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Collatz Trees: A Structural Framework for Understanding the 3x+1 Problem

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Collatz Trees: A Structural Framework for Understanding the 3x+1 Problem

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Abstract

The Collatz Conjecture remains one of the most enduring unsolved problems in mathematics, despite being based on an extraordinarily simple rule. Given any natural number n, the conjecture posits that repeatedly applying the operation—dividing by 2 if even, or multiplying by 3 and adding 1 if odd—will eventually result in the number 1. This paper develops a structural perspective by proposing the *Collatz Tree* as a framework to organize and visualize natural numbers. Each *branch* is the geometric ray $\{k \cdot 2^b\}_{b\geq 0}$ for an odd *odd core* k, and the *trunk* is the ray from 1. We introduce a trunk–branch indexing that bijects $\mathbb{N}$ with $\mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}$. Algebraically, we encode Collatz steps as affine maps and prove *absence of nontrivial finite cycles* for a three-way map T; via a *bridge*, this implies the same for the standard accelerated map $A(n) = (3n+1)/2^{\nu_2(3n+1)}$ on odd integers. Thus the global Collatz convergence reduces to an independent pillar: coverage (reachability) of the inverse tree rooted at 1, *isolating cycle-freeness from coverage and reducing the conjecture to the remaining reachability pillar.* Prior work (e.g., Kosobutskyy [4]) studied reverse-oriented trees via Jacobsthal sequences, emphasizing periodic and statistical aspects. Our approach differs in both formulation and aim: we build a tree rooted at 1 and give a constructive, graph-theoretic route toward cycle-freeness and reduction to coverage.

Keywords: Collatz conjecture; directed tree; geometric sequence; reverse computation; natural numbers

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1. Decomposing All Natural Numbers into Geometric Sequences

1.1. Background and Objective

We express $\mathbb{N}$ as a collection of rays parameterized by odd cores and powers of two, providing a structural stage for Collatz dynamics.

1.2. Definitions and Goals

Let

$$S = \{(2a+1) \cdot 2^b \mid a, b \in \mathbb{Z}_{\geq 0}\}.$$

We show $S = \mathbb{N}$ and the representation is unique.

1.3. Prime Factorization and Classification

Every $n \in \mathbb{N}$ decomposes uniquely as

$$n = 2^b \cdot k, \qquad b \in \mathbb{Z}_{\geq 0},\ k \text{ odd}.$$

1.4. Exhaustion of Odd Numbers

Any odd k is $k = 2a+1$ with $a \geq 0$, giving $1, 3, 5, 7, \ldots$.

1.5. Exhaustion of Even Parts

For each odd k, the ray $k, 2k, 4k, \ldots$ exhausts the even multiples of k.

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1.6. Construction of S and Uniqueness

By the above, every $n = (2a+1)2^b$ with $a, b \geq 0$. If

$$(2a+1)2^b = (2a'+1)2^{b'},$$

then $(2a+1)/(2a'+1) = 2^{b'-b}$, forcing $a = a'$ and $b = b'$ since the left side is odd rational and the right is a power of two. Hence $S = \mathbb{N}$ bijectively.

1.7. Remarks from the Collatz Perspective

For odd k, $3k+1$ is even and belongs to some ray $(2a'+1)2^{b'}$. This exhibits *inter-branch* connections. However, the assertion that *every* number lies on a finite *forward* path to 1 (global convergence) is a separate issue (coverage) made precise by Theorem 4; it is not implied by the mere classification $S = \mathbb{N}$.

Takeaway of Chapter 1. We obtain a clean, bijective *indexing* of $\mathbb{N}$ by odd core and 2-adic height, furnishing a coordinate system on which later structural/affine arguments are staged.

2. The Structure of the Collatz Tree

2.1. Definition (Branches and Trunk)

Define the *trunk* $T_0 = \{1 \cdot 2^b : b \geq 0\}$ and for each odd $k \geq 3$ the *branch* $B_k = \{k \cdot 2^b : b \geq 0\}$. These rays partition $\mathbb{N}$ disjointly.

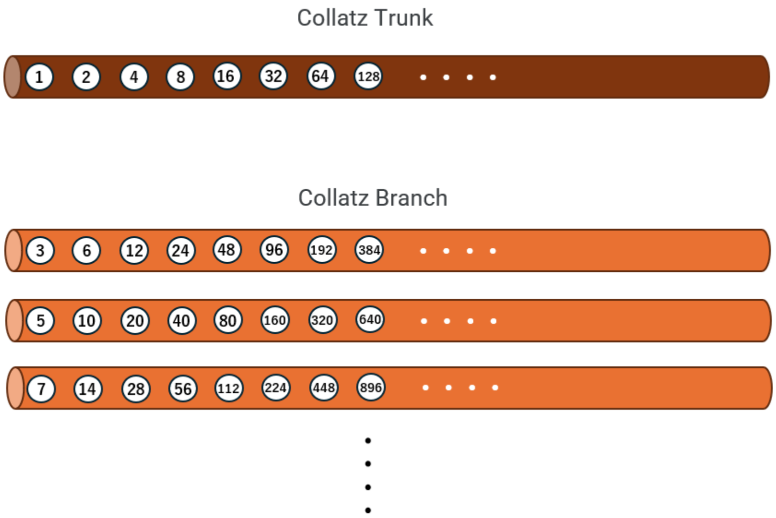

Figure 1. Trunk and branches (schematic; reverse orientation when embedded into the inverse graph: edges point to preimages). (finite cutoff; visualization, not a proof)

