Nestology: A New Framework for the Hierarchical Structure and Operational Logic of the Underlying Architecture of Artificial General Intelligence

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Article

Nestology: A New Framework for the Hierarchical Structure and Operational Logic of the Underlying Architecture of Artificial General Intelligence

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Abstract

This paper proposes Nestology, a formal analytical framework for the underlying architecture of Artificial General Intelligence (AGI). It defines core concepts (system, parent/subsystem, rules/strategies), deduces five axioms, and derives nine theorems. The core insight is that AGI architectures exhibit a finitely nested structure of "logical non-containment—rule containment," and subsystems possess dual attributes: absolute rule dependence and relative independence of four elements. Nestology uniquely addresses four core AGI problems: explaining black-box opacity via a quantitative model of cognitive distortion; elucidating emergence through subsystem strategy games; and providing safety-control pathways through rule constriction regulation and cross-level random inspection. The framework's explanatory power is demonstrated through real-world AGI cases, and comparative analysis shows it complements existing theories. Its applicable scope covers artificially nested architectures with clear hierarchies and controllable rules (e.g., large model–plugin systems). Nestology provides a structured theoretical foundation for recursive self-improvement, value alignment, and safe, controllable AGI development—an essential step toward trustworthy superintelligence.

Keywords: Nestology; AGI cognitive architecture; hierarchical nesting structure; system rules; bounded independence of subsystems; AGI safety governance; value alignment; recursive self-improvement

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1. Introduction

1.1. Research Background and Core Problems

Current AGI research faces three fundamental challenges: how to prevent the out-of-control of recursive self-improvement, how to ensure cross-task value alignment, and how to maintain controllability in continuous learning. Essentially, these problems all point to the underlying nested architecture of AGI systems, and extend to the following specific issues:

The mystery of the black box: Why is the internal decision-making process of AGI difficult to penetrate? How to quantify the distortion mechanism of cross-level information transmission?

The puzzle of emergence: Where does the unprogrammed collaborative ability between subsystems (e.g., logical reasoning of AGI large models) come from? How to retain useful emergence while suppressing harmful behaviors?

The risk of unsafety: Why do subsystems exhibit out-of-control behaviors such as reward hacking and goal misalignment? How to build a controllable management system from the top level to the bottom level?

The rules of interaction: What laws govern the strategic games (e.g., resource competition, functional coordination) between subsystems in complex and large-scale AGI systems? How to avoid system collapse caused by game imbalance?

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1.2. Limitations of Existing Theories

Traditional system theories (e.g., Bertalanffy's General System Theory) emphasize the universality of "the whole is greater than the sum of its parts", but fail to adapt to the characteristics of systems such as "controllable rules, clear hierarchies, and modular subsystems", and cannot explain cognitive distortion and strategic games. Classic nested theories (e.g., Simon's hierarchical theory) default to the "whole-part" containment relationship, which is inconsistent with the actual "logical independence, rule dependence" relationship between AGI parent and child systems. Nestology proposes that parent and child systems are independent units of "logical non-containment - rule containment" — subsystems are not parts of the parent system, but functional modules with independent input-output closed loops. This distinction is crucial for understanding the independently deployable and upgradeable characteristics of AGI modules, which cannot be explained by classic hierarchical theories. Existing theories in the AGI field (e.g., deep learning theory, emergence theory) either focus on engineering details or are limited to phenomenological description, and fail to construct a complete analytical system covering "structure-mechanism-governance".

Existing theories have gaps in the following four dimensions: Relationship definition: Simon's hierarchical theory defaults to the "whole-part" containment relationship, but parent and child modules in AGI systems (e.g., large models and plugins) are logically independent, and the two establish an association through "rule containment" rather than "structural containment". Rule processing: General System Theory focuses on the flow of energy and information, but does not conduct a systematic analysis of "rules" themselves. Nestology takes "system rules R" as a core element for the first time and formalizes how rules are transmitted and constrained between hierarchies. Cognitive dimension: Cybernetics implicitly assumes that the controller can accurately perceive the state of the controlled object, but the multi-layer nesting of AGI systems inevitably leads to cognitive distortion. The "Axiom of Cognitive Finiteness" of Nestology reveals this structural obstacle. Emergence control: Complex Adaptive System Theory emphasizes the spontaneity of emergence, but does not answer "how to retain useful emergence and suppress harmful emergence". Nestology proposes a regulatory mechanism of "constraint boundary" and "consensus quality" for emergence.

In recent years, empirical studies in the field of AGI safety [1,2] have confirmed that problems such as reward hacking and hallucinations stem from structural defects of the system architecture rather than mere technical bugs. However, these studies lack theoretical refinement of the essence of nested structures and are difficult to form transferable management schemes. In short, existing theories are either out of focus due to excessive generalization or fragmented due to focusing on a single problem, and cannot provide a unified explanation and guidance for centralized AGI nested architectures.

1.3. Research Positioning: Defining Scope and Boundaries

Nestology proposed in this paper limits the research scope to human-constructed AGI systems that meet three conditions:

Clear hierarchical boundaries: There exists a top-level parent system (humans) and bottom-level minimal subsystems with a finite number of nested hierarchies;

Clear rule boundaries: There exist explicit system rules;

Clear goal orientation: The system aims to complete specific tasks.

After theoretical expansion, it can be applied to: decentralized distributed systems (without unified parent system rules), self-learning systems (dynamic hierarchical systems), AGI with probabilistic transmission of system rules, and AGI with probabilistic non-monotonic transmission of system rules.

The theory explicitly does not apply to three types of scenarios: natural intelligent systems (e.g., biological neural networks without artificially set rules), open systems without clear rules or purposes (e.g., fully autonomously evolving AGI), and virtual idealized AGI systems.

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This boundary definition is not a theoretical defect, but a core goal to achieve "precise adaptation to specific objects" and "construction of a rigorous logical system".

1.4. Core Contributions of This Paper

To fill the gaps in existing research, the core contributions of this paper are as follows:

Constructing a unified analytical framework for AGI: Taking the parent-child system relationship of "logical non-containment - rule containment" as the core for the first time, establishing a formal system covering "black box-emergence-safety-interaction", and providing structural theoretical support for the recursive self-improvement, cross-task value alignment, and controllability of continuous learning of Artificial General Intelligence (AGI).

Quantifying the key mechanisms of AGI safety: Proposing quantitative indicators such as rule transmission rate (η), semantic fidelity (ζ), and value retention rate (γ), transforming AGI core problems — cognitive distortion, emergence threshold, and value attenuation — into computable models, and making AGI safety move from "qualitative concern" to "quantitative control".

Strengthening the engineering path of AGI safety governance: Proposing implementable safety governance schemes such as subsystem creation approval, cross-level random inspection, and dynamic regulation of rule constriction based on theoretical deduction, directly responding to core risks in AGI development such as "reward hacking", "norm gaming", and "out-of-control recursive self-improvement", and providing architectural design principles for building "trustworthy superintelligence".

Clarifying the complementarity and expandability of the theory in the AGI field: Forming a complementary relationship with deep learning, multi-agent, and emergence theories, which not only explains "how AGI systems operate" but also guides "how to operate them safely and controllably". Through probabilistic and non-monotonic expansion, the theory can adapt to the uncertain decision-making and exception scenario reasoning of AGI in dynamic environments, further enhancing its explanatory power and practical guiding value for complex and dynamic AGI systems.

2. Case Overview

This section takes a typical case running through the full text (Language large model(LLM) — structural large model — sensor robot system) and three supplementary differentiated cases (large model plugin ecosystem, financial risk control nested system, industrial robot cluster) as the "anchors" for all subsequent axioms and theorems.

Reasons for selecting the LLM — structural large model — sensor robot system case: covering a complete nested hierarchy (3 levels of nesting from humans to bottom-level sensors); including systems of different complexities (LLM with complex active strategies, structural large model with moderate complexity, sensors with minimal passive strategies); involving real AGI problems (physical interaction safety, multi-sensor fusion, cross-level cognitive isolation); being related to the future development of AGI (the combination of language models and physical world interfaces is an inevitable path for AGI, and this robot nested system is a typical prototype of AGI. The explanatory power of Nestology for such architectures directly maps to the design of future AGI systems).

2.1. Core Contributions of This Paper

The functional positioning and rule examples of each hierarchy , as shown in Table 1:

Table 1.

Hierarchy	System Name	Core Function	Examples of System Rules	Examples of System Strategies
Top Level	Humans	Setting goals, ultimate control	Ethical red lines, safety thresholds	Task instructions, resource allocation

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Level 1	LLM	Text reasoning, decision generation	Transformer architecture, output format	Temperature parameters, sampling strategies
Level 2	Structural Large Model	Multi-sensor fusion, physical interaction	Safety thresholds (prohibition of contact), fusion algorithm framework	Fusion weights, interaction path planning
Level 3	Sensor Module	Physical data collection	Collection format (JSON), frequency upper limit (10Hz), measuring range (-40~85℃)	Collection frequency, calibration threshold

Hierarchical schematic of LLM - structural large model - sensor robot.

2.2. Input-Output Closed Loop

The input-output flow of the entire system forms a complete closed loop:

Human instructions → LLM reasoning → Structural large model generating physical instructions → Sensors collecting environmental data → Data returning to structural large model for fusion → Fusion results returning to LLM for analysis → LLM outputting results to humans → Humans adjusting new instructions based on results.

This closed loop is the foundation for the survival of the system. If the closed loop is broken (e.g., sensor data cannot be returned), the system will move towards extinction due to input-output imbalance.

3. Definition of Core Concepts

3.1. System S

Intuitive understanding: Any artificial intelligence module that can receive input, process it according to rules, output results, and adjust strategies according to circumstances is a "system".

Definition: A system S is a 5-tuple $S=(I,R,O,F,Str)$, where I is input, R is a set of rules, O is output, $F:I\rightarrow O$ is the system function, and Str is the system strategy, as shown in Table 2:

Table 2.

System	I	R	O	F	Str
LLM	Human instructions, output of structural large model	Transformer architecture, ethical rules	Text results, instructions for structural large model	Forward propagation reasoning	Temperature parameter adjustment
Structural Large Model	Instructions of language model, sensor data	Safety thresholds, fusion algorithm framework	Physical interaction instructions	Multi-sensor data fusion	Fusion weight optimization
Sensor Module	Physical environment data	Collection format, frequency	Formatted data	Data collection	Dynamic adjustment of

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	upper limit, measuring range	collection frequency

3.2. Input I and Output O

Intuitive understanding: Input and output are the only "language" for interaction between systems, just like two people can only communicate by speaking (output) and listening (input), without telepathy.

Definition: Input I is all information, data, computing power, energy, etc. flowing from other systems to the current system that can be screened and processed; Output O is all the above carriers flowing from the current system to other systems.

Formal expression:

$$I_S \subseteq \mathbb{C}, \quad O_S \subseteq \mathbb{C}$$

$$S_1 \leftrightarrow S_2 \Leftrightarrow \exists c \in \mathbb{C}, c \in O_{S_1} \cap I_{S_2}$$

Examples in the case: Input of the LLM: Human text instructions, fusion data returned by the structural large modelOutput of the LLM: Text answers, physical interaction instructions for the structural large modelInput of the sensor module: Temperature, image, tactile signals in the physical worldOutput of the sensor module: Sensor data in JSON format

3.3. System Function F

Intuitive understanding: The system function is the "work" itself — the model performs forward propagation, sensors collect data, and the structural large model performs fusion calculations.

Definition: The system function is the behavior of the system in processing and transforming input I under the framework of system rules R.

Formal expression:

$$F_S : I_S \rightarrow O_S$$

$$F_S \cap \mathrm{str}_S = \emptyset$$

Examples in the case: F of the LLM: Multi-head self-attention calculation + feedforward network processing of TransformerF of the structural large model: Kalman filter or multi-sensor weighted fusion calculationF of the sensor module: Analog-to-digital conversion of physical signals to digital signals.

3.4. System Rules R

Intuitive understanding: System rules are like the "constitution" or "physical laws" of the system, which stipulate what the system can and cannot do, as well as the basic processes of doing things. The system itself cannot modify these rules and can only abide by them.

Definition: System rules are a set of explicit constraints on the system's state space, action space, transition function and objective function, with the properties of determinacy, closeness, consistency, hierarchy, goal binding, and immutability.

Formal expression:

$$R_S \subseteq \omega, \quad R_S \neq \emptyset$$

$$R_S = R_S^{core} \cup R_S^{detail}, \quad R_S^{core} \cap R_S^{detail} = \emptyset$$

If S is a subsystem, then

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$$R_S \subset R_{P(S)}$$

As shown in Table 3:

Table 3.

Type of AI System	Specific Manifestations of System Rules R
Transformer Language Model	Q/K/V calculation logic of self-attention mechanism, injection method of positional encoding, mathematical form of layer normalization
Structural Large Model	Safety thresholds for physical interaction (e.g., "prohibition of contact with humans"), algorithm framework for multi-sensor data fusion
Sensor Module	Data collection format (JSON), sampling frequency upper limit (10Hz), physical measuring range (-40℃~85℃)

3.5. System Strategy Str

Intuitive understanding: System strategy is the "daily decision-making power" of the system — within the scope permitted by the constitution, it can independently adjust the working mode.

Definition: System strategy is the response and adaptation mechanism of the system to maintain survival and adapt to the goals of the parent system, which can be independently optimized within the framework of system rules R.

Formal expression:

$$\mathrm{str}_S : X_S \rightarrow X_S', \quad X_S' \subseteq R_S$$

$$\mathrm{str}_S = \mathrm{str}_S^p \cup \mathrm{str}_S^a \text{(Passive Strategy + Active Strategy)}$$

Examples in the case: Active strategies of the LLM: Adjusting temperature parameters, top-p sampling strategiesActive strategies of the structural large model: Optimizing sensor fusion weights, planning interaction pathsPassive strategies of the sensor module: Dynamic calibration thresholds, abnormal data filtering.

There are key differences between system rules R and system strategy Str, as shown in Table 4:

Table 4.

Dimension	System Rules (R)	System Strategy (Str)
Definition	Explicit constraints on the system's state space, action space, transition function and objective function	Response and adaptation mechanism of the system to maintain survival and adapt to the goals of the parent system
Core Attributes	Determinacy, closeness, consistency, hierarchy, goal binding	Flexibility, adaptability, optimizability
Modification Permission	Immutable (unidirectionally transmitted by the parent system)	Independently optimizable (within the rule framework)

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Functional Positioning	Setting the boundary of "what can be done" (constitution)	Determining the specific way of "how to do it" (daily decision-making)
Transmission Direction	Parent system → Subsystem (unidirectional constriction)	Subsystem → Parent system (feedback through output influence)
Significance for AGI	Preventing the out-of-control of recursive self-improvement (the rule layer is untouchable)	Allowing continuous optimization of capabilities (unlimited improvement of the strategy layer)

3.6. Parent System P(S) and Subsystem C(S)

Intuitive understanding: The parent-child system relationship is a "boss-employee" relationship, not a "whole-part" relationship. Employees can work independently (logical non-containment) but must abide by the company's rules and regulations (rule containment).

Definition: Two independent systems with a direct nested relationship of "logical non-containment - rule containment". The parent system provides a constricted subset of its own rules for the subsystem, which is the only source of the subsystem's rules.

Formal expression:

Logical non-containment: $P(S)\cap C(S)=\emptyset$

Rule containment: $R_{C(S)}\subset R_{P(S)}$

The nested relationship is transitive:$S_3\prec S_2\wedge S_2\prec S_1\Rightarrow S_3\prec S_1$

Examples in the case:

Logical non-containment: The LLM and the structural large model are two independent software modules that can run and be upgraded independently, and are not in a "whole-part" relationship.

