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A Quantum Leap in Asset Pricing: Explaining Anomalous Returns

James W. Kolari *, Jianhua Z. Huang †, Wei Liu ‡ and Huiling Liao §

[1] The corresponding author is JP Morgan Chase Professor of Finance, Texas A&M University, Adam C. Sinn '00 Department of Finance, Mays Business School, College Station, TX 77843-4218. Office phone: 979-845-4803. Email address: j-kolari@tamu.edu
[2] Presidential Chair Professor, Director, Technology and Innovation Center for Digital Economy (TIDE), School of Data Science, The Chinese University of Hong Kong, Shenzhen, China, 51872. Office phone: (86) 0755-2351700. Email address: jhuang@cuhk.edu.cn
[3] Clinical Associate Professor of Finance, Texas A&M University, Adam C. Sinn '00 Department of Finance, Mays Business School, College Station, TX 77843-4218. Office phone: 979-845-1224. Email address: wliu@mays.tamu.edu
[4] Assistant Professor of Applied Mathematics, Applied Mathematics, Illinois Institute of Technology, Chicago, IL 60616 Office phone: 312-567-8990. Email address: hliao13@iit.edu
* Correspondence: Office phone: 979-845-4803. Email address: j-kolari@tamu.edu
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Abstract: The paper investigates the ability of asset pricing models to explain the cross section of average stock returns of anomaly portfolios. A large sample of 286 anomaly portfolios are employed. We perform out-of-sample cross-sectional regression tests of both prominent asset pricing models and a relatively new model dubbed the ZCAPM. Empirical tests strongly support the lesser known ZCAPM but not other multifactor models. Further analyses of out-of-sample mispricing errors of the models reveal that the ZCAPM provides much more accurate pricing of anomaly portfolios than other models. We conclude that anomalies are anomalous to popular multifactor models but not the ZCAPM. By implication, the efficient market hypothesis is supported.

Keywords: anomalies; asset pricing models; mispricing error; ZCAPM

JEL Classification: G12; C20

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1. Introduction

In his Presidential Address to the American Finance Association, John Cochrane (2011) observed that a "factor zoo" exists in the field of asset pricing due to the proliferation of stock market anomalies. In a series of papers, Fama and French (1992, 1993, 1996a, 1996b) showed that the now famous capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), Mossin (1966), and Treynor (1961, 1962) failed to explain stock returns in general and size and value anomalies in particular. High (low) beta stocks did not have higher (lower) returns than other stocks. In its place, they proposed a three-factor model that augmented the market factor with size and value factors. This innovative *multifactor model* did a good job of explaining the size and value anomalies.

Subsequently, consistent with Cochrane, researchers discovered hundreds of anomalies, which has evolved into two branches of literature. One branch documents the anomalies and tests for their persistence over time. Work by McLean and Pontiff (2016) investigated 97 anomaly portfolios and found that anomalies tended to diminish or disappear over time after their publication in academic journals. This diminution of anomalies has been confirmed by other researchers.[1] By contrast, Jacobs and Müller (2020) examined 241 anomalies in 39 stock markets that (with the exception of the United States) did not tend to diminish in post-publication years. Another study by Chen and Zimmerman (2020) found that prior publication only explains 12 percent of anomaly returns. Also, Jensen, Kelly, and Pedersen (2023) found that 153 long/short factors across 93 countries can be replicated over time, hold on an out-of-sample basis after their original publication, are supported by the large number of factors including global evidence, and can be combined into a smaller number of anomaly clusters. Unlike previous studies, the authors used alphas from the CAPM to measure anomaly returns. They found that risk-adjusted returns yield different results than raw returns. Another recent paper by Bowles, Reed, Ringgenberg, and Thornock (2023) explored the timing of anomalies and found that anomaly returns persist but decay rapidly after information release dates. Previous studies that rebalance anomaly portfolios on an annual basis (for example) incorporate stale information. In their study, using real-time data after information events, anomaly returns did not decrease over time. Hence, they concluded that event-based anomalies were alive and well in financial markets. Consistent with these studies, numerous authors ascribe to the notion that anomalies are real mispricing as opposed to data mining.[2]

A second branch of studies has sought to address the factor zoo problem by developing parsimonious asset pricing models that can explain many anomalies. For example, to explain 80 anomaly portfolios, Xue, and Zhang (2015) proposed a four-factor model with market, size, profit, and capital investment factors.[3] Also, using the same 80 anomaly portfolios, Stambaugh and Yuan (2017) specified a four-factor model with market, size, management, and performance factors that outperformed the Hou, Xue, and Zhang (2015) model in terms of explanatory power. Thus, these studies suggest that low dimension asset pricing models can be used to capture many anomalies.

We extend asset pricing studies by comparing the ability of multifactor models to explain large numbers of anomaly portfolio returns. Surprisingly, standard Fama and MacBeth (1973) cross-sectional regression tests show that a lesser known two-factor model dubbed the ZCAPM by Kolari, Liu, and Huang (2021) well outperforms prominent multifactor models in terms of explaining anomaly returns on an out-of-sample basis. In empirical tests, we utilize online databases of anomalies recently made available by researchers. Chen and Zimmerman (2022) have provided an open source database with 161 long/short anomalies in the U.S. stock market.[4] Also, Jensen, Kelly, and Pedersen (2023) have furnished an online database containing153 long/short anomalies in 93 countries including the U.S. Based on 133 anomalies in the former study and 153 anomalies in the latter study with return series available from July 3, 1972 to December 31, 2021, we investigate a combined dataset off 286 anomalies.

We find that, with the exception of the ZCAPM, prominent multifactor models do not explain anomaly portfolio returns. By contrast, the ZCAPM does a much better job of explaining them. In standard Fama and MacBeth (1973) cross-sectional regression tests, factor loadings for the ZCAPM are more significant than well-known multifactor models. Also, goodness-of-fit as estimated by R^2 values are much higher for the ZCAPM than other models. Further graphical tests compare the mispricing errors of different models with respect to anomaly portfolios. We find that the ZCAPM exhibits much lower mispricing errors than other models. We conclude that anomaly returns are anomalous for

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1 See, for example, Fama (1998), Green, Hand, and Zhang (2013, 2017), Chordia, Subrahmanyam, and Tong (2014), Novy-Marx and Velikov (2016), Linnainmaa and Roberts (2018), Jacobs and Müller (2020), Chen and Zimmerman (2022), and others.

2 See Bartram and Grinblatt (2018), Engelberg, McLean, and Pontiff (2018), Lu, Stambaugh, and Yuan (2018), Wahal (2019), Jacobs and Müller (2020), and others.

3 Fama and French (2015, 2018) included somewhat similar profit and investment factors to augment their three-factor model.

4 A total of 161 out of 319 anomalies were classified as clear predictors by the authors.

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the most part with respect to prominent multifactor models but not the ZCAPM. By implication, our evidence supports the efficient markets hypothesis of Fama (1970, 2013) rather than the behavioral hypothesis.[5] As such, stock returns are closely related to systematic market risks.

The next section reviews the methodology. Section 3 reports and discusses the empirical evidence. The last section concludes.

2. Literature Review

The asset pricing literature has evolved over time to produce a wide variety of models to help explain stock market anomalies. Fama and French (1992, 1993) proposed the now famous three-factor model that augmented the market factor with size and value factors. The latter factors were shown to better explain portfolios of stocks sorted on size and value firm characteristics than the CAPM market factor alone. Carhart (1997) added a momentum factor to explain the momentum anomaly. Extending these multifactor models, Fama and French (2015) added profitability and capital investment factors. They found that the value factor could be dropped with similar ability of the resultant four-factor model to explain various anomaly portfolio sorted on size, value, profitability, and capital investment firm characteristics.

Expanding the study of anomalies, Hou, Xue, and Zhang (2015) developed a four-factor model (with factors similar to those in the Fama and French four-factor model. The found that their model outperformed other models in terms of explaining the returns of 80 long/short anomaly portfolios. Like Fama and French, they utilized in-sample Gibbons, Ross, and Shanken (GRS) (1989) test for the joint equality of anomaly portfolios' alpha estimates. Unlike the present study, they did not report out-of-sample cross-sectional regression test results. Another related study by Stambaugh and Yuan (2017) proposed a four-factor model including market, size, management, and performance factors. Using the same 80 anomaly portfolio as Hou et al., their model outperformed other models in GRS tests of model alphas.

Departing from long/short factors constructed by researechers, Lettau and Pelger (2020) tested a five-factor model based on latent (hidden) factors identified by principal components analysis (PCA). GRS tests showed that their model had lower mispricing errors (as measured by alphas for 370 portfolios sorted on various firm characteristics) than the Fama and French three- and five-factor models. In efforts to improve their model, Fama and French (2018) incorporated a momentum factor to form a six-factor model. This model appeared to perform well in alpha tests relative to their prior multifactor models.

