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Article

The Germany Problem for Beal's Conjecture

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Abstract: In this study, I examined the Beal's conjecture on the basis of formulas that produce composite numbers, and I developed some other conjectures that, if answered, would be an answer to the Beal's conjecture. Apart from this, I have examined the Beal's conjecture from another perspective and posed The Germany Problem.

Keywords: number theory; beal's conjecture; germany problem; composite numbers

1. Introduction

Let $A, B, C \in \mathbb{N}^+$; $x, y, z \in \mathbb{N}^+ > 2$. Beal's conjecture is conjectured that: When $A^x + B^y = C^z$, do A, B, C have to have a common prime divisor?

Definitions and Lemmas

DEFINITION 1. According to fundamental theorem of arithmetic [1]:

Composite numbers are numbers that are greater than 1 and have factors other than themselves and 1.

DEFINITION 2. According to fundamental theorem of arithmetic [1].

Prime numbers are numbers greater than 1 that do not have any factors other than themselves and 1.

DEFINITION 3. According to fundamental theorem of arithmetic [1].

An integer must be either a prime number or a composite number.

LEMMA 1. According to Aysun and Gocgen [2]:

$np + p$ gives all composite numbers where n is a positive natural numbers and p is a prime number.

Proof. $np + p = p(n + 1)$. Then, according to fundamental theorem of arithmetic:

$$(n + 1 \in \mathbb{C}) \oplus (n + 1 \in \mathbb{P})$$

Let $n + 1 \in \mathbb{C}$:

$$n + 1 = p_m \times \cdots \times p_{m+k}$$

Then,

$$p(n + 1) = p \times (p_m \times \cdots \times p_{m+k})$$

Let $n + 1 \in \mathbb{P}$:

$$n + 1 = p_m$$

Then,

$$p(n + 1) = p \times p_m$$

LEMMA 2. According to Aysun and Gocgen [2].

$2np + p$ gives all odd composite numbers where n is a positive natural numbers and p is an odd prime numbers.

Proof. $np + p$ gives odd composite numbers where p is a odd number and n is a even number. Then as already proved $np + p$ gives all composite numbers where n is a positive natural number and p is an prime number. Only possibility for odd composite just specified. Therefore, $np + p$ gives all odd composite numbers where p is a odd number and n is a even number. This equal to: $2np + p$ gives all odd composite numbers where n is a positive natural numbers and p is an odd prime numbers.

2. Theorems and Proofs

Preliminary Theorem: Eğer A, B, C asal ve birbirlerine eşit değil iken $A^x + B^y = C^z$ eşitliği sağlanabiliyorsa, teori yanlıştır. Eğer A, B, C asal ve birbirlerine eşit iken $A^x + B^y = C^z$ eşitliği sağlanabiliyor ya da sağlanamıyorsa, teori bu seçenekten yanlışılanamaz. A, B, C değerlerinden en az birinin composite olduğu ikinci seçenek incelemeye alınır. İkinci seçenekte herhangi bir yanlışlama yapılabilirse, teori yanlıştır. Eğer her iki seçenekte de yanlışlama yapılamazsa, iki seçenek dışında bir seçenek dışında bir seçenek olmadığından ve olamayacağından, teoremin doğruluğu kanıtlanmış olur.

Preliminary Proof: There are 2 basic situations.

The first case is where A, B, C are prime. Let's start by examining this situation:

s is any prime number and z ; Let it be a positive natural number greater than 2.

The result of s^z cannot have a prime factor other than s .

Based on this, we can say the following:

In the expression $A^x + B^y = C^z$, if A, B, C are prime numbers; The prime factor of A is only A , the prime factor of B is only B , the prime factor of C is only C . Therefore, when $A = B = C$, A, B, C do not have a common prime factor.

New Conjecture: Since composite numbers are numbers that can be expressed as the product of more than one prime number, the accuracy of the theory directly depends on the answer to the following question:

$A, B, C \in \mathbb{P}$

$A \neq B \neq C$

$x, y, z \in \mathbb{N}^+ > 2$:

Is $A^x + B^y = C^z$ possible?

well

Where A, B, C are different prime numbers and x, y, z are positive natural numbers greater than 2:

Is $A^x + B^y = C^z$ possible?

If the answer is yes, the Beals Conjecture is clearly false.

If the answer is no, the Beals Conjecture is clearly correct.