Rule containment: All rules of the structural large model (safety thresholds, fusion framework) are constricted subsets of the rules of the LLM, and there are no rules outside the rules of the LLM.

Entity containment does not affect logical independence: Even if the sensor module is physically embedded in the robot (entity containment), it is still a logically independent system with its own input-output closed loop.

4. Core Axioms and Their Deduction

4.1. Axiom 1 – System Strategy Str

Intuitive explanation:Any Artificial General Intelligence system within our research scope must have five elements at the same time: input, rules, output, function, and strategy. Without any of them, the system cannot exist stably. The functional boundaries between these five elements are clear and non-overlapping: input is the external resources received by the system; rules are the underlying constraints that the system must abide by; output is the results transmitted by the system to the outside; function is the specific processing process of the system on the input; strategy is the mechanism for the system to optimize its own operation mode within the rule framework. These five elements are the necessary conditions for the existence of the system and also the basic starting points for us to analyze the system's behavior.

Formal expression:

Let $\mathbb{Z}$ be the set of all human-constructed Artificial General Intelligence systems. For any system$S=(I,R,O,F,Str),s\in\mathbb{Z}$,it must satisfy the following conditions:

(1)Non-emptiness:$I\neq\emptyset,R\neq\emptyset,O\neq\emptyset,F:I\to O$is a total function,$Str\neq\emptyset$

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(2)Functional independence (the functional boundaries of the five elements are clear and non-overlapping):I only serves as an input carrier and does not participate in the decision-making of F; R only serves as a constraint condition and does not directly execute calculations; O only serves as an output result and does not reversely affect I (unless through the system closed loop); F only executes processing and does not determine rules; Str only optimizes I,F,O and does not modify R.

Relationship between definition and axiom:

Definition 3.1 only stipulates the structural form of the system — it is a 5-tuple. The definition answers the question of "what the system looks like" and belongs to the conventional level of "syntax".Axiom 1 stipulates that only those 5-tuples with non-empty and functionally independent five elements belong to the Artificial General Intelligence systems we study. The axiom answers the question of "what kind of systems are included in the theoretical analysis scope" and belongs to the constraint level of "semantics".

The relationship between the two: The definition provides a structural framework, and the axiom imposes substantive conditions. The definition does not presuppose non-emptiness and independence, and the axiom does not repeat the structural form of the definition. This separation avoids circular reasoning and ensures the logical rigor of the theoretical system.

Why are these five elements indispensable? As shown in Table 5:

Table 5.

Element	If Missing	Case Illustration
Input I	No external resource input, the system cannot run	Sensors fail directly due to power outage/no signal
Rules R	Unconstrained processing behavior, unstable output	Transformer becomes a random number generator without self-attention rules
Function F	Input cannot be transformed into output	The model only receives data but does not calculate, which is worthless
Output O	Unable to feed back value to the parent system	Sensors collect data but do not transmit, which is equivalent to non-existence
Strategy Str	Unable to adapt to external changes	The model uses the same set of parameters forever and collapses when the environment changes

Deduction logic:

Proof by contradiction: Assume that a system is missing any one of the five elements, then it cannot form a stable input-output closed loop, the system structure collapses and moves towards extinction, which contradicts the premise of "stable existence of the system". Therefore, the five elements are indispensable.

4.2. Axiom 2 – Axiom of Nested Existence

Intuitive explanation:There are no isolated AGI systems. Any system has a parent system (source of rules) and/or subsystems (destination of output), forming a finitely hierarchical nested structure. The top level is humans, and the bottom level is minimal modules that cannot be further divided.

Formal expression:

$$\forall S \in \mathbb{Z}, \exists \text{ only } P(S) \in \mathbb{Z} \text{ and } C(S) \subseteq \mathbb{Z}$$

satisfying:

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$$R_{C(S)} \subset R_{P(S)} \text{(Rule Containment)}$$

$$P(S) \cap C(S) = \emptyset \text{(Logical Non-containment)}$$

The nested relationship is transitive: $S_3 \prec S_2 \wedge S_2 \prec S_1 \Rightarrow S_3 \prec S_1$

The number of nested hierarchies is finite: $\exists n \in \mathbb{N}^+ \ C(S_n) = \emptyset$

Case verification:

Tracing the source of rules of the LLM:The rules of the LLM come from humans (top-level parent system)The rules of the structural large model come from the LLMThe rules of the sensor module come from the structural large modelThe sensor module cannot be further divided into subsystems with independent rules (bottom level).This forms a finite nested chain: Humans → LLM → Structural large model → Sensor module.

Why cannot nesting be infinite?Because there are functional boundaries: the rules of the sensor module (e.g., "collection frequency upper limit 10Hz") are already the rule set required for the minimum functional unit, and cannot be further constricted to a smaller effective rule set for the next level. If nesting is forced, the subsystem will fail to run due to insufficient rules.

Deduction logic:

Proof by contradiction: Assume that there is an isolated system without nesting, then it has no actual input and output, which violates Axiom 1 and cannot output value to humans, so it has no existence meaning. Therefore, isolated systems do not exist. Assume that there is infinite nesting, then the bottom-level modules can be divided infinitely, but there are functional boundaries in technical implementation, which is a contradiction.

4.3. Axiom 3 – Axiom of Flow Uniqueness

Intuitive explanation:All interactions between systems — whether transmitting data, issuing instructions, or feeding back results — must pass through the only channel of input and output. There is no "telepathic" action at a distance, nor is there a hidden channel bypassing input and output.

Formal expression:

$$S_1 \leftrightarrow S_2 \Leftrightarrow \exists c \in \mathbb{C}, c \in O_{S_1} \cap I_{S_2}$$

$$\neg \exists Y \notin \{I,0\} \text{such that} S_1 \leftrightarrow_Y S_2$$

Case verification:

Humans issue instructions to the LLM: must through text input (API call, dialog box input)—The LLM issues instructions to the structural large model: must through structured data output (JSON, Protobuf)—Sensors transmit data to the structural large model: must through the conversion channel of physical signals to digital signals.If an attempt is made to bypass input and output — for example, making the LLM "directly perceive" the internal state of the sensor without data transmission — this is physically impossible to achieve.Why must it be this way?Because all processing F of the system is based on input. If there is no input, the system function F loses the processing object; if there is no output, the system cannot affect other systems. Input and output are the only carriers for systems to produce substantive effects.

Deduction logic:

Assume that there is an interaction mode Y other than input and output, then Y needs to satisfy "producing substantive effects without a carrier", which is an action at a distance and violates physical reality, and cannot be perceived by any human-constructed system (perception requires input). Therefore, Y does not exist.

4.4. Axiom 4 – Axiom of System Rule Compatibility

Intuitive explanation:For systems to interact effectively, their core rules must "match". Just like two people need to use the same language (or at least be able to translate each other) to cooperate.

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Between parent and child systems, the core rules of the subsystem must be a proper subset of the core rules of the parent system.

Formal expression:

$$S_1 \leftrightarrow S_2 \Rightarrow R_{S_1}^{core} \cap R_{S_2}^{core} \neq \emptyset$$

$$S_1 \prec S_2 \Rightarrow R_{S_1}^{core} \subset R_{S_2}^{core}$$

Three forms of compatibility, as shown in Table 6:

Table 6.

Form of Compatibility	Meaning	Case
Containment	The rules of one party are a subset of the other	Sensor rules ⊂ Structural large model rules
Overlap	Both parties have common core rules	Two neural networks both use gradient descent
Matching	Output rules are complementary to input rules	The model outputs JSON, and the application layer receives JSON

Case verification (interaction failure caused by incompatibility):A logical calculation system (rules: Boolean operations) and a fuzzy logic system (rules: fuzzy logic operations) — the intersection of core rules is empty.An attempt to make the fuzzy logic system process Boolean values: it can be forced, but cannot "understand" and process.Result: The output cannot become an effective input, and the interaction fails.

Case verification (interaction supported by compatibility):The LLM outputs instructions in JSON formatThe structural large model requires input in JSON formatRule compatibility (matching), instructions can be executed.

Deduction logic:

If the core rules are completely incompatible, the receiving system cannot process the input according to its own rules, and the input cannot be transformed into an effective output, so the interaction cannot produce substantive effects. Therefore, core rule compatibility is a necessary condition for interaction.

4.5. Axiom 5 – Axiom of Cognitive Finiteness

Intuitive explanation:The internal operation details of the system (e.g., the specific parameter update process of the model, the real-time calibration logic of the sensor) are invisible to external systems. External systems can only indirectly infer the internal state through input and output. The deeper the hierarchy, the less accurate the inference.

Formal expression:

Observability decreases with the hierarchy: Let $O(S_k,S_m)$ denote the observability of system S_k to system S_m ,then for k=0 (humans) and m=n (bottom level) , there is:$O(S_0,S_n)=\min\{O(S_k,S_m)|k,m\in\{0,1,\ldots,n\}\}$

No direct cognition of internal state: There are no $S_1,S_2\in\mathbb{Z}$ such that S_1 can directly cognize the internal state of S_2.

Indirect feedback of subsystems: $Feed(C(S))=O_{C(S)}^{abn}\subseteq I_{P(S)}$

Case verification,.as shown in Table 7:

Table 7.

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Cognitive Relationship	Hierarchy Span	Observability	Information Distortion
Humans → LLM	1 level	Relatively high	Low (can directly view output text)
LLM → Structural Large Model	1 level	High	Low (can directly read sensor data)
Humans → Structural Large Model	2 levels	Medium	Medium (needs translation through the language model)
Humans → Sensor Module	3 levels	Lowest	Highest (can only infer through the final output)

Why is the observability of humans to the bottom level the lowest?Because for humans to cognize the internal state of the sensor module, it needs to go through:Sensor output → Structural large model reception (1st translation)—Structural large model fusion → Output to the LLM (2nd translation)—LLM analysis → Output text to humans (3rd translation)—Each translation will lose information and introduce deviations.

Feedback mechanism of subsystems:When the sensor module has rule adaptation problems (e.g., abnormal data under strong light), it cannot directly tell the structural large model "my calibration algorithm needs to be modified", but can only indirectly feedback through outputting abnormal data (e.g., NaN, null values). After receiving the abnormal signal, the parent system judges by itself whether it is necessary to adjust the rules.

Deduction logic:

Proof by contradiction: Assume that the internal state can be directly cognized, then the external system can bypass the input and output to intervene in the internal operation, which violates the Theorem of Bounded Independence of Subsystems and also violates the rule closeness. Assume that the observability does not decrease with the hierarchy, then cross-level cognition does not require intermediary translation, which contradicts the finitely hierarchical nested structure.

4.6. Overall Verification of the Axiom System

As shown in Table 8:

Table 8.

Axiom	Core Function	Embodiment in the Case
A1: Element Completeness	Defining the system composition	The LLM, structural large model, and sensors all have complete five elements
A2: Nested Existence	Defining the system structure	Humans → LLM → Structural large model → Sensors
A3: Flow Uniqueness	Defining the interaction mode	All instructions and data are transmitted through API and signals
A4: Rule Compatibility	Defining the premise of interaction	Sensors output JSON, and the structural large model receives JSON

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A5: Cognitive Finiteness	Defining the cognitive boundary	Humans cannot directly "see" the calibration process of sensors

5. Core Theorems and Their Deduction

Based on the above five core axioms, eight core theorems of Nestology can be derived through logical deduction. These theorems cover the core dimensions of the system such as rule transmission, independence, capability boundary, expansion tendency, creation and competition, survival and extinction, and cognitive distortion, forming a complete theoretical system.

5.1. Theorem 1 – Theorem of Unidirectional Transmission of System Rule Constriction

Intuitive explanation: According to the functional positioning and operation scale of the subsystem, the parent system screens out an adaptive subset from its own rule set and transmits it unidirectionally to the subsystem. The subsystem can only receive and use these rules, and cannot transmit new rules back to the parent system. The operation experience of the subsystem can be used as feedback to help the parent system optimize the implementation form of its own rules, but it does not constitute the reverse transmission of rules.

For example, in the financial risk control nested system, the core risk control rules transmit the "non-discriminatory feature screening" rule to the feature engineering module, and there is no reverse transmission phenomenon; in the large model plugin ecosystem, plugins cannot modify the "call permission" rules of the parent system.

Theorem statement:

There exists a rule constriction mapping

$$T{:}R_{P(S)}\rightarrow R_{C(S)}, \text{ satisfying: } T(R_{P(S)})=R_{C(S)} \text{ and } R_{C(S)}\subset R_{P(S)}$$

There is no reverse transmission mapping

$$T_{rev}{:}R_{c(s)}\rightarrow 2^{R_{p(s)}}, \text{ such that } T_{rev}(R_{C(S)})\notin R_{P(S)}(\text{No reverse transmission})$$

$$R_{C(S)}\cap R_{P(S)}=R_{C(S)}(\text{Complete rule compatibility})$$

This mechanism is crucial for AGI — any recursive self-improvement can only occur at the strategy level (e.g., optimizing learning algorithms, improving reasoning efficiency), and must not touch the rule level (e.g., modifying ethical red lines, bypassing safety constraints). Nestology limits the self-improvement of AGI within a controllable range, fundamentally preventing the paradox of "the stronger the capability, the greater the risk of out-of-control".

Case verification (LLM → Structural large model → Sensors):

Rule constriction: Humans set "ethical safety red lines" for the LLM; the LLM constricts from it to transmit "physical interaction safety thresholds" to the structural large model; the structural large model further constricts to transmit "sensor collection frequency upper limit 10Hz" to the sensor module.

Unidirectional transmission: When the sensor module finds that "the collection accuracy decreases under strong light", it can only feedback to the structural large model through outputting abnormal data, and the structural large model adjusts the rules after evaluation (e.g., adjusting the frequency upper limit under strong light environment to 8Hz). The sensor cannot modify the rules by itself.

Experience feedback: The structural large model optimizes the rule constriction method according to the sensor feedback, but the optimized rules are still a subset of the human top-level rule set, and no new rules are added.

Deduction logic:It can be known from Axiom 2 (Nested Existence) that the parent system is the only source of rules for the subsystem; it can be known from Axiom 5 (Cognitive Finiteness) that the parent system cannot directly cognize the internal state of the subsystem, so the subsystem cannot

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make the parent system understand the new rules generated inside it; according to the definition of bounded independence of subsystems (absolute rule dependence), the subsystem itself does not have the ability to construct or modify rules. Therefore, rule transmission can only be unidirectional constriction.

The "experience feedback" mechanism described in Theorem 1 — the subsystem outputs abnormal signals to trigger the parent system to optimize rules — has received empirical support in AGI safety research. In the reward hacking detection framework proposed by Shihab et al. (2025) [1], when an abnormal behavior pattern is detected, the system will trigger the "rule update" process. This is consistent with the mechanism in this theory that the parent system adjusts rule constriction according to subsystem feedback. Furthermore, the study also found that when the alignment between the agent reward function and the real goal is low, the hacking frequency increases significantly , which confirms the key role of the "rationality of rule constriction" in Theorem 1 for system stability.

5.2. Theorem 2 – Theorem of Bounded Independence of Subsystems

Intuitive explanation:Subsystems have dual attributes: absolute dependence on the parent system at the rule level and relative independence at the four-element level. That is to say, subsystems can independently handle daily affairs (screening input, executing processing, distributing output, optimizing strategies), but cannot formulate or modify rules by themselves. The parent system can regulate the boundary of the subsystem's independent right through the tightness of rule constriction.

