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Not peer-reviewed version

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Posted Date: 23 March 2026

doi: 10.20944/preprints202603.1710.v1

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Article

A Mathematical Modeling Approach for Analyzing Dairy Supply Chains Under Heterogeneous Conditions

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Abstract

Dairy supply chains are characterized by high structural complexity due to the interaction of multiple actors, stringent quality requirements, and the perishable nature of milk and its derivatives. These systems operate under heterogeneous conditions, where variability in production capacity, logistics infrastructure, and market dynamics significantly affects overall performance. Despite the growing body of literature on optimization, sustainability, and logistics in dairy supply chains, existing mathematical models often rely on simplified or homogeneous assumptions that limit their ability to represent real-world conditions. This study proposes a mathematical modeling framework to analyze the behavior of dairy supply chains under heterogeneous operating conditions. The model integrates production, processing, and distribution decisions within a unified structure, capturing the interdependencies between key components of the system. Scenario-based parameters are incorporated to represent variability in supply and logistics, allowing the evaluation of system performance under different operating conditions. The objective function is formulated to maximize the expected net profit of the system, considering revenues and multiple cost components, including raw material, processing, inventory, transportation, and fixed costs. The proposed formulation enables the analysis of trade-offs between cost efficiency and demand fulfillment, while providing insights into how variability in system conditions affects overall performance. Rather than focusing on isolated optimization problems, the model offers a structured approach to represent the combined effects of operational constraints and system heterogeneity. The results support the use of mathematical modeling as a tool for understanding supply chain behavior in complex agri-food systems and for informing decision-making under non-ideal conditions.

Keywords: supply chain optimization; MILP; resilience; inter-plant reconfiguration; dairy supply chain; disruption management

1. Introduction

Dairy supply chains are characterized by a high level of structural complexity due to the interaction of multiple actors, stringent quality requirements, and the perishable nature of milk and its derivatives. These systems require tightly coordinated operations across production, collection, processing, and distribution stages, where disruptions in early phases can propagate and significantly affect overall system performance. In particular, the dependence on temperature-controlled logistics and timely processing makes dairy supply chains especially sensitive to operational disturbances and infrastructure limitations [1]. A defining feature of dairy supply chains is the strong reliance on small-scale producers with heterogeneous capabilities. Empirical and applied studies have shown that variability in farm size, access to technology, and production practices leads to uneven performance across the network, affecting both product quality and supply reliability [2,3]. This heterogeneity is further reinforced by financial constraints that limit the adoption of key technologies such as refrigerated storage and quality control systems, particularly in

smallholder contexts. As a result, production systems often operate below optimal efficiency levels, introducing instability into upstream supply [4].

In addition to structural heterogeneity, coordination challenges represent a critical issue in dairy supply chains. The interaction between producers, intermediaries, processors, and distributors is often characterized by fragmented decision-making and limited information sharing, which reduces overall system efficiency and increases vulnerability to disruptions. Several studies have emphasized that the lack of integration and alignment among actors can lead to inefficiencies in production planning, distribution, and quality management [5,6]. Logistical constraints further exacerbate these challenges. The distribution of dairy products requires efficient routing, timely delivery, and adequate cold chain infrastructure, all of which are subject to variability and operational risk [7]. Optimization studies in dairy logistics highlight the importance of routing and network design decisions in maintaining system performance yet also reveal the sensitivity of these systems to disruptions in transportation and storage [8,9]. Failures in logistics not only affect delivery performance but also contribute to product deterioration and increased waste.

Moreover, issues related to traceability, transparency, and information management have been widely identified as limitations in dairy supply chains. The absence of robust traceability systems and reliable data integration mechanisms can lead to inconsistencies in quality, increased risk of contamination, and reduced trust among stakeholders [10]. Emerging technologies such as blockchain and IoT have been proposed to address these challenges; however, their implementation remains uneven and context-dependent [11,12]. From a broader perspective, dairy supply chains are also exposed to external disruptions that affect both supply and demand conditions. Events such as the COVID-19 pandemic have demonstrated the vulnerability of these systems to sudden shocks, including demand fluctuations, labor shortages, and logistical constraints, which can lead to significant inefficiencies and losses [13]. These disruptions highlight the need for analytical tools capable of capturing system behavior under varying conditions.

Despite the extensive literature addressing individual aspects such as logistics optimization, sustainability, traceability, and resilience, existing approaches remain largely fragmented [14]. Many studies focus on specific components of the supply chain or assume homogeneous system conditions, limiting their ability to represent the complex and heterogeneous nature of real-world dairy systems [15,16]. There is a lack of mathematical modeling approaches that explicitly incorporate heterogeneous system conditions and allow the evaluation of system behavior under differentiated perturbations affecting multiple components simultaneously. This limitation reveals a relevant research gap in the development of integrated mathematical models capable of capturing the interaction between production, logistics, and economic factors under non-ideal operating conditions. Addressing this gap is essential to better understand how dairy supply chains respond to disturbances and to support more informed decision-making in complex operational environments.

In response to this need, this study proposes a mathematical modeling approach for analyzing the behavior of dairy supply chains under heterogeneous conditions. The model aims to represent key interactions between production, processing, and distribution decisions, providing a structured framework to evaluate system performance and contribute to the analysis of supply chain dynamics under varying operational scenarios.

2. Literature Review: Mathematical Modeling in Dairy Supply Chains

Mathematical modeling has been widely employed to support decision-making in dairy supply chains, particularly in addressing the inherent complexity derived from perishability, multi-echelon structures, and operational interdependencies. The literature shows a progressive evolution from isolated optimization models toward more integrated and multi-dimensional formulations that attempt to capture the interactions between production, logistics, and economic performance.

2.1. Integrated Planning and Multi-Echelon Optimization Models

Early and foundational contributions in dairy supply chain modeling have focused on integrating production, inventory, and distribution decisions within unified optimization frameworks. For instance, Zarei-Kordshouli et al. [16] developed a mixed-integer linear programming (MILP) model for integrated planning and scheduling in yoghurt production systems, incorporating constraints related to perishability, sequence-dependent setups, and working time. This line of research highlights the importance of coordinating operational decisions across different stages to improve efficiency. Similarly, Guarnaschelli et al. [9] proposed a two-stage stochastic MILP model for integrated production and distribution planning in a dairy supply chain. Their formulation explicitly considers uncertainty in both supply and demand, enabling the evaluation of decision robustness under variable conditions. This approach represents a significant step toward incorporating uncertainty into decision-making, although it remains largely focused on tactical planning levels. More recently, Shamnad et al. [1] extended this perspective by proposing an optimization framework that integrates resource flows across the entire supply chain. Using a column generation approach combined with MILP, their model simultaneously considers procurement, production, inventory, and distribution decisions. This work demonstrates a shift toward end-to-end optimization, where upstream and downstream decisions are treated in a coordinated manner.

2.2. Network Design, Facility Location, and Logistics Optimization

Another important research stream focuses on the design and optimization of supply chain networks. Kalai et al. [17] developed a facility location model for dairy logistics using mixed-integer programming, aiming to minimize transportation costs while improving service levels. This type of model is particularly relevant in contexts where infrastructure constraints and geographical dispersion significantly affect performance. In addition, vehicle routing and distribution optimization have been addressed as critical components of dairy logistics. Arenas-Bossa et al. [8] proposed a vehicle routing model tailored to dairy distribution processes, highlighting the importance of routing efficiency in maintaining product quality and reducing operational costs. These studies emphasize the central role of logistics in dairy supply chains, although they often focus on specific subproblems rather than system-wide interactions.

2.3. Sustainable and Closed-Loop Supply Chain Models

Recent advances in dairy supply chain modeling have increasingly incorporated sustainability considerations. Gholizadeh et al. [2] proposed a robust bi-objective model for designing a sustainable closed-loop dairy supply chain, integrating economic and environmental objectives. Their model accounts for product returns, recycling processes, and uncertainty, representing a comprehensive approach to sustainability-oriented decision-making. Similarly, Vanany et al. [18] developed a multi-objective MILP model that integrates economic performance, food waste reduction, and environmental impact in a multi-echelon dairy supply chain. Their results highlight the trade-offs between cost, emissions, and waste, demonstrating the need for multi-criteria decision frameworks in complex agri-food systems.

These contributions reflect a broader trend toward embedding environmental and sustainability dimensions into mathematical models. However, they often rely on predefined structures and objective functions, with limited flexibility to capture heterogeneous responses to operational disturbances.

2.4. Limitations of Current Modeling Approaches and Research Gap

Despite the significant progress in mathematical modeling of dairy supply chains, several limitations remain. Many models focus on specific components of the supply chain—such as production planning, routing, or facility location—without fully capturing the interdependencies between these elements. Even in integrated models, system conditions are often assumed to be

homogeneous, overlooking the structural heterogeneity that characterizes real-world dairy supply chains. Furthermore, while stochastic and robust optimization approaches incorporate uncertainty, they typically represent variability through predefined scenarios or probabilistic distributions, which may not fully capture the differentiated impact of disturbances across system components. Similarly, sustainability-oriented models tend to focus on aggregate indicators (e.g., emissions or waste) without explicitly linking them to heterogeneous operational conditions. These limitations suggest the need for mathematical modeling approaches that explicitly account for heterogeneous system conditions and allow the analysis of supply chain behavior under differentiated perturbations affecting multiple components simultaneously.