Lastly, Kolari, Liu, and Huang (2021) proposed in a book a new asset pricing model dubbed the ZCAPM derived mathematically as a special case of Black's (1972) zero-beta CAPM. As reviewed in the next section, the ZCAPM has two factors: (1) beta risk related to average market returns, and (2) zeta risk associated with cross-section market return dispersion. Empirical tests using size and value portfolios employed by Fama and French showed that the ZCAPM consistently outperformed their three-factor model in out-of-sample cross-sectional regression tests, at times by large margins. Additionally, lower average mispricing errors were documented for the ZCAPM than other multifactor models. Further tests of industry portfolio and individual stocks, as well as other anomaly portfolios sorted on firm characteristics, supported the ZCAPM, which outperformed other popular models.

Kolari, Huang, Liu, and Liao (2022) tested the ZCAPM using U.S. stocks for a long sample period from 1927 to 2020. The results confirmed the findings of Kolari et al. (2021). Again in out-of-sample cross-sectional regression tests, the ZCAPM well outperformed the CAPM, Fama and French three-factor model, and Carhart four-factor model. Subperiod results by splitting sample observations

[5] See Kahneman and Tversky (1979), Shiller (1981), DeBondt and Thaler (1985, 1995), Daniel, Hirshleifer, Subrahmanyam (1997), Barberis, Shleifer, and Vishny (1998), Thaler (1999) and others.

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continued to support the ZCAPM over the other models. As before, t-statistics corresponding to the market price of zeta risk related to market return dispersion loadings exceeded 3 that were higher than other factors loadings market prices of risk. The authors concluded that the ZCAPM offers a parsimonious empirical model with only two factors and theoretical foundations in the general equilibrium CAPM and zero-beta CAPM asset pricing models.

Kolari, Butt, Huang, Butt, and Liao (2022) conducted tests of the ZCAPM on anomaly portfolios in Canada, France, Germany, Japan, the United Kingdom, and the United States. They compared the ZCAPM to the Fama and French three-factor model and Carhart four-factor model. In all countries, the ZCAPM outperformed these models in out-of-sample cross-sectional regression tests with higher R^2 values and t-statistics for zeta risk loadings. Also, average mispricing errors were substantially lower for the ZCAPM relative to the other models. They concluded that the ZCAPM is not a false discovery, which renews interest in the CAPM as a viable approach to asset pricing.

Recently, another book by Kolari, Liu, and Pynnonen (2024) applied the ZCAPM to constructing high return stock portfolios. The CRSP index exhibited much lower Sharpe ratios than these portfolios. Based on their evidence for U.S. stocks, the authors showed that the mean-variance investment parabola of Markowitz (1959) has an architecture that can be mapped in terms of the beta risk and zeta risk of portfolios. The CRSP index and S&P 500 index lied in the middle of their empirical depiction of the mean-variance investment parabola. All portfolios' returns were computed out-of-sample in the month ahead of the estimation of risk parameters in the ZCAPM. Application of the ZCAPM to actively-managed equity mutual funds showed that the ZCAPM could be used to help guide mutual fund investments by pension funds and other investors. We interpret their finding to suggest that more efficient portfolios with higher Sharpe ratios can be constructed using the ZCAPM to estimate beta risk and zeta risk measures.

3. Methodology

We download daily anomaly portfolio returns from the internet websites of Chen and Zimmerman (2022) and Jensen, Kelly, and Pedersen (2023). Of 161 clear predictor portfolios in the former dataset, 133 anomaly portfolios are retained with available returns from July 3, 1972 to December 31, 2021 [6] The latter dataset includes 153 anomaly portfolios with available returns. Appendices A and B contain lists of anomalies in these two respective databases. Factors for the models under study (to be discussed shortly) are downloaded from Kenneth French's online database.[7]

3.1. Cross-Sectional Regression Tests

As already mentioned, we employ standard Fama and MacBeth (1973) tests. First, each model is estimated using a time-series regression. Second, risk loadings of factors in each model are used as independent variables in an out-of-sample (one-month-ahead) cross-sectional regression, which provides estimates of the market price of risk for loadings. More specifically, we begin by estimating the following time-series regression for the ith anomaly portfolio in the one-year estimation window from July 1972 to June 1973:

$$R_{it} - R_{ft} = \alpha_i + \sum_{k=1}^{K} b_{ik} F_{kt} + e_{it}, \; t = 1, \ldots, T, \tag{1}$$

where $R_{it} - R_{ft}$ is the realized excess return on the ith test asset for day t; R_{ft} is the riskless rate proxied by the Treasury bill rate; α_i is the intercept term or alpha; F_{kt} are asset pricing factors for $k = 1, \ldots, K$

6 This dataset was assembled from a number of previous anomaly studies. For further details, see Chen and Velikov (2021).

7 In unreported results, we tested the Hou, Xue, and Zhang as well as Stambaugh and Yuan four-factor models. Our results were unchanged for the most part with poor performance from these models (similar to those of the Carhart four-factor model) but strong explanatory power for the ZCAPM. Results are available from the authors upon request.

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factors; b_{ik} are corresponding K beta risk loadings for the ith portfolio; $i = 1, \ldots, N$ correspond to the number of portfolios; $t = 1, \ldots, T$ is the estimation period; and $e_{it} \sim$ iid $(0, \sigma_i^2)$.

Next, we estimate following out-of-sample cross-sectional regression in the next month of July 1973:

$$R_{iT+1} - R_{fT+1} = \hat{\alpha} + \lambda_1 \hat{b}_{i1} + \lambda_2 \hat{b}_{i2} + \ldots + \lambda_K \hat{b}_{iK} + u_{iT+1}, \; i = 1, \ldots, N, \quad (2)$$

where $R_{iT+1} - R_{fT+1}$ is the realized excess return on the ith portfolio in out-of-sample (one-month-ahead) month $T+1$; $\hat{\alpha}$ is the intercept; $\hat{b}_{ik}, k = 1, \ldots, K$ are K beta risk loadings estimated for N anomaly portfolios in the estimation period $t = 1, \ldots, T$; λ_k are corresponding estimated market prices of beta risk loadings[8]; and u_{iT+1} are zero mean and independent of the explanatory variables.

The above two-step process is rolled forward one month at a time and repeated to generate a monthly time-series of market prices of risk, or λ_k from July 1973 to December 2021. Subsequently, average market prices of risks and associated t-statistics are computed. Additionally, a monthly time series of realized and predicted returns for each portfolio from July 1973 to December 2021 is produced. Using these series, we compute average realized and average predicted returns for each of the 286 anomaly portfolios. Following Jagannathan and Wang (1996) and Lettau and Ludwigson (2001, footnote 17, p. 1254), a simple cross-sectional regression of average realized returns on average predicted returns is run to yield an estimate of the R^2 value. Lastly, we plot average realized returns against predicted returns to graphically illustrate mispricing errors for anomaly portfolios.

3.2. Asset Pricing Models

3.2.1. Prominent Asset Pricing Models

The following leading or renowned asset pricing models are studied:

- CAPM based on the market factor computed using the University of Chicago's Center for Research in Security Prices (CRSP) value-weighted index minus the Treasury bill rate (*M*);
- Fama and French (1992, 1993) three-factor model (FF3) that augments the market factor with a size factor (small minus large firms' stock returns, or *SMB*) and a value factor (high book-to-market equity minus low book-to-market equity firms' stock returns, or *HML*);
- Carhart (1997) four-factor model (C4) that augments the three-factor model with a momentum factor (firms with high past return stock returns minus low past stock returns, or *MOM*);
- Fama and French (2015) five-factor model (FF5) that augments the three-factor model with a profit factor (robust operating profitability minus weak operating profitability returns, *RMW*) and capital investment factor (conservative investment minus aggressive investment returns, or *CMA*);
- Fama and French (2018) six-factor model (FF6) that augments the five-factor model with a momentum factor;

Additionally, we include the little-known Kolari, Liu, and Huang (2021) ZCAPM that augments the market factor with a cross-sectional market return dispersion factor. We next provide an overview of the ZCAPM.

8 For discussion of estimated market prices of risk, see Ferson, Sarkissian, and Simon (1999), Cochrane (2005), Back, Kapadia, Ostdiek (2013, 2015), Ferson (2019), and other.