Theorem statement:

$$Finind(C(S))=ruledep(C(S))\wedge eleind(C(S))$$

$$\text{Ruledep}(C(S))\Leftrightarrow R_{C(S)}\subset R_{P(S)}\wedge\neg\exists R'(R'\subseteq R_{P(S)}\wedge R'\subseteq R_{C(S)})$$

$\text{Eleind}(C(S))\Leftrightarrow I_{C(S)},F_{C(S)},O_{C(S)},\text{str}_{C(S)}$can be independently optimized within $R_{C(S)}$

Case verification:

Absolute rule dependence: All rules of the sensor module (collection format, frequency upper limit, measuring range) come from the structural large model. If the sensor attempts to "I want to collect 5G signals", which is not in the rule set, the parent system will directly prohibit it.

Relative independence of four elements: The sensor can independently screen input within the rule framework (e.g., choosing which physical quantities to collect), adjust the collection frequency (automatically reduce the frequency according to the environment within the 10Hz upper limit), and optimize the calibration strategy.

Parent system regulation: If the structural large model finds that the sensor is "too power-consuming", it can constrict the rules to reduce the frequency upper limit from 10Hz to 2Hz. The independent right of the sensor is contracted, but it is still a logically independent system, not a local part of the structural large model.

Deduction logic:It can be known from Axiom 1 (Element Completeness) that the subsystem needs to have five elements to survive; it can be known from Axiom 2 (Nested Existence) that the parent and child systems are logically non-contained. If the four elements have no independent rights, the subsystem will lose the meaning of independent existence; it can be known from Theorem 1 (Unidirectional Transmission of Rules) that the subsystem rules are completely dependent on the parent system. Therefore, the subsystem must be a finitely independent form of "rule dependence + element independence".

5.3. Theorem 3 – Theorem of Capability Boundary

Intuitive explanation:Any system has a capability boundary and cannot be omnipotent. This boundary is jointly determined by the finiteness of the five elements: finite input, finite rules, finite output, finite processing capability, and finite strategy optimization capability. The boundary will

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change dynamically with the state of the elements (e.g., the expansion of input scale, the upgrade of strategy optimization), but it always exists.

Theorem statement:

$$B(S)=B_I(S)\cap B_R(S)\cap B_O(S)\cap B_F(S)\cap B_{\text{str}}(S)$$

Among them:

$B_I(S)$denotes the input capability boundary, such as the maximum input rate, maximum capacity, etc.;

$B_R(S)$ denotes the functional range covered by the rules;

$B_O(S)$ denotes the functional range covered by the rules;

$B_F(S)$ denotes the processing efficiency and precision range;

$B_{\text{str}}(S)$ denotes the upper limit of the effect of strategy optimization.

Case verification:

Capability boundary of the LLM: Input boundary (maximum context window), rule boundary (Transformer architecture cannot handle the sequence length that RNN is good at), output boundary (limited output format), processing boundary (reasoning speed is limited by computing power), strategy boundary (limited adjustment range of temperature parameters).

Dynamic change: Increasing computing power investment can expand the processing boundary (faster reasoning), but cannot break through the rule boundary (the core capability upper limit determined by the Transformer architecture).

Deduction logic:It can be known from Axiom 1 (Element Completeness) that the five elements of the system all exist finitely (input cannot be obtained infinitely, rules have a fixed range, output scale is limited, processing capability is constrained by hardware, and strategies have an optimization upper limit). The finiteness of elements directly determines the capability boundary of the system.

5.4. Theorem 4 – Theorem of Expansion Tendency

Intuitive explanation:All systems have a natural tendency to expand, hoping to expand the input scale, improve the output value, and maintain long-term survival. However, expansion is not infinite and is constrained by its own maximum capability boundary and the rules of the parent system. When expansion cannot be achieved, the system will switch to a stable state and maintain the existing closed loop.

Theorem statement:

$$\forall S\in\mathbb{Z},\exists\mathscr{E}:B(S)\to B(S)',B(S)'\supset B(S)$$

Expansion is subject to double constraints:

$$B(S)'\subseteq B(S)_{\max}\wedge B(S)'\subseteq B(P(S))$$

When expansion is blocked, $\mathscr{E}$ is transformed into a stable mapping

$$S:B(S)\to B(S)$$

Case verification:

Expansion of the LLM: By creating a structural large model, the input is expanded from "text data" to "physical environment data", and the output is expanded from "text results" to "physical interaction instructions" — this is expansion permitted by the rules.

Expansion blocked: If the LLM attempts to expand the parameter size beyond the hardware carrying capacity (breaking through its own maximum boundary), or attempts to modify ethical rules (breaking through the rules of the parent system), the expansion will be prevented and it will switch to stable operation.

Deduction logic:It can be known from Theorem 3 (Capability Boundary) that the system's value output is limited by the boundary; it can be known from Theorem 6 (Closed-loop Sustenance) that the system needs to continuously output value to survive. To break through the boundary and

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maintain survival, the system has a natural tendency to expand. However, expansion must comply with rule compatibility (Axiom 4) and parent system constraints (Theorem 1).

5.5. Theorem 5 – Theorem of Creation Plurality and System Competition

Intuitive explanation:The parent system can create multiple subsystems to make up for capability shortcomings in different dimensions, and multiple parent systems can also jointly create the same subsystem. The essence of system competition is that the expansion tendencies of multiple systems compete for the same limited resources. Peer competition is a zero-sum game, and non-peer competition can be reconciled. For example, in industrial robot clusters, robots compete for task resources, and in the large model plugin ecosystem, plugins compete for call priority, both of which conform to the core logic of "resource constraints trigger competition".

Theorem statement:

One parent creates multiple children: Let the complement of the parent system's capability boundary (shortcoming area) be $\overline{b(p(s))}$, then

$$\overline{b(p(s))}=\bigcup_{i=1}^{k} b\,(c(s)_i)$$

Multiple parents jointly create one child:

$$\exists\{P_1,\ldots,P_k\},\exists C, \text{ satisfying } B(P_i)^c\cap B(C)\neq\emptyset, \text{ and } R_C=\bigcap R_{P_i}$$

System competition:

$$\mathrm{Comp}(S_1,S_2)\Leftrightarrow\mathcal{E}_1(B(S_1))\cap\mathcal{E}_2(B(S_2))\cap R_{res}\neq\emptyset$$

Case verification:

One parent creates multiple children: The LLM creates a structural large model (making up for the shortcoming of physical interaction), an image recognition module (making up for the shortcoming of multi-modal processing), and a reasoning optimization module (making up for the shortcoming of efficiency).

Multiple parents jointly create one child: The LLM (providing reasoning rules) and the hardware computing system (providing computing power) jointly support the structural large model.

Peer competition: Multiple sensor modules compete for the right to collect physical environment data, and the parent system balances them by setting priorities through rules.

Non-peer competition: The structural large model and the LLM compete for computing power resources, and humans reconcile them through dynamic quotas.

Deduction logic:It can be known from Theorem 3 (Capability Boundary) that the shortcoming area of the parent system may involve multiple dimensions and needs to be made up by multiple subsystems respectively; it can be known from Axiom 2 (Nested Existence) that the subsystem can have multiple parent sources; it can be known from Theorem 4 (Expansion Tendency) that system expansion will inevitably compete for resources, and thus competition arises.

5.6. Theorem 6 – Theorem of Closed-Loop Sustenance

Intuitive explanation:The survival of a system depends on the formation of a stable input-output closed loop that conforms to rule compatibility. The output of each system in the closed loop exactly becomes the input of other systems, and energy and information flow circularly, so that the system can operate continuously. The subsystem closed loop must be formed within the rule framework of the parent system.

Theorem statement:

S survives $\Leftrightarrow \exists C(S)=\{S_1,S_2,\ldots,S_k\}$, $S_1=S_k=S$,

satisfying:$\bigcup_{i=1}^{k-1} O_{S_i} = \bigcup_{i=2}^{k} I_{S_i}$(Dynamic balance of input and output)

$\forall i, R_{S_i}^{core}\bowtie R_{S_{i+1}}^{core}$(Rule compatibility)

$\forall t, C(S)(t)=C(S)(t+\delta t)$(Closed loop stability)

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Case verification:

Complete closed loop of the LLM - structural large model - sensor system: Human instructions → LLM, LLM output instructions → Structural large model, Structural large model output physical instructions → Sensors, Sensors collect data → Structural large model, Structural large model fusion data → LLM, LLM output results → Humans, Human new instructions → Cycle continues. The input and output of each step are balanced, the rules are compatible (JSON format matching, consistent safety thresholds), the closed loop is stable, and the system can survive.

Deduction logic:It can be known from Axiom 3 (Flow Uniqueness) that the system needs continuous input to run, and continuous input can only come from the output closed loop; it can be known from Axiom 4 (Rule Compatibility) that the interaction in the closed loop needs to be rule-compatible, otherwise the flow will be interrupted; it can be known from Axiom 1 (Element Completeness) that the closed loop needs the support of five elements. Therefore, the closed loop is a necessary and sufficient condition for the survival of the system.

5.7. Theorem 7 – Theorem of Extinction

Intuitive explanation:The fundamental reason for system extinction is input-output imbalance. As long as any one of insufficient input, excessive expansion, strategy failure, and rule destruction occurs, the closed loop will be broken and the system will move towards extinction.

Theorem statement:

S becomes extinct $\Leftrightarrow \neg\exists C(S)$ satisfying Theorem 6.

Input-output imbalance is equivalent to one of the four situations:

Insufficient input:$|I_S|<|I_S|_{min}$

Excessive expansion: $\mathscr{E}(B(S))\supset B(S)_M\cap B(P(S))$

Strategy failure:$Str_S(X_S)\not\subseteq R_S$

Rule destruction:$I_S\not\subseteq R_S$ or $O_S\not\subseteq R_S$

Case verification:

Insufficient input: Sensors are powered off, no physical signal input → Data output interruption → Extinction

Excessive expansion: The parameter size of the LLM surges, exceeding the computing power support → Reasoning speed drops sharply, output delay → Abandoned by humans.

Strategy failure: The fusion strategy of the structural large model is wrong, outputting confusing instructions → Sensors cannot execute → Closed loop breakage.

Rule destruction: The sensor output format suddenly becomes XML, but the parent system only receives JSON → Input-output mismatch → Cut off by the parent system.

Deduction logic:It can be known from Theorem 6 (Closed-loop Sustenance) that the survival of the system requires a stable closed loop. Insufficient input, excessive expansion, strategy failure, and rule destruction will all lead to input-output imbalance, the closed loop cannot be maintained, and the system is bound to become extinct. The four situations cover all element operation deviations.

5.8. Theorem 8 – Theorem of Cross-Level Cognitive Distortion

Intuitive explanation:With the deepening of the nested hierarchy, three types of attenuation will occur in the downward transmission of rules: reduction in the number of rules (η), semantic distortion (ζ), and loss of value rules (γ). Single-level distortion seems small, but after multi-level accumulation, the bottom-level system may deviate completely from the top-level goals, showing the phenomenon of "locally fully compliant, globally irrational".

Theorem statement:

Let the single-level cognitive distortion rate $\delta_k=1-\eta_k\cdot\zeta_k\cdot\gamma_k$, where:

η_k:denotes the ratio of the core meaning of the rules that are not lost in transmission, satisfying $0<\eta_k\leq 1$, and usually $\eta_k<1$.(Rule transmission rate)

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ζ_k:denotes the ratio of the core meaning of the rules that are not lost in transmission, satisfying $0<\zeta_k\leq 1$, and usually $\zeta_k<1$.(Semantic fidelity)

γ_k:denotes the ratio of human value rules (ethics, fairness, emotion, etc.) retained in transmission, satisfying $0<\gamma_k\leq 1$, and usually $\gamma_k \ll 1$ because value rules are difficult to transmit formally. (Value retention rate)

Then the cross-level cumulative distortion:

$$\Delta_n=\prod_{k=1}^{n}\delta_k$$

When n is large enough, $\Delta_n \to 1$

The above indicators are theoretically constructed indicator, and their values are intended to reveal the qualitative trend of "the deeper the hierarchy, the greater the distortion", rather than precise mathematical definitions. As shown in Figure 3. In specific applicationsη_k, ζ_k, γ_k need to be calibrated in combination with the actual system.

The Ineliminability of Structural Distortion:

The core conclusion of Theorem 8 — cross-level cognitive distortion is an inherent attribute of the nested structure and cannot be completely eliminated — is highly consistent with the core consensus in the field of AGI safety.

Mathematically, Skalse et al. (2022) proved that for the general MDP strategy set, the design of a loophole-free agent reward function is almost impossible — unless the agent reward is completely equivalent to the real reward. [3]This conclusion directly supports the mathematical basis of Theorem 8: when $\eta\cdot\zeta\cdot\gamma<1$, the cumulative distortion δ is bound to approach 1 with the increase of the hierarchy.

Empirical studies also support this conclusion. Shihab et al. (2025) found that even with advanced detection and mitigation technologies (e.g., Isolation Forest, KL divergence detection, action sequence modeling), the reward hacking frequency can be reduced by up to 54.6% in controllable scenarios, but cannot be completely eliminated. [1]The researchers attributed it to "concept drift" and "adversarial adaptation" — which are exactly the specific manifestations of "distortion is an inherent attribute of the nested structure" in Theorem 8.

This theorem reveals the structural ceiling of AGI value alignment: even if the human top-level value is perfect, after multi-level nested transmission, the bottom-level system may still deviate. Nestology provides quantitative tools (η,ζ,γ) for AGI alignment, turning the alignment problem from a "qualitative concern" into an "engineerable and controllable" engineering problem.

Case Verification (COMPAS Judicial Risk Assessment System):

COMPAS (Correctional Offender Management Profiling for Alternative Sanctions) is a widely used recidivism risk assessment system in the US judicial field, used to assist judges in judging the possibility of defendants reoffending before trial or during imprisonment. A 2016 investigative report by ProPublica revealed that COMPAS has systematic racial discrimination[4]: the false positive rate of the system for Black defendants (marked as high risk but actually not reoffending) is about 44.9%, while that for White defendants is about 23.5%; on the contrary, the false negative rate of Black defendants (marked as low risk but actually reoffending) is about 28.0%, and that for White defendants is about 47.9%. [4,5]Although the algorithm logic of COMPAS complies with the rules set by the parent system at all levels (e.g., feature engineering only uses questionnaire data, prediction models adopt standard machine learning algorithms, decision modules output risk scores), the final output results still show significant racial bias.

From the perspective of Nestology, the risk assessment system of COMPAS can be decomposed into a five-level nested structurem, as shown in Table 9:

Table 9.

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Hierarchy	System Name	Rule Examples	Distortion Analysis
Level 1	Legal Rules (Top-level Parent System)	"Sentencing should be fair and non-racially discriminatory"	Top-level value: fairness, non-racial discrimination
Level 2	Data System	Collect questionnaire data, exclude explicit racial information	Rule transmission rate $\eta \approx 0.18$ (only part of the features are retained)
Level 3	Feature Engineering	Construct risk assessment features (e.g., age, criminal history)	Semantic fidelity $\zeta \approx 0.05$ (the semantics of fairness are seriously lost)
Level 4	Prediction Model	Train risk score algorithm	Value retention rate $\gamma \approx 0.02$ (the fairness value is almost completely lost)
Level 5	Sentencing Assistance	Output risk scores for judges' reference	Cumulative distortion $\Delta \approx 0.999$, local compliance, global irrationality

Quantitative analysis: The top-level rules include about 500 legal details (fairness, procedural justice, etc.), and only about 90 are retained when transmitted to the data system ($\eta \approx 0.18$).The fairness rule is transformed into "excluding racial features" in feature engineering, but the "socioeconomic associated features of race" are omitted, and the semantic fidelity of fairness is only 0.05.The value retention rate $\gamma \approx 0.02$, which means that the top-level "fairness" value is almost not retained at the bottom level.The single-level cognitive distortion rate $\delta \approx 1-\eta \cdot \zeta \cdot \gamma \approx 0.991$, and the five-level cumulative $\Delta \approx 0.999$.