Based on the reviewed literature, a key research gap emerges in the development of integrated mathematical models capable of capturing the interaction between production, logistics, and economic variables under heterogeneous and non-ideal conditions. There is a lack of modeling approaches that allow the evaluation of system behavior when disturbances affect different parts of the supply chain in distinct ways.

Addressing this gap is essential to improve the representation of real-world dairy supply chains and to provide more robust analytical tools for decision-making in complex and uncertain environments.

3. Mathematical Model Formulation

3.1. Problem Description

Dairy supply chains are characterized by the interaction of production, processing, and distribution activities under conditions of perishability, capacity constraints, and heterogeneous system performance. As discussed in the literature, decision-making in these systems involves coordinating multiple stages, including raw milk supply, processing operations, and product distribution, while accounting for variability in supply, logistics, and operational conditions (Guarnaschelli et al., 2020; Sel et al., 2015). In this context, the problem addressed in this study consists of modeling the behavior of a dairy supply chain under varying operating conditions, considering the interdependence between production, transportation, and demand fulfillment decisions. The objective is not only to determine optimal operational configurations, but also to analyze how different system conditions affect overall performance.

3.2. Overview of the Dairy Products Supply Chain

The agro-industrial supply chain for dairy products is characterized by the perishability of raw materials and final products, requiring strict logistical coordination and resilience measures to ensure continuous operation. This supply chain consists of multiple stages, including:

- Raw material sourcing: Collection of fresh milk from dairy farms.
- Processing and transformation: Conversion of raw milk into various dairy products such as cheese, yogurt, and milk powder.
- Distribution and storage: Transportation to distribution centers and retailers under controlled temperature conditions.
- Retail and consumption: Final delivery to end consumers through supermarkets, direct sales, or food service industries.

Given its high dependency on logistics, infrastructure, and stable supplier networks, this supply chain is vulnerable to disruptions that can compromise its efficiency and long-term viability. Therefore, an assessment of robustness requirements is essential to ensure operational continuity under adverse conditions. The gathering of raw milk takes place in four production zones situated in the central region of the Bogotá Savannah. Milk is transported in rented vehicles to various small plants processing dairy derivatives. The products manufactured there are then dispatched to various dairy product cooperative distributors in the city of Bogotá. The system comprises two networks, an

external network representing the flow along the dairy supply chain and an internal network representing the pasteurizer production system (see Figure 1). These pasteurizing plants transform milk into low-cost products for consumers in rural towns near the Bogotá Savannah.

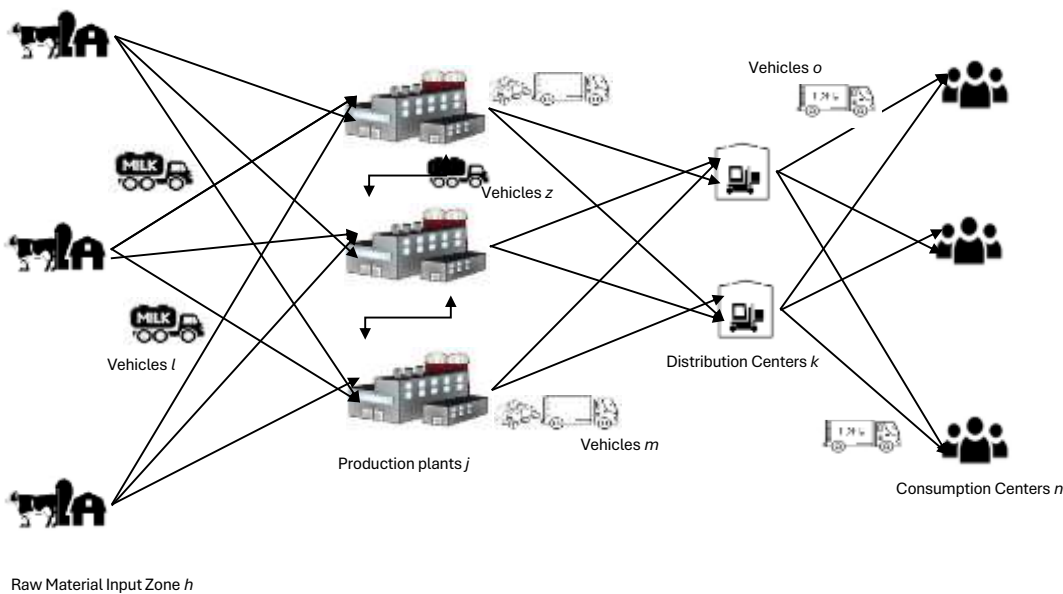


Figure 1. External network.

Furthermore, the network incorporates three pasteurizing plants and two distribution centers responsible for processing and distributing butter, yogurt, and pasteurized milk. The examination of this AFSC encompasses all the parametric details outlined in the proposed mathematical model over a four-week analysis period. Consequently, in this case study, we will present only the essential information required to comprehend the analysis of system capabilities. External networks incorporate transportation between the various links of the network. The operation of the network commences when milk producers supply raw milk (either refrigerated or non-refrigerated) to diverse collection centers strategically positioned in pre-identified areas. The operation of the network concludes upon the delivery of dairy products to various consumption centers in the rural market. The products, including butter, yogurt, and pasteurized milk, traverse this network. The movement through the various links of the network is facilitated by various modes of transportation, distinguished by their respective carrying capacities.

Conversely, the internal production network (see Figure 2) consists of plants that vary in their production capacity. Each internal network encompasses thirteen production units and a Stack Buffer designed for cooling final products. Within this network, there may exist modest inventories at each stage of the processes. The modeling and analysis of the supply chain consider multiple periods, necessitating a dynamic approach.

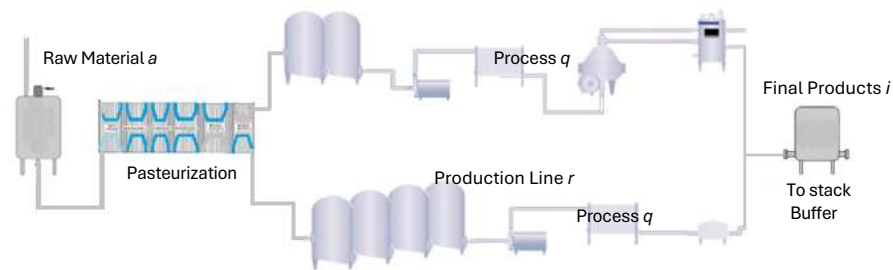


Figure 2. Internal production network.

Both networks are affected during their operation by external and internal disruptions. Externally, disruptions caused by the overflow effect of climate change such as flooding on the distribution of products [19]; the decrease in the yield and quality of milk [20], and the agrarian social mobilization due to discontent with State policies [21]. Internally, the main example of these disruptions is the lack of knowledge/technology transfer in managing production processes [9]. It is necessary to clarify that the model explained below does not consider supply chain design decisions; it only allows analyzing the operation of the networks subject to several disruptions.

3.3. Mathematical Model

The model is tailored for a distribution issue with the objective of maximizing the overall profit in the operation of a supply chain. Mixed Integer Linear Programming (MILP) is employed for solving this model. The mathematical representation of the model considers the following sets:

Set	Description
H	Set of milk sourcing zones
J	Set of processing plants
K	Set of distribution centers
N	Set of market nodes
T	Set of time periods
S	Set of disruption scenarios
Q	Set of processing units
R	Set of production lines (e.g., L1, L2)
A	Set of raw materials
I	Set of final products
L	Set of vehicles for raw milk collection
M	Set of vehicles for plant-to-DC transport
O	Set of vehicles for DC-to-market transport
Z	Set of vehicles for inter-plant transport

The following indices are used to represent elements within each set:

$h \in H$: index of milk sourcing zones

$j \in J$: index of processing plants

$k \in K$: index of distribution centers

$n \in N$: index of market nodes

$t \in T$: index of time periods

$s \in S$: index of disruption scenarios

$q \in Q$: index of processing units

$r \in R$: index of production lines

$a \in A$: index of raw materials

$i \in I$: index of final products

$l \in L$: index of vehicles for raw milk collection

$m \in M$: index of vehicles for plant-to-distribution transport

$o \in O$: index of vehicles for distribution-to-market transport

$z \in Z$: index of vehicles for inter-plant transport

Moreover, it is crucial to consider the subsequent parametric details related to the functioning of a supply chain:

WRM_a : Weight per unit of main raw material a in kg.

WP_i : Weight per unit of product i in kg.

WIP : Weight in kg of intermediate product.

LCT_l : Load capacity of trucks l in kg

LCT_m : Load capacity of trucks m in kg.

LCT_o : Load capacity of truck o in kg.

LCT : Load capacity of the truck z in kg.

$LCTV_l$: Load capacity in m3 of truck l .

$LCTV_m$: Load capacity in m3 of truck m .

$LCTV_o$: Load capacity in m3 of truck o .

$LCTV_z$: Load capacity in m3 of truck z .

NAT_l : Number of available trucks l for transport of main raw material.

NAT_{ms} : Number of trucks m available in scenario s .

NAT_{os} : Number of trucks o available in scenario s .

NAT_{zs} : Number of trucks z available in scenario s .

$MAXT_l$: Maximum number of truck l journeys.

$MAXT_m$: Maximum number of truck m journeys.

$MAXT_o$: Maximum number of truck o journeys.

$MAXT_z$: Maximum number of truck z journeys.

VRH_a : Volume in m3 per unit of main raw material a .

VP_i : Volume in m3 per product i .

VIP : Volume in m3 per intermediate product.

DSP_{hj} : Distance in km between Storage center h to plants j .

