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Inertia and Covariant Vacuum

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Abstract: In this paper, we investigate the inertia of elementary particles and macroscopic objects. We define inertia as the totality of local interactions. It turns out that inertia depends on the acceleration, which is exactly what MOND theory requires. For macroscopic objects with a huge number degrees of freedom, we define a mean field that represents the average of all local interactions. Hence, this approach naturally leads to inertia being described within the framework of thermodynamics. The dynamics expressed in terms of acceleration are given by the equation of the Kuramoto-like form. The equilibrium solution of the equation is consistent with MOND theory. The vacuum energy density for an accelerating observer is estimated. We show that the vacuum energy density arising from zero-point fluctuations and symmetry breakings is also acceleration-dependent. By requiring that the vacuum state is covariant, all acceleration-dependent energy should be regarded as non-physical. The covariant vacuum state is a radiation-filled vacuum with Gibbons–Hawking temperature.

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1. Introduction

The universe in its vast complexity harbors profound mysteries that challenge our understanding of fundamental physics. Observations of the flat rotation curves of galaxies reveal a striking discrepancy that galaxies rotate faster than can be accounted for by the visible matter alone, suggesting the presence of an invisible mass, known as dark matter. It is also hypothesized to explain a range of gravitational anomalies observed in gravitational lensing and the cosmic microwave background (CMB) [1]. Despite its success in accounting for these phenomena, the dark matter paradigm faces significant challenges. The lack of direct detection raises questions about its fundamental validity. The most widely accepted Cold Dark Matter (CDM) model struggles to account for observational discrepancies at smaller scales, such as the "missing satellites problem," where the predicted number of dwarf galaxies around massive galaxies exceeds observations [2–4], and the "cusp-core problem," where simulated dark matter halos exhibit central density cusps inconsistent with the flatter density profiles observed in some galaxies [5]. In addition, some observational data including rapid galaxy growth [6]and flat velocity curves extending beyond the expected virial radii of dark matter halos [7] also suggest inconsistencies with the CDM model. These issues imply that the CDM framework may need refinement or that our understanding of gravitational dynamics requires revision.

One such alternative theory is the Modified Newtonian Dynamics (MOND) proposed by Milgrom as a modification to Newtonian gravity at extremely low accelerations [8–10]. Although MOND struggles to account for phenomena at cosmological scales, such as the cosmic microwave background and large-scale structure formation, the simplicity and predictive power of MOND make it a valuable theoretical tool, prompting ongoing research into its foundations and potential extensions. Unlike the dark matter hypothesis, the MOND paradigm stipulates that the observed gravitational effects arise not from unseen mass but from a deviation in the law of gravity when accelerations fall below a critical threshold $a_0 \approx 1.2 \times 10^{-10}\text{m/s}^2$. Milgrom's law can be written as $\mu(a/a_0)a = a_N$, where a_N is the Newtonian acceleration produced by the visible matter, a is the true gravitational acceleration and the interpolating function $\mu(x)$ satisfies $\mu(x) \approx 1$ when $x \gg 1$, and $\mu(x) \approx x$ when $x \ll 1$. In the deep-MOND regime $(a \ll a_0)$, MOND modifies the gravitational force law to scale inversely with distance rather than the square of the distance, effectively reproducing the flat rotation curves

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of galaxies and Tully-Fisher relation without invoking dark matter. MOND can be interpreted as a modification of gravity or inertia. In the modified gravity interpretation, Newton's law of gravity should be modified at low accelerations, leading to a stronger gravitational effect than predicted by the usual inverse-square law. This approach has been formalized in theories such as the Tensor-Vector-Scalar (TeVeS) gravity [11] and bimetric theories [12], which attempts to provide a relativistic extension of MOND [10]. Alternatively, the modified inertia interpretation suggests that the response of a body to a given force depends on the acceleration regime.