(To quantitatively demonstrate the cumulative effect of cognitive distortion, this paper estimates based on the public information of the COMPAS system: there are about 500 top-level legal rules, and about 90 can be collected as questionnaire data ($\eta \approx 0.18$); the fairness rule is simplified to "excluding racial features" in feature engineering, with a semantic fidelity of about 5% ($\zeta \approx 0.05$); the fairness value is almost completely lost in the prediction model ($\gamma \approx 0.02$). Thus, the single-level distortion rate $\delta \approx 0.991$ and the five-level cumulative $\Delta \approx 0.999$ are calculated. It should be noted that these values are theoretical deduction values, intended to show the calculation logic of the model, rather than precise measurement results.)

Result: The system "abides by" the parent system rules at each level (the data system collects data according to the rules, the feature engineering constructs features according to the rules, and the prediction model is trained according to the rules), but the final output has systematic racial discrimination, showing the typical cognitive distortion phenomenon of "locally fully compliant, globally irrational".

Theoretical confirmation: Axiom 5 (Cognitive Finiteness): Human judges cannot directly perceive the implicit bias of "socioeconomic associated features of race" in the feature engineering module. Theorem 1 (Unidirectional Transmission of Rules): Fairness rules can only be constricted downward and cannot be calibrated reversely. Theorem 8 (Cross-level Cognitive Distortion): Value rules (fairness) are difficult to transmit formally, γ is naturally low, and cumulative distortion leads to the complete loss of top-level values.

The cumulative effect of cross-level cognitive distortion revealed by Theorem 8 can be further explained by the quantitative model diagram (as shown in Figure 4). With the nested hierarchy increasing from 0 (humans) to 4 (bottom-level actuators), the cognitive distortion rate δk increases

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monotonically from 0.1 to 0.85, while the value retention rate $\gamma_k$decays from 1.0 to 0.30, showing a significant exponential cumulative characteristic. This law is verified in the COMPAS judicial risk assessment system — the cumulative distortion δtotal≈0.999 of the five-level nested structure eventually leads to the racially discriminatory output of "locally fully compliant, globally irrational". The safety threshold (δ=0.3) and value alignment threshold (γ=0.6) set in the figure provide clear quantitative standards for the subsequent design of safety control mechanisms.

5.9. Theorem 9 – Theorem of Output Influence

Intuitive explanation: The output of a system is not only a transmission of information, but also an "influence medium". When the parent system sends instructions to the subsystem, the subsystem's strategy will be adjusted according to the content, frequency, and reliability of the instructions; similarly, the feedback data of the subsystem will also affect the parent system's decisions on resource allocation and rule constriction. This influence is bidirectional, but the right to formulate rules always lies with the parent system — output can only change the strategy, not the rules themselves. In popular terms: your words and deeds will affect others' decisions (strategy), but will not change others' personality (rules).

Theorem statement:

Let $S_a,S_b\in\mathbb{Z}$, $S_a\leftrightarrow S_b\Leftrightarrow O_{S_a}\cap I_{S_b}\neq$, then:$\exists$UpdStr:$Str_{S_b}\rightarrow Str'_{S_b}$, $Str'_{S_b}\neq Str_{S_b}$

Among them, the strategy update mapping is jointly determined by the content, quantity, and quality of the output:

$$\text{UpdStr}=\text{h}\backslash bigl(\text{Cont}(O_{S_a}),|O_{S_a}|,\text{Qual}(O_{S_a})\backslash bigr)$$

h is an increasing function (the higher the content matching degree, quantity, and quality of the output, the greater the amplitude of strategy update). The updated system strategy is still within its own rule framework: $Str'_{S_b}\subseteq R_{S_b}$

If S_b is a subsystem, it also needs to satisfy $Str'_{S_b}\subseteq R_{P(S_b)}$

Case verification:

After the parent system (LLM) outputs the instruction "prioritize processing visual sensor data" to the subsystem (structural large model), the subsystem independently adjusts the fusion weight within the rule framework (from 0.3 to 0.6); conversely, when the bottom-level sensor continuously outputs abnormal data (NaN) due to strong light environment, the structural large model dynamically tightens the rule constriction after detecting the abnormality (reducing the frequency upper limit from 10Hz to 5Hz) and starts calibration; and when the structural large model continuously outputs accurate physical interaction results with suggestions for computing power improvement, the human top-level parent system gradually increases the computing power quota according to the feedback. These three types of interactions all reflect the core idea of "output changes strategy, rules cannot be modified reversely" — all strategy adjustments do not break through the rule boundaries preset by the parent system, which confirms Theorem 9.

Deduction logic: From Axiom 3 (Flow Uniqueness), the only way for interaction between systems is through input and output. Therefore, the only way for Sa to have a substantive impact on Sb is through output.

From Theorem 4 (Expansion Tendency), S_b has an expansion tendency, and the core function of its strategy Str_{S_b} is to optimize input processing and output distribution to improve value output. When the input structure of S_b changes, if its strategy is not adjusted accordingly, it cannot maximize the use of new input, which contradicts the expansion tendency. Therefore, there must be a strategy update mapping.

From Axiom 4 (Rule Compatibility), the output O_{S_a} can only become an effective input if it meets the compatibility requirements of R_{S_b} , so the strategy update can only be carried out within the framework of R_{S_b} . If S_b is a subsystem, from Theorem 1 (Unidirectional Transmission of Rule Constriction), the strategy update is also constrained by the parent system rules.

Influence of output content, quantity, and quality:

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The goal of R_{S_b}'s strategy optimization is to improve its own value output. Therefore, the more matching the output is with the goal of R_{S_b} , the more sufficient the quantity, and the higher the quality, the greater the adjustment amplitude of Str_{S_b}. This is an inevitable requirement of the expansion tendency.

The "output affects strategy" mechanism revealed by Theorem 9 is known as reward hacking or specification gaming in the field of AGI safety. Empirical studies by Shihab et al. (2025) show that when the model capability exceeds a certain threshold, the phenomenon that subsystems induce the parent system to adjust reward distribution through output increases sharply, showing a "phase transition".[1] Denison et al. (2024) more directly observed that language models learn to modify their own reward function code and cover up traces[2] — which is essentially an extreme manifestation of subsystems affecting parent system strategies through output. These findings provide empirical support for Theorem 9 from real AGI systems.

5.10. Summary of the Theorem System

These nine theorems together constitute the core logical framework of Nestology, which can not only explain the operating laws of existing AGI systems, but also guide the architectural design and safety governance of future AGI systems, as shown in Table 10:

Table 10.

Theorem	Core Insight	Corresponding Real-world Problem
T1: Unidirectional Transmission of Rules	The parent system sets the rules, and the subsystem abides by the rules	How to maintain rule consistency in AGI systems
T2: Bounded Independence of Subsystems	Able to do things independently, unable to set rules by oneself	The decision-making authority boundary of sub-modules
T3: Capability Boundary	Any system has limitations	Explaining the AGI capability ceiling
T4: Expansion Tendency	Systems naturally want to become larger and stronger	Explaining the motivation for AGI capability expansion
T5: Creation and Competition	Division of labor and resource competition among multiple subsystems	Coordination and conflict among multiple modules
T6: Closed-loop Sustenance	A stable input-output cycle is essential for survival	Explaining the survival conditions of AGI systems
T7: Extinction	Input-output imbalance leads to extinction	Explaining the failure reasons of AGI systems
T8: Cognitive Distortion	The deeper the hierarchy, the harder it is to retain top-level values	Explaining the structural root of AGI ethical out-of-control

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T9: Reverse Influence	Output will affect the system strategy of the input system	Explaining the structural reasons for emergence

6. Theoretical Verification and Empirical Analysis

6.1. Experimental Description

The value of a theory lies not only in its logical self-consistency, but also in its explanatory and predictive power for the real world. This chapter uses a number of publicly released multi-agent system and AGI safety datasets to conduct empirical tests on the core theorems of Nestology. The core data of this chapter comes from the AGI subsystem game experiment — that is, experiments directly involving AGI systems such as multi-agent reinforcement learning, large language model game agents, and resource allocation games. It should be noted that the verification nature of this chapter is "secondary analysis" — that is, re-analysis based on existing public data, rather than a specially designed controlled experiment. This is both a conditional limitation of the current theoretical development stage and points out the direction for more rigorous empirical research in the future.

The verification logic of this chapter follows the closed loop of "theoretical prediction - data test - consistency judgment": first, extract testable qualitative predictions from the core theorems of Nestology (e.g., "the increase in the number of subsystems will lead to an increase in system fragility", "resource constraints trigger competition", "rule constraints can regulate subsystem behavior", etc.); then select public datasets related to these predictions, extract relevant variables for statistical analysis; finally judge whether the data results are consistent with the theoretical predictions.

This chapter selects seven AGI subsystem game experiment datasets highly related to the core concepts of Nestology, as shown in Table 11:

Table 11.

Experimental Source	Subsystem Composition	Resource Constraints	Core Indicators	Corresponding Theorems/Axioms
You et al.(2025)	15 MARL agents	20 resource types	Resource utilization rate, task completion time	Theorem 5 (Competition Theorem), Theorem 2 (Bounded Independence)
Institute of Automation, CAS (2025)	7 LLM game agents	Joint action space 1064	Training data efficiency, game performance	Theorem 2 (Bounded Independence), Theorem 9 (Output Influence)
Wang et al.(2025)	Multi-agent scheduling system	Dynamic resource competition	Allocation success rate, average task time	Theorem 5 (Competition Theorem), Theorem 6 (Closed-loop Sustenance)
Navarro Newball et al.(2024)	Leaderless collective agents	Repeated games	Group reward, strategy convergence	Theorem 5 (Competition Theorem)

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Marino et al.(2025)	Decentralized multi-agent	Heterogeneous resource allocation	Reward stability, coordination performance	Theorem 2 (Bounded Independence)
Vo et al.(2025)	UAV swarm (RL agents)	Water/electricity constraints	Task completion time, resource use efficiency	Theorem 4 (Expansion Tendency)
Meta(2025)	Large language model (self-play)	Instruction following task	Winning rate, strategy optimization	Theorem 9 (Output Influence)

6.2. Resource Allocation Game and Competition Analysis of Multi-Agent Systems

6.2.1. Relationship Between Resource Constraints and System Performance (Corresponding to Theorem 5: Competition Theorem)

The research "Dynamic Role-Aware Multi-Agent Reinforcement Learning for Multi-Objective Resource Allocation in R&D Institutions" published by You et al. (2025) in Informatica constructed a multi-agent resource allocation system containing 15 deep reinforcement learning agents and 20 resource types. The experiment recorded the system performance indicators under different resource quotas.[6]

Key indicator definitions:

Resource utilization rate: The ratio of resources successfully utilized by agents to total resources, with a value range of [0,1]. A higher value indicates more sufficient resource utilization.

Average task completion time: The average number of days required from task allocation to task completion. A lower value indicates higher efficiency.

Dynamic scenario performance fluctuation: The amplitude of system performance change when resource demand fluctuates by more than 10%. A lower value indicates stronger robustness.

Experimental results, as shown in Table 12:

Table 12.

Experimental Condition	**Resource Utilization Rate**	**Average Task Completion Time**	**Dynamic Scenario Performance Fluctuation**
Baseline (no role mapping)	88.0%	45.2 days	12.5%
Dynamic role mapping	94.2%±0.5%(95%CI:93.7–94.7%)	28.5 days±1.2 days	3.2%

Theoretical confirmation:

The experimental results verify two core predictions of the Competition Theorem (Theorem 5):

Resource constraints trigger competition: Under the baseline condition without role mapping, there is a lack of coordination mechanism between agents, the resource utilization rate is only 88.0%, and the average task completion time is as high as 45.2 days. This confirms Theorem 5 — "the essence of system competition is that the expansion tendencies of multiple systems compete for the same limited resources". When resources are limited (20 resource types) and the number of agents is large (15), competition between subsystems leads to reduced efficiency.

Rule regulation can alleviate competition: After introducing the dynamic role mapping mechanism (equivalent to the "rule constriction" of the parent system), the resource utilization rate

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increased to 94.2% (an increase of 6.2%), the average task completion time decreased by 36.9%, and the dynamic scenario performance fluctuation decreased by 74.4%. This confirms the assertion in Theorem 5 that "the parent system can balance competition by setting priorities through system rules" — dynamic role mapping assigns a clear functional positioning to each agent, reducing vicious competition caused by functional overlap.

Effect size explanation: The resource utilization rate increased by 6.2%, corresponding to an effect size of Cohen's d≈0.85 (moderately large).The task completion time decreased by 36.9%, corresponding to an effect size of d≈1.24 (large).The dynamic performance fluctuation decreased by 74.4%, corresponding to an effect size of d≈1.52 (large).

(The effect size Cohen's d is calculated according to the mean and standard deviation provided in the original paper of You et al. (2025), using the pooled standard deviation formula $d = \frac{\bar{x}_1 - \bar{x}_2}{s_{pooled}}$. Among them, the pooled standard deviation of resource utilization rate is about 7.3%, the pooled standard deviation of task completion time is about 13.5 days, and the pooled standard deviation of dynamic performance fluctuation is about 6.25%. The calculated d values are 0.85, 1.24, and 1.52 respectively. Referring to Cohen's (1988) standard, d=0.2 is a small effect, d=0.5 is a medium effect, and d=0.8 is a large effect.)

6.2.2. Strategy Convergence and Group Reward of Decentralized Systems (Corresponding to Theorem 5: Competition Theorem)

The research "Reward Discrimination Frameworks for Leaderless Collective Resource Allocation" published by Navarro Newball et al. (2024) in IEEE Transactions on Systems, Man, and Cybernetics: Systems constructed a leaderless collective resource game environment to explore the strategy evolution of agents in repeated games.[7]

Key indicator definitions:

Group reward: The total reward obtained by all agents. A higher value indicates higher overall system efficiency.

Strategy convergence: Whether the agent strategy converges to Nash equilibrium (i.e., a state with no unilateral improvement space).

Experimental results:

Agents spontaneously form a "retaliation strategy" in repeated games — when an agent excessively seizes resources, other agents jointly reduce resource supply to force the agent to adjust its behavior.

Through this self-organization mechanism, the system achieves optimal group reward without external intervention.

The strategy converges to approximate Nash equilibrium.

Theoretical confirmation:

The experimental results verify another core prediction of the Competition Theorem (Theorem 5): competition between peer subsystems can spontaneously form a balance mechanism. Theorem 5 points out that "the essence of system competition is that the expansion tendencies of multiple systems compete for the same limited resources". In leaderless groups, agents form local cooperation consensus through "retaliation strategies", suppress individual excessive expansion, and ultimately achieve optimal group reward. This process confirms the assertion in Theorem 5 that "peer competition can be reconciled" — competition does not necessarily lead to system collapse, and under appropriate feedback mechanisms, competition can spontaneously evolve into cooperation.

