Spatio-Temporal Systems

Masoud Ataei

Department of Mathematical and Computational Sciences, University of Toronto, Ontario, Canada; masoud.ataei@utoronto.ca

Abstract

We construct a spatio-temporal model of information dynamics in which fundamental mathematical constants arise from analytically defined observational mechanisms. The model describes a transient field propagating radially with exponential intensity and geometric attenuation, yielding a point of maximal observability at radius Ω, the Omega constant. Temporal detection is modeled via a harmonic acquisition process, with cumulative inefficiency asymptotically approaching the Euler-Mascheroni constant γ. These two scales are shown to satisfy the approximate balance law $\gamma + \Omega \approx \log \pi$, interpreted as a structural resonance between spatial embedding and temporal accumulation. A refined expression, $\frac{e^\gamma}{\Omega} + \frac{\alpha}{2\pi} \approx \pi$, incorporating the fine-structure constant α, achieves greater numerical precision. The formulation highlights an interaction between growth, attenuation, and sampling efficiency, and suggests a deeper geometric-informational correspondence among different mathematical constants.

Keywords: information geometry; spatio-temporal dynamics; Euler-Mascheroni constant; Omega constant; Pi constant; fine-structure constant; Lambert-W function; harmonic series

---

1. Introduction

This work develops an analytically defined model in which three fundamental mathematical constants, Euler-Mascheroni's constant γ, the Omega constant Ω, and the circular constant π, arise as structural features of a coupled spatio-temporal process. Each constant traditionally originates from a distinct analytic domain: γ characterizes the limiting discrepancy between the harmonic series and the natural logarithm, Ω is the unique real solution to the transcendental equation $xe^x = 1$, and π governs intrinsic geometric periodicity [1–3]. We show that, within a unified model of information dynamics, these constants are linked through a balance law that emerges from well-defined spatial and temporal mechanisms.

The model considers a transient process emitting an informational field that expands radially with exponential intensity while undergoing geometric attenuation. This competition yields an extremal observation radius, analytically identified as Ω, corresponding to maximal detectability. Temporal observation is modeled as a sequential sampling process governed by harmonic accumulation, whose asymptotic efficiency is captured by γ. From these two mechanisms, we derive a resonance quotient e^γ/Ω, which closely approximates π, suggesting a structural synchronization between spatial observability and temporal acquisition.

We further refine the model by incorporating a small dimensionless correction term $\alpha/2\pi$, where α denotes the fine-structure constant [4]. This yields a numerically sharper approximation, $\frac{e^\gamma}{\Omega} + \frac{\alpha}{2\pi} \approx \pi$, with a residual error of order 10^{-6}. While this refinement is primarily empirical, it motivates further investigation into the relation between dimensionless geometric-information models and physically grounded scale parameters.

The following section formalizes this framework. We derive the spatial and temporal observation principles, establish the role of each constant, and interpret the resulting relations as evidence of an underlying structure linking information flow, geometric embedding, and sampling efficiency.

 doi:10.20944/preprints202506.1780.v1

2. Spatio-Temporal Observation Model

We model the phenomenon as emitting an informational field that expands radially within a three-dimensional spherical medium. The informational intensity at radius r is assumed to grow exponentially, reflecting entropic dispersion or volumetric signal proliferation in the embedding space as follows:

$$I(r) \sim e^r. \tag{1}$$

This models scenarios where each additional unit of radial extension exponentially increases the potential informational content, as observed in diffusion-limited or radiative systems.

Simultaneously, the likelihood of successful observation diminishes with distance due to geometric attenuation as follows:

$$A(r) \sim \frac{1}{r}. \tag{2}$$

This attenuation reflects the dilution of informational flux across expanding spherical shells. The competing effects of exponential growth and inverse-area decay determine a balance point for optimal detectability.

Equating the two factors yields the following transcendental condition

$$e^r = \frac{1}{r}. \tag{3}$$

This equation admits a unique solution in $(0, 1)$ as follows:

$$r^* = W(1) = \Omega, \tag{4}$$

where $W(\cdot)$ denotes the principal branch of the Lambert-W function. The solution $\Omega \approx 0.5671$ characterizes the critical spatial scale at which the signal-to-noise ratio is maximized and hence defines the surface of maximal observational coherence.

We interpret this shell $r = \Omega$ as a geometric locus of informational resonance, a manifestation horizon beyond which dispersal dominates and within which field strength remains subcritical.