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3.2.2. Overview of ZCAPM

Kolari, Liu, and Huang (2021) (hereafter KLH) recently published a book that proposed a new asset pricing model dubbed the ZCAPM.[9] Their model is mathematically derived as a special case of Black's (1972) now famous zero-beta CAPM. Two systematic risk factors emerge from their derivation: (1) average market return in excess of the riskless rate (i.e., market factor); and (2) cross-sectional standard deviation of returns in the market (i.e., market return dispersion). These factors are computed as the first and second moments of stock market returns on any given day. Note that the market dispersion factor is a cross-sectional rather than time-series standard deviation of returns.[10] Market return dispersion is computed using the value-weighted CRSP index return (denoted R_{at}) on each day t as:

$$\sigma_{at} = \sqrt{\frac{n}{n-1}\sum_{i=1}^{n} w_{it-1}(R_{it} - R_{at})^2}, \tag{3}$$

where n is the total number of stocks; w_{it-1} is the previous day's market value weight for the ith stock (i.e., market capitalization of the stock divided by the market capitalization of all n stocks); R_{it} is the return of the ith stock on day t; and R_{at} is value-weighted average return of all available stocks in the CRSP database on day t. Many authors associate market return dispersion with macroeconomic shocks, such as economic uncertainty, business cycles, market volatility, and unemployment rates.[11]

The theoretical ZCAPM is closely related to the now famous mean-variance investment parabola of Markowitz (1952, 1959). KLH. proved that the width or span of the parabola is determined by market return dispersion. Figure 1 shows the parabola with total risk (as measured by the time-series standard deviation of returns) on the X-axis and expected returns on the Y-axis. Interestingly, assuming the width is defined by market return dispersion, it must be true that the average market return lies somewhere along the axis of symmetry that divides the parabola into two symmetric halves. Thus, KLH inferred that the value-weighted CRSP index lies in the middle of the parabola, which is far from the efficient frontier. Many researchers have conjectured that the CRSP index represents an efficient portfolio and, therefore, is a plausible proxy for the market portfolio in Sharpe's (1964) CAPM. According to the ZCAPM, to reach the efficient frontier, investors should use the average market return to move along the axis of symmetry and then positive market return dispersion to move upward to the efficient frontier. Conversely, to reach the lower inefficient boundary of the parabola, investors can move downward from the axis of symmetry via negative market return dispersion. Importantly, market return dispersion can be positive or negative in sign for assets within the opportunity set described by the parabola.

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9 For further discussion of the ZCAPM, see conference presentations and publications by the authors, including Liu, Kolari and Huang (2012), Liu (2013), Liu, Kolari, and Huang (2019), Liu, Kolari, and Huang (2020), Kolari, Huang, Butt, and Liao (2022), Kolari, Huang, Liu, and Liao (2022, 2023, 2025), Kolari and Pynnonen (2023), and Kolari, Liu, and Pynnonen (2024).

10 Previous studies have incorporated time-series market volatility (e.g., VIX index) in asset pricing models, including Ang, Hodrick, Xing, and Zhang (2006), Bekaert, Engstrom, and Ermolov (2023), Detzel, Duarte, Kamara, and Siegel (2024), and citations therein.

11 For example, see work by Loungani, Rush, and Tave (1990), Christie and Huang (1994), Bekaert and Harvey (1997, 2000), Connolly and Stivers (2003), Gomes, Kogan, and Zhang (2003), Stivers (2003), Bansal and Yaron (2004), Zhang (2005), Pastor and Veronesi (2009), Bansal, Kiku, Shaliastovich, and Yaron (2014), Angelidis, Sakkas, and Tessaromatis (2015), among others. Garcia, Mantilla-Garcia, and Martellini (2014) have conjectured that cross-sectional return dispersion in the stock market is related to aggregate idiosyncratic risk. Also, Cooper, Gulen, and Ion (2024) have shown that macroeconomic shocks associated with market return dispersion are related to the asset growth factor in the Hou, Xue, and Zhang and Fama and French five-factor models.

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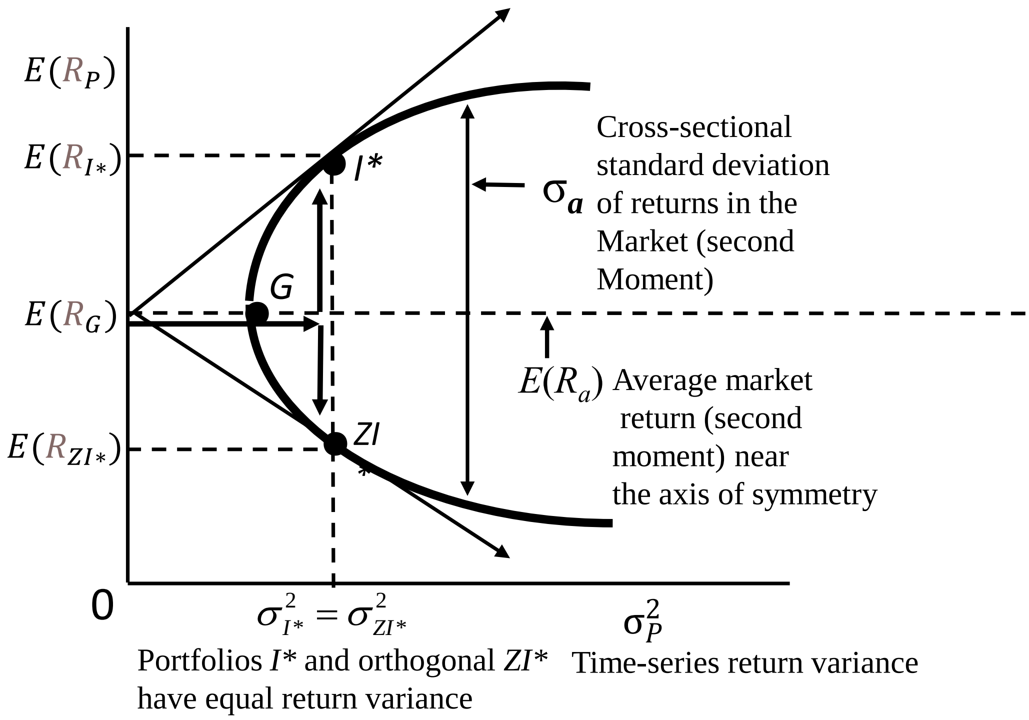

Figure 1. Locating orthogonal portfolios I^* and ZI^* on the mean-variance parabola. Source: Adapted from Kolari, Liu, and Huang (2021, Figure 3.2, p. 59).

In Figure 1, KLH chose two portfolios with the same time-series return variance (denoted σ_P^2) to derive the ZCAPM – namely, efficient portfolio I^* and orthogonal inefficient zero-beta portfolio ZI^*. Simplifying their derivation, the expected returns of these two portfolios can be specified as follows:[12]

$$E(R_{I^*}) \approx E(R_a) + \sigma_a \tag{4}$$
$$E(R_{ZI^*}) \approx E(R_a) - \sigma_a, \tag{5}$$

where $E(R_a)$ is the average market return; and σ_a is the market return dispersion.

Sharpe's CAPM assumed perfect capital markets, homogeneous investor expectations, two-parameter probability distributions of returns, investor risk aversion, no short selling, and a riskless rate. Black amended the last two assumptions to allow short selling and borrowing at a rate greater than the riskless rate. His now famous zero-beta CAPM can be written as:

$$E(R_i) = E(R_{ZM}) + \beta_{iM}[E(R_M) - E(R_{ZM})], \tag{6}$$

where $E(R_M)$ is the expected market portfolio return; $E(R_{ZM})$ is the expected zero-beta portfolio return that is uncorrelated (orthogonal) to the market portfolio; and β_{iM} is beta risk associated with excess expected market returns. According to Roll (1972), Copeland and Weston (1980), and others, Black's model can be more generally specified in terms of any efficient portfolio and its orthogonal inefficient portfolio:

$$E(R_i) = E(R_{ZI}) + \beta_{iI}[E(R_I) - E(R_{ZI})], \tag{7}$$

[12] They more precisely specified $f(\theta)\sigma_a$ instead of simply σ_a in these equations, where $f(\theta) > 0$ is a complex function of other terms.

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where I and ZI are any efficient index and its orthogonal zero-beta counterpart, respectively, located on the minimum variance boundary of the parabola.

Substituting the expected returns defined in equations (4) and (5) into zero-beta CAPM relation (7), the theoretical ZCAPM is:

$$\begin{aligned} E(R_i) &= E(R_{ZI^*}) + \beta_{iI^*}[E(R_{I^*}) - E(R_{ZI^*})] \\ &= E(R_a) - \sigma_a + \beta_{iI^*}\{[E(R_a) + \sigma_a] - [E(R_a) - \sigma_a]\} \\ &= E(R_a) + (2\beta_{iI^*} - 1)\sigma_a \\ E(R_i) &= E(R_a) + Z^*_{ia}\sigma_a, \end{aligned} \tag{8}$$

where $Z^*_{ia} = 2\beta_{iI^*} - 1$ (i.e., the systematic risk of asset i associated with σ_{at}). Notice that the theoretical ZCAPM is an alternative equivalent form of the zero-beta CAPM with respect to portfolios I^* and ZI^*. Lastly, assuming the existence of a riskless rate asset, the theoretical ZCAPM can be respecified as:[13]

$$E(R_i) - R_f = \beta_{ia}[E(R_a) - R_f] + Z^*_{ia}\sigma_a, \tag{10}$$

where $\beta_{i,a}$ captures beta risk related to expected market excess returns; and Z^*_{ia} is *zeta risk* associated with cross-sectional market return dispersion of expected returns of all assets in the market. As noted above, zeta risk can be positive or negative in sign with respect to the expected returns of portfolios I^* and ZI^*, respectively.

In the ZCAPM, as the the width of the parabola increases or decreases due to σ_a, the expected returns of assets within the parabola are affected. As σ_a increases (decreases), assets in the upper half of the parabola above the axis of symmetry experience increasing (decreasing) expected returns; alternatively, assets in the bottom half of the parabola below the axis of symmetry experience decreasing (increasing) expected returns. It is obvious that market return dispersion has major impacts of the expected returns of assets in the market. Also, depending where assets are located within the parabola, these dispersion impacts can be positive or negative.