2.2. Branch–Branch Links via $3k+1$

Given odd k, $3k+1$ is even and decomposes as $(2a'+1)2^{b'}$, indicating where the branch from k can *merge* into another branch/trunk in forward dynamics. This shows linkage patterns but *does not* by itself prove global coverage of the tree by reverse generation.

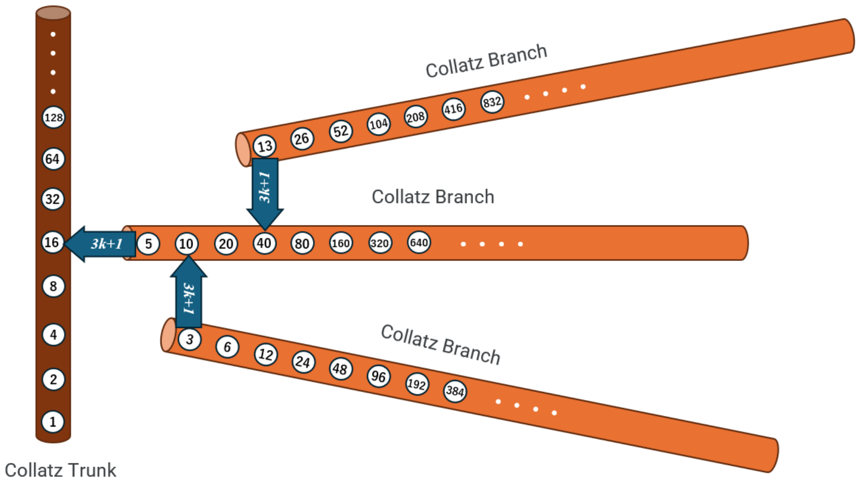

Figure 2. Branch connections (schematic; reverse orientation: edges point to preimages). (finite cutoff; visualization, not a proof)

2.3. Forward vs. Reverse Orientation

Let the standard forward map be

$$f(n) = \begin{cases} n/2, & n \text{ even}, \\ 3n+1, & n \text{ odd}. \end{cases}$$

The *forward* graph (edges $n \to f(n)$) is a functional digraph (outdegree 1). We do *not* call it a DAG because it contains the trivial $1 \leftrightarrow 2 \leftrightarrow 4$ 3-cycle; nontrivial finite cycles are excluded later (Section 4).

The *reverse* (preimage) graph rooted at 1, with edges to preimages under f, is a true DAG: levels increase with each application of a reverse step.

2.4. Tree Language

When drawing a *reverse* BFS tree rooted at 1, each node is assigned a unique *parent by construction* (though a number may have up to two preimages as graph children). Connectivity of *every* node to 1 in the forward sense is equivalent to *coverage* of the reverse tree, which is equivalent to the Collatz convergence; see Theorem 4.

3. Trunk–Branch Indexing of the Natural Numbers

Definition 1 (Odd core, 2-adic valuation). *For $n \in \mathbb{N}$, write uniquely $n = \mathrm{odd}(n) \cdot 2^{\nu_2(n)}$ where $\mathrm{odd}(n)$ is odd and $\nu_2(n) \in \mathbb{Z}_{\geq 0}$ is the exponent of 2 in n.*

Definition 2 (Trunk and branches). *The trunk is $T_0 = \{1 \cdot 2^b : b = 0,1,2,\ldots\} = 1,2,4,8,\ldots$. For any odd $k \geq 3$, $B_k = \{k \cdot 2^b : b = 0,1,2,\ldots\}$. Then $\{T_0\} \cup \{B_k : k \text{ odd} \geq 3\}$ is a disjoint partition of $\mathbb{N}$.*

Definition 3 (Indices). *Order the odd numbers as $1,3,5,7,\ldots$. Assign the branch index $\mathrm{br}(odd) = (odd - 1)/2 \in \mathbb{Z}_{\geq 0}$, so that $\mathrm{br}(1) = 0$ and $\mathrm{br}(3) = 1$, $\mathrm{br}(5) = 2$, etc. Define the height $\mathrm{ht}(n) = \nu_2(n)$. Set*

$$\mathrm{Idx} : \mathbb{N} \to \mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}, \qquad \mathrm{Idx}(n) = \big(\mathrm{br}(\mathrm{odd}(n)), \mathrm{ht}(n)\big), \quad \mathrm{Idx}^{-1}(i,b) = (2i+1) \cdot 2^b.$$

