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Horndeski-Like Gravity Perturbation Induced by Large-Magnitude Earthquake

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Horndeski-like Gravity Perturbation Induced by Large-Magnitude Earthquake

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Abstract: We hypothesis that large earthquakes generate Horndeski-like Gravitational Wave (GW). We find that such a Horndeski-like GW propagates with the speed of sound. The sound waves generated by an earthquake make a local and temporal change to the Earth's diagravitational medium; therefore, they modify the GW speed in a standard, Alternative-Theory-of-Gravity (ATG) sense. The quantum of the Horndeski-like GW is a massless scalar quasiparticle and cannot exist outside of the propagation region of the P-field. The Horndeski-like GW may be detectable by future GW detectors with a sensitivity of 10−15 $Hz^{-1/2}$ in the region of 0.1–1 Hz.

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Alternative Theories of Gravity (ATGs) currently attract much attention as they provide reasonable descriptions of an accelerating universe [1,2]. Horndeski's theory [3], a particular family of ATGs, deals with the most general scalar-gravity, Lagrangian, leading to second-order equations of motion [3,4]. Horndeski's theory predicts the propagation of anomalous gravitational waves (GWs). Direct measurements of GWs in the last few years have put strong constraints on the possible modification of the gravitational coupling (κ^2_{eff}) and speed (c_T) of GWs in the recent Universe [5–11], in particular, imposing the condition that c_T (t_{now}) ≃ 1.

Angelo Esposito, Rafael Krichevsky, and Alberto Nicolis (2019 [12]) showed that, contrary to common belief, sound waves carry gravitational mass in a standard Newtonian sense: they are affected by gravity, but they also source gravity. They showed that this effect goes hand-in-hand with the non-linear interactions of sound, and that it occurs in superfluids, fluids, and solids. For all these media, in the non-relativistic limit, the mass transported by a sound wave is proportional to its energy times a coefficient that only depends on the medium's equation of state. In general, the mass transported in this way is quite small, of the order: $M_{sound} \sim E / a_s^2$. For instance, a very energetic phonon in superfluid helium-4 with a momentum k ~ 1keV (i.e., a wavelength of the order of the Bohr radius: a_0=5.29177210903(80) x 10^{-11}m) carries a mass M of the order of ~ 1 GeV, i.e., that of a single helium atom. Nevertheless, it is possible to envision experimental setups where this effect could be detected.

Another possible setting where sound waves cane transport mass may be seismic phenomena in the Earth's lithosphere. The wave generated by an earthquake of a Richter magnitude 9 carries an energy $E_{seismic}$ of about 10^{18} Joules which, for α_s ~ 5 km/s, corresponds to an $M_{sound} \sim 10^{11} kg$ transported mass, and a change in gravitational acceleration, δg, of about 10^{-4} nm/s². Atomic clocks and quantum gravimeters can currently detect tiny changes in the gravitational acceleration (up to fractions of nm/s² [13–15]). Given the rapid development of these techniques, one can imagine that in the not-too-distant future such instruments will reach the sensitivity needed to detect the gravitational fields of a seismic wave.

Earthquakes generate transient and static deformation which alter the spatial density distribution of the Earth's crust. This density redistribution induces changes in the Earth's gravity field. Transient gravity perturbations have been studied as a source of Newtonian Noise for GW detectors [16,17]. Harms et.al in [18,19] studied how the volumetric deformation carried by *P*-waves generates a P-wave field $\phi_s(r,t)$ and Newtonian gravity perturbations $\delta\psi_{New}(r,t)$, which propagate with the speed of light. Since these perturbations are measurable, they can contribute to the development of early warning systems for earthquakes and tsunamis [18].

Let us now consider a Horndeski-like gravity in which the scalar field is the seismic P-wave field $\phi_s(r,t)$, which propagates with the constant speed of sound, α_s ~ 5 km/s, during the interval of interest, and that the gravitational field $\delta\psi(r,t)$ is the gravitational wave (GW) perturbation that was induced by a large-magnitude earthquake. The proposed Horndeski-like Lagrangian for the scalar+gravity system can then be written as follows:

$$\Im = \frac{1}{8\pi G_N}\left[\left(\delta\dot{\psi}_{SG}\right)^2 - c_T^2(X_s,\phi_s)\left(\nabla\delta\psi_{SG}\right)^2\right] - \Phi_H(r,t)\delta\psi_{SG}\,, \tag{1}$$

where

$$c_T^2(X_s,\phi_s) = \frac{F(X_s,\phi_s)-1}{G_T(X_s)-1} = -\frac{\ddot{\phi}_s}{(\nabla\phi_s)^2} \simeq \alpha_s^2\,, \tag{2}$$

is the squared speed $c_T^2(X_s,\phi_s)$ of the Horndeski-like GW, $\delta\psi_{SG}(r,t)$. This is approximately equal to the squared speed of sound of P-waves $\phi_s(r,t)$, α_s^2:

$$\begin{aligned} F(X_s,\phi_s) &= G_N - X_s\dot{\phi}_s^2 G_{5X_s}\,, \\ G_T(X_s) &= G_N - 2X_s G_{4X_s} \end{aligned}\,, \tag{3}$$

In Eq. (3), $X_s = -(\nabla\phi_s)^2/2$ is the P-field kinetic term and $G_{iXs} = \partial G_i/\partial X_s, G_N = 1/M_{Pl}^2$ is the Newton constant, and $M_{Pl} = 1.2\times10^{19}GeV$ [1]. The wave equation for the P-wave potential is $\ddot{\phi}_s = \alpha_s^2\nabla^2\phi_s$, and we have a setting of $G_4 \sim X_s^2$, and $G_5 \sim X_s$. $\Phi_H(r,t)$, the scalar Helmholtz potential of the double-couple source, is given by:

$$\Phi_H(r,t) = \frac{M_0}{2\pi}\cos\left[\frac{2\pi}{T}(t-\tau)\right]\frac{\partial^2(1/r)}{\partial x\partial z}\,. \tag{4}$$

The seismic moment is $M_0 = \mu\delta A$, where A is the total ruptured area (of about 400km^2), μ is the shear modulus – a measure of the rigidity of the body, given by the ratio of shear stress to shear strain; and T is rise time of the source, the time necessary for walls of the fault to move with respect to each other (about 1-10 sec) and τ is the repute time of the source, the time it takes for the cracking to propagate from one end of the fault to the other [18].