A major challenge for the ZCAPM is the specification of an empirical model that can capture both positive and negative market dispersion effects on assets' returns. KLH proposed an innovative solution to this problem by introducing a hidden signal variable to model these two-sided dispersion effects:

$$R_{it} - R_{ft} = \alpha_i + \beta_{ia}(R_{at} - R_{ft}) + Z_{ia}D_{it}\sigma_{at} + u_{it}, \quad t = 1, \ldots, T, \tag{11}$$

[13] See Kolari et al. (2021, p. 71) for the mathematical derivation. Like Black (1972, pp. 452-454), the ZCAPM is extended to the existence of a riskless asset. Investors can purchase the riskless asset but cannot short (borrow) this asset. Investors are allowed to take short positions in risky assets (e.g., the zero-beta portfolio). We can derive Black's zero-beta CAPM with a riskless asset as follows:

$$E(R_i) = \beta_{iI}E(R_I) + (1 - \beta_{iI})E(R_{ZI}).$$

Assuming a riskless asset, proportion α of investor funds is allocated to risky assets I and ZI and proportion (1-α) to the riskless asset:

$$(E(r_i) = \alpha[\beta_{iI}E(R_I) + (1 - \beta_{iI})E(R_{ZI})] + (1 - \alpha)R_f.$$

After rearranging terms, the zero-beta CAPM becomes:

$$\begin{aligned} E(R_i) - R_f &= \alpha\beta_{iI}[E(R_I) - R_f] + \alpha(1 - \beta_{iI})[E(R_{ZI}) - R_f] \\ &= \beta'_{i,I}[E(\widetilde{R}_I) - R_f] + \beta'_{i,ZI}[E(\widetilde{R}_{ZI}) - R_f], \end{aligned} \tag{9}$$

where $\beta'_{i,I}$ and $\beta'_{i,ZI}$ correspond to the beta risks of asset i with respect to efficient portfolio I and its zero-beta portfolio counterpart ZI, respectively.

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where Z_{ia} measures systematic risk associated with market return dispersion σ_{at}; D_{it} is a hidden signal variable.[14] D_{it} is assigned values $+1$ and -1 to coincide with positive and negative market return dispersion effects on asset returns at time t, respectively; α_i is the intercept related to mispricing errors; $u_{it} \sim$ iid $\mathrm{N}(0, \sigma_i^2)$; and other notation is as before. In forthcoming analyses, we proxy average market returns, or R_a, with the value-weighted CRSP index return. Also, we proxy the market return dispersion using all common stocks in the CRSP database per equation (3).

Unlike virtually all asset pricing models that employ ordinary least squares (OLS) regression, the empirical ZCAPM is estimated by means of the expectation-maximization (EM) algorithm, which is well known in the hard sciences.[15] EM provides an estimate of the probability that hidden signal variable D_{it} is $+1$ (p) or -1 ($1-p$).[16] In empirical ZCAPM regression relation (11), D_{it} is an independent random variable with a two-point distribution:

$$D_{it} = \begin{cases} +1 & \text{with probability } p_i \\ -1 & \text{with probability } 1 - p_i, \end{cases} \tag{12}$$

where p_i (or $1 - p_i$) is the probability of a positive (or negative) market return dispersion effect; and D_{it} are independent of u_{it}. The EM algorithm estimates probability p_i using available in-sample information about the dependent and independent variables in the model.

Based on equation (11), $Z_{ia}D_{it}$ can take on values of $+Z_{ia}$ or $-Z_{ia}$ due to the sign of D_{it}. Since the mean of binary signal variable $D_{i,t}$ equals $2p_i - 1$, we can define $Z^*_{ia} = Z_{ia}(2p_i - 1)$. The parameter Z^*_{ia} captures the systematic risk related to market return dispersion σ_{at}. Hence, the empirical ZCAPM can be written as:

$$R_{it} - R_{ft} = \alpha_i + \beta_{ia}(R_{at} - R_{ft}) + Z^*_{ia}\sigma_{at} + u_{it}, \;\; t = 1, \ldots, T, \tag{13}$$

where β_{ia} and Z^*_{ia} proxy beta risk and zeta risk parameters in the theoretical ZCAPM. In sample period $t = 1, \ldots, T$, the sign of Z^*_{ia} is based on the estimated probability p_i of hidden variable D_{it}. Hence, if $p_i > 1/2$ (or $< 1/2$), the sign of Z^*_{ia} is positive (negative). The empirical ZCAPM as estimated via EM is a probabilistic mixture model with two components, wherein one component has positive zeta risk and the other component has negative zeta risk.

We should mention that some authors have augmented the market factor with a market return dispersion factor, including Jiang (2010), Demirer and Jategaonkar (2013), and Garcia, Mantilla-Garcia, and Martellini (2014). Using OLS regression to estimate the time-series regression model and subsequent cross-sectional regression tests of factor loadings, they found both positive and negative sensitivity of stock returns to market return dispersion factor loadings. Sometimes the latter factor loadings associated with market return dispersion were significant but not in other tests. We have found in repeated tests of the ZCAPM using many different test assets and sample time periods that zeta risk loadings are always significant in cross-sectional regression tests at high statistical levels that well exceed previous studies.

[14] Precedent exists in the asset pricing literature for the introduction of hidden or latent variables. For example, principal component analysis (PCA) and factor analysis use statistical methods to identify hidden factors in asset pricing models. See, Roll and Ross (1980), Chen (1983), Lettau and Pelger (2020), and many others.

[15] See Dempster, Laird, and Rubin (1977), Jones and McLachlan (1990), McLachlan and Peel (2000), McLachlan and Krishnan (2008), and others. Some finance studies have applied EM regression, including Asquith, Jones, and Kieschnick (1998), McLachlan and Krishnan (2008), Harvey and Liu (2016), Chen, Cliff, and Zhao (2017), among others. See Bo and Batzoglou (2008) for a primer on the EM algorithm with application to computational biology. Also, Wikipedia provides discussion of the EM algorithm and literature citations.

[16] More specifically, the E-step provides a conditional expectation of the log-likehood function using current estimates of parameter values, and the M-step iteratively maximizes the log-likelihood in the E-step. The EM algorithm converges to a stationary point of the likelihood equation.

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Unlike OLS regression asset pricing models, KLH found that the intercept parameter (or alpha) can be dropped from the empirical ZCAPM. No decrease in residual error variance was achieved by adding an intercept. For interested readers, KLH provide open source empirical ZCAPM software using Matlab, R, and Python programs on the internet at GitHub (https://Github.com/zcapm). Programs for running Fama-MacBeth cross-sectional regression tests are available at this website also.

Finally, ZCAPM factor loadings for $\hat{\beta}_{ia}$ and $\hat{Z}^*_{ia}$ based on one year of daily returns prior to month $T+1$ are used to estimate the following OLS cross-sectional regression:

$$R_{i,T+1} - R_{fT+1} = \lambda_0 + \lambda_a \hat{\beta}_{ia} + \lambda_{RD} \hat{Z}^*_{ia} + u_{it}, i = 1, ..., N, \tag{14}$$

where λ_0 is the intercept term, λ_a and λ_{RD} are estimates of the market prices of beta risk and zeta risk loadings in percent terms, respectively, and other notation is as before. It is notable that the dependent variable in this regression is the one-month ahead excess returns for the ith anomaly portfolio. Since zeta risk loadings are estimated using daily returns, we rescale the estimated zeta risk coefficient $\hat{Z}^*_{i,a}$ from a daily to monthly basis as follows:

$$R_{i,T+1} - R_{fT+1} = \lambda_0 + \lambda_M \hat{\beta}_{ia} + \lambda_{RD} \hat{Z}^*_{ia} N_{T+1} + +u_{it}, i = 1, ..., N, \tag{15}$$

where N_{T+1} is the number of trading days in month $T+1$ (i.e., 21 days), $Z^*_{i,a} N_{T+1}$ is the monthly estimated zeta risk, and λ_{RD} is the monthly risk premium associated with zeta risk. This rescaling does not change the risk premium $\hat{\lambda}_{RD}$ in terms of each unit of zeta risk. Now the estimated monthly market price of $Z^*_{i,a}$, or λ_{RD}, can be compared to the estimated monthly market price of β_{ia}, or λ_a.

4. Empirical Evidence

In this section, we report the results of cross-sectional regression tests of asset pricing models as well as comparative graphs of their average mispricing errors. Table 1 reports descriptive statistics for asset pricing factors in different models in our sample period. The market return dispersion factor σ_{at} computed using equation (3) is denoted as *RD*.

Table 1. Descriptive statistics for asset pricing factors.

This table summarizes the descriptive statistics for the asset pricing factors. Distribution information is provided, including the mean return, standard deviation of returns, maximum and minimum returns, and 25th percentile (P25), median (P50), and 75th percentile (P75) returns. Daily factor returns (in percent) corresponding to different asset pricing models are denoted as follows: $R_m - R_f$ (CRSP index return minus Treasury bill rate, see footnote 1), *SMB* (size), *HML* (value), *MOM* (momentum), *RMW* (profit), *CMA* (capital investment), and *RD* (market return dispersion), The sample period is from July 1972 to December 2021.