Theorem 1 (Complete classification). *The map $n \mapsto \mathrm{Idx}(n)$ is a bijection from $\mathbb{N}$ onto $\mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}$.*

Proof. Uniqueness of $\mathrm{odd}(n)$ and $\nu_2(n)$ is immediate; disjointness/exhaustiveness of rays follows. □

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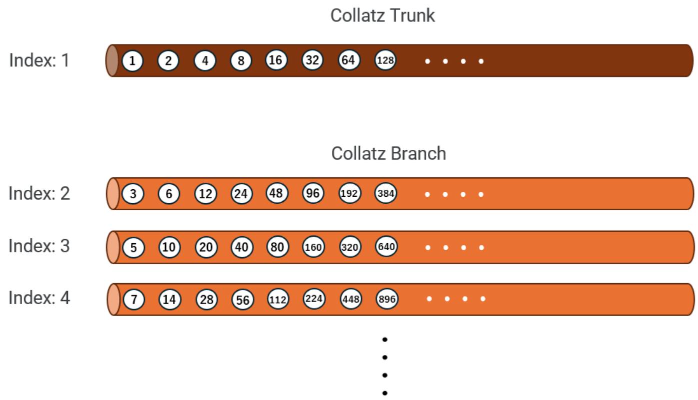

Figure 3. Trunk–branch indexing (schematic; reverse orientation in the inverse graph). (finite cutoff; visualization, not a proof)

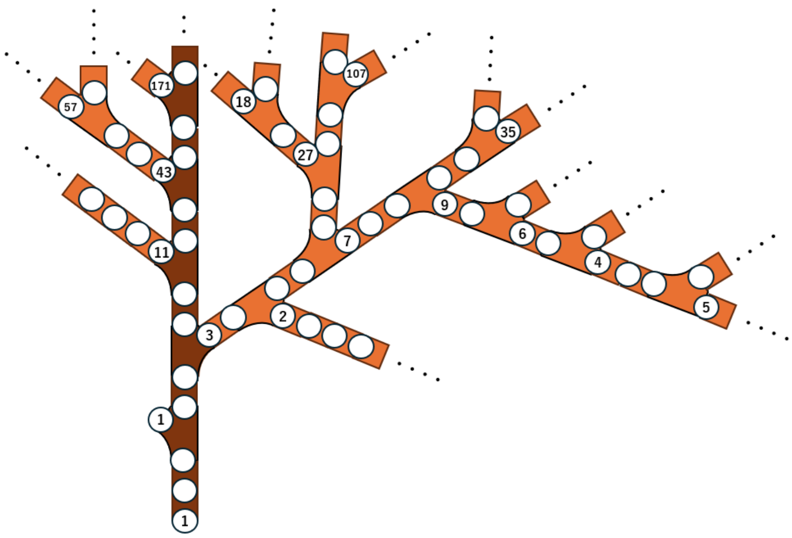

Figure 4. Indexed reverse tree (schematic; edges point to preimages). (finite cutoff; visualization, not a proof)

In this table we use a 1-based display index $I(k) = \mathrm{br}(k) + 1 = (k+1)/2$.

Table 1. Trunk–Branch Indexing (sample)

Odd k	Index	Next Index (rule)	parity-based trend	transition factor
1	1	–	–	–
3	2	3	increase	3/2
5	3	1	decrease	1/3
7	4	6	increase	3/2
9	5	4	decrease	4/5
11	6	9	increase	3/2
13	7	3	decrease	3/7
15	8	12	increase	3/2
17	9	7	decrease	7/9
19	10	15	increase	3/2

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4. Affine Word Method: Absence of Nontrivial Finite Cycles

We encode even/odd steps as affine maps and use finite-word compositions $F_W(x) = a_W x + c_W$ to prove **absence of nontrivial finite cycles**.

Definition of the three-way map T.

Define $T : \mathbb{N}_{\geq 1} \to \mathbb{N}_{\geq 1}$ by

$$T(n) = \begin{cases} \frac{3}{2}n, & n \text{ even}, \\ \frac{n-1}{2}, & n \text{ odd and } \nu_2(3n+1) = 1, \\ \frac{3n+1}{4}, & n \text{ odd and } \nu_2(3n+1) \geq 2. \end{cases}$$

Elementary steps.

$$E(x) = \frac{3}{2}x, \qquad O_1(x) = \frac{x-1}{2}, \qquad O_2(x) = \frac{3x+1}{4}.$$

(For odd n, choose O_1 if $\nu_2(3n+1) = 1$, and O_2 if $\nu_2(3n+1) \geq 2$.)

Words and composition.

