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Article

Stability and Gas Diffusion: A Theoretical Approach to Evanescence and Permanence in Oxyhydrogen Nanobubbles

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Abstract: Nanobubbles, gas-filled entities that possess a size of less than 200 nm, have notably emerged as critical components within various domains such as environmental science and medicine, primarily due to their remarkable stability and distinctive diffusion characteristics that differentiate them from larger bubbles. This comprehensive study meticulously explores the diffusion rates and stability of hydrogen and oxygen when encapsulated in oxyhydrogen nanobubbles, specifically within aqueous environments, utilizing the Chapman-Enskog theory, which is a well-established framework for understanding gas diffusion processes, alongside a modified Arrhenius equation to effectively evaluate activation volume and stability parameters. The findings from this investigation reveal that the diffusion processes of both gasses experience a significant reduction in rate due to the influence of high internal pressure, as well as the molecular interactions occurring with water, whereby hydrogen is observed to exhibit a marginally accelerated diffusion rate in comparison to oxygen. It is crucial to understand that elements like surface charges, zeta potential, and the idea of activation volume significantly influence the enduring stability of nanobubbles, particularly in specific environmental scenarios that could boost their longevity. These significant findings pose a challenge to classical diffusion models, thereby suggesting a promising potential for the sustained application of nanobubbles in various fields such as water treatment processes and therapeutic medical interventions. Future research endeavors should prioritize the refinement of theoretical models while also aiming to experimentally validate the mechanisms of stability that have been observed across a range of diverse applications to further substantiate these findings.

Keywords: nanobubble; ultrafine bubble; gas diffusion; oxygen; hydrogen; oxyhydrogen; Chapman-Enskog theory; Arrhenius equation

I. Introduction

Nanobubbles, small gas-filled entities under 200 nm, are of great interest in chemistry for their properties and applications. Understanding their stability and diffusion is essential for their utilization in diverse fields like environmental science and medicine. Nanobubbles' enhanced stability arises from factors such as surface tension and charge distribution. Surface nanobubble stability is modulated by threshold current density (Zhang et al., 2024). The stability of bulk nanobubbles is contingent upon their liquid environment and gas transport parameters. These factors can prolong nanobubbles' lifetimes, extending up to 100 seconds for 200 nm radius bubbles (Wang et al., 2022). Surface charges on nanobubbles are critical for stability, preventing collisions and maintaining size distribution (Kuang et al., 2022).

Nanobubbles possess distinct diffusion properties due to their diminutive size and elevated internal pressure. Smaller nanobubbles exhibit a thicker liquid-gas interface and heightened liquid forces, improving diffusivity (Lei et al., 2024). The stability of nanobubbles is also governed by local gas oversaturation, affecting their growth. Molecular simulations indicate that nanobubbles can achieve equilibrium at current densities below a specific threshold, preserving their nanoscopic size (Zhang et al., 2024). The gas diffusion in nanobubbles involves a dynamic interplay of dissolution

and release, essential for comprehending their stability across varying environments (Wang et al., 2022).

Nanobubbles are investigated for applications in water treatment, remediation, and medical therapies. Their stability and reactivity enhance mass transfer efficiency in these areas (Cho et al., 2023) (Foundas et al., 2022). In medicine, nanobubbles facilitate improved delivery of antibacterial agents and drugs, showing increased efficacy against infections like those caused by *Staphylococcus aureus* (Senthilkumar & Kumar, 2023) (Kulkarni et al., 2022). Methods such as compression-decompression and varied gas usage enable control over nanobubble concentration and stability, expanding their industrial and chemical application potential (Xu et al., 2022). Despite their promise, understanding stability mechanisms and optimizing production for scalability remains a challenge. Ongoing research focuses on the interactions between gas saturation, surface charge, and environmental factors, which are vital for maximizing nanobubbles' potential across various domains. Further investigation is required to ascertain the long-term stability and behavior of nanobubbles under diverse conditions, crucial for their practical application.

The complexity associated with grasping the stability and dissipation of H_2 and O_2 within HHO nanobubbles arises from their distinctive characteristics. The stability of these nanobubbles contradicts classical theories like the Young-Laplace equation, which suggests quick dissolution from high internal pressures. Recent research has illuminated the mechanisms and factors affecting their stability and gas dissipation. In oxyhydrogen nanobubbles, the dissociation of H_2 and O_2 generates radicals which affect the bubble's chemical reactions. The production of hydrogen peroxide (H_2O_2) as a reaction byproduct can also influence gas stability and dissipation (Jain & Qiao, 2018).

High initial gas concentrations and densities enhance nanobubble stability by creating a supersaturated environment that hampers gas diffusion and dissolution (Hewage & Meegoda, 2022). Furthermore, a dual-layer surface charge reduces surface tension, which helps in further stabilizing the nanobubble (Gao et al., 2021). A partial hydrophobic coating on the bubble surface, according to the dynamic equilibrium model, can enhance nanobubble stability by decreasing gas permeability (Yasui et al., 2018). This is corroborated by evidence that a gas-impermeable layer on surface nanobubbles provides significant resistance to diffusion (Wang et al., 2013). The interaction between gas and water molecules disrupts interfacial hydrogen bonds, lowering surface tension and aiding stability, while electrostatic stress from surface charges contributes to nanobubble integrity (Gao et al., 2021). Increased temperatures accelerate gas diffusion, resulting in quicker dissipation from nanobubbles. Elevated internal pressures can destabilize nanobubbles, leading to their shrinkage or dissolution (Hewage & Meegoda, 2022). Reactive molecular dynamics simulations indicate that higher initial pressures and radical concentrations amplify the consumption rates of H_2 and O_2 , influencing stability and dissipation dynamics (Jain & Qiao, 2018).