The observation process unfolds temporally as a sequence of probabilistic detection attempts. Each attempt yields diminishing marginal returns, with the kth contribution given by $\frac{1}{k}$. The cumulative gain over n trials is captured by the harmonic series $\sum_{k=1}^{n} \frac{1}{k}$. This gain is offset by a logarithmic observational cost $\log n$, modeling entropic growth in the search space. The asymptotic difference then converges to the Euler-Mascheroni constant as follows:

$$\lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{1}{k} - \log n \right) = \gamma. \tag{5}$$

The exponential e^γ thereby quantifies the intrinsic efficiency of sequential acquisition, representing the asymptotic rate at which information is accrued per logarithmic unit of effort.

We now consider the interaction between temporal sampling efficiency, governed by γ, and spatial optimality, governed by Ω. Define the dimensionless ratio $\frac{e^\gamma}{\Omega}$, which satisfies numerically the following relation

$$\frac{e^\gamma}{\Omega} \approx 3.14043 \approx \pi.$$

The residual discrepancy, approximately 1.1651×10^{-3}, indicates a remarkable alignment between harmonic accumulation and spherical embedding. This near-equality suggests a resonance condition wherein the observer's intrinsic sampling rhythm becomes phase-synchronized with the geometric periodicity of the manifestation shell.

Taking logarithms yields the additive approximation as follows:

$$\gamma + \Omega \approx \log \pi. \tag{6}$$

We interpret this as a logarithmic balance law: the cumulative efficiency of temporal observation added to the spatial scale of optimal manifestation approximates a fundamental geometric invariant. The additive structure reflects a more transparent and interpretable form of the underlying resonance.

In this formulation, γ quantifies the inefficiency of harmonic sampling, Ω characterizes the spatial embedding of the observable structure, and $\log \pi$ serves as a geometric binding constant. Although this relation is heuristic rather than derived from axiomatic foundations, the minimal residual and the natural emergence of each constant suggest it captures a deeper structural regularity governing the interaction of space, time, and information flow.

We further note that the balance law may admit a refined form through the incorporation of a quantum correction. Specifically, the following relation

$$\frac{e^{\gamma}}{\Omega} + \frac{\alpha}{2\pi} \approx \pi, \tag{7}$$

yields an even more accurate approximation, with the residual reduced to approximately 3.6868×10^{-6}. Here, $\alpha \approx \frac{1}{137}$ denotes the fine-structure constant, a dimensionless physical constant characterizing the strength of electromagnetic interaction. The correction term $\frac{\alpha}{2\pi}$ corresponds to the leading-order contribution in the anomalous magnetic moment of the muon, suggesting a potential coupling between the geometric-harmonic structure of the observational model and subtle quantum electrodynamic effects.

The appearance of this term within the refined balance may indicate that the emergent spatio-temporal resonance encodes not only classical informational dynamics but also traces of quantum field-theoretic structure. This observation invites further inquiry into whether such precision refinements can be systematically linked to fundamental interaction constants, thereby bridging abstract mathematical constructs with empirically grounded physical scales.

3. Conclusion

We have presented a formally motivated model in which three analytically distinct constants, γ, Ω, and π, arise as natural invariants of a coupled spatio-temporal observation process. By modeling the propagation of an informational field with exponential intensity and geometric attenuation, and pairing it with a temporal acquisition process governed by harmonic accumulation, we derived a balance law linking these constants. The resulting expressions, including the additive form $\gamma + \Omega \approx \log \pi$ and the multiplicative form $e^{\gamma}/\Omega \approx \pi$, exhibit structural coherence and a high degree of numerical precision.

We also examined a refined version of the balance law involving a small correction term $\alpha/2\pi$, where α is the fine-structure constant. While the inclusion of this term is introduced empirically, its role as a precision enhancement invites further study into whether dimensionless physical parameters may play a role in regulating spatio-temporal informational efficiency.

References

1. J. Spanier and K. B. Oldham, *An Atlas of Functions*, Taylor & Francis/Hemisphere, 1987.
2. R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth, "On the Lambert W function," *Advances in Computational Mathematics*, vol. 5, pp. 329–359, Springer, 1996.
3. K. Knopp, *Theory and Application of Infinite Series*, Courier Corporation, 1990.
4. L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory*, vol. 3, Elsevier, 2013.