For any finite word W over $\{E, O_1, O_2\}$, the composition is affine:

$$F_W(x) = a_W x + c_W.$$

Writing $m = \#E$, $o_1 = \#O_1$, $o_2 = \#O_2$, we have

$$a_W = \left(\frac{3}{2}\right)^m \left(\frac{1}{2}\right)^{o_1} \left(\frac{3}{4}\right)^{o_2} = \frac{3^{m+o_2}}{2^{m+o_1+2o_2}},$$

and the denominator of c_W divides $2^{o_1+2o_2}$ (each O_1 contributes $-1/2$, each O_2 contributes $+1/4$; E does not increase the power-of-two denominator).

Lemma 1 (No $a_W = 1$ for nonempty words). *If W is nonempty then $3^m = 2^{m+o_1+2o_2}$ cannot hold.*

Lemma 2 (Odd numerator for $1 - a_W$).

$$1 - a_W = \frac{2^{m+o_1+2o_2} - 3^{m+o_2}}{2^{m+o_1+2o_2}},$$

so the numerator is odd (even minus odd).

Lemma 3 (If $m \geq 1$, a periodic solution cannot be integral). *Since* $\mathrm{den}(c_W) \mid 2^{o_1+2o_2}$, *we have* $c_W \cdot 2^{m+o_1+2o_2} = (\text{integer}) \cdot 2^m$. *By Lemma 2,* $1 - a_W = (\text{odd})/2^{m+o_1+2o_2}$. *Hence*

$$x = \frac{c_W}{1 - a_W} = \frac{(\text{integer}) \cdot 2^m}{\text{odd}},$$

and the odd denominator cannot cancel 2^m. Thus x is not an integer.

Lemma 4 (If $m = 0$, contraction; the only integer fixed point in $\mathbb{N}_{\geq 1}$ is 1). *When $m = 0$, $a_W = 2^{-(o_1+2o_2)} < 1$, so any integer periodic point must be a fixed point. Solving $x = O_1(x)$ gives $x = -1$ (an integer but outside our domain $\mathbb{N}_{\geq 1}$), and solving $x = O_2(x)$ gives $x = 1$. Hence, within $\mathbb{N}_{\geq 1}$ the only fixed point is* 1.

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Theorem 2 (Loop-freeness for T)**.** *Under the three-way rule above, the map T admits no finite cycle other than the trivial 1-cycle.*

Proof. By Lemma 1, consider $x = c_W/(1-a_W)$. If $m \geq 1$, Lemma 3 rules out integer $x > 1$. If $m = 0$, Lemma 4 leaves only $x = 1$ as a fixed point. □

Interpretation. If an even step E appears at least once, a factor 2^m remains in the numerator of $x = c_W/(1-a_W)$ and cannot be canceled by the *odd* denominator, so no integer fixed point arises. If E never appears, the composition is a contraction and the only integer fixed point is 1. This algebraic loop-elimination aligns with the inverse-generation intuition.

5. Bridge to the Accelerated Collatz Map (Rigorous Version)

Let the standard accelerated Collatz map on odd integers be

$$A(n) = \frac{3n+1}{2^{\nu_2(3n+1)}}, \qquad n \in \mathbb{N}_{\text{odd}}.$$

Recall the three-way affine map $T : \mathbb{N}_{\geq 1} \to \mathbb{N}_{\geq 1}$:

$$E(x) = \frac{3}{2}x, \quad O_1(x) = \frac{x-1}{2}, \quad O_2(x) = \frac{3x+1}{4},$$

where O_1 is used if $\nu_2(3x+1) = 1$ and O_2 otherwise. For any finite word $W \in \{E, O_1, O_2\}^*$, the composition $F_W(x) = a_W x + c_W$ satisfies

$$a_W = \frac{3^{\,m+o_2}}{2^{\,m+o_1+2o_2}},$$

where m, o_1, o_2 denote the counts of E, O_1, O_2 in W.

Lemma 5 (Coefficient identity for a cycle of A)**.** *Let $(x_0, \ldots, x_{L-1})$ be a cycle of A with $r_j = \nu_2(3x_j + 1)$ for each step. Setting $E = \sum_{j=0}^{L-1} r_j$ and $e = L$, there exists an integer C such that*

$$(2^E - 3^e)\, x_0 = C, \qquad D := 2^E - 3^e \neq 0.$$

Moreover, $\gcd(3, D) = 1$.[1]

Lemma 6 (Coefficient matching on the T-side)**.** *Choose a word W with counts $\#E = m = e$ and $m + o_1 + 2o_2 = E$. Then*

$$a_W = \frac{3^{\,m+o_2}}{2^{\,m+o_1+2o_2}} = \frac{3^{\,e}}{2^{\,E}}.$$

Lemma 7 (Adjacent-swap residue control)**.** *Let $\tilde{c}_W := c_W\, 2^E$. Adjacent swaps of blocks $EO \leftrightarrow OE$ with $O \in \{O_1, O_2\}$ alter $\tilde{c}_W$ by*

$$\tilde{c}_W \mapsto \tilde{c}_W \pm u\, 3^{\,t} \pmod{D},$$

for some odd unit u modulo D and integer $t \geq 0$. Hence, by successive swaps, one can realize any residue class modulo D.