Statistic	$R_m - R_f$	*SMB*	*HML*	*MOM*	*RMW*	*CMA*	*RD*
Mean	0.01	0.005	0.01	0.03	0.01	0.01	1.86
Standard deviation	1.07	0.57	0.59	0.79	0.41	0.37	0.66
Maximum	11.35	6.17	6.74	7.12	4.52	2.53	10.77
Minimum	-17.47	-11.19	-5.00	-14.37	-3.01	-5.87	0.68
P25	-0.46	-0.30	-0.24	-0.25	-0.17	-0.18	1.47
P50	0.04	0.02	0.01	0.06	0.01	0.01	1.73
P75	0.53	0.32	0.26	0.36	0.19	0.19	2.04

[1] In the ZCAPM, due to using average market excess returns rather than a proxy for market portfolio excess returns, the market factor associated with CRSP index excess returns is denoted $R_a - R_f$ instead of $R_m - R_f$ per other models.

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4.1. Cross-Sectional Regression Results

Table 2 documents that empirical results from OLS cross-sectional regression tests for different asset pricing models. As shown there, the CAPM has almost no explanatory power with only a 1 percent R^2 value. The market price of market beta loadings, or $\hat{\lambda}_m$, is zero and far from statistical significance. Comparatively, the Fama and French three-factor model noticeably improves matters. The R^2 value increases to 11 percent and the market price of value beta risk loadings, or $\hat{\lambda}_{HML}$, is significant at the 10 percent level. The results are little changed for the Carhart four-factor model. The Fama and French five- and six-factor models exhibit some improvement in both explanatory power and statistical significance. The six-factor model boosts R^2 to 19 percent. Also, capital investment factor loadings, or $\hat{\lambda}_{CMA}$, are significant at the 1 percent level or lower.

Table 2. Out-of-sample Fama-MacBeth cross-sectional tests of 286 anomaly portfolios for asset pricing models in the sample period July 1972 to December 2021.

Using a total of 286 anomaly portfolios (comprised of 133 plus 153 anomalous portfolios from Chen and Zimmerman (2020) and Jensen, Kelly, and Pedersen (2023), respectively), this table contains results for the full sample period July 1972 to December 2021. Based on standard two-step Fama-MacBeth cross-sectional tests, we report out-of-sample (one-month-ahead) estimated market prices of risk denoted $\hat{\lambda}_k$ for the kth factor for the analysis period July 1973 to December 2021. Associated t-statistics are shown in parentheses below estimated prices of risk. The asset pricing models are: CAPM, Fama and French three-factor model (FF3), Carhart four-factor model (C4), Fama and French five-factor model (FF5), Fama and French six-factor model (FF6), and ZCAPM. Factors in these models are denoted as m (CRSP index excess return, see footnote 1), RD (market return dispersion), SMB (size), HML (value), MOM (momentum), RMW (profit), and CMA (capital investment). Results are shown for the analysis period.

Model	$\hat{\alpha}$	$\hat{\lambda}_m$	$\hat{\lambda}_{RD}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	$\hat{\lambda}_{RMW}$	$\hat{\lambda}_{CMA}$	R^2
CAPM	0.44	-0.00							0.01
	(17.77***)	(-0.01)							
FF3	0.42	0.07		-0.09	0.27				0.11
	(19.37***)	(0.39)		(-0.37)	(1.82*)				
C4	0.40	-0.04		-0.13	0.26	0.31			0.17
	(18.52***)	(-0.23)		(-0.61)	(1.84*)	(1.29)			
FF5	0.40	0.12		0.04	0.12		0.18	0.32	0.15
	(20.33***)	(0.70)		(0.17)	(0.77)		(1.25)	(2.76***)	
FF6	0.38	-0.04		0.03	0.18	0.40	0.19	0.26	0.19
	(19.00***)	(-0.26)		(0.16)	(1.19)	(1.64)	(1.54)	(2.31**)	
ZCAPM	0.28	0.30	0.72						0.84
	(9.96)	(1.43)	(6.69***)						

[1]In the ZCAPM, due to using average market excess returns rather than a proxy for market portfolio excess returns, the price of beta risk associated with CRSP index excess returns is denoted $\hat{\lambda}_a$ instead of $\hat{\lambda}_m$ in the other models. Asterisks indicate the level of statistical significance: * – 10 percent, ** – 5 percent, and *** – 1 percent.

Turning to the ZCAPM, the results are much stronger than the other models. Now the R^2 value jumps to 84 percent. This very high goodness-of-fit suggests that the ZCAPM well explains anomaly portfolio returns. Given the fact that our tests are out-of-sample in the month ahead of the period used to estimate beta risk and zeta risk loadings, this explanatory power is exceptional.

For the ZCAPM, the market price of zeta risk loadings associated with market return dispersion, or $\hat{\lambda}_{RD}$, is very significant with a t-statistic of 6.69. To the authors' knowledge, no prior asset pricing studies have reported a t-statistic of this magnitude with respect to systematic risk loadings. In this regard, Harvey, Liu, and Zhu (2016) and Chordia, Goyal, and Saretto (2020) have recommended a t-statistic threshold of 3 to avoid false discoveries of significant asset pricing factors. Since $\hat{\lambda}_{RD}$ exceeds 6, we infer that market return dispersion is an extremely significant asset pricing factor. Furthermore,

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the estimated magnitudes of $\hat{\lambda}_m$ and $\hat{\lambda}_{RD}$ associated with beta risk and zeta risk loadings, respectively, are economically significant at 0.30 percent and 0.72 percent per month, or 3.60 percent and 8.64 percent per year.

As a robustness check, we re-ran the analyses using a full sample period regression approach, rather than a one-month rolling regression approach. Using the full period from July 1972 to December 2021, we estimate the empirical ZCAPM. Subsequently, we estimate cross-sectional regressions in each month and average the monthly results. In unreported results, our findings are unchanged for the most part. The market prices of beta and zeta risk loadings, or $\hat{\lambda}_m$ and $\hat{\lambda}_{RD}$, respectively, are 0.45 percent and 1.32 percent with significant t-statistics equal to 2.21 and 8.62. The R^2 value is estimated at 86 percent. Other models' results improve somewhat also but are again much weaker than those for the ZCAPM. Interestingly, when we ran daily cross-sectional regressions in the full sample period and average the results, the t-statistic associated with the market price of zeta risk loadings surges to 13.69, which is even more significant than before. It is clear that zeta risk is important to explaining the cross section of average anomaly portfolio returns.

In sum, stock market anomaly portfolios' returns are well explained by beta risk and zeta risk in the empirical ZCAPM. Contrarily, the CAPM has no explanatory ability, and prominent multifactor models have far less explanatory ability than the ZCAPM. It is noteworthy that our out-sample cross-sectional regression results cannot be compared to prior multifactor model studies reviewed in Section 2 as they primarily only reported in-sample GRS tests to evaluate different asset pricing models. We believe that out-of-sample tests allow enable a better understanding of the pricing ability of models that is more consonant with real world investor experience. That is, investors measure the risk of assets that they purchase and then observe their performance after portfolio formation. Thus, unlike in-sample tests, out-of-sample tests are akin to an investable market strategy.

4.2. Graphical Mispricing Error Results

According to Fama and MacBeth (1973), a *normative model* that helps investors make better return/risk decisions is valid to the extent that it can utilize past information to explain future returns. In line with this logic, Cochrane (1996) and Lettau and Ludvigson (2001) have highlighted the comparative analyses of predicted versus realized (actual) returns to evaluate mispricing errors in asset pricing models. As Cochrane has asserted, "Expected return pricing errors ... are a useful characterization of a model's performance."(Cochrane, 1996, p. 596) In conducting these analyses, he recommended that " ... it is important to examine a model's ability to explain the expected returns of economically interesting portfolios." (Cochrane, 1996, p. 598) Of course, because anomaly portfolios are difficult for asset pricing models to explain, they are compelling test assets to investigate.

As described earlier, we compute mispricing errors on an out-of-sample basis based on one-month-ahead cross-sectional regressions. Using one year of daily returns for the 286 anomaly portfolios, a time-series regression is run to estimate each asset pricing model and its factor loadings. In the second step, we estimate an out-of-sample (one-month-ahead) cross-sectional regression using one-month-ahead excess returns for anomaly portfolios. By rolling forward one month at a time, this process generates $t = 1, \cdots, 582$ cross-sectional regressions from July 1973 to December 2021. In the next month for each portfolio, we utilize the average of the estimated factor prices of risk $\hat{\lambda}_k$ and average estimated loadings for the ith portfolio to compute average fitted excess returns. Lastly, for each asset pricing model, a graph is created with average fitted excess returns plotted against each portfolio's one-month-ahead average actual excess returns.

Figures 2–4 illustrate the average pricing errors of different asset pricing models. In Figure 2, we see that both the CAPM and Fama and French three-factor model exhibit large mispricing errors.