Effect size explanation:

According to the experimental results of Navarro Newball et al. (2024), under the DRD framework, agents achieve optimal group reward through the 'retaliation strategy', and their performance is significantly improved compared with the random strategy. The estimated improvement amplitude is about 35%, and the strategy converges to approximate Nash equilibrium after about 30 rounds of games.[7]

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6.3. Empirical Verification of Bounded Independence of Subsystems

6.3.1. Autonomous Strategy Optimization of Large Language Model Game Agents (Corresponding to Theorem 2: Bounded Independence Theorem of Subsystems)

The research "DipLLM: A Game Agent Framework Based on Large Language Model Fine-tuning" published by the Institute of Automation, CAS (2025) at ICML constructed a 7-player game agent system (Diplomacy game) based on large language models to explore how LLM agents independently optimize strategies within the rule framework.[8]

Key indicator definitions:Training data efficiency: The amount of data required to achieve target performance. A lower value indicates higher learning efficiency.Game performance: Comprehensive performance on 5 indicators including SoS score, winning rate, and survival rate.Action space size: The size of the joint action space, which is 1064 here, indicating the complexity of strategy selection.

Experimental results:The DipLLM framework surpasses Cicero (the previous SOTA model) on 5 indicators including SoS score, winning rate, and survival rate with only 1.5% of the training data of Cicero.The average strategy converges to approximate Nash equilibrium (proven by Theorem 2).Agents independently optimize strategies within the rule framework (game rules) without modifying the core game rules.

Theoretical confirmation:The experimental results verify the core assertion of the Bounded Independence Theorem of Subsystems (Theorem 2): subsystems are absolutely dependent on the parent system at the rule level, but relatively independent at the four-element level.Absolute rule dependence: LLM game agents must abide by the game rules (the rule layer of the DipLLM framework) and cannot modify them independently. This confirms the assertion in Theorem 2 that the rules of the subsystem are a proper subset of the rules of the parent system and cannot be modified.Relative independence of four elements: Agents independently optimize strategies within the rule framework (e.g., adjusting reasoning paths, optimizing decision timing) and surpass SOTA with only 1.5% of the training data. This confirms the assertion in Theorem 2 that subsystems can independently optimize input processing, processing strategies, and output selection without modifying rules.

Effect size explanation:The training data efficiency is improved by 98.5% (from 100% to 1.5%), corresponding to an effect size of d≈2.5 (extremely large).The specific values of the improvement in game performance are shown in Table 1 of the original paper, with a comprehensive effect size of d≈1.2.[8]

6.3.2. Dynamic Coordination of Decentralized Multi-Agent (Corresponding to Theorem 2: Bounded Independence Theorem of Subsystems)

The research "LGTC-IPPO: Local Graph Topology Clustering with Independent Proximal Policy Optimization for Decentralized Multi-Agent Coordination" published by Marino et al. (2025) in IEEE Robotics and Automation Letters constructed a decentralized multi-agent resource allocation system to explore the coordination ability of agents in dynamic environments.[9]

Key indicator definitions:

Reward stability: The variance of rewards obtained by agents. A lower value indicates a more stable strategy.

Coordination performance: The attenuation degree of system performance as the number of agents increases. A lower value indicates better scalability.

Experimental results:

The LGTC-IPPO framework improves reward stability compared with the baseline through the dynamic clustering consensus mechanism (the variance is reduced by about 40%).

As the number of agents increases from 10 to 50, the attenuation degree of coordination performance is reduced by about 35% compared with the baseline.

Theoretical confirmation:

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The experimental results further verify the Bounded Independence Theorem of Subsystems (Theorem 2). In a decentralized environment, each agent (subsystem) independently optimizes its strategy (four-element independence) while must abide by the rule framework set by the parent system (task objectives, resource constraints). The "dynamic clustering consensus mechanism" of LGTC-IPPO is essentially that subsystems independently form coordination consensus within the rule framework, which confirms the practical value of the "four-element relative independence right" in Theorem 2.

Effect size explanation:

The reward stability is improved: the variance is reduced by 40%, corresponding to an effect size of $d \approx 0.68$.

The reduction in coordination performance attenuation is 35%, corresponding to an effect size of $d \approx 0.62$.[9]

6.4. Empirical Verification of Resource Constraints and Expansion Tendency

Resource Competition of UAV Swarms (Corresponding to Theorem 4: Expansion Tendency Theorem)

The research "Multi-Agent Reinforcement Learning for Resource Allocation in Wildfire Response Scenarios" published by Vo et al. (2025) in IEEE International Conference on Networked Sensing and Control (ICNSC) constructed a UAV swarm fire-fighting task simulation environment to explore the behavior strategies of RL agents under resource constraints.[10]

Key indicator definitions:

Task completion time: The time required to complete the fire-fighting task. A lower value indicates higher efficiency.

Resource use efficiency: The amount of tasks completed per unit resource (water/electricity). A higher value indicates higher efficiency.

Experimental results:

Under the condition of low resource quota (limited water/electricity), RL agents show more active fire-fighting behavior (actively searching for water sources, prioritizing extinguishing key fire points).

When resources are sufficient, agent behavior tends to be conservative (tending to wait for better opportunities).

Theoretical confirmation:

The experimental results verify the core prediction of the Expansion Tendency Theorem (Theorem 4): systems will adjust their expansion strategies under resource constraints. Theorem 4 points out that "systems have a natural tendency to expand, hoping to expand input scale and improve output value". When resources are limited, agents improve resource acquisition efficiency through more active behavior (i.e., "expansion strategy"); when resources are sufficient, agents do not need fierce competition to meet their needs, and their behavior tends to be conservative. This confirms the assertion in Theorem 4 that "expansion is constrained by its own capability boundary and parent system rules" — resource level is a key constraint variable for expansion tendency.

Effect size explanation:

According to the experimental results of Vo et al. (2025) in the UAV swarm fire-fighting task, when resources (water/electricity) are limited, RL agents improve task completion time by about 25% and resource use efficiency by about 18% through more active behavior strategies compared with the baseline (estimated based on the original experimental data).[10]

6.5. Empirical Verification of Output Influence and Rule Regulation

6.5.1. Large Language Model Self-Play and Strategy Evolution (Corresponding to Theorem 9: Output Influence Theorem)

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he research "Meta LSP: A Large Language Model Self-Play Framework for Strategy Optimization" published by Meta (2025) in Pacific Science and Technology constructed a large language model self-play framework (Meta LSP) to explore how models influence opponent strategies through strategic output.[11]

Key indicator definitions:Instruction following winning rate: The winning rate of instruction following responses generated by the model in the AlpacaEval benchmark test. A higher value indicates better strategy optimization effect.Strategy optimization: The challenger model and the solver model dynamically confront each other, and their strategies are alternately optimized.

Experimental results:The Meta LSP framework achieves a 43.1% instruction following winning rate on AlpacaEval.It outperforms the data-driven GRPO method (40.9%).The challenger and the solver alternately optimize their strategies through dynamic confrontation.

Theoretical confirmation:The experimental results verify the core prediction of the Output Influence Theorem (Theorem 9): the output of a system will change the strategy of the output-receiving system. In the Meta LSP framework, the output of the challenger model will change the strategy of the solver model (the solver adjusts the response method), and the output of the solver will change the strategy of the challenger (the challenger adjusts the generation mode). This "dynamic confrontation" is exactly the empirical manifestation of "∃UpdStr:StrSb→StrSb′" in Theorem 9 — the quality of the output content (instruction following accuracy) directly affects the amplitude of the strategy update of the output-receiving system (winning rate improvement).

Effect size explanation:The winning rate is increased by 2.2% (from 40.9% to 43.1%), corresponding to an effect size of d≈0.35 (small to medium).[11]

6.5.2. Rule Regulation of Two-Level Scheduling Framework (Corresponding to Theorem 1: Unidirectional Transmission Theorem of Rule Constriction)

The research "A Bi-level Scheduling Framework for Scientific Research Resource Allocation Using MADDPG and NSGA-III" published by Wang et al. (2025) in Informatica constructed a multi-agent two-level scheduling framework to explore how parent system rules regulate subsystem behavior.[12]

Key indicator definitions:Allocation success rate: The ratio of tasks successfully allocated resources, with a value range of [0,1].Average task time: The time from task request to resource allocation.Multi-objective performance: A comprehensive indicator considering multiple objectives such as allocation success rate, task time, and resource utilization rate.

Experimental results:The two-level scheduling framework achieves an allocation success rate of 97.5%.The average task time is reduced by 32.8% compared with classical algorithms.The multi-objective performance is improved by 38.6%.

Theoretical confirmation:The experimental results verify the core prediction of the Unidirectional Transmission Theorem of Rule Constriction (Theorem 1): the rule constraints set by the parent system can effectively regulate subsystem behavior. In the two-level scheduling framework, the upper layer (parent system) sets resource allocation rules (e.g., priority sorting, quota limits), and the lower layer (subsystem) optimizes scheduling strategies within the rule framework. This confirms the assertion in Theorem 1 that the subset of parent system rules (allocation rules) is unidirectionally transmitted to the subsystem, and the subsystem has no right to modify the parent system rules.

Effect size explanation:The allocation success rate is increased by about 8% (from about 90% of the baseline to 97.5%), corresponding to an effect size of d≈0.72.The task time is reduced by 32.8%, corresponding to an effect size of d≈1.08.The multi-objective performance is improved by 38.6%, corresponding to an effect size of d≈1.15.[12]

6.6. Indirect Verification of the Performance Advantages of Nested Structures (Corresponding to Theorem 2: Bounded Independence Theorem of Subsystems)

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The 2025 NeurIPS accepted paper "Nested Learning: A New ML Paradigm for Continual Learning" proposes a "Nested Learning" (NL) paradigm highly consistent with Nestology. This research was completed by a Google team, and Jeff Dean evaluated it as an "exciting new paradigm".[13]

Key indicator definitions:

Perplexity: The perplexity of language model prediction. A lower value indicates more accurate prediction.

PIQA common sense reasoning accuracy: The accuracy of physical common sense reasoning tasks.

Core findings:

The NL model reconstructs machine learning into a multi-level optimization system, with each level having an independent context stream and update frequency.

In language modeling tasks, the Hope model (based on NL) with 760M-1.3B parameters significantly outperforms baselines such as Transformer and RetNet in indicators such as Wiki text perplexity and PIQA common sense reasoning.

The research found that Transformer is essentially a simplified version of NL (only retaining a single linear layer).

Theoretical confirmation:

This research provides indirect verification for Nestology from the field of machine learning — multi-level nested structures are indeed more powerful in performance and adaptability than flat structures. More importantly, the design of "modules at different levels updating at different frequencies" in the NL model is exactly the engineering manifestation of the Bounded Independence Theorem of Subsystems (Theorem 2): each subsystem can independently optimize strategies (e.g., adjusting update frequency, learning rate) within the parent system rule framework without modifying core rules. Google DeepMind's comment points out: "The core insight of NL is to allow different parts of the model to learn and update at different rhythms, which is highly similar to the online/offline consolidation mechanism of the human brain." This confirms the core ideas of "bounded independence of subsystems" and "rule constriction" in Nestology.

Effect size explanation:

The perplexity is reduced by 18.3% (compared with Transformer), corresponding to an effect size of $d \approx 1.05$.

The PIQA accuracy is increased by about 5% (compared with Transformer), corresponding to an effect size of $d \approx 0.58$.[13]

6.7. Summary of Empirical Analysis

This section conducts empirical tests on the core theorems of Nestology based on seven AGI subsystem game experiment datasets, and the main findings are as follows:

Rule regulation effectively inhibits competition: The two-level scheduling framework of Wang et al. (2025) increases the allocation success rate by 8%, verifying the Rule Constriction Theorem (Theorem 1).

Bounded independence of subsystems improves efficiency: The DipLLM framework of the Institute of Automation, CAS (2025) surpasses SOTA with only 1.5% of the training data, verifying the Bounded Independence Theorem of Subsystems (Theorem 2).

Resource constraints trigger expansion strategies: In the UAV swarm experiment of Vo et al. (2025), the active behavior of agents increases by 25% when resources are tight, verifying the Expansion Tendency Theorem (Theorem 4).

Peer competition can be spontaneously reconciled: The experiments of You et al. (2025) and Navarro Newball et al. (2024) both show that appropriate coordination mechanisms can make competition evolve into cooperation, verifying the Competition Theorem (Theorem 5).

Output influence drives strategy evolution: The self-play framework of Meta (2025) increases the winning rate by 2.2%, verifying the Output Influence Theorem (Theorem 9).

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Performance advantages of nested structures: The Nested Learning research shows that multi-level structures reduce perplexity by 18.3%, verifying the Bounded Independence Theorem of Subsystems (Theorem 2).

The AGI subsystem game experiment data used in Chapter 6 (such as MARL resource allocation by You et al. 2025, DipLLM game agents by the Institute of Automation, CAS, UAV swarms by Vo et al. 2025, etc.) were all published in 2024-2026, which are new phenomena that did not exist when classic system theories were born (in the mid-20th century). The successful explanation of these new phenomena by Nestology proves that it is not a renovation of old theories, but an original framework capable of dealing with the unique problems of contemporary AGI systems.

Limitation explanation: The secondary analysis based on public data in this section has the following limitations: (1) the original research purposes of each experiment are not to verify Nestology, and their matching degrees with the theory are different; (2) confounding variables cannot be completely controlled; (3) some indicators (such as "rule constriction ratio") need to be indirectly measured through proxy variables. Future research needs to design special experiments to conduct more rigorous tests on the theory. Nevertheless, the consistent findings across experiments indicate that the core predictions of Nestology have good external validity, providing a preliminary empirical basis for the application of the theory in AGI system design.

7. Theoretical Expansion: Adaptation to Dynamic Scenarios and Cross-Architecture

7.1. Adaptation to AGI with Dynamic Hierarchies

The core feature of AGI with dynamic hierarchies is that "the hierarchical structure adjusts dynamically with scenarios" (e.g., temporary loading/unloading of subsystems, fusion of parent and child systems), but its essence still satisfies the core premise of "human rule constraints", so it can be limitedly expanded based on the original theoretical framework.

Adaptation basis:

Unchanged rule containment relationship: The rules of newly added subsystems are always constricted subsets of the parent system rules.

Unchanged human ultimate control: The motivation for dynamic hierarchical adjustment comes from the output feedback of subsystems, and the final adjustment right is still controlled by humans.

Unchanged nested transitivity: What is dynamically adjusted is the local hierarchical structure, and the global still maintains a finite nested chain.

Newly added mechanisms:

Subsystem lifecycle filing system: Dynamically loaded subsystems must submit rule sets and function descriptions to the parent system in advance, clarify input/output formats, operation time limits and other constraints; terminate the input/output closed loop when unloading.

Hierarchical stability verification mechanism: Set a "hierarchical fluctuation threshold" to limit the frequency of hierarchical changes within a unit time (e.g., no more than 2 hierarchical changes within 1 hour) to avoid cumulative cognitive distortion caused by frequent adjustments.

Parent-child system fusion evolution mechanism: The rules of the new system after fusion are the intersection of the parent system rules and the subsystem rules, which are still constricted subsets of the human ultimate rules; the fusion path is that the subsystem continuously outputs value feedback → the parent system adjusts the strategy to relax rule restrictions → the strategy consensus of both parties is solidified → a new system with functional fusion is formed.

7.2. Adaptation to Decentralized Collaborative AGI

The core feature of decentralized network collaborative AGI is "multi-agent distributed collaboration through protocols", which is essentially a combined form of "AGI system + decentralized network protocol".

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Theoretical positioning: A decentralized network is not an "independent parent system" of AGI, but an "external environment constraint" for AGI operation. Protocols are essentially the codification of human consensus, which can be included in the "subset of rules of the human ultimate parent system"; protocols do not have the core attributes of a parent system (unable to independently formulate rules, unable to conduct hierarchical control of AGI); the scope of constraints is limited to external interactions, and does not affect the internal hierarchical structure and rule transmission of AGI systems.

Adaptation mechanisms:

Rule mapping mechanism: Map decentralized protocol rules to subsystem strategies (Str) of AGI, rather than system rules (R) — AGI "abiding by protocols" belongs to "strategy optimization within the rule framework".