Inference: Let's start examining the question given in Question 1. The basic examination to start with will be to examine the odd/even status of the values in the equation. So let's start reviewing. Since there is only one even prime number, in the analysis, there can only be one even prime number, that is, no more than one of the values in the equation can be even:

Option 1) $A^x = \text{odd}, B^y = \text{odd}, C^z = \text{odd}$. In this case the equality would look like this: odd + odd = odd. This option is impossible because the sum of two odd numbers cannot equal an odd number.

Option 2) $A^x = \text{odd}, B^y = \text{odd}, C^z = \text{even}$. In this case the equality would look like this: odd + odd = even. This option is possible because the sum of two odd numbers will always equal an even number.

Option 3) $A^x = \text{odd}, B^y = \text{even}, C^z = \text{odd}$. In this case the equality would look like this: odd + even = odd. This option is possible because the sum of an odd number and an even number always equals an odd number.

Option 4) $A^x = \text{even}, B^y = \text{odd}, C^z = \text{odd}$. In this case the equality would look like this: even + odd = odd. This option is possible because the sum of an even number and an odd number will always equal an odd number.

The only option that is not possible is Option 1, in which A^x and B^y and C^z are not even. However, in the possible options A^x or B^y or C^z are even. Then, if the equality in Question 1 can be satisfied, A or B or $C = 2$. Let's call this situation "love triangle for 2" or simply "love triangle". Then we will activate this term.

Therefore, we can reduce the question to the following question:

$A, B \in \mathbb{P} \cap \mathbb{O}$

$A \neq B$

$x, y, z \in \mathbb{N}^+ > 2$:

Is $A^x + B^y = 2^z$ or $2^x + A^y = B^z$ or $A^x + 2^y = B^z$ possible? well

Can the sum of any exponent of 2 greater than two be equal to any exponent of 2 greater than two, or can the sum of any exponent of 2 greater than two and any exponent of an odd prime greater than two be equal to any exponent of 2 greater than two?

Observation: When we removed the condition that x, y, z must be greater than 2 for the equality in Question 2, it was seen that x or y or z was always less than or equal to 2 in the resulting equations. For example:

$$2^4 = 3^1 + 13^1$$

$$2^5 = 3^3 + 5^1$$

$$2^1 + 3^1 = 5^1$$

$$2^3 + 17^1 = 5^2$$

$A, B \in \mathbb{P} \cap \mathbb{O}$

$A \neq B$

$x, y, z \in \mathbb{N}^+$:

x or y or z in cases where the equations $A^x + B^y = 2^z$ or $2^x + A^y = B^z$ or $A^x + 2^y = B^z$ are satisfied Can it be proven that \leq must be 2?

If the answer is yes, since there is a condition $x, y, z > 2$ in Question 2, equality will not be achieved and Beals Conjecture will be proven correct.

If the opposite is proven, the Beals Conjecture will be proven false, since the equality in Question 2 is possible.

New Observation: The expression p^m can be expressed with the formulas $np + p$ and $2np + p$. Therefore, examining these statements together with these formulas may be useful to offer a new perspective.

$$f(x) = \begin{cases} pn + p, & p = 2 \\ 2pn + p, & p \neq 2 \end{cases}$$

Because while the $np + p$ formula produces all composite numbers, the $2np + p$ formula produces only single composite numbers. Since any exponent of a single prime number cannot be even, it would be inefficient and useless to examine the exponent of any odd prime through a formula that also produces even composite numbers. For this reason, the formula $np + p$ for even primes and $2np + p$ for odd primes should be examined.

In the $np + p$ and $2np + p$ formulas, p must be the p value in the p^m expression. However, another method must be followed to determine n :

$$f(x) = \begin{cases} x - 1, & m \leq 2 \\ 1 + \sum_{k=1}^{x-2} A^k, & m > 2 \end{cases}$$

Although the above n determination method has been confirmed by experiments, it should be fully proven in another article. According to this determination:

$x, y, z \leq 2$:

$$2(x-1) + 2 + 2A(y-1) + A = 2B(z-1) + B$$

or

$$2A(x-1) + A + 2(y-1) + 2 = 2B(z-1) + B$$

or

$$2A(x-1) + A + 2B(y-1) + B = 2(z-1) + 2 \text{ olmaktadır.}$$

sc.