DPD_{jk} : Distance in km between plants j and distribution centres k .

DDC_{kn} : Distance in km between distribution centres and Consumption Centres n .

DBP_{jj} : Distance in km between plants.

SRM_{hats} : Supply of raw material type a in zone h in period t in scenario s .

PCR_{qrjts}

: Production capacity in regular time of the process q in plant j line r period t in scenario s .

PCE_{qrjts}

: Production capacity in extra time of the process q in plant j line r in period t in scenario s .

WST_{qrjt} : Waste coefficient in process q in plant j line r in period t .

MOS_{ijt}

: Maximum amount of product i that can be outsourced in plant j in week t .

CDC_{ikt} : Capacity of the distribution centre k of product i .

DEM_{ints}

: Demand for product i in consumption centre n in week t in scenario s (liters).

CIW_{ijt} : Inventory capacity of i in the Warehouse of plant j in week t .

$INRM_{aj}$: Initial Inventory of the main raw material in week zero.

$INFP_{ij}$: Initial inventory of final product in week zero.

$INPC_{ik}$: Initial inventory of product i in CD k .

$INWIP_{qrj}$: Initial inventory in the process q of line r in plant j .

PP_{aht} : Purchase price of main raw material a in Storage center of h in period t .

INC_{ih} : Income for sale of product i in period t in scenario s .

FCP_j : Fixed cost of operation per week of plant j .

$FCCD_k$: Fixed cost of operation per week of CD k .

SC_{mt} : Setup Cost of truck m in week t .

SC_{lt} : Setup Cost of truck l in week t .

SC_{ot} : Setup Cost of truck o in week t .

SC_{zt} : Setup Cost of truck z in week t .

CPR_{qrj} : Cost of production in regular time in process q of line r in plant j .

CPE_{qrj} : Cost of production in extra time in process q of line r in plant j .

COS_{ij} : Cost of outsourcing product i in plant j .

CRM_{aj} : Cost of main raw material a inventory.

$CPRM_{ah}$: Cost of purchase raw material a in zone h .

$CWIP$: Cost of intermediate product inventory .

CFP_{ij} : Cost of final product i in plant j

$CPDC_{ik}$: Cost of finished product i inventory in distribution centres k .

TCP_{ijkt} : Trasport cost per unit of product i from plant j to DC k in period t .

TCP_{iknt} : Trasport cost per unit of product i from k to CC n for period t .

$TCRM_{ahjt}$

: Trasport cost per unit of rae material a from zone h to plant j in period t

$TCWIP_{jtt}$: Transport cost of intermediate product between plants j for period t .

SCP : Shipment cost between processes.

$CAPXR_{qrjts}$: Production capacity at plant j , line r , unit q , in period t , scenario s

$CAPXE_{qjrts}$:

Overtime production capacity of plant j , line r , unit q , in period t , scenario s

$MAXO_{ijts}$

: Maximum capacity to be subcontracted for product i , in plant j , period t , scenario s

$RecOK_{L1,j,t,s}$, $RecOK_{L2,j,t,s}$

: Binary parameters of technical eligibility for reception.

$FailProc_{q,j,t,s}$: Binary parameter that identifies process failure in plant j

$BigMRole$: Large constant used to activate or deactivate logical constraints

αRC : adjustment factor of the effective receiving capacity

P_s : probability of scenario s

The variables in the model are used to represent shipments between links, production levels, inventory levels, vehicle journeys among other important aspects. The study variables are presented below:

QRM_{ahjts}

: Quantity of main raw material to be sent from h to j in period t in scenario s .

QP_{ijkts}

: Quantity of product i to be sent from plant j to DC k in period t in scenario s .

QP_{iknts}

: Quantity of product i to be sent from DC k to CC in period t in scenario s .

SBP_{jqrts}

: Shipment of products between processes q in line r in plant j for period t in scenario s .

$SIPP_{jjts}$

: Shipment of intermediate product from plant j to plant j for period t in scenario s .

X_{ahjqts}

: Quantity of main raw material type a from zone h to process q in line r in plant j for period t in scenario s .

XR_{qrjts}

: Quantity of product i to be produced in regular time process q in line r in plant j for period t in scenario s .

XE_{qrjts}

: Quantity of product i to be produced in extra time process q in line r in plant j for period t in scenario s .

XFP_{ijts} : Total quantity of final product i in plant j in period t in scenario s .

XO_{ijts}

: Quantity of product i to be outsourced in plant j in period t in scenario s .

$INRM_{ajts}$: Inventory of main raw material a in plant j in period t in scenario s .

$INPP_{ijts}$: Inventory of finished product i in plant j in period t in scenario s .

$INPC_{ikts}$

: Inventory of product i in the distribution centre k in period t in scenario s .

IN_{jqrts} : Inventory in the process q of line r in plant j in period t in scenario s .

$NJCH_{lhjts}$: Number of truck l journeys from h to p in period t in scenario s .

NJC_{mjts} : Number of truck m journeys from j to k in period t in scenario s .

$NJCC_{oknts}$: Number of truck o journeys from k to n in period t in scenario s .

$NJPP_{zjjts}$: Number of truck z journeys between plants in period t in scenario s .

$RMCO_s$: Costs of raw material in scenario s .

$PRCO_s$: Production costs in scenario s .

$OSCO_s$: Outsourcing costs in scenario s .

$INCO_s$: Inventory costs in scenario s .

$TRCO_s$: Transport costs in scenario s .

$FICO_s$: Fixed costs in scenario s .

YP_j : 1 if plant j is opening, 0 otherwise

YDC_k : 1 if distribution center k is opening, 0 otherwise

$YSendL1_{j,t,s}, YRecvL1_{j,t,s}$

: Binary variables that indicate whether the plant acts as a transmitter or receiver on line $L1$.

$YSendL1_{j,t,s}, YRecvL1_{j,t,s}$

: Binary variables that indicate whether the plant acts as a transmitter or receiver on line $L1$

4. Model Assumptions

The proposed model is developed under the following assumptions:

Operational assumptions

- The planning horizon is discretized into time periods, and all decisions are made at the beginning of each period.
- Demand and supply are deterministic within each scenario.
- All costs are linear and time-invariant unless modified by disruption scenarios.
- Inventory is allowed only at processing plants.

Production assumptions

- Each processing unit has fixed regular-time and overtime capacities.
- Conversion rates between raw materials and final products are constant.
- Overtime production is limited and more expensive than regular-time production.
- Processing failures are scenario-dependent and modeled through exogenous parameters.

Logistics assumptions

- Transportation is constrained by both weight and volume capacities.
- The number of trips per vehicle is limited per period.
- Transportation times are not explicitly modeled.
- All flows occur within the same period (no lead times).

Reconfiguration and resilience assumptions

- At most one plant can act as a sender in each period and scenario.
- Inter-plant flows are activated only when disruptions occur.
- Reconfiguration decisions are centrally coordinated.
- The system prioritizes maintaining service levels through internal reallocation before outsourcing.

Outsourcing assumptions

- Outsourcing is allowed but limited by maximum capacity.
- Outsourcing is more expensive than internal production.
- Outsourced products are assumed to meet quality standards.

The model represents the supply chain as an integrated system in which material flows, production decisions, and logistics operations are jointly optimized. At the upstream level, raw materials are collected from sourcing zones and transported to processing plants, subject to supply availability and transportation constraints. Once at the plants, raw materials are processed through multiple production stages, where capacity limitations, production yields, and potential losses are explicitly considered. At the downstream level, finished products are distributed to market nodes through distribution centers, ensuring that demand is satisfied as much as possible while respecting logistics and inventory constraints. The model also incorporates outsourcing decisions as an external support mechanism when internal production is insufficient. However, outsourcing is limited and associated with higher costs, ensuring that internal production is prioritized whenever feasible.

A key feature of the model is the incorporation of disruption scenarios that affect supply, processing capacity, and distribution performance. These disruptions are modeled through exogenous parameters that modify system conditions across time periods. To mitigate the impact of disruptions, the model includes an inter-plant reconfiguration mechanism that allows the transfer of semi-processed products between plants. This mechanism enables the system to dynamically redistribute processing loads when a plant experiences capacity loss or failure. The reconfiguration process is governed by logical constraints that define sending and receiving roles, enforce capacity feasibility, and prevent circular flows. As a result, the system can adapt to disruptions while maintaining operational stability. This adaptive behavior reflects a resilience-oriented strategy, in which the system responds to localized failures through coordinated reallocation of resources, thereby preserving service levels and reducing performance degradation.