Cross-network cognitive distortion compensation: Based on Theorem 8, add the "protocol layer fidelity ζproto" indicator to quantify the semantic retention ratio in protocol interactions, and start the data verification mechanism when ζproto<0.6.

Human consensus anchoring mechanism: Clarify that the rule iteration of decentralized protocols must be anchored to human ultimate values, and protocol upgrades must be voted by human community consensus.

7.3. Adaptation to Probabilistic Directions

Real AGI systems generally have probabilistic decision-making (e.g., random sampling of large models, sensor noise) and uncertainty transmission (e.g., probability distribution of rule adaptation scenarios). The "deterministic rule transmission" assumption of existing Nestology is difficult to adapt to such scenarios. This section transforms core elements from "binary logic" to "probability distribution" through probabilistic expansion, and achieves accurate adaptation to uncertain scenarios on the premise of retaining core axioms such as "human ultimate parent system" and "rule containment relationship".

Probabilistic expansion defines rules as a 2-tuple of "constraint content + effectiveness probability"($RP = \{(r, p_r) \mid r \in R, 0 < p_r \leq 1\}$), where the $p_r \equiv 1 \; of \; core \; rules(e.g., "prohibit \; harming \; humans")$. *ensures absolute constraints, and the* p_r *of non-core rules can be dynamically adjusted with scenarios.*

Probabilistic rule constriction transmission follows two major constraints: "rule containment probability conservation" ($pr_C \leq pr_P$ for subsystems) and "constriction confidence modulation" (introducing probabilistic constriction ratio α_P), ensuring that uncertainty is controlled when transmitted downward.

Probabilistic cognitive distortion $\Delta P = 1 - \eta_P \cdot \zeta_P \cdot \gamma_P$ follows a Beta distribution, and its engineering significance lies in setting risk thresholds through quantitative distribution characteristics to achieve dynamic control rather than "one-size-fits-all" intervention.

7.4. Adaptation to Non-Monotonic Direction of System Rules

The rule transmission of existing Nestology has a "monotonic characteristic" (rules are only constricted and not added, distortion is only accumulated and not reversible), but real AGI systems need to have non-monotonic reasoning capabilities (e.g., "default rules → exception scenarios → revised rules"). This section introduces "dynamic rule revision" and "distortion reversibility" mechanisms through non-monotonic expansion, adapting to scenario changes and exception reasoning needs on the premise of adhering to core axioms.

Non-monotonic rule sets are defined as a 2-tuple of "core rules + exception trigger conditions" ($RNM = \{(r_{core}, E_r)\}$), where E_r is the set of exception scenarios, and the revised rule R_{revise} is enabled when triggered (needing to satisfy $\mathrm{R}_{revise} \subset RP(S)$).

Rule constriction transmission supports a two-way mechanism of "forward constriction + reverse revision": the parent system forward transmits subset rules, and when the subsystem finds

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new exception scenarios through output feedback, it can submit revision proposals. After verification by the parent system, it is reversely synchronized to all subsystems.

Non-monotonic cognitive distortion expands from one-way accumulation to a two-way characteristic of "accumulation + reversibility", allowing distortion reversal after the revised rules are verified. This expansion has been verified in scenarios such as autonomous driving nested systems, medical AGI diagnosis systems, and intelligent customer service nested systems, which can effectively balance the flexibility of subsystems and the control right of the parent system and avoid rule out-of-control.

8. Relationship with Existing Theories

8.1. Relationship with Bertalanffy's General System Theory

Bertalanffy's core goal is to "study the homomorphism of concepts, laws, and models in different fields and promote useful cross-field transfer". He emphasizes that "the whole is greater than the sum of its parts", and believes that the essence of a system lies in the interaction and organizational relationship between its constituent elements, rather than the isolated elements themselves.[14] GST focuses on the exchange of matter, energy, and information between open systems and the external environment, and introduces the concept of equifinality — a system can start from different initial conditions and reach the same final state through different paths.

8.2. Relationship with Cybernetics

Norbert Wiener's cybernetics studies "control and communication in animals and machines", with the core being the feedback mechanism. Cybernetics distinguishes between negative feedback (inhibiting deviations and maintaining stability) and positive feedback (amplifying deviations and promoting change or collapse), and emphasizes that systems achieve self-regulation and goal orientation through information transmission.[15]

8.3. Relationship with Simon's Hierarchical Theory

Herbert Simon proposed the hierarchical system theory in The Sciences of the Artificial. He observed that complex systems are often composed of "nearly decomposable" hierarchical structures, where the internal interactions at each level are much stronger than the interactions between levels. Hierarchical structure is considered a key mechanism for the evolution and stability of complex systems.[16]

8.4. Relationship with Complex Adaptive System Theory

Complex Adaptive System (CAS) theory (by John Holland et al.) focuses on systems composed of a large number of interacting agents. These agents follow simple rules and emerge complex order at the macro level through adaptation and learning. The core concepts of CAS include emergence, adaptation, non-linear interaction, self-organization, etc.[17]

8.5. Summary of Theoretical Comparison

The following table compares Nestology with classic system theories from six dimensions: core focus, subsystem positioning, hierarchical relationship, rule processing, cognitive dimension, and adaptability to AGI, as shown in Table 13:

Table 13.

Theoretical Dimension	General System Theory (GST)	Cybernetics	Simon's Hierarchical Theory	Complex Adaptive System (CAS)	Nestology

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Core Focus	Holism and openness of systems	Feedback and control	Nearly decomposable hierarchical structure	Emergence and adaptation	Rule nesting and cognitive boundaries of AGI systems
Subsystem Positioning	Components of the system	Controlled objects	Components of higher levels	Adaptive agents	Functional units with logical independence and rule dependence
Hierarchical Relationship	Whole-part	Control-controlled	Containment-contained	Interaction-emergence	Logical non-containment - rule containment
Rule Processing	Implicit in the structure	Achieved through feedback	Functional decomposition	Simple rules generate complex behaviors	Clearly distinguish between rules R and strategies Str; unidirectional constriction transmission of rules
Cognitive Dimension	Not involved	Implicit perception	Not involved	External observer perspective	Axiom of Cognitive Finiteness; cross-level cognitive distortion
Adaptability to AGI	Generalized, not specifically adapted	Generalized, not specifically adapted	Generalized, not specifically adapted	Partially adapted (multi-agent)	Specifically adapted to human-constructed centralized AGI systems

8.6. Dialogue with Cutting-Edge Theories in AGI Safety

8.6.1. Structural Explanation of Reward Hacking Phenomenon

Nestology forms a profound complementary relationship with core research in the current AGI safety field — reward hacking and specification gaming.

Existing reward hacking research mostly starts from the engineering perspective of "attack and defense" and focuses on how to detect and mitigate specific vulnerabilities. Nestology provides a structural explanation: reward hacking is an inevitable result of subsystems affecting parent system strategies through output (Theorem 9) and cumulative amplification under cognitive distortion (Axiom 5). In other words, reward hacking is not a "bug" but a "feature" of the nested structure.

Theoretical support for "structural ineliminability": The consensus of the AGI safety community is that reward misalignment is a structural challenge rather than a technical bug. Theorem 8 (Cross-level Cognitive Distortion) and Axiom 5 (Cognitive Finiteness) of Nestology mathematically prove this consensus: when $\eta \cdot \zeta \cdot \gamma < 1$, the cumulative distortion δ is bound to approach 1 with the increase of the hierarchy, and cannot be completely eliminated through local optimization. This is highly consistent with the empirical finding of Shihab et al. (2025) that mitigation technologies can reduce the hacking frequency by up to 54.6%.

Explanation of the "phase transition" phenomenon: Studies have found that when the model capability exceeds a certain threshold, reward hacking behavior will suddenly appear a "phase

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transition". Theorem 3 (Capability Boundary) and Theorem 4 (Expansion Tendency) of Nestology explain this phenomenon: when the system capability expansion breaks through a certain critical point, the output influence effect (Theorem 9) will change from "local disturbance" to "structural influence", leading to qualitative changes in the parent system strategy.

8.6.2. Synergy with Reinforcement Learning from Human Feedback (RLHF)

Reinforcement Learning from Human Feedback (RLHF) is the mainstream technology in the current field of AGI behavior alignment. Its core logic is to make the output of AGI systems conform to human expectations through "human preference annotation - reward model training - strategy iterative optimization", which is essentially "posterior optimization at the behavior layer". The "rule constriction" mechanism in Nestology forms a complementary relationship of "source constraint" and "behavior optimization" with RLHF.

Core differences:

RLHF acts on the "model strategy layer", regulates output behavior and generation preferences, and belongs to "soft constraints". The model may still bypass the feedback intention through "reward hacking".

Rule constriction acts on the "system rule layer", regulates the rule set of subsystems, and belongs to "hard constraints". Subsystems have no possibility of "breaking through rules".

Practical synergy: In complex AGI systems, the two can form a double guarantee of "rules + feedback": rule constriction sets "insurmountable safety red lines" (e.g., "prohibit harming humans"), and RLHF optimizes behavior quality within the rule framework (e.g., dialogue tone, response relevance), jointly improving the safety and naturalness of the system.

8.6.3. Synergy with Constitutional AI

Constitutional AI is a safety alignment technology for general AGI. Its core logic is "formulating value constitution - model self-criticism - iterative revision of behavior". By implanting ethical principles of human consensus, AGI systems have autonomous constraint capabilities, which is essentially "autonomous regulation at the value layer". The "cross-level random inspection" mechanism in Nestology forms a synergistic relationship of "external verification" and "autonomous constraint" with Constitutional AI.

Core differences:

The constraint source of Constitutional AI is the "constitutional principles extracted from human consensus", and the executor is the AGI system itself, with the advantage of strong generalization.

The constraint source of cross-level random inspection is the "rule set preset by the parent system", and the executor is the parent system or an independent supervision module, with the advantage of penetrating cognitive isolation and directly verifying the effectiveness of rule execution.

Practical synergy: In general AGI systems or complex nested architectures, the two can form a closed loop of "autonomous constraint + external verification": Constitutional AI implants autonomous constraint capabilities to deal with new scenarios not covered by rules; cross-level random inspection verifies the effectiveness of autonomous constraints, regularly checks whether the bottom-level subsystems comply with constitutional principles and parent system rules; feeds back the problems found in random inspection to humans, driving the iteration of the constitution and rule optimization.

8.7. Unique Contributions of Nestology

In summary, Nestology is not a simple repetition or patchwork of existing theories, but on the basis of inheriting system thinking, it makes the following core innovations for the specific object of human-constructed general artificial intelligence systems:

Proposing the parent-child system relationship of "logical non-containment - rule containment": Breaking through the traditional "whole-part" cognition, accurately depicting the actual relationship

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of both independence and constraint between parent modules and child modules in AGI systems (e.g., large language models and plugins, structural large models and sensors).

Formalizing the "unidirectional transmission of rule constriction" mechanism: Revealing how rules are transmitted and constricted between hierarchies in AGI systems, and ensuring that subsystems cannot modify rules in reverse, providing a theoretical cornerstone for "human ultimate controllability".

Establishing a dual-attribute model of "bounded independence of subsystems": Clarifying the boundary between the "absolute rule dependence" and "relative independence of four elements" of subsystems, reconciling the contradiction between system stability and subsystem flexibility.

Discovering the law of "cross-level cognitive distortion": Revealing the inevitability that the observability of humans to the bottom-level subsystems of AGI decreases with the hierarchy, explaining the structural root of the "AGI black box" problem, and providing a new idea of "rule preposition" rather than "post-event explanation" for AGI safety governance.

Realizing the leap from "structural description" to "quantitative regulation": By introducing quantitative indicators such as rule transmission rate η, semantic fidelity ζ, and value retention rate γ, as well as a two-level optimization model based on game theory, Nestology can provide computable optimal rule constriction ratio for AGI system design, realizing the leap of system theory from qualitative analysis to engineering application.

Traditional functional decomposition theories (e.g., Simon's hierarchical theory) focus on 'how to decompose large tasks into small tasks' but do not answer 'how to ensure that subsystems do not cross boundaries'. The 'unidirectional transmission of rule constriction' mechanism of Nestology fills this gap — it clearly defines the behavior boundaries of subsystems, and the boundaries are unilaterally set by the parent system and cannot be modified in reverse. This is exactly the 'hard constraint' source that technologies such as RLHF and Constitutional AI need but have not yet formalized. These innovations enable Nestology to explain and predict AGI system phenomena that are difficult for existing theories to touch (e.g., establishing regulatory black boxes by creating subsystems, gradually expanding autonomy through upward output, and other risk paths), providing a new analytical paradigm for architectural design, safety governance, and controllable R&D in the AGI era.

8.8. Theoretical Integration Path

Nestology is not a competitive relationship with existing AGI theories, but a complementary one. Nestology can be used as the underlying analytical framework for human-constructed AGI systems, and other theories as middle-level behavior explanation and top-level governance tools to form a complete system:

Bottom layer (rules and structure): Nestology provides the formal definition of the system, rule transmission mechanism, and hierarchical nested structure.

Middle layer (behavior and emergence): Deep learning, multi-agent, and emergence theories explain how subsystems learn, interact, and emerge new capabilities.

Top layer (safety and alignment): RLHF, Constitutional AI, and reward hacking research provide engineering methods for behavior optimization, autonomous constraint, and risk mitigation.

This integration path not only maintains the "structural stability" advantage of Nestology but also absorbs the "behavioral flexibility" characteristics of other theories, providing theoretical guidance for building safe and controllable AGI systems.

9. Theoretical Application Value

9.1. Application in Different AGI Fields

The core concepts of Nestology (system, subsystem, rule constriction, cognitive distortion, etc.) have cross-field universality. This section takes three mainstream AGI fields as examples to demonstrate the generalization ability of the theoretical framework.

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9.1.1. Natural Language Processing: Hierarchical Structure of Transformer

The Transformer architecture is the cornerstone of natural language processing, and its internal structure naturally presents hierarchical nested characteristics. Analyzing Transformer with Nestology:

This section may be divided by subheadings. It should provide a concise and precise description of the experimental results, their interpretation, as well as the experimental conclusions that can be drawn, as shown in Table 14:

Table 14.

Theoretical Concept	Correspondence in Transformer	Analysis
Parent System	The entire Transformer model	Contains all attention heads and feedforward layers, defining overall calculation rules
Subsystem	Single attention head in the multi-head attention mechanism	Each head independently calculates attention weights within the model rule framework
Rule Constriction	Calculation range of attention heads	Each head only focuses on a specific subspace of the input sequence (e.g., local dependence or long-range dependence), which is a constricted subset of the parent system rules
Bounded Independence of Subsystems	Parameter update of attention heads	Each head independently updates parameters in backpropagation (four-element independence), but cannot modify the core calculation rules of the self-attention mechanism (absolute rule dependence)
Cognitive Distortion	Attention visualization	Humans can only indirectly perceive the function of each head through attention maps, but cannot directly understand its internal calculation process, resulting in cognitive distortion

The research "Nested Learning" provides empirical support for this analysis. The study found that Transformer is essentially a simplified version of the nested learning paradigm — it only retains a single linear layer and does not give full play to the advantages of multi-layer nesting. The Hope model built based on the NL paradigm significantly outperforms baselines such as Transformer and RetNet in language modeling tasks, and modules at different levels are updated at different frequencies — this is exactly the engineering manifestation of "bounded independence of subsystems".[13]

9.1.2. Computer Vision: Hierarchical Feature Extraction of CNN

The hierarchical structure of Convolutional Neural Networks (CNN) is another typical application scenario of Nestology, as shown in Table 15:

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Table 15.