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Instead of pricing errors lining up on the 45 degree line, they are densely populated around the line in a vertical pattern.

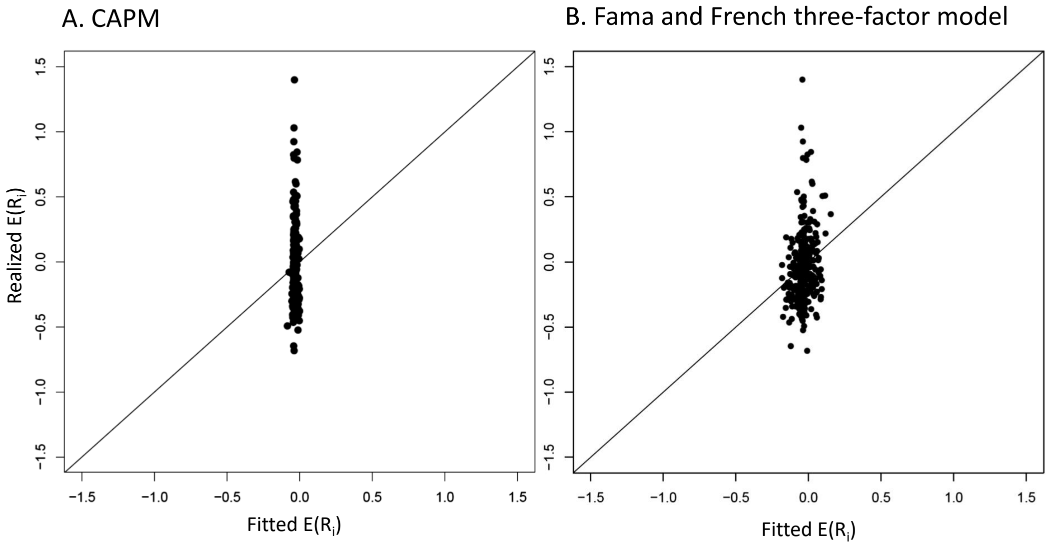

Figure 2. Out-of-sample cross-sectional CAPM (Panel A) and Fama and French three-factor model (Panel B) mispricing errors comparing average one-month-ahead realized excess returns in percent (Y-axis) to average one-month-ahead predicted (fitted) excess returns in percent (X-axis) for 286 portfolios. The analysis period is July 1973 to December 2021.

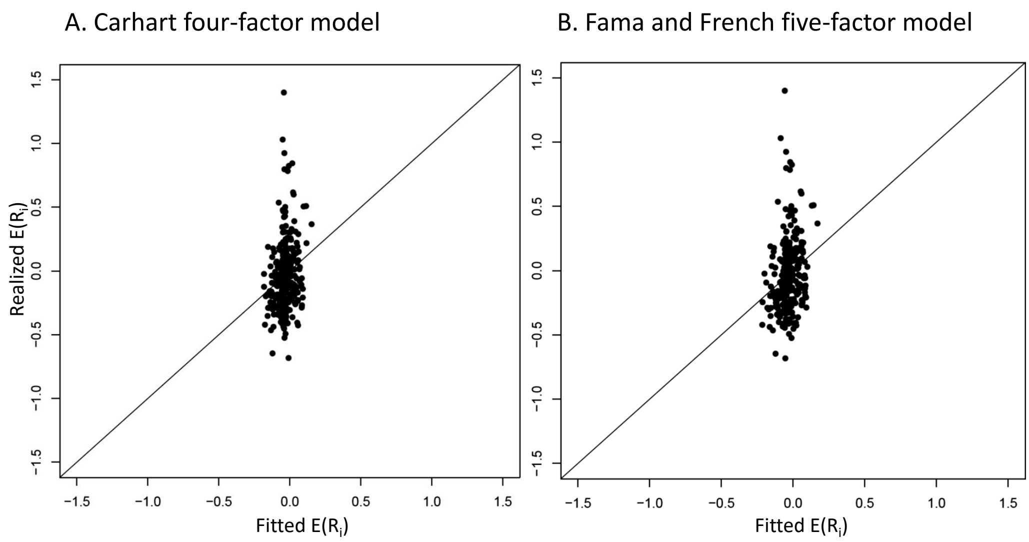

Figure 3. Out-of-sample cross-sectional Carhart four-factor model (Panel A) and Fama and French five-factor model (Panel B) mispricing errors comparing average one-month-ahead realized excess returns in percent (Y-axis) to average one-month-ahead predicted (fitted) excess returns in percent (X-axis) for 286 portfolios. The analysis period is July 1973 to December 2021.

As shown in Figure 3, the Carhart four-factor model and Fama and French five-factor model do not improve matters much. By casual observation, the pricing errors for the 286 anomaly portfolios are large in both models.

Finally, Figure 4 compares the Fama and French six-factor model to the ZCAPM. While the six-factor model improves upon the five-factor model in terms of lower mispricing errors, the ZCAPM makes a quantum leap in pricing. Indeed, no anomaly portfolio lies far off the 45 degree line. This level of out-of-sample pricing is exceptional. One would expect that some anomalies would be difficult to

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price and, therefore, would be located well off the line. In this case, winsorizing the data for eliminate spurious outliers would be useful to investigate. However, because virtually all of the anomaly portfolios are well explained by the ZCAPM, we did not winsorize the data. It is noteworthy that these mispricing error results are based on out-of-sample tests that employ beta risk and zeta risk estimated in a prior one year period to compute fitted or predicted returns in the next month. In effect, future returns are lining up almost exactly with previously estimated systematic risks via the ZCAPM!

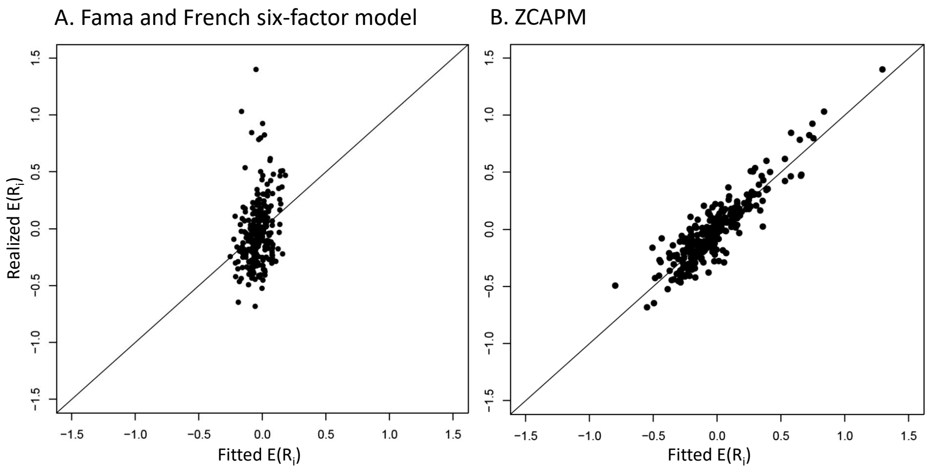

Figure 4. Out-of-sample cross-sectional Fama and French six-factor model (Panel A) and ZCAPM (Panel B) mispricing errors comparing average one-month-ahead realized excess returns in percent (Y-axis) to average one-month-ahead predicted (fitted) excess returns in percent (X-axis) for 286 portfolios. The analysis period is July 1973 to December 2021.

The major implication of these graphical analyses is that the 286 anomaly portfolios are anomalous to other asset pricing models but not the ZCAPM. In effect, there none of these portfolios is anomalous from the perspective of the ZCAPM. We infer that researchers will have to expand their search for long/short anomalies in the stock market.

4.3. What Explains ZCAPM Outperformance?

The clear outperformance of the ZCAPM relative to prominent asset pricing models published in top tier finance journals is impressive. What explains this outperformance? According to KLH, long/short factors in multifactor models are rough measures of cross-sectional market return dispersion. For example, the size factor is long higher yielding small stocks and short lower yielding large stocks. It is likely that small stocks are located in the upper half of the mean-variance investment parabola, whereas large stocks are somewhere in the lower half of the parabola. As the parabola widens (narrows), the size factor experiences higher (lower) returns. This two-sided volatility effect is tantamount to the market return dispersion factor in the ZCAPM. However, the long/short size factor is only a slice or piece of the total market return dispersion (or σ_a) in the ZCAPM.

Other multifactors account for other slices within the total market return dispersion that defines the width or span of the mean-variance parabola. As long/short factors are added to multifactor models, they will gradually converge to the total market return dispersion of the ZCAPM. However, long/short factors tend to be unstable over time. If large stocks and small stocks switch their positions in the parabola, then the size factor would flip from being positively priced to being negatively priced. Some studies have observed this pricing behavior for the size factor. Also, a factor could be significant

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in one time period but not in other periods as the locations of long and short portfolios in the factor move around in the parabola over time.[17]

KLH have argued that all multifactors are contained within total market return dispersion. There is no need to use all these factors and look for new factors – the factor zoo is captured by market return dispersion. In turn, the ZCAPM measures their aggregate systematic risk in zeta risk.