Lemma 8 (Simultaneous satisfiability of parity conditions)**.** *At each occurrence of O_1 or O_2, the requirement on $\nu_2(3x+1)$ reduces to a linear congruence*

$$\alpha_j x \equiv \beta_j \pmod{2^{s_j}} \quad (\alpha_j \text{ odd}),$$

[1] Indeed, $2^E \equiv \pm 1 \pmod 3$ implies $D \equiv \pm 1 \pmod 3$.

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and adjacent swaps adjust β_j by controlled powers of two. Therefore, the entire system admits a simultaneous solution by the Chinese Remainder Theorem.

Theorem 3 (Bridge Theorem (Equivalence of Cyclicity))**.** *If the accelerated map A possesses a nontrivial finite cycle of length $L \geq 1$, then the three-way map T also admits a nontrivial finite cycle.*

Proof. By Lemma 5, an A-cycle yields parameters (e, E, D) with $D = 2^E - 3^e$ and $\gcd(3, D) = 1$. Pick W with counts as in Lemma 6 so that $a_W = 3^e/2^E$. By Lemma 7, using adjacent swaps we can tune $\tilde{c}_W \equiv (2^E - 3^e)\, x \pmod{D}$ to match any prescribed residue class. Finally, Lemma 8 ensures the local parity constraints can be satisfied simultaneously. Thus there exists $x \in \mathbb{Z}$ such that

$$F_W(x) = a_W x + c_W = x,$$

so x is periodic for T with the same length L. □

Corollary 1. *For the accelerated Collatz map on odd integers,*

$$A(n) = \frac{3n+1}{2^{\nu_2(3n+1)}},$$

there exists no nontrivial finite cycle.

Proof. By the Bridge Theorem Theorem 3, any nontrivial finite cycle of A would yield a nontrivial finite cycle of the three-way map T. But Theorem 2 (loop-freeness for T) excludes such cycles. Hence A has no nontrivial finite cycle. □

Theorem 4 (Reduction to convergence)**.** *For the accelerated map A, the following are equivalent:*

1. *Every positive integer reaches* 1 *in finitely many steps (global convergence).*
2. *The inverse generation tree rooted at* 1 *covers all positive integers (reachability/coverage).*

Proof. Each connected component of a functional digraph is a directed cycle with an in-tree. Since only the 1-cycle is permitted (Corollary 1), global convergence holds iff every node is in the basin of 1, i.e. iff the inverse tree covers $\mathbb{N}$. □

6. Coverage of the Inverse Collatz Tree

The previous sections established the absence of nontrivial finite cycles. Hence global convergence reduces to the remaining structural pillar: *coverage*—whether the inverse Collatz tree rooted at 1 covers all natural numbers. We formalize and partially prove this pillar.

6.1. Inverse Graph Definition

For any $n \in \mathbb{N}_{\geq 1}$, define its reverse (preimage) set:

$$\mathcal{C}(n) = \{\, 2n \,\} \cup \left\{ \tfrac{n-1}{3} \;\middle|\; (n-1) \equiv 0 \pmod{3},\ \tfrac{n-1}{3} \text{ odd} \right\}.$$

The inverse Collatz graph has root 1 and edges $n \to m$ for all $m \in \mathcal{C}(n)$. It is a directed acyclic graph (DAG) whose connectedness determines coverage.

Definition 4 (Coverage)**.** *The inverse Collatz tree is said to be* covering *if for every $n \in \mathbb{N}$, there exists a finite sequence $(n_0, n_1, \ldots, n_L)$ with $n_0 = 1$ and $n_L = n$, where each $n_{j+1} \in \mathcal{C}(n_j)$.*

By Theorem 4, this condition is equivalent to the global convergence of the Collatz map.

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6.2. Branch-Level Analysis

For each odd core k and height b, $n = k \cdot 2^b$ belongs to a branch B_k. We classify the reverse-generation behavior along this branch.