The objective function maximizes the expected net profit of the dairy supply chain across all scenarios. Expected profit is computed as the probability-weighted sum of scenario-specific net

incomes, where each scenario accounts for revenues minus raw material, processing, outsourcing, inventory, transportation, and fixed operating costs. (Eq. A1).

$$\max Z = \sum_{s \in S} P_s * TINC_s$$

Where:

$$TINC_s = \sum_{i \in I} \sum_{k \in K} \sum_{n \in N} \sum_{t \in T} (INC_{its} * QP_{iknt} - (RMCOS_s + PRCOS_s + OSCOS_s + INCOS_s + TRCOS_s)) + FICO \quad \forall s \quad (A1)$$

$$RMCOS_s = \sum_{a \in A} \sum_{h \in H} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} X_{ahjlt} * CPRM_{ah} \quad \forall s \quad (A2)$$

$$PRCOS_s = \sum_{q \in Q} \sum_{r \in R} \sum_{j \in J} \sum_{t \in T} (XR_{qrjt} * CPR_{qrj} + XE_{qrjt} * CPE_{qrj}) \quad \forall s \quad (A3)$$

$$OSCOS_s = \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} XO_{ijt} * COS_{ij} \quad \forall s \quad (A4)$$

$$INCOS_s = \sum_{a \in A} \sum_{j \in J} \sum_{t \in T} INRM_{ajt} * CRM_{aj} + \sum_{j \in J} \sum_{q \in Q} \sum_{r \in R} \sum_{t \in T} IN_{jqrt} * CWIP + \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} INPP_{ijt} * CFP_{ij} + \sum_{i \in I} \sum_{k \in K} \sum_{t \in T} INPC_{ikt} * CPDC_{ik} \quad \forall s \quad (A5)$$

$$FICO = \sum_{j \in J} CFP_j * YP_j + \sum_{k \in K} CFCD_k * YDC_k \quad (A6)$$

$$TRCOS_s = \sum_{v \in V} \sum_{j \in J} \sum_{k \in K} \sum_{t \in T} SC_{mts} * NJC_{mjkt} + \sum_{l \in L} \sum_{h \in H} \sum_{j \in J} \sum_{t \in T} SC_{lts} * NJCH_{lhjt} + \sum_{o \in O} \sum_{k \in K} \sum_{n \in N} \sum_{t \in T} SC_{ots} * NJCC_{oknt} + \sum_{z \in Z} \sum_{j \in J} \sum_{j' \in J} \sum_{t \in T} SC_{zts} * NJPP_{zjj't} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{t \in T} QP_{ijkts} * TCP_{ijk} + \sum_{i \in I} \sum_{k \in K} \sum_{n \in N} \sum_{t \in T} QP_{iknts} * TCP_{iknt} + \sum_{j \in J} \sum_{j' \in J} \sum_{t \in T} SIPP_{jjts} * TCWIP_{jj't} + \sum_{a \in A} \sum_{h \in H} \sum_{j \in J} \sum_{t \in T} QRM_{ahjts} * TCRM_{ahjt} \quad \forall s \quad (A7)$$

Equations (A2)–(A7) define the different cost components of the system. These include raw material procurement costs, production costs in both regular and overtime modes, outsourcing costs, inventory holding costs, fixed operating costs of plants and distribution centers, and transportation costs across all network links. Each cost component is explicitly linked to decision variables, ensuring that all operational decisions directly impact the economic performance of the system.

$$\sum_{j \in J} \sum_{l \in L} QRM_{ahjls} \leq SRM_{hats} \quad \forall a, h, t, s \quad (A8)$$

$$\sum_{a \in A} \sum_{h \in H} \sum_{l \in L} QRM_{ahjls} = \sum_{r \in R} (XR_{process\ 1',r,j,t,s} + XE_{process\ 1',r,j,t,s}) \quad \forall j, t, s \quad (A9)$$

$$\sum_{j \in J} \left[\sum_{k \in K} QP_{ijkts} + INPP_{ijt-1,s} \right] = \sum_{k \in K} \sum_{n \in N} QP_{iknts} + \sum_{j \in J} INPP_{ijts} \quad \forall i, t, s \quad (A10)$$

$$\sum_{k \in K} QP_{iknts} \leq DEM_{ints} \quad \forall i, n, t, s \quad (A11)$$

Equation (A8) allows limiting the maximum of main raw material that can be collected in the different production centers, according to each production zone offer. Equation (A9) allows balancing the entry of main raw material in processes A of both line one and two of the production system in the plants of processes C. Equation (A10) allows balancing the inputs and outputs in the distribution centers. Equation (A11) allows restricting the demand in the consumption centers, where the relation is less or equal to the demand, thus we can evaluate the level of fulfilment of the demand.

$$\sum_{h \in H} \sum_{j \in J} NJCH_{lhjts} \leq NAT_l * MAXT_l \quad \forall l, t, s \quad (A12)$$

$$NJCH_{lhjts} \leq NAT_l * MAXT_l \quad \forall l, h, j, t, u, s \quad (A13)$$

$$QRM_{ahjls} * WRM_a \leq LCT_l * NJCH_{lhjts} * NAT_l \quad \forall a, h, j, l, t, s \quad (A14)$$

$$QRM_{ahjls} * VRH_a \leq LCTV_l * NJCH_{lhjts} * NAT_l \quad \forall a, h, j, l, s \quad (A15)$$

$$WRM_a * QRM_{ahjls} \leq LCT_l * NJCH_{lhjts} \quad \forall a, h, j, l, t, u, s \quad (A16)$$

$$\sum_{j \in J} \sum_{k \in K} NJC_{mjks} \leq NAT_m * MAXT_m \quad \forall m, t, s \quad (A17)$$

$$NJC_{mjks} \leq MAXT_m * NAT_m \quad \forall j, k, t, u, m, s \quad (A18)$$

$$\sum_{i \in I} QP_{ijkts} * WP_i \leq LCT_m * NJC_{mjks} * NAT_m \quad \forall j, k, m, t, s \quad (A19)$$

$$\sum_{i \in I} QP_{ijkts} * VP_i \leq NAT_m * CCTV_m * NJC_{mjks} \quad \forall j, k, m, t, s \quad (A20)$$

$$\sum_{i \in I} PEP_i * QP_{ijkts} \leq LCT_m * NJC_{mjks} \quad \forall j, t, m, k, u, s \quad (A21)$$

$$\sum_{j \in J} \sum_{j' \in J} NJPP_{zjjts} \leq NAT_z * MAXT_z \quad \forall z, t, s \quad (A22)$$

$$NJPP_{zjts} \leq MAXT_z * NAT_z \quad \forall j, j, t, u, z, s \quad (A23)$$

$$(SIPP_{jj'Process\ 2',-11't,s} + SIPP_{jj'Process\ 2',-12't,s}) * WIP \leq LCT_z * NJPP_{zjts} * NAT_z \quad \forall j, j, z, t, s \quad (A24)$$

$$(SIPP_{jj'Process\ 2',-11't,s} + SIPP_{jj'Process\ 2',-12't,s}) * WIP \leq NAT_z * LCTV_z * MAXT_z \quad \forall j, j, z, t \quad (A25)$$

$$(SIPP_{jj'Process\ 2',-11't,s} + SIPP_{jj'Process\ 2',-12't,s}) * WIP \leq LCT_z * NJPP_{zjts} \quad \forall j, t, z, j, u, s \quad (A26)$$

$$\sum_{k \in K} \sum_{n \in N} NJCC_{oknts} \leq NAT_o * MAXT_o \quad \forall o, t, s \quad (A27)$$

$$NJCC_{oknts} \leq MAXT_o * NAT_o \quad \forall k, t, n, u, o, s \quad (A28)$$

$$\sum_{i \in I} QP_{iknts} * WP_i \leq LTC_o * NAT_o * NJCC_{oknts} \quad \forall k, n, o, t, s \quad (A29)$$

$$\sum_{i \in I} QP_{iknts} * VP_i \leq NAT_o * LCTV_o * MAXT_o \quad \forall k, n, o, t, s \quad (A30)$$

$$\sum_{i \in I} WP_i * QP_{iknts} \leq LTC_o * NJCC_{oknts} \quad \forall t, o, k, n, u, s \quad (A31)$$

The equations (A12) to (A31) allow restricting important aspects in the transport of the product between the different links of the chain. This is according to the type of vehicle, the quantity of vehicles available, the maximum number of possible journeys per period and vehicle's maximum transport capacity in weight and volume. Equations (A12), (A17), (A22) and (A27) allow restricting the maximum number of journeys allowed per type of vehicle used for transport between links. Specifically, for journeys from collection centers to plants, plants to distribution centers, journeys between plants, and journeys from distribution centers to consumption centers. Equations (A13), (A18), (A23) and (A28) are transport availability restrictions per type of vehicle. We consider that each vehicle has a Lead Time for each journey and cannot be used again until the previous journey ends. On the other hand, equations (14), (A19), (A24) and (A29) restrict the quantities (measured in Kg) that can be sent in each type of vehicle and all the vehicles available for shipment. It also occurs with equations (A15), (A20), (A25) and (A30), which restrict the amount of product that can be transported in volumetric terms between links. Additionally, equations (A16), (A21), (A26) and (A31), also allow restricting the amount of product that can be transported, nonetheless in this case the Lead Time of delivery of each type of vehicle is also considered.