$$2x - 2 + 2 + 2Ay - 2A + A = 2Bz - 2B + B$$

or

$$2Ax - 2A + A + 2y - 2 + 2 = 2Bz - 2B + B$$

or

$$2Ax - 2A + A + 2By - 2B + B = 2z - 2 + 2 \text{ olmaktadır.}$$

sc.

$$2x + 2Ay - A = 2Bz - B$$

or

$$2Ax - A + 2y = 2Bz - B$$

or

$$2Ax - A + 2By - B = 2z.$$

$x, y, z \leq 2$:

$$2(1 + \sum_{k=1}^{x-2} A^k) + 2 + 2A(1 + \sum_{k=1}^{y-2} A^k) + A = 2B(1 + \sum_{k=1}^{z-2} A^k) + B$$

or

$$2A(1 + \sum_{k=1}^{x-2} A^k) + A + 2(1 + \sum_{k=1}^{y-2} A^k) + 2 = 2B(1 + \sum_{k=1}^{z-2} A^k) + B$$

or

$$2A(1 + \sum_{k=1}^{x-2} A^k) + A + 2B(1 + \sum_{k=1}^{y-2} A^k) + B = 2(1 + \sum_{k=1}^{z-2} A^k) + 2 \text{ olmaktadır.}$$

sc.

$$2 + 2 \sum_{k=1}^{x-2} A^k + 2 + 2A + 2A \sum_{k=1}^{y-2} A^k + A = 2B + 2B \sum_{k=1}^{z-2} A^k + B$$

or

$$2A + 2A \sum_{k=1}^{x-2} A^k + A + 2 + 2 \sum_{k=1}^{y-2} A^k + 2 = 2B + 2B \sum_{k=1}^{z-2} A^k + B$$

or

$$2A + 2A \sum_{k=1}^{x-2} A^k + A + 2B + 2B \sum_{k=1}^{y-2} A^k + B = 2 + 2 \sum_{k=1}^{z-2} A^k + 2 \text{ olmaktadır.}$$

sc.

$$2 \sum_{k=1}^{x-2} A^k + 4 + 2A \sum_{k=1}^{y-2} A^k + 3A = 2B \sum_{k=1}^{z-2} A^k + 3B$$

or

$$2A \sum_{k=1}^{x-2} A^k + 3A + 2 \sum_{k=1}^{y-2} A^k + 4 = 2B \sum_{k=1}^{z-2} A^k + 3B$$

or

$$2A \sum_{k=1}^{x-2} A^k + 3A + 2B \sum_{k=1}^{y-2} A^k + 3B = 2 \sum_{k=1}^{z-2} A^k + 4.$$

The clear difference between the two situations may affect the feasibility of the equation, but this difference could not be understood in this study and was left to another article.

The Germany Problem:

Lover Ex.

$$\text{Ex 1. } 3^2 + 2^4 = 5^2$$

for $z \leq 2$:

$$z^2 \leq 25.$$

Potential values for z : 2, 3, 4, 5.

for $z > 2$:

$$z^3 \leq 25.$$

Potential values for z : 2.

As can be seen, the potential values found for $z > 2$ are values that cannot be used because they are already used in the equation. Let's call these values lover-values. Because of the love triangle, 2 always has to be lover-value. If all potential values found for $z > 2$ consist of previously used values, let's call these examples lover examples. For example above is a lover ex. It is understood that it is. Then it becomes clear why lover ex.s cannot satisfy the equation for $z > 2$.

Non-lover Ex.

$$\text{Ex. 1 } 2^7 + 17^3 = 71^2$$

for $z \leq 2$:

$$z^2 \leq 5041.$$

Potential values for z : $p \leq 71$

for $z > 2$:

$$z^3 \leq 5041.$$

Potential values for z : 2, 3, 5, 7, 11, 13, 17

As can be seen, all of the potential values found for $z > 2$ are not values that cannot be used since they are already used in the equation. Let's call these values non-lover-values. If all potential values found for $z > 2$ do not consist of previously used values, let's call these examples non-lover examples. The above example is a non-lover ex. It is understood that it is. In this case, it is not understood why non-lover ex. cannot satisfy the equation for $z > 2$.

Therefore, it should be explained why non-lover exes cannot satisfy the equation. With explanation and clear evidence, the Beals Conjecture will be solved.

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