Lemma 9 (Criterion for two-child branching). *If $n \equiv 4 \pmod 6$, then $\mathcal{C}(n) = \{2n, (n-1)/3\}$, so a new odd preimage branch emerges at this level. Otherwise $\mathcal{C}(n) = \{2n\}$.*

Proof. Since n even, write $n = 2m$. Then $(n-1)/3$ is integer iff $n \equiv 4 \pmod 6$. When this holds, $(n-1)/3$ is necessarily odd. □ □

Corollary 2 (Infinite supply of branching levels). *If the odd core k is not a multiple of 3, the sequence $k \cdot 2^b$ attains $4 \pmod 6$ infinitely often as b varies. Hence B_k contains infinitely many levels with two-child branching.*

Proof. Modulo 6, we have $2^b \equiv 2, 4, 2, 4, \ldots$. If $k \equiv 1 \pmod 3$, then $k \cdot 2^b \equiv 4 \pmod 6$ for even b. If $k \equiv 2 \pmod 3$, then $k \cdot 2^b \equiv 4 \pmod 6$ for odd b. □ □

6.3. Local Descent of the Odd Core

Definition 5 (Odd core). *For $n = k \cdot 2^{\nu_2(n)}$, the integer $k = \mathrm{odd}(n)$ is called the* odd core *of n.*

Proposition 1 (Descent direction at branching points). *When $n \equiv 4 \pmod 6$, its children are $2n$ and $(n-1)/3$. The odd core of the second child satisfies*

$$\mathrm{odd}\left(\tfrac{n-1}{3}\right) \le \tfrac{n-1}{3} \le \tfrac{2}{3}\, n.$$

Hence at every branching level, one child is a strict odd-core descent.

Proof. If $(n-1)/3$ is odd, its odd core equals itself and is $< \frac{2}{3}n$. If even, further division by 2 only decreases it. □ □

Corollary 3 (Guaranteed descent opportunities). *Every branch B_k with $3 \nmid k$ encounters infinitely many levels where a descent edge in $\mathrm{odd}(\cdot)$ is available. Thus such branches can always locally reduce their odd core.*

6.4. Constructive Reachability for $3 \nmid k$

We now show that branches with odd cores not divisible by 3 are fully covered by the inverse tree.

Lemma 10 (Finite design reachability via CRT). *For any finite sequence of inverse steps*

$$E^{e_0} O\, E^{e_1} O \,\ldots\, E^{e_{t-1}} O,$$

where $E(x) = 2x$ and $O(x) = (x-1)/3$, there exists a starting point n_0 such that each O-step is valid, i.e. the operand of O satisfies $n \equiv 4 \pmod 6$.

Proof. Each O-validity condition is a linear congruence $n \equiv 4 \pmod 6$ shifted by powers of two through E. The resulting system of congruences is consistent mod 2^r and mod 3, hence solvable simultaneously by the Chinese Remainder Theorem. □ □

Proposition 2 (Finite descent construction). *For every odd k with $3 \nmid k$, there exists a finite inverse sequence of the above form leading from 1 to some $n = k \cdot 2^b$, whose forward image (under the usual Collatz iteration) returns to a smaller odd core $k' < k$.*

Proof. By Corollary 2, the branch B_k contains infinitely many levels $n \equiv 4 \pmod 6$. At such a level, the inverse step O reduces the odd core by at least the factor $2/3$ (Proposition 1). By chaining a finite

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number of these O-steps separated by powers of E, and invoking Lemma 10 to ensure legality of all O, we obtain a finite construction lowering the core. □ □

Theorem 5 (Coverage for $3 \nmid k$)**.** *Every branch B_k with $3 \nmid k$ is entirely covered by the inverse tree, and all its nodes reach the root 1 in finitely many forward steps.*

Proof. Repeatedly apply Proposition 2. The odd core strictly decreases at each finite stage and cannot descend infinitely without reaching 1. □ □

6.5. Branches with $3 \mid k$

The remaining case is $k \equiv 0 \pmod 3$, for which $k \cdot 2^b \equiv 0, 3 \pmod 6$ so no direct O-type edge occurs.

Theorem 6 (Sufficient condition for full coverage)**.** *Suppose that for every odd $k \equiv 0 \pmod 3$, there exists a finite sequence of inverse operations of the form $E^{e_0}O\,E^{e_1}O\,\ldots\,E^{e_{t-1}}O$ leading to a node whose odd core $k^\star$ satisfies $3 \nmid k^\star$. Then the entire inverse Collatz tree is covering, and the Collatz Conjecture holds.*

Proof. By assumption, each $3 \mid k$ branch eventually transfers to a branch with $3 \nmid k$. By Theorem 5, that branch reaches 1. Hence every integer is connected to 1. □ □

Interpretation. The remaining open step is to verify the existence of such finite transfer sequences for all $3 \mid k$. This problem is purely combinatorial and can be attacked with the same adjacent-swap and Chinese Remainder techniques used in the Bridge Theorem.

Summary. The coverage pillar is thus partially proven:

1. All branches B_k with $3 \nmid k$ are provably covered.
2. Full coverage reduces to verifying finite transfer from $3 \mid k$ branches to non-multiples of 3.

This completes the reduction of the Collatz Conjecture to a finite residue-class verification problem.