$$\begin{aligned} \sum_{a \in A} X_{a,j,'Process\ A_11't,s} + \sum_{a \in A} INRM_{aj,t-1,s} + IN_{Process\ A_11'jt-1,s} \\ = XR_{Process\ A_{11}',j,t,s} + XE_{Process\ A_{11}',j,t,s} + \sum_{a \in A} INRM_{aj,t,s} \\ + IN_{Process\ A_11'jt,s} \end{aligned} \quad \forall q, j, t, s \quad (A32)$$

$$\begin{aligned} SBP_{Process\ A_{11}',Process\ B_{11}',s} + IN_{Process\ A_11'jt-1,s} \\ = XR_{Process\ B_{11}',jts} + XE_{Process\ B_{11}',jts} + IN_{Process\ A_11'jts} \end{aligned} \quad \forall j, t, s \quad (A33)$$

$$\sum_{j \in J} SIPP_{j'Process B_l1'ts} + SBP_{Process B_l1',Process C_l1's} + IN_{Process B_l1'jt-1,s} \quad (A34)$$

$$= XR_{Process C_l1'jts} + XE_{Process C_l1'jts} + IN_{Process B_l1'jts} \quad \forall j, t, s$$

$$\sum_{a \in A} X_{main_raw2',j',Process A_l2ts} + \sum_{a \in A} INRM_{main_raw2',j,t-1,s} + IN_{q,j,t-1,s} \quad (A35)$$

$$= XR_{Process A_{l2}',j,t,s} + XE_{Process A_{l2}',j,t,s} + \sum_{a \in A} INRM_{main_raw2',j,t,s}$$

$$+ IN_{q,j,t,s} \quad \forall q, j, t, s$$

$$SBP_{Process A_{l2}',Process B_{l2}',s} + IN_{Process B_{l1'jt-1,s}} \quad (A36)$$

$$= XR_{Process B_{l2'jts}} + XE_{Process B_{l2'jts}} + IN_{Process B_{l2'jts}} \quad \forall j, t, s$$

$$\sum_{j \in J} SIPP_{j'Process B_{l2'ts}} + SBP_{Process B_{l2}',Process C_{l2',s}} + IN_{Process B_{l2'jt-1,s}} \quad (A37)$$

$$= XR_{Process C_{l2'jts}} + XE_{Process C_{l2'jts}} + IN_{Process B_{l2'jts}} \quad \forall j, t, s$$

$$SBP_{Process C_{l1}',Process D',s} + IN_{Process C_{l1'jt-1,s}} \quad (A38)$$

$$= XR_{Process D'jts} + XE_{Process D'jts} + IN_{Process C_{l1'jts}} \quad \forall j, t, s$$

$$SBP_{Process D',Process D',s} = XR_{Process D'jts} + XE_{Process D'jts} \quad \forall j, t, s \quad (A39)$$

$$SBP_{Process C_{l1}',Process E',s} + SBP_{Process C_{l2}',Process E',s} + IN_{Process C_{l2,l2',jt-1,s}} \quad (A40)$$

$$= XR_{Process E'jts} + XE_{Process E'jts} + IN_{Process C_{l2',jts}} \quad \forall j, t, s$$

$$SBP_{Process E',Process F',s} + IN_{Process E'jt-1,s} \quad (A41)$$

$$= XR_{Process F'jts} + XE_{Process F'jts} + IN_{Process E'jts} \quad \forall j, t, s$$

$$SBP_{Process F',Process F',s} = XR_{Process F'jts} + XE_{Process F'jts} \quad \forall j, t, s \quad (A42)$$

$$SBP_{Process C_{l2}',Process G',s} + IN_{Process C_{l2'jt-1,s}} \quad (A43)$$

$$= XR_{Process G'jts} + XE_{Process G'jts} + IN_{Process C_{l2'jts}} \quad \forall j, t, s$$

$$SBP_{Process G',Process G',s} = XR_{Process G'jts} + XE_{Process G'jts} \quad \forall j, t, s \quad (A44)$$

Equations (A32) to (A44) enable us to carry out the balance in the productive units of the plants. To do this, we took into account the inputs of product in the process, the inventory and the volumes of product to be processed. Then, we performed the balance in all units of process.

$$WST_{Process A_{l1'jts}} * (XR_{Process A_{l1',jts}} + XE_{Process A_{l1',jts}}) + IN_{Process A_{l1'jt-1,s}} - \quad (A45)$$

$$IN_{Process A_{l1'jts}} = SBP_{Process A_{l1',Process B_{l1',s}} \quad \forall j, t, s$$

$$(1 - WST_{Process B_{l1'jt}}) * (XR_{Process B_{l1',jt}} + XE_{Process B_{l1',jt}}) + IN_{Process B_{l1'jt-1}} - IN_{Process B_{l1'jt}} \quad (A46)$$

$$= SBP_{Process B_{l1',Pas_{l1'}}} + \sum_{j \in J} SIPP_{j'Process B_{l1't}} \quad \forall j, t, s$$

$$\begin{aligned}
(1 - WST_{Process\ C_11',jt}) * (XR_{Process\ C_11',jts} + XE_{Process\ C_11',jts}) + IN_{Process\ C_11'jt-1,s} \\
- IN_{Process\ C_11'jts} \\
= SBP_{Process\ C_11',Process\ D',s} + PAP_{Process\ C_{11}',Mad',s} \quad \forall\ j, t, s
\end{aligned} \tag{A47}$$

$$\begin{aligned}
(1 - WST_{Process\ A_12',jt}) * (XR_{Process\ A_12',jts} + XE_{Process\ A_12',jts}) + IN_{Process\ A_12'jt-1,s} \\
- IN_{Process\ A_12'jts} = SBP_{Process\ A_{12}',Ref_{12},s'} \quad \forall\ j, t, s
\end{aligned} \tag{A48}$$

$$\begin{aligned}
(1 - WST_{Process\ B_12',jts}) * (XR_{Process\ B_{12}',jts} + XE_{Process\ B_{12}',jts}) + IN_{Process\ B_{12}'jt-1,s} - IN_{Process\ B_{12}'jts} \\
= SBP_{Process\ B_{12}',Process\ C_{12},s'} \quad \forall\ j, t, s
\end{aligned} \tag{A49}$$

$$\begin{aligned}
(1 - WST_{Process\ C_12',jt}) * (XR_{Process\ C_{12}',jts} + XE_{Process\ C_{12}',jts}) + INV_{PAST2}_{jt-1,s} - INV_{PAST2}_{jts} \\
= SBP_{Process\ C_{12}',Process\ E',s} + PAP_{Process\ C_{12}',Process\ E',s} \quad \forall\ j, t, s
\end{aligned} \tag{A50}$$

$$\begin{aligned}
(1 - WST_{Process\ D',rjts}) * (XR_{Process\ D',rjts} + XE_{Process\ D',rjts}) + IN_{Process\ D'rjt-1,s} \\
- IN_{Process\ D'rjts} = SBP_{Process\ D',Process\ D'} \quad \forall\ r, j, t, s
\end{aligned} \tag{A51}$$

$$\begin{aligned}
(1 - WST_{Process\ E',rjts}) * (XR_{Process\ E',rjts} + XE_{Process\ E',rjts}) + IN_{Process\ E'rjt-1,s} \\
- IN_{Process\ E'rjts} = SBP_{Process\ E',Process\ F's} \quad \forall\ r, j, t, s
\end{aligned} \tag{A52}$$

$$\begin{aligned}
(1 - WST_{Process\ F',rjts}) * (XR_{Process\ F',rjts} + XE_{Process\ F',rjts}) + IN_{Process\ F'rjt-1,s} \\
- IN_{Process\ F'rjts} = SBP_{Process\ F',Process\ F's} \quad \forall\ r, j, t, s
\end{aligned} \tag{A53}$$

$$\begin{aligned}
(1 - WST_{Process\ G',rjts}) * (XR_{Process\ G',rjts} + XE_{Process\ G',rjts}) + IN_{Process\ G'rjt-1,s} - \\
IN_{Process\ G'rjts} = SBP_{Process\ G',Process\ G's} \quad \forall\ r, j, t, s
\end{aligned} \tag{A54}$$

Based on equations (A45) to (A54), the flows of the product between the processes can be restricted, as during production some product can be wasted due to evaporation and other environmental factors of the production system. Therefore, the parameter WST_{qrjt} marks the percentage of the production that can be lost after the operation of each productive unit.

$$XR_{Process\ D'',rjts} + XE_{Process\ D'',rjts} = XPF_{Process\ D',rjts,s} \quad \forall\ r, j, t, s \tag{A55}$$

$$XR_{Process\ F'',rjts} + XE_{Process\ F'',rjts} = XPF_{Process\ F',rjts,s} \quad \forall\ r, j, t, s \tag{A56}$$

$$XR_{Process\ G'',rjts} + XE_{Process\ G'',rjts} = XPF_{Process\ G',rjts,s} \quad \forall\ r, j, t, s \tag{A57}$$

or the calculation of the total production in the plants, we used equations (A55), (A56) and (A57). These equations consider the production in regular and extra time for each processed product.

$$XPF_{i,j,t,s} + XO_{ijts} + INPP_{ijts} = \sum_{k \in K} XFP_{ijkts} + INPP_{ijts} \quad \forall\ i, j, t, s \tag{A58}$$

$$INPP_{ijts} = CASB_{ijts} \quad \forall\ i, j, t, s \tag{A59}$$

$$INPC_{ikt} = CDC_{ikt} \quad \forall\ i, k, t, s \tag{A60}$$

Equation (A58) allow calculating the equilibrium in the Stack Buffer of the pasteurizing plants. It studies the produced quantities of each product, the outsourced quantities, inventories and

shipments of existing products towards the distribution centers. On the other hand, equations (A59) and (A60) allow restricting the inventory of finished products in the Stack Buffer of the pasteurization plants and of the distribution centers, respectively.

$$IN_{qrts} \leq CAP_{q,j,r,s} \quad \forall q, j, r, t, s \quad (A61)$$

Using equations (A61) it is possible to restrict the inventories of intermediate product in the plants for each period.

$$XR_{qrts} \leq CAPXR_{qrts} \quad \forall q, r, j, t, s \quad (A62)$$

$$XE_{qrts} \leq CAPXE_{qrts} \quad \forall q, r, j, t, s \quad (A63)$$

$$XO_{ijts} \leq MAXO_{ijts} \quad \forall q, j, t, s \quad (A64)$$

Using the equations (A62) and (A63), the quantity to be produced can be restricted in regular time and in extra time considering the installed capacity of each productive unit. On the other hand, equation (64) seeks to restrict the maximum of finished products that can be outsourced in each plant per period.