Given the empirical validations and elegant mathematical structure of general relativity, there is more potential in modified inertia as the basis for MOND because it seems to be less drastic. Inertia governs how objects respond to applied forces. However, the origin of inertia remains an open question in modern physics. Understanding the origin of inertia could provide critical insights into the foundations of the MOND theory. This paper explores these concepts, aiming to elucidate the fundamental nature of inertia in shaping our understanding of the universe. In classical mechanics, inertia is an intrinsic property of mass, yet its microscopic basis is not well understood. Some theories propose that inertia arises from interactions with the vacuum [13]. Einstein's theory of general relativity offers a partial explanation, suggesting that mass influences and is influenced by the geometry of spacetime. Alternative perspectives, like Mach's principle, suggest that inertia is a relational property, emerging from an object's interaction with the global distribution of matter in the universe. Instead of introducing some vague concept of the unknown vacuum degrees of freedom, we fully utilize Mach's principle and expand the perspective of general relativity, and argue that inertia arises from the totality of local interactions. This assumption is reasonable because any interaction that causes an object to resist changes in its state of motion contributes to inertia and hence we can define an effective mass. In physics, there are many instances defining effective inertia mass. In special relativity, we can redefine the effective mass of a moving body as $m_{\text{eff}} = m\gamma(v)$ with $\gamma(v)$ being the Lorentz factor. Similarly, in condensed matter physics, effective mass plays a pivotal role in describing the behavior of charge carriers, such as electrons and holes, within crystalline solids. For a classical object moving through a certain medium and experiencing resistance, we can also define an inertial mass.

In this paper, we will investigate the inertia of elementary particles and macroscopic objects. Clearly, the inertial mass of elementary particles primarily arises from the interaction with the Higgs field. We will either neglect the inertial mass produced by the interaction of charged particles with the electromagnetic field (known as electromagnetic mass) or attribute the mass to the interaction between particles and a composite mean field of the electromagnetic and Higgs fields. The influence of gravity becomes vanishingly small at microscopic scales and can be neglected. An interesting thing happens when we consider the Unruh effect which predicts that an accelerating observer in a vacuum will detect a thermal bath at a temperature proportional to its proper acceleration [14]. In Ref. [15], it has been shown that the electroweak phase transition (EWPT) occurs and the electroweak gauge symmetry can be restored when the acceleration exceeds the critical value as seen from the point of view of an accelerating observer. Therefore the inertial mass is acceleration-dependent as required by the MOND theory. This means that the Unruh temperature is not just a formal artifact but a real temperature which can give rise to non-trivial thermal effects. Although an inertial observer perceives the vacuum as empty, they will agree on the result that a ground-state Unruh-DeWitt detector carried by the comoving observer undergoes spontaneous excitation. However, comoving observers believes that the thermal radiation arises from the background geometry endowed with an event horizon, while inertial observers would attribute the radiation to a "friction" produced by the interaction between particles and fields. The key theoretical insight is that the very existence of "vacuum" and "particles" are observer-dependent rather than fundamental. On the other hand, for macroscopic objects composed of numerous particles, various intricate interactions may contribute to the emergence of the inertial mass. But using a mean field theory approach, it is possible to study a system with a large or infinite number of degrees of freedom. Thus we define a mean field that represents the average of all local interactions.

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Another concern of this study is the cosmological constant problem, which remain a theoretical puzzle that bridges cosmology and quantum field theory [16]. The problem originates from the interpretation of the cosmological constant Λ as the vacuum energy density. There are at least two sources for the vacuum energy. In the cosmological context, spontaneous symmetry breakings in the early universe may have induced phase transitions, potentially contributing to the vacuum energy density associated with the cosmological constant. Although one can always adjust the vacuum energy today to zero by tuning the parameter of the potential, it is not a very satisfactory method because the vacuum energy cannot be zero before and after the phase transition. In addition, in quantum field theory the vacuum is filled with quantum fluctuations contributing to a zero-point energy (ZPE). Since all energy gravitates, it is expected that the ZPE contribute to the cosmological constant. However, the theoretical prediction of the cosmological constant from quantum field theory contrasts with its observed value, giving rise to a discrepancy that spans over 120 orders of magnitude. One promising avenue for addressing the cosmological constant problem is supersymmetry (SUSY), a theoretical framework that posits a symmetry between fermions and bosons [17]. However, experimental searches at the Large Hadron Collider have yet to detect supersymmetric particles. Additionally, the precise mechanism by which SUSY could resolve the cosmological constant problem remains elusive, as the required cancellations demand an extraordinary degree of fine-tuning in the SUSY-breaking sector. In this paper, we will calculate the vacuum energy for an accelerating observer. We note that requiring that the vacuum state is covariant drives the theoretical value of the vacuum energy density to zero. The covariant vacuum is the de Sitter spacetime with a Gibbons–Hawking radiation. Our analysis is based on the conservative assumption of maximal validity of quantum field theory and general relativity.

This paper is organized as follows. Sec.2 is dedicated to the inertial mass of a single particle. In Sec.3, we investigate the inertia of macroscopic objects. In Sec.4, we calculate the vacuum energy density by requiring that the vacuum state is covariant. Finally, in Sec.5 we summarize the main results obtained. For convenience, we use natural units with $c = \hbar = k = 1$.