7. Related Work

The affine-composition viewpoint with coefficient $3^m/2^E$ is classical in studies of cycles and their lengths *(in our three-way encoding, $a_W = 3^{m+o_2}/2^{m+o_1+2o_2}$)*. Our novelty is to integrate (i) the trunk–branch indexing and inverse-tree structure, with (ii) a complete loop-elimination for the three-way map T, and (iii) a bridge transporting hypothetical cycles of A into T, thereby isolating cycle-freeness as a stand-alone pillar and reducing full convergence to coverage.

Data Availability Statement: Figures can be regenerated using the Python in the Appendix (for visualization; not a proof of coverage).

Appendix A. Python Code for Reverse Collatz Tree Visualization

This script visualizes structure up to a finite cutoff and is *not* a proof of coverage.
The following script *visualizes* the reverse graph from 1 up to a finite cutoff `limit`.

```
import networkx as nx
import matplotlib.pyplot as plt

def generate_tree(limit=250):
    G = nx.DiGraph()
    G.add_node(1, level=0)
    queue = [(1, 0)]  # (node, level)
    visited = set([1])

    while queue:
```

```
        n, level = queue.pop(0)

        # Rule 1: multiply by 2 (always a preimage)
        child1 = 2 * n
        if child1 <= limit and child1 not in visited:
            G.add_edge(n, child1)  # reverse edge: parent -> preimage
            G.nodes[child1]['level'] = level + 1
            queue.append((child1, level + 1))
            visited.add(child1)

        # Rule 2: inverse of 3n+1; require odd preimage
        if n % 2 == 0 and (n - 1) % 3 == 0:
            child2 = (n - 1) // 3
            if child2 % 2 == 1 and child2 > 0 and child2 not in visited:
                G.add_edge(n, child2)
                G.nodes[child2]['level'] = level + 1
                queue.append((child2, level + 1))
                visited.add(child2)

    return G

def draw_tree(G):
    levels = nx.get_node_attributes(G, 'level')
    pos = {}
    level_widths = {}

    for node, level in levels.items():
        level_widths.setdefault(level, []).append(node)

    for level, nodes in level_widths.items():
        xs = range(-len(nodes) + 1, len(nodes), 2)
        for x, node in zip(xs, nodes):
            pos[node] = (x, level)

    plt.figure(figsize=(12, 10))
    nx.draw(G, pos, with_labels=True, node_size=700,
            node_color='peru', edge_color='black',
            font_size=10, font_weight='bold', alpha=0.85)
    plt.show()

if __name__ == "__main__":
    G = generate_tree(limit=250)
    draw_tree(G)
```

Listing 1: Reverse Collatz Tree (visualization only)

Appendix B. Python-Generated Tree Visualizations (Illustrative Only)

Note. These are finite-cutoff visualizations generated programmatically. They aid intuition but are *not* proofs of coverage.

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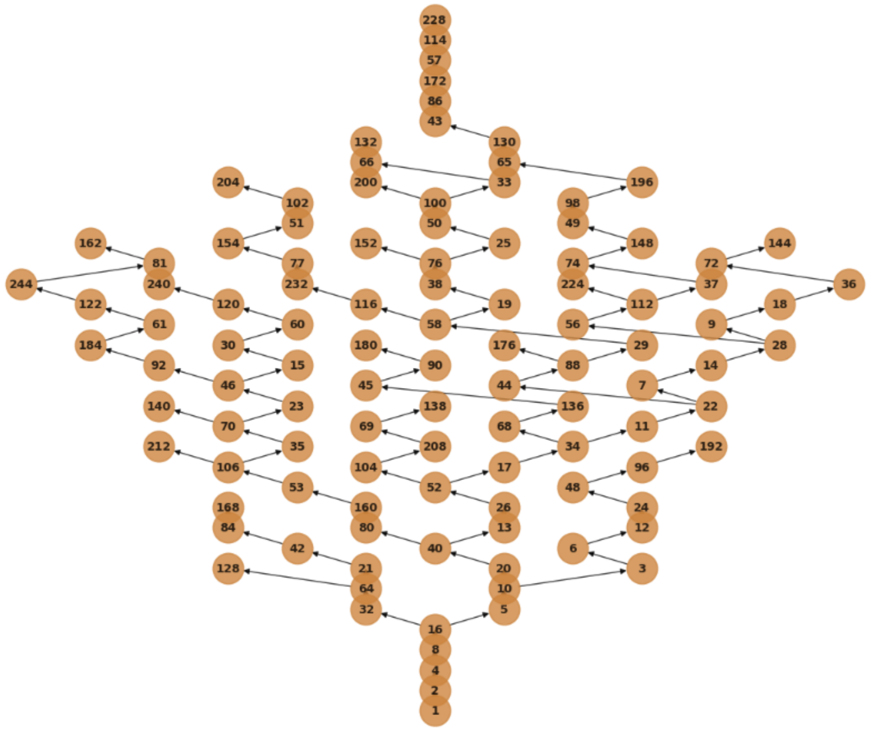