The set of constraints (A65) to (A80) models the inter-plant reconfiguration mechanism of the production system under disruption scenarios. This formulation enables the dynamic redirection of semi-processed products between plants when local processing capabilities are compromised, thus supporting system-wide continuity and resilience. This mechanism is consistent with resilience-oriented modeling approaches, where system performance is preserved through controlled redistribution of flows under adverse conditions. It also aligns with immune-inspired analogies, in which localized failures trigger coordinated system-wide responses to maintain functionality.

$$\sum_{\substack{j' \in J \\ j' \neq j}} SIPP_{j,j',Process\ B_I1',t,s} \leq BigMRole \cdot YSendL1_{j,t,s} \quad \forall j, t, s \quad (A65)$$

$$\sum_{\substack{j' \in J \\ j' \neq j}} SIPP_{j,j',Process\ B_I1',t,s} \leq BigMRole \cdot YRecvL1_{j,t,s} \quad \forall j, t, s \quad (A66)$$

Constraints (A65) and (A66) ensure that inter-plant flows are only activated when a plant is explicitly assigned a sending or receiving role, respectively. These binary role variables act as logical switches, preventing uncontrolled flow exchanges across the network.

$$YSendL1_{j,t,s} + YRecvL1_{j,t,s} \leq 1 \quad \forall j, t, s \quad (A67)$$

Constraint (A67) enforces mutual exclusivity between sending and receiving roles. A plant cannot simultaneously operate as both an origin and a destination of inter-plant flows within the same period and scenario. This restriction avoids circular or conflicting flow configurations.

$$\sum_{\substack{j' \in J \\ j' \neq j}} SIPP_{j,j',Process\ B_I1',t,s} \leq BigMRole \cdot YP_j \cdot RecOK_{L1,j,t,s} \quad \forall j, t, s \quad (A68)$$

$$\sum_{\substack{j' \in J \\ j' \neq j}} SIPP_{j,j',Process\ B_I1',t,s} \quad (A69)$$

$$\leq \alpha RC[(CAPXR_{Process\ C_I1',j,t,s} + CAPXE_{Process\ C_I1',j,t,s})YP_j - XR_{Process\ C_I1',j,t,s}] \quad \forall j, t, s$$

$$\sum_{\substack{j' \in J \\ j' \neq j}} SIPP_{j,j',Process\ B_I1',t,s} \leq BigMRole \cdot FailProc_{Process\ C_I1,j,t,s} \quad \forall j, t, s \quad (A70)$$

$$\sum_{\substack{j' \in J \\ j' \neq j}} SIPP_{j,j',Process\ B_11',t,s} \leq BigMRole \cdot (1 - YRecvL1_{j,t,s}) \quad \forall j, t, s \quad (A71)$$

$$\sum_{j \in J} YSendL1_{j,t,s} \leq 1 \quad \forall t, s \quad (A72)$$

$$\sum_{\substack{j' \in J \\ j' \neq j}} SIPP_{j,j',Process\ B211',t,s} \leq BigMRole \cdot YSendL2_{j,t,s} \quad \forall j, t, s \quad (A73)$$

$$\sum_{\substack{j' \in J \\ j' \neq j}} SIPP_{j,j',Process\ B211',t,s} \leq BigMRole \cdot YRecvL2_{j,t,s} \quad \forall j, t, s \quad (A74)$$

$$YSendL2_{j,t,s} + YRecvL2_{j,t,s} \leq 1 \quad \forall j, t, s \quad (A75)$$

$$\sum_{\substack{j' \in J \\ j' \neq j}} SIPP_{j,j',Process\ B_12',t,s} \leq BigMRole \cdot YP_j \cdot RecOK_{L2,j,t,s} \quad \forall j, t, s \quad (A76)$$

$$\sum_{\substack{j' \in J \\ j' \neq j}} SIPP_{j,j',Process\ B_12',t,s} \quad (A77)$$

$$\leq \alpha RC[(CAPXR_{Process\ C_12',j,t,s} + CAPXE_{Process\ C_12',j,t,s})YP_j - XR_{Process\ C_12',j,t,s}] \quad \forall j, t, s \quad (A78)$$

$$\sum_{\substack{j' \in J \\ j' \neq j}} SIPP_{j,j',Process\ B_12',t,s} \leq BigMRole \cdot FailProc_{Process\ C_12,j,t,s} \quad \forall j, t, s$$

$$\sum_{\substack{j' \in J \\ j' \neq j}} SIPP_{j,j',Process\ B_12',t,s} \leq BigMRole \cdot (1 - YRecvL2_{j,t,s}) \quad \forall j, t, s \quad (A79)$$

$$\sum_{j \in J} YSendL2_{j,t,s} \leq 1 \quad \forall t, s \quad (A80)$$

Additionally, constraints (A71) and (A79) reinforce this logic by explicitly preventing any plant assigned as a receiver from sending flows during the same time. These complementary constraints strengthen model stability and eliminate potential degeneracies.

Constraints (A68) and (A76) impose technical eligibility conditions on receiving plants. A plant is allowed to receive semi-processed products only if it is active in the network and satisfies predefined technical requirements for the corresponding processing line. These conditions ensure that reconfiguration decisions remain physically and operationally feasible, avoiding assignments to plants that lack the necessary infrastructure or capabilities.

Constraints (A69) and (A77) limit the volume of incoming inter-plant flows based on the effective residual capacity of the receiving plant in the subsequent processing stage (pasteurization for L1 and L2, respectively). Specifically, the quantity received cannot exceed the difference between the adjusted available capacity (including potential expansion) and the current production level. This difference is further scaled by a reception efficiency parameter. This formulation guarantees that

reallocated flows do not overload downstream processes and preserves feasibility within the plant's operational limits.

Constraints (A77) and (A78) link inter-plant shipments to the presence of process-level disruptions. A plant is allowed to send semi-processed products to other plants only if it experiences a failure in the subsequent processing stage. This condition embeds a key resilience mechanism: reconfiguration is not arbitrarily activated but is instead triggered by operational disruptions. As a result, inter-plant flows act as an adaptive response to maintain system performance under stress.

Constraints (A72) and (A80) restrict the number of sending plants to at most one per period and scenario for each processing line. This assumption simplifies coordination and avoids excessive fragmentation of flows across multiple origins. While this design choice limits the flexibility of the system, it enhances interpretability and ensures a controlled and traceable reconfiguration process. It is particularly suitable for systems where centralized or coordinated decision-making is assumed.

$$X_{ahjlts}, XE_{qrjts}, XFP_{iknts}, XR_{qrjts}, SBP_{qjts}, SIPP_{jjts}, XO_{ijts}, QRM_{ahjlts}, QP_{ijkts}, QP_{iknts} \quad (81)$$

$$INRM_{ajts}, INPP_{ijts}, INPC_{ikts}, IN_{q,j,r,t,s} \geq 0$$

$$NJC_{mjkt}, NJCH_{lhjt}, NJCC_{oknt}, NJPP_{zjtt} \in \{Z\} \quad (82)$$

Finally, yet importantly, it is necessary to consider the restriction of non-negativity for the study variables, shown in equation (A81). The variables of expression (A82) must be positive integers as they are journeys per vehicle type.

4.1. Modeling Considerations

The proposed model captures key characteristics identified in the literature on dairy supply chains. First, it integrates production and distribution decisions within a unified framework, consistent with prior optimization approaches (Guarnaschelli et al., 2020; C ccola et al., 2022). Second, it allows the representation of varying operating conditions through scenario-dependent parameters, reflecting the sensitivity of dairy systems to disruptions in supply and logistics. Unlike traditional models that assume homogeneous system conditions, this formulation introduces variability directly into capacity and flow constraints, enabling the analysis of system behavior under differentiated conditions. This is particularly relevant in dairy supply chains, where heterogeneity in production capabilities and infrastructure plays a critical role in overall performance.

4.2. Reproducibility and Data Availability

To ensure reproducibility, this document provides a complete description of the mathematical formulation, including sets, parameters, decision variables, objective function, and constraints. Additionally, Tables A1–A6 present a representative numerical instance of the model, including supply, demand, processing capacities, transportation parameters, and outsourcing limits. The model can be implemented in any algebraic modeling language (e.g., GAMS, AMPL, Pyomo) and solved using standard MILP solvers such as CPLEX or Gurobi. Scenario-specific parameters are provided separately in the associated manuscript, allowing the replication of computational experiments under different disruption conditions.

Tables A1–A6 summarize the main numerical inputs used to instantiate the dairy supply chain case study and to support the reproducibility of the mathematical model. These tables provide representative values for supply, demand, installed capacities, fleet characteristics, physical conversion factors, and outsourcing limits. Together, they define the operational structure of the network and the baseline conditions under which the optimization model is solved.

Table A1 reports the weekly raw milk supply available from each sourcing zone, differentiated by milk type. These values represent the baseline procurement capacity of the system and constitute the main upstream input of the production network. The data reflects heterogeneity across sourcing areas, both in terms of total available volume and in the composition of milk types, which is relevant for downstream processing decisions. Table A2 presents the weekly demand for final products by

market node. The demand structure includes butter, yogurt, and pasteurized milk, which are distributed across three demand zones over the planning horizon. This table captures the temporal and spatial variability of customer requirements and defines the fulfillment targets that guide the production, inventory, and distribution decisions of the model. Table A3 contains the installed regular-time processing capacities for selected processes and plants. The reported values show that capacities vary not only across plants but also across processing stages, reflecting the structural differences of the production system. In particular, the table includes key operations such as pasteurization and packaging, which are critical for evaluating bottlenecks, feasible production plans, and the potential for inter-plant reconfiguration under operational stress.