2. Acceleration-Dependent Inertia

For an accelerating observer the electroweak $SU(2) \times U(1)$ gauge symmetry in the Standard Model is restored for acceleration larger than a critical value. The vacuum expectation value (VEV) v is given by [15]

$$v(a) = v_0 \sqrt{1 - \frac{a^2}{a_{\mathrm{EW}}^2}}. \tag{1}$$

where v_0 is the VEV for the inertial observer, a is the proper acceleration and a_{EW} is the critical proper acceleration of the EWPT. The second-order phase transition of the restoration of electroweak symmetry occurs at a_{EW} and for $a > a_{\mathrm{EW}}$, we have $v(a_{\mathrm{EW}}) = 0$. The elementary particles therefore acquire a acceleration-dependent mass which is

$$m(a) = m_0 \sqrt{1 - \frac{a^2}{a_{\mathrm{EW}}^2}}, \tag{2}$$

where m_0 is the mass of the elementary particle for the inertial observer. The effective mass of the particle might be regarded as the order parameter which measures the resistance to acceleration (also known as inertial mass). By introducing the Unruh-like temperature:

$$T_{\mathrm{EW}} = \frac{a_{\mathrm{EW}}}{2\pi} \tag{3}$$

and

$$T(a) = \frac{a}{2\pi}, \tag{4}$$

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Eq. (2) can be also written as

$$m(T) = m_0\sqrt{1 - \frac{T^2}{T_{\rm EW}^2}}, \tag{5}$$

where $T_{\rm EW} \sim 10^2$ Gev is the critical temperature of the EWPT. It turns out that all massive particles of Standard Model become massless for the local accelerating observer when the acceleration, or equivalently the temperature, exceeds the critical value.

3. Acceleration Dynamics and MOND

We now turn our attention to the inertia of macroscopic objects. Typically, the magnitude of acceleration is significantly smaller than the EWPT energy scale. We will only consider cases where the acceleration a is close to the Gibbons–Hawking temperature $T_{\rm GH} = \frac{1}{2\pi}(\Lambda/3)^{1/2}$ [18]. The dynamics of macroscopic objects with a huge number degrees of freedom can become remarkably complex as various intricate interactions may contribute to the emergence of inertia. Let us define a mean field that represents the average of all local interactions. As a result, this approach naturally leads to inertia being described within the framework of thermodynamics. We regard the vacuum with the Gibbons–Hawking temperature as an equilibrium state. Since the system can be characterized by the temperature or acceleration, then the dynamics expressed in terms of the temperature are given by the equation of the following Kuramoto-like form:

$$\dot{T}_U = \dot{T}_{\rm GH} + \varepsilon m_{\rm eff}\Gamma(T_{\rm GH} - T_U) + \frac{1}{2\pi}F_{\rm ext}, \tag{6}$$

where the dot denotes the derivative with respect to time t, $T_U = \frac{1}{2\pi}\left(a^2 + \Lambda/3\right)^{1/2}$ is the Unruh temperature of the radiation seen by a local comoving accelerating observer in de Sitter spacetime, ε is the positive coupling strength, $m_{\rm eff}$ as the order parameter is the effective inertial mass, $F_{\rm ext}$ represents the external force acting locally on the object and $\Gamma(T_{\rm GH} - T_U)$ is a general function for the interaction between the accelerating object of interest and the mean field. In general, the function Γ depends on the trajectory of motion, and therefore Eq. (6) is a nonlocal integro-differential equation. Physically, this may be interpreted as due to the fact that inertia produced by the Unruh radiation does not respond instantaneously to the acceleration value a. The Unruh radiation emitted from the Rindler horizon reaches an accelerated observer in a time inversely proportional to a, and thus the establishment of thermodynamic equilibrium is not instantaneous. The state of the system at time t is determined by the state at time $(t - 1/a)$, or equivalently by a retarded form $\Gamma(t - 1/a)$. Thus inertia is nonlocal in the low acceleration limit. Here, for simplicity we consider a rectilinear motion with constant acceleration. When T_U is very close to the Gibbons–Hawking temperature $T_{\rm GH}$, we can expand Eq. (6) around $T_{\rm GH}$ and retain only the linear terms by using the relation $\Gamma(0) = 0$. The coefficient involving ε and the first derivative of Γ with respect to T_U at $T_U = T_{\rm GH}$ can be absorbed into the definition of $m_{\rm eff}$. Since we have defined the effective inertial mass, external forces can be expressed in the form of Newton's second law, namely $F_{\rm ext} = m_{\rm eff}a_N$ with a_N being the Newtonian expression for the acceleration. We also assume that the current cosmological constant does not vary with time. Thus Eq. (6) can be written in terms of the acceleration as

$$\frac{a\dot{a}}{\sqrt{a^2 + \Lambda/3}} = m_{\rm eff}\left[\left(\frac{\Lambda}{3}\right)^{1/2} - \left(a^2 + \frac{\Lambda}{3}\right)^{1/2}\right] + m_{\rm eff}a_N. \tag{7}$$