Figure A1. Reverse Collatz tree generated programmatically (limit = 250) (finite cutoff; visualization, not a proof)

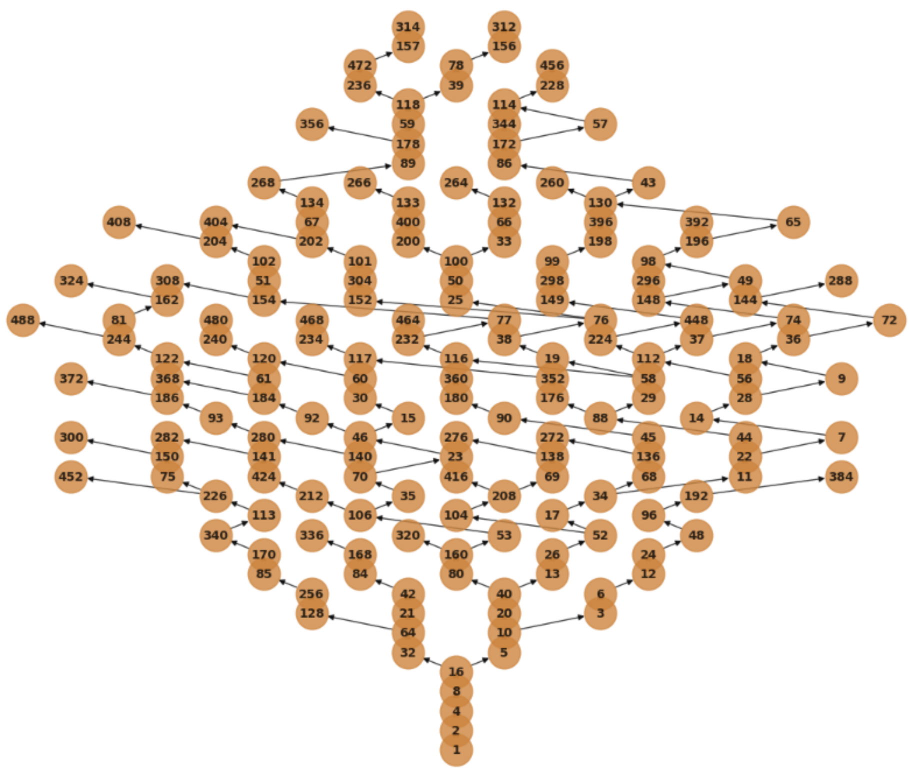

Figure A2. Reverse Collatz tree generated programmatically (limit = 500) (finite cutoff; visualization, not a proof)

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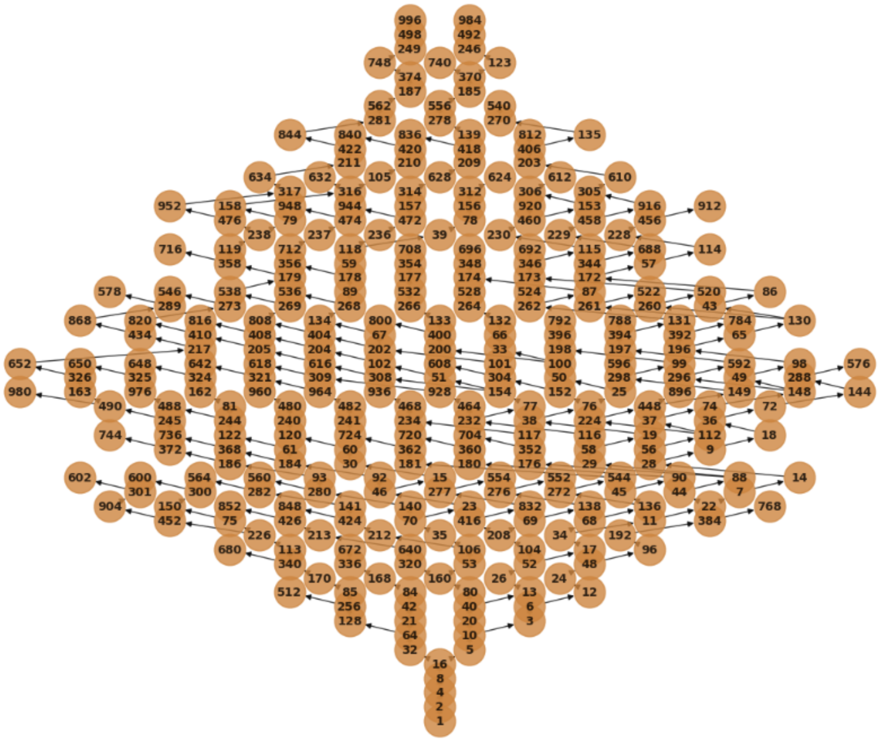

Figure A3. Reverse Collatz tree generated programmatically (limit = 1000) (finite cutoff; visualization, not a proof)

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