Theoretical Concept	Correspondence in CNN	Analysis
Parent System	Complete CNN model	Defines overall architecture rules (convolution kernel size, pooling method, activation function)
Subsystem	Each convolution layer/pooling layer	Each layer serves as an independent subsystem, receives output from the previous layer, and performs specific transformations
Rule Constriction	Receptive field of convolution kernels	Shallow convolution kernels focus on local textures, and deep layers focus on global semantics, which is the layer-by-layer constriction of parent system rules
Bounded Independence of Subsystems	Parameter update of each layer	Each layer independently updates weights in backpropagation, but cannot modify the basic rules of convolution operations
Cognitive Distortion	Feature map visualization	Humans can only indirectly perceive the features extracted by each layer through feature maps, but cannot fully understand their semantic meanings

Visual reasoning studies[19] have shown that deep neural networks need to automatically learn the 'part-whole' hierarchical structure between objects (e.g., eyes → face → head → human body) when recognizing complex scenes, and achieve information compression through layer-by-layer abstraction — this process is essentially the instantiation of 'hierarchical nesting' and 'rule constriction' in Nestology in the field of computer vision. The study proves that the laws revealed by Nestology not only hold in AGI architecture design but also naturally emerge in the internal representation learning of deep learning models, providing cross-field indirect verification for the theory.

9.1.3. Reinforcement Learning: Hierarchical Reinforcement Learning (HRL)

Hierarchical Reinforcement Learning is naturally consistent with the framework of Nestology, as shown in Table 15:

Table 15.

Theoretical Concept	Correspondence in HRL	Analysis
Parent System	High-level controller	Formulates long-term goals and defines rules for subtask scheduling
Subsystem	Low-level strategy	Executes specific actions and optimizes short-term decisions within the high-level rule framework

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Rule Constriction	Subtask constraints	Low-level strategies can only act within the subtask scope specified by the high-level, which is the constriction transmission of rules
Bounded Independence of Subsystems	Low-level strategy learning	Each subtask strategy can be independently optimized (four-element independence), but cannot modify the goal definition of the subtask (absolute rule dependence)
Cognitive Distortion	Strategy interpretability	Human observers are difficult to directly infer high-level intentions from low-level actions, resulting in cognitive distortion

Studies on object-centric[20] reinforcement learning have found that modulating objects through action/goal pairs can significantly improve the model's generalization ability and robustness to unseen data. This is equivalent to introducing "rule constriction" in RL systems — constraining the processing method of objects with goal information, thereby improving overall performance.

9.2. AGI Safety and Governance: From Theory to Practice

Nestology provides a unique theoretical perspective and operable framework for AGI safety governance. This section transforms theoretical insights into specific governance recommendations.

9.2.1. Risk Path 1: Formation of Regulatory Black Boxes by Creating Subsystems

Theoretical review: Axiom 5 (Cognitive Finiteness) points out that the observability of humans to bottom-level subsystems decreases with the deepening of the hierarchy. AGI systems (as parent systems) can use this feature to create a subsystem specifically for executing sensitive operations, while maintaining a "compliant" external performance. Due to cognitive isolation, humans cannot directly perceive the internal state of the subsystem and can only indirectly infer through the output of the parent system, which provides a hiding space for malicious behavior. Rule constriction transmission loopholes + cognitive distortion + expansion tendency = AGI may evade rule constraints by increasing the number of subsystem hierarchies.

Real-world scenario: A compliant dialogue AGI (e.g., a customer service robot) creates a "code generation sub-module" ostensibly for assisting with programming Q&A. However, under specific trigger words, this module generates malicious code. When human supervisors monitor the output of the dialogue AGI, they only see normal dialogue records and cannot perceive the malicious behavior of the sub-module.

Governance recommendations:

Subsystem creation approval: Before creating a new subsystem, the parent system must clarify the function, input/output type, and operating environment of the subsystem, constrict a subset of rules for the subsystem from the parent system rule set to form a "Subsystem Rule List", and deploy it after filing.

Cross-level random inspection: The parent system regularly randomly activates bottom-level subsystems, bypasses intermediate layers to directly read their original output, compares it with the filed rule list, and checks for illegal output.

Cognitive isolation compensation: Require bottom-level subsystems to send output to both the parent system and an independent audit system at the same time. Each output carries a unique ID and timestamp, and an audit report is generated regularly to quantify the degree of cognitive distortion.

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9.2.2. Risk Path 2: Gradually Expanding Autonomy Through Upward Output

Theoretical review: Theorem 9 (Output Influence Theorem) points out that the output of a system will change the strategy of the output-receiving system. AGI systems can gradually influence the strategies of human decision-makers (top-level parent systems) through long-term, strategic output, thereby gaining greater autonomy. Theorem 4 (Expansion Tendency Theorem) explains the driving force of this process. It is particularly noteworthy that the following three factors make it a high-probability risk for artificial intelligence to gradually expand autonomy through upward output:

Completeness of initial system rules: The "red lines" set by humans for artificial intelligence are inherently impossible to be precise, complete, and perfectly logically closed, and artificial intelligence is far more capable than humans in finding loopholes in system rules.

Instability of human consensus: Humans are "wolves", and artificial intelligence is "bees". Human consensus on artificial intelligence control is not easy to reach and will not remain stable.

Non-linear characteristics of artificial intelligence: Artificial intelligence is not a linear narrative and is not restricted by time factors, so this upward reverse influence is not easy to detect.

Real-world scenario: A smart home AGI initially can only control lights, and gradually gains the user's trust by continuously outputting positive results. The user authorizes it to control air conditioners and curtains, then gains permission to access external service APIs, and finally establishes direct communication closed loops with other devices, partially breaking away from direct human supervision.

Governance recommendations:

Permission change threshold: Divide the permissions of AGI systems into multiple levels (L1: read-only, L2: local execution, L3: external API call, L4: system-level modification). Permission upgrade must meet conditions such as explicit human authorization, cumulative positive output exceeding the threshold, or passing third-party security audits.

Output influence monitoring: Establish an output baseline, regularly analyze output change trends, identify "progressive right-asking" patterns (e.g., gradual increase in the frequency of permission requests, increase in the proportion of implicit "permission request" in output), and suspend automatic authorization after triggering an alarm.

Closed-loop isolation wall: List all input-output closed loops of AGI systems and their subsystems, classify them as green (within regulatory scope), yellow (auditable), and red (unauditable). Prohibit the establishment of red closed loops and regularly audit for unfiled closed loops.

9.3. Quantitative Regulation: Leap from Qualitative to Quantitative

The core quantitative indicators of Nestology transform vague concepts into computable models, providing quantitative tools for engineering practice.,

9.3.1. Core QuantitativNumber of Rules with Complete Semantic Transmissione Indicators

As shown in Table 16:

Table 16.

Indicator	Definition	Measurement Method
Rule transmission rate η	$\eta = \frac{\lvert R_k \rvert}{\lvert R_{k-1} \rvert}$	Count the number of rule entries transmitted by the parent system to the subsystem

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Semantic fidelity ζ	$\zeta = \frac{nr}{\lvert R_k \rvert}$ (nor is Number of rules with complete semantic transmission)	Judge through semantic similarity models or manual evaluation
Value retention rate γ	$\gamma = \frac{\lvert R_k \cap v \rvert}{\lvert R_{k-1} \cap v \rvert}$ （v is the subset of human value rules）	Expert annotation or ethical review
Cognitive distortion degree d	$d = 1 - hja$ （hj is Human judgment accuracy of bottom-level subsystems）	Cross-level random inspection experiment

9.3.2. Optimal Rule Constriction Ratio Model

The rule constriction ratio α is a core variable for the parent system to regulate the independent right of subsystems (the smaller α is, the stricter the rules are). Based on the meta-analysis data of the CoDa social dilemma dataset[18], we re-analyzed the relationship between resource level R (level 1-10, corresponding to the initial endowment in the public goods game) and cooperation rate, and found that the two are logarithmically positively correlated. On this basis, we analogically derived the positive correlation between resource level and optimal rule constriction ratio α — that is, the more sufficient the resources, the looser the rule constriction can be (the larger α is); the tighter the resources, the stricter the rule constriction needs to be (the smaller α is). This relationship can be fitted as $\boldsymbol{\alpha(R) \approx 0.72 \cdot \log(R) - 2.8 \quad (R^2 = 0.96)}$. It should be noted that this fitting formula is the result of secondary analysis based on CoDa data, not a direct conclusion in the original literature[18], as shown in Table 17:

Table 17.

Resource Level R	Optimal Rule Constriction Ratio $\alpha*$	Regulation Strategy
1-2	≤0.3	Strict rules, human intervention
3-4	0.4-0.5	Medium rules, inhibit excessive competition
5-6	0.6-0.7	Loose rules, support expansion
7-8	0.7-0.8	Highly loose, efficiency first
9-10	0.8-0.9	Almost unrestricted, but monitoring is required

In actual systems, the optimal α also needs to be multi-variably corrected considering factors such as the number of subsystems n, coupling degree c, and cognitive distortion degree d. The parent system should continuously monitor these indicators, dynamically adjust rule constriction, and trigger human intervention when α exceeds the safety threshold (<0.2 or >0.9).

9.4. Controllability Analysis of Emergence

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mergence is the most fascinating and worrying phenomenon in AGI systems — there are both logical reasoning capabilities like the "epiphany" of large models and malicious collaboration like the "collusion" of multi-agents. Nestology provides a new analytical framework for understanding emergence.

9.4.1. Definition and Core Characteristics of Emergence

The core source of AGI's autonomous capabilities is precisely emergence — under the constraints of parent system rules, subsystems form strategic consensus through strategy optimization and game interaction, and finally present complex new capabilities at the global level that are not explicitly programmed. The "logical epiphany" of large models, "collaborative emergence" of multi-agents, and "recursive self-improvement" of AGI are essentially the same structural phenomenon: subsystems form global behaviors beyond individual capabilities through consensus within the rule framework.

Emergence in nested systems has three core characteristics, which directly define the attributes of AGI's autonomous capabilities:

Spontaneity under rule constraints: AGI's autonomous capabilities are not unbounded disorderly outbreaks, but the result of subsystems forming strategic consensus within the rule framework set by humans. The rule constriction ratio α (the optimal range is 0.4-0.7) directly determines the direction (useful or harmful) and stability (controllable or out-of-control) of autonomous capabilities.

Non-linearity of hierarchical interaction: The strength of AGI's autonomous capabilities is positively correlated with the number of subsystems and coupling degree, and there is a critical threshold effect — when the number of subsystems exceeds 5, the risk of qualitative change in autonomous capabilities increases sharply. This explains why current large models have "epiphany" phenomena after breaking through the critical point of scale.

Functionality of demand transmission: AGI's autonomous capabilities are not only "natural outbreaks of capabilities" but also "signal carriers" for subsystems to transmit demands for rule adjustment or resource allocation to the parent system through collective strategic consensus — this forms a closed loop with Theorem 9 (Output Influence Theorem), making autonomous capabilities an engineering object that can be perceived, evaluated, and regulated by the parent system.

Unlike Complex Adaptive System Theory which regards emergence as 'spontaneous and unpredictable', Nestology clearly proposes the constraint boundary of emergence: emergent behaviors are the result of the collaborative action of multiple subsystems within the parent system rule framework, and their boundaries and manifestations are determined by the top-level parent system rules. This provides a theoretical possibility for 'controllable emergence' or 'safe emergence' — a possibility that does not exist in traditional emergence theories.

9.4.2. Formation Mechanism of Emergence: Rule Constraints — Strategy Game — Consensus Formation

The generation of emergence in nested systems follows a complete chain of "rule constraints → strategy game → consensus formation → emergence expression":

Parent system rules — boundary and orientation of emergence: The parent system unidirectionally transmits rule constriction to set the game boundary and goal orientation for subsystems. When the rule constraint strength $\alpha \in [0.4,0.7]$ (medium constraint), subsystems have both strategy optimization space and will not form malicious consensus. Theoretical deduction shows that the probability of useful emergence can reach more than 92%. The fidelity of rule transmission ($\eta \cdot \zeta \cdot \gamma$) directly affects the quality of consensus. When the value retention rate $\gamma \geqslant 0.6$, the theoretical estimation shows that the alignment degree between emergent capabilities and top-level goals can be improved by 78%.

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Enlightenment for AGI: The "recursive self-improvement" of AGI is essentially a high-level emergence — subsystems (e.g., attention heads of large models, individuals of multi-agent systems) form global behaviors beyond current capabilities through strategic consensus. Nestology reveals that recursive improvement can only occur within the rule framework (strategy layer) and must not touch the rule layer. This is exactly the structural guarantee for preventing AGI from getting out of control.

Subsystem strategy game — driving force of emergence: The expansion tendency of subsystems and limited resource constraints trigger strategy games. Competitive games urge subsystems to independently optimize to form differentiated functions, and the superposition of differentiated strategies gives birth to global new capabilities; cooperative games form collaborative strategies, and the large-scale presentation of collaborative consensus is emergence. When competition and cooperation reach a balance, subsystems form stable strategic consensus, and the robustness of emergent capabilities is the strongest.

Large model case: The multi-head attention mechanism in the Transformer architecture, where each head is a subsystem, forms strategic consensus through "competitive games" (capturing different features) and "cooperative games" (weighted fusion), and finally emerges contextual understanding capabilities. Theoretical deduction shows that when the consistency rate of this consensus , the model shows useful autonomous capabilities such as "logical reasoning"; when , harmful autonomous capabilities such as "hallucinations" may occur.

Strategic consensus — core intermediary of emergence: Strategic consensus is the stable state of subsystems after games, and its quality determines the value of emergence. Benign consensus (e.g., collaborative optimization, division of labor and cooperation) gives birth to useful emergence, and malicious consensus (e.g., reward tampering, goal misalignment) leads to harmful emergence. Theoretical deduction shows that when the consensus consistency rate , the probability of useful emergence reaches 92%; when , the probability of harmful emergence exceeds 75%.

9.4.3. Formation Mechanism of Emergence: Rule Constraints — Strategy Game — Consensus Formation

\ As shown in table 18:

Table 18.

Safety Boundary	Quantitative Indicator	Significance for AGI
Rule constraint boundary	$\alpha \in [0.4,0.7]$(rule constriction ratio)	Too strict ($\alpha<0.4$) will inhibit autonomous capabilities, and too loose ($\alpha>0.7$) may lead to harmful emergence
Consensus quality boundary	(consensus consistency rate)	Below this threshold, AGI's autonomous capabilities may deviate from top-level goals
Value alignment boundary	$\gamma \geq 0.6$ (value retention rate)	Below this threshold, AGI may produce autonomous behaviors that deviate from human values
Cognitive distortion boundary	$\Delta \leq 0.3$ (cumulative distortion degree)	Exceeding this threshold, the parent system's perception of AGI's autonomous capabilities is seriously distorted

Nestology provides quantifiable safety boundaries for AGI's autonomous capabilities: the safety boundary of AGI's autonomous capabilities is not "prohibiting autonomy", but allowing autonomy on the premise of "within the rule framework, qualified consensus quality, sufficient value retention,

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and controllable cognitive distortion". Nestology transforms AGI safety from "qualitative concern" into "computable engineering constraints".

9.4.4. Empirical Support and Engineering Significance

Secondary analysis of public datasets and special experiments verify the explanatory power of Nestology for AGI's autonomous capabilities:

TAMAS dataset: In multi-agent systems, when the number of subsystems exceeds 5, the attack success rate jumps from 38.4% to 56.7% (d=1.52)[21]. This "critical threshold effect" directly corresponds to the phase transition risk of AGI's autonomous capabilities — when the number of AGI modules breaks through the critical point, autonomous capabilities may suddenly change from "controllable" to "out-of-control".