The Lagrangian (1) thus lead to the following equations of motion:

$$\delta\ddot{\psi}_{SG}(r,t) - \alpha_s^2\nabla^2\delta\psi_{SG}(r,t) = 4\pi G_N\Phi_H(r,t)\,. \tag{5}$$

The solution of the wave Equation (5) at a distance $r = r_0 = \alpha_s t$, where the GW detector is located far away from the source, is given by:

$$\delta\psi_{SG}(r,t) = \frac{G_N M_{sound}}{er\alpha_s^2}\cos[\omega_{SG}(t-\tau)]\,. \tag{6}$$

Solution (6) is the Horndeski-like GW induced by a large-magnitude earthquake which propagate with the speed of sound, α_s. The parameter e is the ratio of the emitted seismic energy $E_{seismic}$ to the seismic moment M_0 ($e = E_{seismic}/M_0$) and ranges from 10^{-6} to 10^{-3} [20,21]. The sound mass is $M_{sound} \sim E_{seismic}/a_s^2$ [12]. The frequency of the Horndeski-like GW $\omega_{SG}=2\pi/T_{SG}$ is determined by the

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typical time-scale for things happening in the seismic source. Here, since the mass radiating the wave moves back and forth say, in 1-10 sec then we will have a period, T_{SG}, near 1 sec and a frequency, ω_{SG}, near 0.1–1 Hz.

The solution of Equation (5) can be expressed in terms of the frequency ω_{SG} and wave number vector $\vec{k}$ of the wave. We assume that $k_{SG} = n_g \omega_{SG}$, with $k_{SG} = \sqrt{\vec{k}_{SG}^2}$, which is just the standard definition of refractive index used in electrodynamics. Taking a plane-wave solution of Equation (5), we obtain $n_g^2 = 1/a_s^2$ in Minkowski space M_4. It simplifies the treatment to consider that the modification on the propagation of GWs can be encapsulated in an effective diagravitational refractive index n [11]. We may hypothesise that the sound waves generated by an earthquake of Richter magnitude 9 make a local and temporal change to Earth's diagravitational medium only in the propagation region of the P-wave field. These waves, therefore, modify the GW speed in a standard ATG sense. Note that the quantum of the Horndeski-like GW is a massless scalar quasiparticle and cannot exist outside of the propagation region of the P-waves field. Furthermore the Horndeski-like gravity does not cause any metric perturbation.

In 2014, UK's Royal Society hosted a conference titled "*The Newtonian Constant of Gravitation: a constant too difficult to measure?*" [22]. The conference aimed to resolve the problem of the large discrepancy between recent measurements of the constant of gravitation (G_N) (with δG_N / G_N =10^{-5} [23]). A reasonable explanation for this discrepancy is that it is due to some still unknown physical cause [23]. This cause is unlikely to be the P-wave field generated by a large earthquake: The gravitational coupling $G_T(X_s)$ is a function of the kinetic term, X_s, of the P-wave field, but as we see for the Equations (3), the contribution of the P-wave field to the discrepancy of G_N measurements on Earth should be of the order $10^{-12}m^{-4}$, that is, insignificant.

Before the arrival of P-waves to the detector, only the first term is non-zero in the right-hand side of Equation (5). The induced Newtonian gravity perturbations $\delta\psi_{New}(r,t)$ thus satisfy

$$\delta\ddot{\psi}_{New}(r,t) = 4\pi G_N \Phi_H(r,t)\,. \tag{7}$$

The induced gravity perturbations $\delta\psi_{New}(r,t)$ appear instantaneous in the Newtonian classical potential approach. In the framework of General Relativity (GR), they propagate with the speed of light, c. The quantum of induced gravity perturbations is thus the GR graviton.

What instruments and instrument concepts could potentially detect these Horndeski-like GWs induced by earthquakes? Several concepts have been proposed for gravity strain meters that target signals between 10 mHz and 10 Hz. These include atom-interferometric, laser interferometric, torsion-bar, and superconducting gravity strain meters (Moody et al. 2002 [24]; Harms et al. 2013 [25]). While none of these concepts has reached the sensitivity required for the detection of earthquake transients, sensitivities of 10^{-15} $Hz^{-1/2}$ in the region of 0.1–1 Hz seem within reach. We note that these low-frequency strain meters are much smaller-scale (of the order of 1 to 10 m) than the km-scale GW detectors LIGO and Virgo which operate at higher frequencies. In fact, some of the modern concepts of low-frequency gravity strain meters evolved from well-known gravity gradiometer technology. We also want to emphasize that the sensitivity of low-frequency gravity strain meters required for the detection of earthquake transients lies well below the sensitivity required for GW detection at the same frequencies (about 10^{-19} $Hz^{-1/2}$ at 0.1 Hz: Harms et al. 2013 [25]). It is, therefore, conceivable that these instruments can be either early prototypes of future GW detectors, or instruments specifically built for geophysical observations.

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