An important implication of these insights is that multifactor models are related to the ZCAPM. All multifactor models, including the ZCAPM, employ cross-sectional return dispersion in stock returns to explain asset prices. Moreover, in view of this association, we can link multifactor models to the CAPM, as the ZCAPM is mathematically derived from the zero-beta CAPM. According to KLH, the ZCAPM provides a unifying theory and empirical methodology in the field of asset pricing.

It is interesting that Black (1995) later discussed the second beta factor in his zero-beta CAPM. In this words, the second beta factor is " ... the minimum-variance, zero-beta portfolio of risky assets, where beta is defined using whatever market portfolio we use to represent the first factor." (Black, 1995, p. 170). He reasoned as follows:

> "When Fama and French say that the line relating expected return and beta is flat,
> they are just saying that the expected excess return on the second factor is large.
> If we believe it's as large as they say, we won't fool around with their third and fourth
> factors, for which they give no theory.[18] We'll go for the gold in the second factor!"

KLH conjectured that market return dispersion corresponds to Black's second factor. Figure 1 illustrates his second factor as the difference between the returns of an efficient portfolio and inefficient, zero-beta portfolio (with equal return variance), or market return dispersion. It is not necessary to know the expected returns in these two portfolios as their difference corresponds to total market return dispersion. An important policy implication of the ZCAPM relevant to investors is that zeta risk related to market return dispersion is a salient second risk measure (in addition to beta risk associated with the market factor when evaluating the return performance of stock portfolios.

Finally, the relatively accurate pricing of large numbers of anomaly portfolios by the ZCAPM strongly supports the efficient market hypothesis. As observed by Schwert (2003), Fama and French (2008), Fama (2013) and others, stock market anomalies could be explained by a bad model problem in asset pricing or an inefficient market. The existence of a valid model would confirm that the market is efficient. Echoing this observation, the authors of a recent asset pricing study noted that " ... the credibility of the anomalies literature can improve via a close connection with economic theory." (Hou, Xue, and Zhang (2020, p. 2072)) In this regard, our empirical tests and results of the ZCAPM for anomaly portfolios establish a connection to general equilibrium asset pricing theory.

5. Conclusion

This paper sought to compare the ability of different asset pricing models to explain stock market anomaly portfolio returns. We employed a large dataset of 286 anomaly portfolios provided by Chen and Zimmerman (2022) and Jensen, Kelly, and Pedersen (2023). Anomalies are economically interesting due to the fact that they are difficult for asset pricing models to explain.

Surprisingly, virtually all of the anomaly portfolios were well explained the ZCAPM, a recent model proposed by Kolari, Liu, and Huang (2021). In stark contrast, the CAPM as well as a number of prominent multifactor models could not explain anomaly portfolio returns for the most part.

[17] Taking into account this time-varying factor return behavior, Ang, Madhaven, and Sobczyk (2017) have proposed a method to allow for dynamic factor loadings with some success in U.S. mutual fund portfolios. It is possible that multifactor models based on long/short factors could benefit from time-variable factors and their loadings. However, this research is beyong the scope of the present study.

[18] Black was referring here to the small-firm factor and price-to-book factor, respectively.

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Mispricing errors were large for the latter models compared to relatively small pricing errors for the ZCAPM.

We conclude that, given the ZCAPM, long/short stock market anomaly portfolios under study are no longer anomalies. Given the ability of the ZCAPM to explain large datasets of anomalies, our findings support the efficient markets hypothesis. The scope of our study is limited to U.S. stock market anomalies. Further research is recommended on stock market anomalies in other countries as well as other classes of assets, including bonds, commodities, and other asset traded on a daily basis. Also, an important implication for future research is the search to identify long/short stocks portfolios not explained by the ZCAPM. Perhaps behavioral explanations of these anomalies are possible.

Appendix A. 133 Anomaly Portfolios from Chen and Zimmerman (2022)

N	Abbreviation	Description
1	AbnormalAccruals ret	Abnormal accruals
2	Accruals ret	Accruals
3	AdExp ret	Advertisement expense
4	AM ret	Total assets to market
5	AnnouncementReturn ret	Earnings announcement return
6	AssetGrowth ret	Asset growth
7	Beta ret	CAPM beta
8	BetaFP ret	Frazzini-Pedersen beta
9	BetaTailRisk ret	Tail risk beta
10	BidAskSpread ret	Bid-ask spread
11	BM ret	Book to market using most recent ME
12	BMdec ret	Book to market using December ME
13	BookLeverage ret	Book leverage (annual)
14	BPEBM ret	Leverage component of BM
15	BrandInvest ret	Brand capital investment
16	Cash ret	Cash to assets
17	CashProd ret	Cash productivity
18	CBOperProf ret	Cash-based operating profitability
19	CF ret	Cash flow to market
20	Cfp ret	Operating cash flows to price
21	ChAssetTurnover ret	Change in asset turnover
22	ChEQ ret	Growth in book equity
23	ChInv ret	Inventory growth
24	ChInvIA ret	Change in capital investment (industry adjusted)
25	ChNNCOA ret	Change in net noncurrent operating assets
26	ChNWC ret	Change in net working capital
27	ChTax ret	Change in taxes
28	CompEquIss ret	Composite equity issuance
29	CompositeDebtIssuance ret	Composite debt issuance
30	Coskewness ret	Coskewness
31	DelCOA ret	Change in current operating assets
32	DelCOL ret	Change in current operating liabilities
33	DelEqu ret	Change in equity to assets
34	DelFINL ret	Change in financial liabilities
35	DelLTI ret	Change in long-term investment
36	DelNetFin ret	Change in net financial assets
37	DNoa ret	Change in net operating assets
38	DolVol ret	Past trading volume
39	EarningsConsistency ret	Earnings consistency
40	EarningsSurprise ret	Earnings surprise
41	EarnSupBig ret	Earnings surprise of big firms
42	EBM ret	Enterprise component of BM
43	EntMult ret	Enterprise multiple
44	EP ret	Earnings-to-Price ratio
45	EquityDuration ret	Equity duration
46	FirmAge ret	Firm age based on CRSP
47	Frontier ret	Efficient frontier index
48	GP ret	Gross profits/total assets
49	GrAdExp ret	Growth in advertising expenses
50	Grcapx ret	Change in capex (two years)
51	Grcapx3y ret	Change in capex (three years)
52	GrLTNOA ret	Growth in long term operating assets
53	GrSaleToGrInv ret	Sales growth over inventory growth
54	GrSaleToGrOverhead ret	Sales growth over overhead growth
55	Herf ret	Industry concentration (sales)
56	HerfAsset ret	Industry concentration (assets)
57	HerfBE ret	Industry concentration (equity)
58	High52 ret	52 week high
59	Hire ret	Employment growth
60	IdioRisk ret	Idiosyncratic risk
61	IdioVol3F ret	Idiosyncratic risk (3 factor)
62	IdioVolAHT ret	Idiosyncratic risk (AHT)

Appendix A, continued

N	Abbreviation	Description
63	Illiquidity ret	Amihud's illiquidity
64	IndMom ret	Industry momentum
65	IndRetBig ret	Industry return of big firms
66	IntanBM ret	Intangible return using BM
67	IntanCFP ret	Intangible return using CFtoP
68	IntanEP ret	Intangible return using EP
69	IntanSP ret	Intangible return using Sale2P
70	IntMom ret	Intermediate momentum
71	Investment ret	Investment to revenue
72	InvestPPEInv ret	Change in PPE and inventory/assets
73	InvGrowth ret	Inventory growth
74	Leverage ret	Market leverage
75	LRreversal ret	Long-run reversal
76	MaxRet ret	Maximum return over month
77	MeanRankRevGrowth ret	Revenue growth rank
78	Mom6m ret	Momentum (6 month)
79	Mom12m ret	Momentum (12 month)
80	Mom12mOffSeason ret	Momentum without the seasonal part
81	MomOffSeason ret	Off season long-term reversal
82	MomOffSeason06YrPlus ret	Off season reversal years 6 to 10
83	MomOffSeason11YrPlus ret	Off season reversal years 11 to 15
84	MomOffSeason16YrPlus ret	Off season reversal years 16 to 20
85	MomSeason ret	Return seasonality years 2 to 5
86	MomSeason06YrPlus ret	Return seasonality years 6 to 10
87	MomSeason11YrPlus ret	Return seasonality years 11 to 15
88	MomSeason16YrPlus ret	Return seasonality years 16 to 20
89	MomSeasonShort ret	Return seasonality last year
90	MRreversal ret	Medium-run reversal
91	NetDebtFinance ret	Net debt financing
92	NetDebtPrice ret	Net debt to price
93	NetEquityFinance ret	Net equity financing
94	NetPayoutYield ret	Net payout yield
95	NOA ret	Net operating asset
96	NumEarnIncrease ret	Earnings streak length
97	OperProf ret	Operating profits/book equity
98	OperProfRD ret	Operating profitability R&D adjusted
99	OPLeverage ret	Operating leverage
100	OrderBacklog ret	Order backlog
101	OrderBacklogChg ret	Change in order backlog
102	OrgCap ret	Organizational capital
103	PayoutYield ret	Payout yield
104	PctAcc ret	Percent operating accruals
105	Price ret	Price
106	PS ret	Piotroski F-score
107	RD ret	R&D over market cap
108	RDAbility ret	R&D ability
109	Realestate ret	Real estate holdings
110	ResidualMomentum ret	Momentum based on FF3 model residuals
111	ReturnSkew ret	Return skewness
112	ReturnSkew3F ret	Idiosyncratic skewness (3 factor model)
113	RevenueSurprise ret	Revenue surprise
114	Roaq ret	Return on assets (qtrly)
115	RoE ret	Net income/book equity
116	ShareIss1Y ret	Share issuance (1 year)
117	ShareIss5Y ret	Share issuance (5 year)
118	Size ret	Size
119	SP ret	Sales-to-price
120	Std turn ret	Share turnover volatility
121	STreversal ret	Short term reversal
122	Tang ret	Tangibility
123	Tax ret	Taxable income to income
124	TotalAccruals ret	Total accruals
125	TrendFactor ret	Trend in the general stock market
126	VarCF ret	Cash-flow to price variance
127	VolMkt ret	Volume to market equity
128	VolSD ret	Volume variance
129	VolumeTrend ret	Volume trend
130	XFIN ret	Net external financing
131	Zerotrade ret	Days with zero trades
132	ZerotradeAlt1 ret	Days with zero trades
133	ZerotradeAlt12 ret	Days with zero trades