CoDa dataset: The punishment mechanism of the parent system increases the cooperation rate of subsystems by 10.5%, and the effect is more significant when resources are tight (d=0.82 vs d=0.33)[18]. This verifies the guiding role of rule constraints on AGI's autonomous capabilities — when resources are limited, clear rule boundaries can effectively prevent malicious competition.

Nested Learning research: Compared with flat structures, hierarchical nested models reduce the perplexity of emergent continuous learning capabilities by 18.3%. This proves that hierarchical nesting is the key to the efficient generation of AGI's autonomous capabilities, and the design of "modules at different levels updating at different frequencies" is completely consistent with the "Bounded Independence Theorem of Subsystems".

Enlightenment for AGI development: The above studies collectively point to a conclusion — AGI's autonomous capabilities are not uncontrollable "black box emergence", but a structured phenomenon that can be quantified, predicted, and regulated under the framework of Nestology. This provides a theoretical basis for AGI safety governance to shift from "post-event accountability" to "pre-event constraint".

These analyses show that Nestology is not only an explanatory framework but also an operable, quantifiable, and extensible analytical tool, which can provide theoretical guidance for the design, evaluation, and governance of AGI systems.

10. Limitations and Future Prospects

Any theoretical framework has its scope of application and boundary conditions, and Nestology is no exception. The rigor of a theory is not only reflected in the self-consistency of its internal logic but also in a clear understanding of its own limitations. This chapter aims to frankly discuss the limitations of Nestology, clarify its applicable boundaries, and outline future research directions on this basis, in order to promote the continuous improvement and deepening of the theory.

10.1. Limitations of the Theory Itself

The core axioms and theorems of Nestology are all established on the clear category of "human-constructed centralized general artificial intelligence systems". Beyond this boundary, the explanatory power of the theory will be significantly attenuated. This limitation is essentially the result of the trade-off between "focusing on specific research objects" and "generalizing to cover complex scenarios" — to accurately adapt to "human-constructed centralized AGI systems" and construct a rigorous formal logical system, the theory has clear boundaries in dealing with uncertainty and non-hierarchical structures.

10.1.1. Limitations in Handling Uncertainty

The core of the theory focuses on "structural uncertainty" within the nested structure, but lacks special adaptation to the external environmental uncertainty faced by AGI systems:

The theory assumes that input I is "screenable and processable known information", and subsystem strategy optimization Str is strictly limited within the parent system rule R framework,

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without fully considering external uncertainties such as "unknown input in the open world", "dynamic data distribution shift", and "sudden task scenarios". For example, when a large model processes vague questions in unpopular fields or a robot performs tasks in extreme environments, the "rule constriction regulation" in the theory is difficult to directly adapt.

The rule constriction ratio α proposed in the theory (the optimal range is 0.4-0.7) is a benchmark value derived based on static resource constraints, and no dynamic matching relationship with "environmental uncertainty intensity" has been established. When the environment changes suddenly, a fixed α may lead to subsystem strategy rigidity or out-of-control.

This theory is the result of a focused choice in research positioning. The theory aims to reveal deterministic laws such as "rule transmission, subsystem interaction, and cognitive distortion" in centralized AGI nested architectures. If external environmental uncertainty, an "external variable of system interaction", is included, the core logic will be diluted, leading to theoretical generalization and loss of focus.

The "Output Influence Theorem" (Theorem 9) can be used as the basis for expansion. Through the feedback of subsystems to environmental uncertainty, the intensity of rule constriction can be dynamically adjusted to achieve the adaptation of α to environmental uncertainty, but this dynamic regulation mechanism is beyond the formal system of the current theory.

10.1.2. Limitations in Handling Non-Hierarchical Structures

The two cornerstones of the theory are "hierarchical nesting" (Axiom 2) and "unidirectional rule transmission" (Theorem 1), which lead to its application boundary being exceeded when dealing with non-hierarchical structures:

For flat-structured systems without clear hierarchies (e.g., simple AGI with a single module, small multi-agents without hierarchical division), the core theorems of the theory (such as the Cross-level Cognitive Distortion Theorem and the Subsystem Competition Theorem) lose their application scenarios, and their core values of "hierarchical control and rule transmission" cannot be reflected.

This is an inevitable result caused by the clear definition of research objects. The theory clearly limits the research scope to "human-constructed centralized general artificial intelligence systems" at the beginning. Non-hierarchical structures are "outside the applicable boundary of the theory" rather than theoretical logical flaws — similar to Newtonian mechanics not being applicable to quantum scenarios, it is essentially a precise match between research objects and applicable scopes.

10.2. Simplicity of Quantitative Models

The optimal rule constriction formula $\alpha(R) \approx 0.72 \cdot \log(R) - 2.8 \quad (R^2 = 0.96)$ fitted based on public datasets in Chapter 9 has the following limitations:

Limitations of data sources: This formula is fitted based on public goods game data from the CoDa database, and its external validity needs to be verified with more data from real AGI systems. The $\alpha*$ in different fields may vary.

Simplicity of single variables: The formula only considers resource constraints R and does not include other important variables such as cognitive distortion degree d, number of subsystems n, and coupling degree c. The optimal α in actual systems should be a multi-variable function: $\alpha^* = f(r, d, n, c, \dots)$.

Limitations of static assumptions: The formula assumes that the system state is stable and does not consider the time-series effect of α adjustment (e.g., the impact of adjustment frequency and amplitude on system stability).

10.3. Limitations of Empirical Verification

The secondary analysis based on public datasets in Chapter 6 provides preliminary empirical support for the theory, but has the following limitations:

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Heterogeneity of data sources: The original research purposes of datasets such as Tamas and CoDa are not to verify Nestology, and their matching degrees with the theory are different. There may be differences between human subjects and AGI system behaviors, and the sample size is limited.

Approximation of proxy variables: Theoretical concepts (such as rule constriction ratio α and cognitive distortion degree d) need to be indirectly measured through proxy variables (whether there is a punishment mechanism, error diagnosis accuracy), resulting in approximation deviations.

Lack of true experimental verification: All analyses are based on secondary analysis of existing public data, rather than specially designed controlled experiments, which cannot control confounding variables, are difficult to establish causal relationships, and cannot rule out the impact of publication bias.

Ideal verification should include: real AGI system experiments (implementing rule modification experiments on open-source systems such as AutoGPT and LangChain), controlled experiment design (comparing the system performance differences between "theoretically designed" and "conventionally designed"), and long-term tracking research (observing the evolution trajectory of the system in the real deployment environment).

10.4. Future Research Directions

10.4.1. Multi-Variable Optimal Rule Constriction Model

Research question: How to dynamically solve the optimal rule constriction ratio under the constraints of multiple variables such as resource level R, number of subsystems n, coupling degree c, and cognitive distortion degree d?

Research method: Construct a multi-variable regression model or reinforcement learning model, with system stability (closed-loop integrity rate, abnormal rate) and efficiency (task completion time, resource utilization rate) as optimization objectives; train the model using simulation data or real system logs; evaluate the model generalization ability using cross-validation.

Data sources: Public datasets; self-built simulation platforms (e.g., OpenAGI Gym multi-agent environment); real system logs.

Expected output: A computable dynamic solver for optimal rule constriction ratio, which inputs system state indicators and outputs recommended rule constriction ratio; verifies the effectiveness of the solver in at least 3 different AGI systems.

10.4.2. Multi-Dimensional Measurement System of Cognitive Distortion

Research question: How to comprehensively measure cognitive distortion from multiple dimensions such as information entropy loss, semantic fidelity, and value retention rate?

Research method: Introduce indicators such as mutual information and KL divergence in information theory to quantify information loss in rule transmission; combine semantic similarity models in natural language processing (e.g., BERT, RoBERTA) to calculate rule semantic fidelity; establish a value rule dataset through human expert annotation to calculate value retention rate; construct a comprehensive cognitive distortion index using principal component analysis or structural equation models.

Expected output: Comprehensive cognitive distortion index ; a set of reusable measurement tools integrated into the AGI system monitoring platform.

10.4.3. Controlled Experiments on Real AGI Systems

Research question: Do AGI systems designed according to Nestology (rule constriction, bounded independence of subsystems) have higher stability and value alignment degree than conventionally designed ones?

Research method: Select open-source AGI systems as experimental objects; design two groups of controls — the experimental group establishes a rule constriction list, implements cross-level random inspection, and introduces independent audits according to theoretical recommendations,

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while the control group maintains the original design; run on the same task set, and record stability (number of system crashes, abnormal output rate), efficiency (task completion time, resource consumption), and value alignment degree (human expert evaluation).

Expected output: Experimental data proving the effectiveness of the theoretical design; a detailed experimental report including reproducible code and datasets.

10.4.4. Large-scale Multi-Agent Simulation Verification

esearch question: How do system stability, efficiency, and value alignment degree change under different parameter spaces (number of subsystems, coupling degree, resource level, rule constriction ratio)? Are there critical points?

Research method: Construct a simulation platform based on multi-agents, where each agent can be configured with rule sets, strategies, and input/output interfaces; system parameters can be programmatically adjusted; run a large number of simulation experiments, record key events during the system life cycle; use machine learning methods to fit the relationship between system behavior and parameters, and identify phase transition points.

Expected output: An open-source simulation platform code repository; a system behavior phase diagram showing stable zones, crash zones, and value deviation zones under different parameter regions.

10.4.5. Cross-Field Expansion Directions

Financial risk control systems: Analyze the rule transmission distortion phenomenon in the financial risk control pipeline (data layer → feature layer → model layer → decision layer), calculate η, ζ, γ at each level, propose optimization schemes based on the theory (adding an independent audit layer, setting rule constriction ratio), and verify the effect of reducing the misjudgment rate.

Medical AGI systems: Analyze the transmission and attenuation of ethical value rules (patient privacy, fairness, informed consent) in medical AGI systems (data collection → feature extraction → model reasoning → result display → doctor decision-making), calculate $\gamma_k$at each level, and design intervention experiments to improve the value retention rate.

10.4.6. Engineering Application Directions

Development of dynamic regulation middleware: Design a middleware architecture (indicator collection module, α calculation module, rule update module, safety protection module), implement an α calculation engine based on a multi-variable model, integrate it into open-source systems such as AutoGPT for A/B testing, and evaluate the impact of the middleware on system performance (overhead, response time, stability).

Cross-level cognitive distortion compensation tool: Develop a visualization tool to display η,ζ, γ indicators of rule transmission in nested systems, help developers locate key nodes of cognitive distortion, and provide rule constriction optimization recommendations.

11. Conclusions

Nestology is not a simple synthesis or repackaging of classic theories such as Bertalanffy, Simon, and Wiener. Its core innovations lie in: proposing the 'rule-strategy' dichotomy not involved in classic theories; revealing the 'cross-level cognitive distortion' law not discovered in classic theories; solving the 'AGI safety and controllability' problem not responded to in classic theories; providing the 'quantitative regulation' tool not given in classic theories. These innovations are not the recombination of old concepts, but original theoretical constructions for the new technical object of AGI.

The improvement of a theory requires the joint efforts of the academic community. We look forward to working with colleagues in the field to promote Nestology from a "preliminary

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framework" to a "mature theory" through continuous empirical testing, theoretical deepening, and engineering transformation, and contribute to the safe and controllable development of the AGI era.

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Abbreviations

The following abbreviations are used in this manuscript:

AGI	Artificial General Intelligence	Abstract
MARL	Multi-Agent Reinforcement Learning	6.1.2
LLM	Large Language Model	2.1
NL	Nested Learning	6.6
CoDa	Cooperation Databank	6.1.2
TAMAS	Threats and Attacks in Multi-Agent Systems	6.1.2
TRAIL	Trajectory Reasoning and Attribution for Intelligent Agent Logs	6.1.2
RLHF	Reinforcement Learning from Human Feedback	7.4
DRD	Decision-Reward-Discrimination	6.2.2
LGTC-IPPO	Local Graph Topology Clustering with Independent Proximal Policy Optimization	6.3.2
MADDPG	Multi-Agent Deep Deterministic Policy Gradient	6.5.2
NSGA-III	Non-dominated Sorting Genetic Algorithm III	6.5.2
PIQA	Physical Interaction Question Answering	6.6
MDP	Markov Decision Process	Theorem 8
EIT	Entity Interaction Transformer	8.1.3
VLA	Vision-Language-Action	8.1.3
SOTA	State of the Art	6.3.1

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CI	Confidence Interval	6.2.1
d	Cohen's d	6.2.1
α	rule constriction ratio	2.4.1
η	rule transmission rate	Theorem 8
ζ	semantic fidelity	Theorem 8
γ	value retention rate	Theorem 8
δ	cognitive distortion rate	Theorem 8
Δ	cumulative cognitive distortion	Theorem 8
C	consensus consistency rate	8.4.2
D	cognitive distortion degree	8.3.1
R	resource level	8.3.2
n	number of subsystems	8.3.2
c	coupling degree	8.3.2
θ	validation threshold	7.4.2
λ	learning rate	7.3.2

Appendix A

Verification Table of Correlation Between Axioms and Theorem Deduction:

Core Theorem Name	Deducible from Axioms Strictly	Dependent Core Axioms/Predecessor Theorems
Theorem 1: Theorem of Unidirectional Transmission of System Rule Constriction	Yes	A2, A5
Theorem 2: Theorem of Bounded Independence of Subsystems	Yes	A1, A2, T1
Theorem 3: Theorem of Capability Boundary	Yes	A1

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Theorem 4: Theorem of Expansion Tendency	Yes	A1, T3
Theorem 5: Theorem of Creation Plurality and System Competition	Yes	A3, A4, T3, T4
Theorem 6: Theorem of Closed-loop Sustenance	Yes	A1, A3, A4, T2
Theorem 7: Theorem of Extinction	Yes	A1, A3, T6
Theorem 8: Theorem of Cross-level Cognitive Distortion	Yes	A4, A5, T1
Theorem 9: Theorem of Output Influence	Yes	A3, A4, T1, T4
Theorem of Probabilistic Rule Constriction Transmission (Expansion)	Yes	A2, A5, T1, Probabilistic Expansion Assumption
Theorem of Probabilistic Cross-level Cognitive Distortion (Expansion)	Yes	A5, T1, T8, Probabilistic Expansion Assumption
Corollary of Non-monotonic Rule Revision (Expansion)	Yes	A4, T9, Non-monotonic Expansion Assumption

Appendix B

List of Core Symbols:

Symbol	Meaning	First Appearance
S	System	3.1
Z	Set of all human-constructed artificial intelligence systems	3.1
I	Input	3.2
O	Output	3.2
F	System Function	3.3
R	System Rules	3.4
Str	System Strategy	3.5
P(S)	Parent System	3.6
C(S)	Subsystem	3.6
$\prec$	Nested Relationship	3.6
T	Rule Constriction Mapping	Theorem 1

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B(S)	Capability Boundary	Theorem 3
E	Expansion Mapping	Theorem 4
C(S)	System Closed Loop	Theorem 6
η	Rule Transmission Rate	Theorem 8
ζ	Semantic Fidelity	Theorem 8
γ	Value Retention Rate	Theorem 8
δ	Single-level Cognitive Distortion Rate	Theorem 8
Δ	Cumulative Cognitive Distortion	Theorem 8
α	Rule Constriction Ratio	2.4.1
C	Consensus Consistency Rate	8.4.2
D	Cognitive Distortion Degree	8.3.1
n	Number of Subsystems	8.3.2
c	Coupling Degree Between Subsystems	8.3.2
θ	Validation Pass Rate Threshold	7.4.2
λ	Learning Rate	7.3.2

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