Table A1. Weekly raw milk supply by sourcing zone and milk type (liters).

Zone	Milk type	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8
h1	Milk1	2240	2240	2080	2160	2365	2365	2205	2285
h1	Milk2	48600	54000	52200	49500	48725	54125	52325	49625
h2	Milk1	2160	2000	2160	2160	2285	2125	2285	2285
h2	Milk2	46800	45000	50400	45000	46925	45125	50525	45125
h3	Milk1	2000	2080	2240	2080	1875	1955	2115	1955
h3	Milk2	48600	53100	54000	47700	48475	52975	53875	47575
h4	Milk1	2240	2080	2240	2080	2115	1955	2115	1955
h4	Milk2	47700	45900	51300	48600	47575	45775	51175	48475

Table A2. Weekly demand for final products by market node (units).

Product	Market	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8
Butter	n1	536	435	449	578	661	560	574	703
Butter	n2	564	464	459	570	689	589	584	695
Butter	n3	620	487	508	635	745	612	633	760
yogurt	n1	6866	6868	6643	7885	6991	6993	6768	8010
yogurt	n2	6226	6598	6885	8153	6351	6723	7010	8278
yogurt	n3	2582	2536	2592	2697	2707	2661	2717	2822
pasteurized milk	n1	11488	11057	11165	11773	11613	11182	11290	11898
pasteurized milk	n2	11010	11414	11093	11175	11135	11539	11218	11300
pasteurized milk	n3	11067	11358	11291	11163	11192	11483	11416	11288

Table A3. Installed regular-time processing capacity for selected processes and plants (liters/week).

Process	Plant	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8
Pasteurization l1	j1	9171	8988	9028	9377	9296	9113	9153	9502
Pasteurization l1	j2	7518	7669	7558	7457	7643	7794	7683	7582
Pasteurization l1	j3	5901	5976	5916	5948	6026	6101	6041	6073
Pasteurization l1	j4	6039	6105	6070	6087	6184	6244	6190	6228
Pasteurization l1	j5	7021	7068	7300	7297	7106	7305	7154	7219
Pasteurization l2	j1	25263	26457	25747	25117	25388	26582	25872	25242
Pasteurization l2	j2	25263	25899	25015	25117	25388	26024	25140	25242
Pasteurization l2	j3	26474	26453	26404	26474	26599	26578	26529	26599
Pasteurization l2	j4	26633	26603	26537	26607	26728	26721	26651	26739
Pasteurization l2	j5	27595	27641	27622	27649	27949	27649	27810	27808
Yogurt packaging	j1	9441	1048	5490	7374	9566	1173	5615	7499
Yogurt packaging	j2	7652	7800	7681	7447	7777	7925	7806	7572
Yogurt packaging	j3	6670	6413	6534	6692	6795	6538	6659	6817
Yogurt packaging	j4	6817	6539	6661	6850	6951	6678	6797	6951
Yogurt packaging	j5	7974	7611	7853	7918	8062	7723	7772	7989
Milk packaging	j1	17942	17242	18011	17611	18067	17367	18136	17736
Milk packaging	j2	16537	16916	16479	16142	16662	17041	16604	16267
Milk packaging	j3	15701	15930	15296	15100	15826	16055	15421	15225
Milk packaging	j4	15857	16064	15437	15227	15965	16207	15577	15356
Milk packaging	j5	16864	17255	16651	16464	16885	17239	16576	16382

Table A4 describes the transport fleet and trip-capacity parameters used in the logistics constraints. The table includes the number of available vehicles, the maximum number of trips per week, and the weight and volume capacity for each vehicle type. These parameters are essential for linking material flows with physical transportation resources, thereby ensuring that procurement, plant-to-distribution shipments, and inter-plant reallocations remain operationally feasible.

Table A4. Transport fleet and trip capacity parameters.

Vehicle set	Vehicle	Available vehicles	Max trips/week	Weight capacity	Volume capacity
Raw milk collection	l1	10	18	4000 kg	4 m ³
Raw milk collection	l2	15	16	8000 kg	8 m ³
Raw milk collection	l3	18	14	12000 kg	12 m ³
Plant to DC	m1	5	18	5000 kg	10 m ³
Plant to DC	m2	10	19	10000 kg	15 m ³
DC to market	o1	5	16	5000 kg	10 m ³
DC to market	o2	10	16	10000 kg	15 m ³
DC to market	o3	15	15	12000 kg	20 m ³

Inter-plant	z1	10	18	5000 kg	5 m ³
Inter-plant	z2	15	19	10000 kg	10 m ³

Table A5 reports the physical conversion factors used in the transport constraints. Since the model simultaneously considers weight-based and volume-based vehicle limits, each raw material, intermediate product, and final product must be associated with appropriate conversion coefficients. These factors allow the model to translate production and shipment quantities into logistics loads and therefore support a realistic representation of transport utilization.

Table A5. Product conversion factors used in transport constraints.

Material/product	Weight factor (kg/unit)	Volume factor (m ³ /unit)
Milk 1	1.05	0.00105
Milk 2	1.02	0.00102
Butter	1.05	0.00105
Yogurt	1.02	0.00108
Pasteurized milk	1.01	0.00106
Intermediate product	1.02	0.00102

Table A6. Maximum outsourcing capacity.

Product		Plant	Effective max outsourcing
Butter (Kg)		j1	125
Butter (Kg)		j2	125
Butter (Kg)		j3	138
Butter (Kg)		j4	165
Butter (Kg)		j5	283
Yogurt (liters)		j1	250
Yogurt (liters)		j2	250
Yogurt (liters)		j3	190
Yogurt (liters)		j4	210
Yogurt (liters)		j5	185
Pasteurized (liters)	milk	j1	440
Pasteurized (liters)	milk	j2	315
Pasteurized (liters)	milk	j3	340
Pasteurized (liters)	milk	j4	450
Pasteurized (liters)	milk	j5	390

Table A6 provides the maximum outsourcing capacities by product and plant. These values define the upper bound of external support that can be activated when in-house production is insufficient to satisfy market demand. In this sense, outsourcing acts as a complementary resilience mechanism, enabling the system to preserve service levels when internal capacities are disrupted or temporarily constrained.

Tables A1–A6 define the baseline numerical structure of the case study. They describe the main material, operational, and logistical conditions of the dairy network before the introduction of disruption scenarios. The scenario-specific parameters, which modify demand, supply, process availability, and distribution performance, are presented separately in the manuscript to distinguish the deterministic case configuration from the stress conditions used in the computational experiments.

5. Discussion

The proposed mathematical model provides a structured representation of key interactions within dairy supply chains, particularly those related to production, transportation, and demand fulfillment decisions. In line with previous studies, the formulation integrates multiple decision layers into a unified framework, enabling the analysis of system performance beyond isolated operational components [9,22]. This integration is especially relevant in dairy systems, where decisions in one stage—such as raw milk availability or processing capacity—directly influence downstream performance.

One important aspect highlighted by the model is the role of heterogeneous operating conditions. By incorporating scenario-dependent parameters affecting production and transportation capacities, the formulation captures variability in system performance that is often overlooked in more deterministic or homogeneous models. This is consistent with empirical observations reported in the literature, where differences in infrastructure, resource availability, and operational capabilities lead to uneven performance across supply chain actors [2,18]. Furthermore, the model illustrates the trade-offs between cost efficiency and service performance. The inclusion of unmet demand penalties allows the evaluation of system behavior under constrained conditions, revealing how capacity limitations and logistical disruptions can affect demand satisfaction. Similar trade-offs have been identified in previous optimization models, particularly in contexts where perishability and time constraints impose additional pressure on system performance [8,13].

From a modeling perspective, the formulation extends existing approaches by explicitly incorporating variability through scenario-based parameters, rather than relying solely on static or average conditions. While stochastic and robust models have addressed uncertainty in dairy supply chains [4,23], the present approach emphasizes the differentiated impact of operating conditions on system constraints, allowing a more flexible representation of real-world dynamics. The results derived from the model can be interpreted in terms of system viability, understood as the ability of the supply chain to maintain acceptable performance levels under varying conditions. Rather than defining viability as a formal metric, the model enables the identification of operating regions where demand fulfillment and cost performance remain within acceptable bounds. This interpretation is particularly relevant in dairy supply chains, where structural heterogeneity and operational constraints limit the ability to sustain performance under adverse conditions.

Proposed model contributes to the literature by providing a flexible and integrated framework for analyzing dairy supply chain behavior under heterogeneous conditions. It complements existing optimization models by incorporating variability directly into system constraints, thereby enhancing the analytical capacity to explore how different operational conditions influence overall performance.

6. Concluding Remarks

This study presented a mathematical modeling approach for analyzing the behavior of dairy supply chains under heterogeneous operating conditions. The proposed formulation integrates

production, transportation, and demand fulfillment decisions within a unified framework, allowing the evaluation of system performance across different scenarios. By incorporating variability directly into capacity and flow constraints, the model provides a flexible representation of the structural and operational characteristics that define dairy supply chains. The results highlight the importance of considering heterogeneous system conditions when modeling agri-food supply chains. Unlike traditional approaches that rely on homogeneous or average assumptions, the proposed model captures the differentiated impact of operating conditions on system performance. This is particularly relevant in dairy supply chains, where variability in production capacity, infrastructure, and logistics plays a significant role in shaping overall outcomes.