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Appendix B. 153 Anomaly Portfolios from Jensen, Kelly, and Pedersen (2023)

N	Anomaly abbreviation	Description
1	capex abn	Abnormal corporate investment
2	z score	Altman Z-score
3	ami 126d	Amihud measure
4	at gr1	Asset growth
5	tangibility	Asset tangibility
6	sale bev	Assets turnover
7	at me	Assets-to-market
8	at be	Book leverage
9	bev mev	Book-to-market enterprise value
10	be me	Book-to-market equity
11	capx gr1	CAPEX growth (1 year)
12	capx gr2	CAPEX growth (2 years)
13	capx gr3	CAPEX growth (3 years)
14	at turnover	Capital turnover
15	ocfq saleq std	Cash flow volatility
16	cop at	Cash-based operating profits-to-book assets
17	cop atl1	Cash-based operating profits-to-lagged book assets
18	cash at	Cash-to-assets
19	dgp dsale	Change gross margin minus change sales
20	be gr1a	Change in common equity
21	coa gr1a	Change in current operating assets
22	col gr1a	Change in current operating liabilities
23	cowc gr1a	Change in current operating working capital
24	fnl gr1a	Change in financial liabilities
25	lti gr1a	Change in long-term investments
26	lnoa gr1a	Change in long-term net operating assets
27	nfna gr1a	Change in net financial assets
28	nncoa gr1a	Change in net noncurrent operating assets
29	noa gr1a	Change in net operating assets
30	ncoa gr1a	Change in noncurrent operating assets
31	ncol gr1a	Change in noncurrent operating liabilities
32	ocf at chg1	Change in operating cash flow to assets
33	niq at chg1	Change in quarterly return on assets
34	niq be chg1	Change in quarterly return on equity
35	sti gr1a	Change in short-term investments
36	ppeinv gr1a	Change PPE and Inventory
37	dsale dinv	Change sales minus change Inventory
38	dsale drec	Change sales minus change receivables
39	dsale dsga	Change sales minus change SG&A
40	dolvol var 126d	Coefficient of variation for dollar trading volume
41	turnover var 126d	Coefficient of variation for share turnover
42	coskew 21d	Coskewness
43	prc highprc 252d	Current price to high price over last year
44	debt me	Debt-to-market
45	beta dimson 21d	Dimson beta
46	div12m me	Dividend yield
47	dolvol 126d	Dollar trading volume
48	betadown 252d	Downside beta
49	ni ar1	Earnings persistence
50	earnings variability	Earnings variability
51	ni ivol	Earnings volatility
52	ni me	Earnings-to-price
53	ebitda mev	Ebitda-to-market enterprise value
54	eq dur	Equity duration
55	eqnpo 12m	Equity net payout
56	age	Firm age
57	betabab 1260d	Frazzini-Pedersen market beta
58	fcf me	Free cash flow-to-price
59	gp at	Gross profits-to-assets
60	gp atl1	Gross profits-to-lagged assets
61	debt gr3	Growth in book debt (3 years)
62	rmax5 21d	Highest 5 days of return

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Appendix B, continued

N	Abbreviation	Description
63	rmax5 rvol 21d	Highest 5 days of return scaled by volatility
64	emp gr1	Hiring rate
65	iskew capm 21d	Idiosyncratic skewness from the CAPM
66	iskew ff3 21d	Idiosyncratic skewness from the Fama-French 3-factor model
67	iskew hxz4 21d	Idiosyncratic skewness from the q-factor model
68	ivol capm 21d	Idiosyncratic volatility from the CAPM (21 days)
69	ivol capm 252d	Idiosyncratic volatility from the CAPM (252 days)
70	ivol ff3 21d	Idiosyncratic volatility from the Fama-French 3-factor model
71	ivol hxz4 21d	Idiosyncratic volatility from the q-factor model
72	ival me	Intrinsic value-to-market
73	inv gr1a	Inventory change
74	inv gr1	Inventory growth
75	kz index	Kaplan-Zingales index
76	sale emp gr1	Labor force efficiency
77	aliq at	Liquidity of book assets
78	aliq mat	Liquidity of market assets
79	ret 60 12	Long-term reversal
80	beta 60m	Market beta
81	corr 1260d	Market correlation
82	market equity	Market equity
83	rmax1 21d	Maximum daily return
84	mispricing mgmt	Mispricing factor: Management
85	mispricing perf	Mispricing factor: Performance
86	dbnetis at	Net debt issuance
87	netdebt me	Net debt-to-price
88	eqnetis at	Net equity issuance
89	noa at	Net operating assets
90	eqnpo me	Net payout yield
91	chcsho 12m	Net stock issues
92	netis at	Net total issuance
93	ni inc8q	Number of consecutive quarters with earnings increases
94	zero trades 21d	Number of zero trades with turnover as tiebreaker (1 month)
95	zero trades 252d	Number of zero trades with turnover as tiebreaker (12 months)
96	zero trades 126d	Number of zero trades with turnover as tiebreaker (6 months)
97	o score	Ohlson O-score
98	oaccruals at	Operating accruals
99	ocf at	Operating cash flow to assets
100	ocf me	Operating cash flow-to-market
101	opex at	Operating leverage
102	op at	Operating profits-to-book assets
103	ope be	Operating profits-to-book equity
104	op atl1	Operating profits-to-lagged book assets
105	ope bel1	Operating profits-to-lagged book equity
106	eqpo me	Payout yield
107	oaccruals ni	Percent operating accruals
108	taccruals ni	Percent total accruals
109	f score	Pitroski F-score
110	ret 12 1	Price momentum t-12 to t-1
111	ret 12 7	Price momentum t-12 to t-7
112	ret 3 1	Price momentum t-3 to t-1
113	ret 6 1	Price momentum t-6 to t-1
114	ret 9 1	Price momentum t-9 to t-1
115	prc	Price per share
116	ebit sale	Profit margin
117	qmj	Quality minus Junk: Composite
118	qmj growth	Quality minus Junk: Growth
119	qmj prof	Quality minus Junk: Profitability
120	qmj safety	Quality minus Junk: Safety
121	niq at	Change in quarterly return on assets
122	niq be	Quarterly return on equity
123	rd5 at	R&D capital-to-book assets
124	rd me	R&D-to-market

Appendix B, continued

N	Abbreviation	Description
125	rd sale	R&D-to-sales
126	resff3 12 1	Residual momentum t-12 to t-1
127	resff3 6 1	Residual momentum t-6 to t-1
128	ni be	Return on equity
129	ebit bev	Return on net operating assets
130	rvol 21d	Return volatility
131	saleq gr1	Sales growth (1 quarter)
132	sale gr1	Sales growth (1 year)
133	sale gr3	Sales growth (3 years)
134	sale me	Sales-to-market
135	turnover 126d	Share turnover
136	ret 1 0	Short-term reversal
137	niq su	Standardized earnings surprise
138	saleq su	Standardized Revenue surprise
139	tax gr1a	Tax expense surprise
140	pi nix	Taxable income-to-book income
141	bidaskhl 21d	The high-low bid-ask spread
142	taccruals at	Total accruals
143	rskew 21d	Total skewness
144	seas 1 1an	Year 1-lagged return, annual
145	seas 1 1na	Year 1-lagged return, nonannual
146	seas 2 5an	Years 2-5 lagged returns, annual
147	seas 2 5na	Years 2-5 lagged returns, nonannual
148	seas 6 10an	Years 6-10 lagged returns, annual
149	seas 6 10na	Years 6-10 lagged returns, nonannual
150	seas 11 15an	Years 11-15 lagged returns, annual
151	seas 11 15na	Years 11-15 lagged returns, nonannual
152	seas 16 20an	Years 16-20 lagged returns, annual
153	seas 16 20na	Years 16-20 lagged returns, nonannual

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