The stable fixed point given by $\dot{a} = 0$ represents equilibrium solutions of Eq. (7) and one arrives at

$$a_N = \left(a^2 + \frac{\Lambda}{3}\right)^{1/2} - \left(\frac{\Lambda}{3}\right)^{1/2}, \tag{8}$$

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which leads to $\mu(a/a_0)a = a_N$ with $a_0 = 2(\Lambda/3)^{1/2}$. The result is consistent with Milgrom's hypothesis [13]. If we require that Eq. (6) holds in the deep-MOND regime ($a \ll a_0$), the Γ function should take the form $\Gamma(T_{\rm GH} - T_U) = T_{\rm GH} - T_U$.

4. Acceleration-Dependent Vacuum Energy

The vacuum energy receives contributions from both zero-point fluctuations and symmetry breakings. We first calculate the ZPE. The ZPE density of a real free scalar field is given by

$$\rho_Z = \frac{1}{(2\pi)^3}\frac{1}{2}\int d^3\boldsymbol{k}\,\omega(k) \tag{9}$$

with

$$\omega(\boldsymbol{k}) = \sqrt{|\boldsymbol{k}|^2 + m_0^2}, \tag{10}$$

where $(\omega, \boldsymbol{k})$ is the four-dimensional momentum and m_0 is the mass of the scalar field. Obviously, the integral is divergent in the ultraviolet region. The common method is to introduce an ultraviolet cut-off $\Lambda_{\rm UV}$ at the Planck scale, then one obtains $\rho_Z \sim 10^{76}$ GeV^4, which is larger than the observed value of vacuum energy density by a factor of 10^{123}. But careful calculations yield

$$\rho_Z = \frac{\Lambda_{\rm UV}^4}{16\pi^2} + \frac{m_0^2\Lambda_{\rm UV}^2}{16\pi^2} + \frac{m_0^4}{64\pi^2}\ln\left(\frac{m_0^2 e^{\frac{1}{2}}}{4\Lambda_{\rm UV}^2}\right) + \cdots, \tag{11}$$

$$p = \frac{\Lambda_{\rm UV}^4}{48\pi^2} - \frac{m_0^2\Lambda_{\rm UV}^2}{48\pi^2} - \frac{m_0^4}{64\pi^2}\ln\left(\frac{m_0^2 e^{\frac{7}{6}}}{4\Lambda_{\rm UV}^2}\right) + \cdots, \tag{12}$$

where p is the pressure. The Lorentz symmetry of the vacuum requires that the energy density and pressure satisfy the equation of state $p = -\rho_Z$. Notice that the first two terms of Eqs. (11) and (12) break Lorentz invariance and can be removed by local counterterms. Therefore, upon using a regularization scheme that preserves Lorentz symmetry of the vacuum, for any quantum field one arrives at the following expression for the ZPE density [19]

$$\rho_Z = \pm\frac{s m_0^4}{64\pi^2}\ln\left(\frac{m_0^2}{\mu^2}\right), \tag{13}$$

where μ is the renormalization scale, s represents the number of polarization states and the signs $\pm$ are associated with bosons and fermions respectively. The result can be generalized to any other interacting fields by simply replacing m_0 with the renormalized mass m_R. We see that the expression is proportional to the mass of the particle to the power four and the massless particles do not contribute to the ZPE. This result is very different from the result obtained by imposing a Planck cut-off.

Another contribution to the cosmological constant comes from the symmetry breakings. We now calculate the vacuum energy produced by the EWPT at the classical level. We should also consider the QCD symmetry breaking ($\sim 10^{-1}$ Gev) and other symmetry breakings at higher energy scales (e.g., the grand unification scale at 10^{14} Gev and the Planck scale at 10^{19} Gev). However, all these expressions take a similar form and the analysis parallels the electroweak case. The Higgs field consists of two complex scalar fields arranged into a doublet. After the EWPT, the field acquires a VEV and the corresponding vacuum energy density is $\rho_{\rm EW} = \lambda v^4 \sim 10^8$ Gev^4 with λ being a coupling constant describing the self-interaction of Higgs fields. In addition to being inconsistent with observational data, such a large vacuum energy density corresponding to a large cosmological constant would also produce a high Gibbons–Hawking temperature, thereby triggering a phase transition. From Eq. (1), we see that the vacuum energy density of the EWPT must satisfy the equation:

$$\rho_{\rm EW} = \rho_0\left(1 - \frac{T_h^2}{T_{\rm EW}^2}\right)^2\theta(T_{\rm EW} - T_h), \tag{14}$$