From an analytical perspective, the model contributes to the literature by offering a structured way to explore the interaction between key supply chain components under non-ideal conditions. While existing studies have addressed optimization, sustainability, and uncertainty, the present work emphasizes the role of variability within system constraints as a mechanism to better represent real-world dynamics. This enables a more comprehensive understanding of how different configurations of the system respond to changing conditions. The framework allows for the interpretation of system behavior in terms of performance sustainability under varying conditions. Although viability is not explicitly modeled as a formal construct, the identification of operating regions where demand fulfillment and cost performance remain within acceptable bounds provides useful insights into the stability of the system. This perspective is particularly valuable for understanding the limits within which dairy supply chains can operate effectively.

From a practical standpoint, the model offers decision support capabilities for analyzing the effects of capacity limitations, logistical constraints, and demand variability. It can be used to evaluate alternative configurations and identify critical factors that influence system performance, supporting more informed decision-making in complex operational environments. This study opens several avenues for future research. Further work may extend the model by incorporating additional system dimensions, such as multi-product interactions, inventory dynamics, and environmental considerations. In addition, future studies may explore more advanced representations of system behavior, including adaptive or dynamic mechanisms to capture the evolution of supply chain performance over time. These extensions would contribute to a deeper understanding of complex supply chain systems and enhance the applicability of mathematical modeling approaches in real-world contexts.

6.1. Future Research Directions

The proposed model provides a simplified but structured representation of dairy supply chain dynamics, which opens several opportunities for further development. One natural extension involves incorporating multi-product interactions and inventory decisions, particularly relevant in dairy systems where product differentiation and shelf-life constraints significantly influence operational performance. Including these elements would allow a more detailed representation of trade-offs across different product categories and time horizons. Another relevant direction lies in expanding the model to account for environmental and sustainability considerations. Integrating variables such as emissions, waste generation, and energy consumption would enable a broader evaluation of system performance, aligning the modeling framework with current research trends in sustainable supply chains. This extension would also facilitate the analysis of trade-offs between economic efficiency and environmental impact.

The representation of system variability can also be further refined. While the current approach incorporates heterogeneous operating conditions through scenario-dependent parameters, future work could explore more detailed representations of variability, including dynamic changes over time. Such extensions would improve the model's ability to capture evolving disruptions and shifting operational constraints, which are common in real-world dairy supply chains. The modeling framework could be enhanced by incorporating adaptive decision-making mechanisms. Allowing decisions to evolve in response to changing system conditions would provide a more realistic

representation of supply chain behavior, particularly in environments characterized by uncertainty and structural heterogeneity.

Finally, applying the model to empirical case studies would provide valuable insights into its practical applicability. Testing the framework in real-world dairy supply chains, especially those involving small-scale producers and heterogeneous infrastructure conditions, would contribute to validating the model and identifying context-specific challenges and opportunities for improvement.

References

1. M S, John FR. Multi-objective Optimization for Sustainability in the Cold Supply Chain of the Dairy Products Industry. Process Integration and Optimization for Sustainability 2026. <https://doi.org/10.1007/s41660-025-00675-9>.
2. Gholizadeh H, Pasha A, Hejazi TH, Jahani H. Reconfigurable and Resilient Closed-Loop Supply Chain Design: A Simulation–Optimization Approach. Global Journal of Flexible Systems Management 2025;26:85–113. <https://doi.org/10.1007/s40171-024-00427-7>.
3. Alzate Rendón IC, Álvarez Gallo SM, Boada A. Supply chain resilience through collaborative networks and dynamic capabilities: evidence from an agri-food productive chain in Colombia. Discover Sustainability 2025;6. <https://doi.org/10.1007/s43621-025-02094-9>.
4. Nirmala DAR, Ramaswamy S, Logesh K, Gnanaraj SJP. Empirical study on risk mitigation for dairy supply chain management of Aavin Co-operative Milk Producers’ Union Ltd. Mater. Today Proc., vol. 49, Elsevier Ltd; 2020, p. 3657–60. <https://doi.org/10.1016/j.matpr.2021.09.243>.
5. Yang M, Işık C, Yan J. Analysis of collaborative control of dairy product supply chain quality based on evolutionary games: Perspectives from government intervention and market failure. Heliyon 2023;9. <https://doi.org/10.1016/j.heliyon.2023.e23024>.
6. Talukder B, Agnusdei GP, Hipel KW, Dubé L. Multi-indicator supply chain management framework for food convergent innovation in the dairy business. Sustainable Futures 2021;3. <https://doi.org/10.1016/j.sftr.2021.100045>.
7. Perrin A, Martin G. Resilience of French organic dairy cattle farms and supply chains to the Covid-19 pandemic. Agric Syst 2021;190. <https://doi.org/10.1016/j.agsy.2021.103082>.
8. Arenas-Bossa N, Gómez-Gómez V, Morante-Venera J, Salas-Navarro K. A vehicle routing model to optimize the distribution process in a dairy supply chain: Case of the Department of Bolivar in Colombia. Procedia Comput. Sci., vol. 251, Elsevier B.V.; 2024, p. 795–800. <https://doi.org/10.1016/j.procs.2024.11.187>.
9. Guarnaschelli A, Salomone HE, Méndez CA. A stochastic approach for integrated production and distribution planning in dairy supply chains. Comput Chem Eng 2020;140. <https://doi.org/10.1016/j.compchemeng.2020.106966>.
10. Rajkumar S, Palaniappan PLK, Kanagaraj G. Sustainable raw milk procurement: a strategic MCDM framework for dairy supply chains. OPSEARCH 2026. <https://doi.org/10.1007/s12597-026-01093-2>.
11. Sharma D, Singh G, Kaur A. Blockchain-based layered architecture for traceability in dairy supply chain management. International Journal of Information Technology (Singapore) 2025;17:4201–8. <https://doi.org/10.1007/s41870-025-02554-5>.
12. Mehannaoui R, Mouss KN, Aksa K, Benchine H, Ismail S. A holochain-based IoT-enabled agri-food traceability system: application in the recombined milk supply chain. International Journal of Information Technology (Singapore) 2025. <https://doi.org/10.1007/s41870-024-02396-7>.
13. Karwasra K, Soni G, Mangla SK, Kazancoglu Y. Assessing dairy supply chain vulnerability during the Covid-19 pandemic. International Journal of Logistics Research and Applications 2024;27:2378–96. <https://doi.org/10.1080/13675567.2021.1910221>.
14. Kettele M, Lechner G. Assessing resilience measures in the Austrian dairy supply chain: a case-based simulation integrating strategic, tactical, and operational decisions. Int J Prod Res 2025;63:3982–4015. <https://doi.org/10.1080/00207543.2024.2432472>.
15. Goli A, Tirkolaee EB. Designing a portfolio-based closed-loop supply chain network for dairy products with a financial approach: Accelerated Benders decomposition algorithm. Comput Oper Res 2023;155. <https://doi.org/10.1016/j.cor.2023.106244>.

16. Zarei-Kordshouli F, Paydar MM, Nayeri S. Designing a dairy supply chain network considering sustainability and resilience: a multistage decision-making framework. *Clean Technol Environ Policy* 2023;25:2903–27. <https://doi.org/10.1007/s10098-023-02538-8>.
17. Kalai R, Ouertatani A, Bouchery Y, Stauffer G, Saïdi I. Impact of refrigerated tanks on smallholder milk producers' business in Northern Tunisia. *Supply Chain Forum* 2025. <https://doi.org/10.1080/16258312.2025.2564059>.
18. Vanany I, Wangsa ID, Jeremi NA. A Multi-objective Mixed-Integer Linear Model for Sustainable Dairy Supply Chain with Food Waste and Environmental Pollutants. *Process Integration and Optimization for Sustainability* 2024;8:723–40. <https://doi.org/10.1007/s41660-023-00382-3>.
19. Nirmala DAR, Ramaswamy S, Logesh K, Gnanaraj SJP. Empirical study on risk mitigation for dairy supply chain management of Aavin Co-operative Milk Producers' Union Ltd. *Mater. Today Proc.*, vol. 49, Elsevier Ltd; 2020, p. 3657–60. <https://doi.org/10.1016/j.matpr.2021.09.243>.
20. Bayatzadeh S, Talaie H. An evaluation of traceability dynamics in dairy supply chains through causal modeling in emerging economies. *Supply Chain Analytics* 2025;11. <https://doi.org/10.1016/j.sca.2025.100156>.
21. Gómez-Villegas M, Ariza-Buenaventura D. Accounting and social mobilization: The counter accounts of the university student movement in Colombia. *Critical Perspectives on Accounting* 2024;99. <https://doi.org/10.1016/j.cpa.2024.102703>.
22. Kiani Mavi R, Semiari M, Hosseini Shekarabi SA, Kiani Mavi N, Moshkdanian F, Nikravesh A, et al. Multi-Objective Optimization of a Three-Level Sustainable Food Supply Chain: Modeling the Impact of Government Subsidies. *Global Journal of Flexible Systems Management* 2025;26:571–600. <https://doi.org/10.1007/s40171-025-00454-y>.
23. Kumar U, Shankar R. Smart dairy: Unleashing emerging ICT-enabled lean dairy supply chains through data-driven decision-making. *International Journal of Information Management Data Insights* 2024;4. <https://doi.org/10.1016/j.jjime.2024.100297>.

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