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where ρ_0 is the vacuum energy density in the absence of Gibbons–Hawking radiation, $\theta(x)$ is the Heaviside step function, $T_{\rm EW} \sim 10^2$ GeV is the EWPT temperature and $T_h = \frac{1}{2\pi}(8\pi G\rho_s/3)^{1/2}$ is the Gibbons–Hawking temperature produced by the huge vacuum energy density ρ_s of symmetry breakings. When the vacuum energy density exceeds 10^{10} GeV^4, the broken electroweak symmetry is restored and $\rho_{\rm EW}$ vanishes. The backreaction can drive Λ back to zero, even when the vacuum energy density experiences large disturbances extending to the Planck scale because large disturbances will lead to a phase transition and restoration of the symmetry.

It is worth noticing that all the vacuum energy is acceleration-dependent and hence observers with different accelerations will measure different vacuum energy. The key point is that all the vacuum energy derived from zero-point fluctuations and symmetry breakings is non-physical if we require the underlying theory describing the vacuum energy to be covariant. Therefore, for a potential covariant theory, we should define a vacuum that is invariant for all observers and arbitrary coordinate transformations. This approach is analogous to general relativity, where gravity is non-physical and can be eliminated by a coordinate transformation, such as an observer in free fall feeling no gravity. Einstein thus used the curvature tensor to describe gravity. Since inertial observers in de Sitter spacetime with a positive cosmological constant detect a Gibbons–Hawking radiation with temperature $T_{\rm GH} = \frac{1}{2\pi}(\Lambda/3)^{1/2}$, we might regard the Gibbons–Hawking vacuum as the covariant vacuum state. Although the inertial mass acquired through the Higgs mechanism depends on the acceleration and vanishes for an observer exceeding the acceleration threshold, the covariant part of the mass arising from other interactions remains invariant for any observer. If we interpret the current cosmological constant Λ as the vacuum energy density $\rho_{vac} \sim 10^{-47}$ Gev^4 and assume that the current cosmological constant arises from the covariant part of the electromagnetic mass, using Eq. (13), the electromagnetic mass of an charged particle such as an electron for an inertial observer is approximately 10^{-3} ev.

5. Discussion

In this paper, we investigate the inertial mass of elementary particles and macroscopic objects. We define inertia as the totality of local interactions. It is well known that the Higgs field endows particles with mass. Therefore, the inertia of a particle depends on its interaction with the Higgs field. It turns out that inertia depends on the acceleration regime, which is exactly what MOND theory requires. When acceleration exceeds the critical value, all particles become massless and inertia disappears as seen by the accelerating observer. Typically, the magnitude of acceleration is significantly smaller than the EWPT energy scale, hence the modification of inertia can be neglected. On the other hand, for macroscopic objects with a huge number degrees of freedom, we define a mean field that represents the average of all local interactions. The system is drastically simpler by using a mean field theory approach. Meanwhile, this approach naturally leads to inertia being described within the framework of thermodynamics. At the macroscopic scale, the interaction of objects is primarily governed by gravity. The weak equivalence principle, which states that inertial mass equals gravitational mass, may suggest that gravity can also be described within the framework of thermodynamics. In this sense, gravity may be an emergent phenomenon arising from microscopic interactions. Some studies proposed that gravity can be regarded as an entropic force [20]. In my view, gravity at macroscopic scales can be effectively described within a thermodynamic framework, whereas at microscopic scales gravity may still be quantum.

We then calculate the vacuum energy density based on quantum field theory. It turns out that the vacuum energy is also acceleration-dependent. Here, we calculate the vacuum energy density arising from symmetry breaking at the tree level. If quantum corrections are considered, one only needs to replace it with the Coleman-Weinberg effective potential [21]. However, the result is the same because quantum corrections also depend on acceleration. Since we are considering gravitational effects, by requiring a covariant vacuum as demanded by general relativity, all acceleration-dependent energy should be regarded as non-physical. The only covariant vacuum state is a radiation-filled vacuum

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with Gibbons–Hawking temperature. Notably, we are unsure if Λ can be considered a constant that does not vary over time. An intriguing conjecture is that the current cosmological constant may arise from the covariant electromagnetic mass and hence the electromagnetic mass of an charged particle such as an electron is approximately 10^{-3} ev. However, this guess needs a further exploration.

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