The characterization of nanobubbles on rough surfaces is insufficient for understanding their stability and dynamics. Advanced techniques like synchrotron-based microscopy have provided valuable insights into nanobubbles at hydrophilic interfaces. Current models fail to accurately represent nanobubble response times. A model combining gas transport dynamics and environmental factors is essential for predicting nanobubble behavior. Numerical simulations are vital for investigating nanobubble stability and dissolution processes. Despite progress, challenges persist in fully understanding gas dynamics in nanobubbles. Theoretical and experimental methods must advance for a complete understanding of nanobubble dissipation, facilitating applications in energy, water treatment, and medicine.

This study aims to assess the diffusion and stability of H_2 and O_2 gases in oxyhydrogen nanobubbles in water. Two theoretical models are applied: Chapman-Enskog for diffusion coefficients and the modified Arrhenius equation for assessing activation volume and stability. The study quantifies gas diffusion rates and examines conditions affecting nanobubble dissipation and stability. This research enhances the understanding of nanobubble behavior in gas-liquid systems.

II. Theoretical Framework

II.1. Chapman-Enskog Theory for Gas Diffusion

The Chapman-Enskog theory was created to address the Boltzmann equation concerning transport properties in gasses. It employs a series expansion, referred to as the Chapman-Enskog expansion, to derive formulas for transport coefficients (Tipton et al., 2009) (Loyalka et al., 2007). Initially focused on simple gasses, the theory has been adapted to encompass more intricate systems, such as binary gas mixtures and fluids with continuous potentials like the Lennard-Jones fluid (Miyazaki et al., 2001).

The Enskog theory, a variation of the Chapman-Enskog framework, has been utilized to investigate the diffusion of intruders within granular suspensions. This entails simulating the motion of particles in a granular gas enveloped by a molecular gas that functions as a thermal bath (González et al., 2023). The theory applies the Chapman-Enskog expansion to resolve the Enskog-Lorentz kinetic equation, yielding insights into the mean square displacement of particles and the diffusion coefficient. The incorporation of Sonine polynomial approximations aids in achieving precise predictions, particularly when accounting for the mass and diameter ratios of the particles (González et al., 2023) (Abad & Garzó, 2023).

Although the Chapman-Enskog theory serves as a robust instrument for comprehending gas diffusion in liquids, its application is largely confined to systems where the assumptions of the Boltzmann equation are valid. In highly dense or strongly interacting systems, deviations from ideal behavior may arise, necessitating adjustments or alternative methodologies. Furthermore, while the model has been refined to address more complicated structures, its fidelity can be notably influenced by the selection of approximations and the specific qualities of the system being evaluated. However, the theory continues to be a fundamental aspect of the study of transport phenomena, providing essential insights into the microscopic dynamics of particles across various media.

Mathematical formulation

The Chapman-Enskog equation is used to calculate the diffusion coefficient of a gas in terms of its molecular properties. This equation is derived from kinetic theory and is particularly useful for estimating the diffusion of gasses in mixtures at low densities, where interactions between molecules are modeled statistically.

Chapman-Enskog equation for diffusion coefficient:

$$D_{AB} = \frac{3}{16} \cdot \frac{(k_B T)^{3/2}}{p \sigma_{AB}^2 \Omega_{D,AB}} \cdot \sqrt{\frac{2\pi}{\mu_{AB}}} \quad [\text{equation 1}]$$

Where D_{AB} is the *diffusion coefficient* of species A in species B, k_B *Boltzmann constant*, typically $1.38 \cdot 10^{-23} \text{ J/K}$, T the *temperature* in Kelvin (K), p the *pressure* in pascals (Pa), σ_{AB} the *average collision diameter* of molecules A and B (in meters), $\Omega_{D,AB}$ the *collision integral* (a function of the reduced temperature), accounting for intermolecular forces, and μ_{AB} the *reduced mass* of the system, given by:

$$\mu_{AB} = \frac{m_A m_B}{m_A + m_B}$$

where m_A and m_B are the molecular masses of species A and B, respectively.

The diffusion coefficient indicates the speed at which one gas moves through another. Factors like temperature, pressure, molecular size, and interaction forces affect it. The temperature is crucial in diffusion due to its relationship with molecular kinetic energy. Higher pressures hinder diffusion as crowded molecules collide more often. σ_{AB} measures molecular size and helps determine the probability of intermolecular collisions, while $\Omega_{D,AB}$ considers the complexity of these interactions across temperatures. The reduced mass impacts the relative movement of the two interacting molecules. The Chapman-Enskog equation integrates molecular properties and thermodynamic variables to describe diffusion in gas mixtures. It helps predict gas mixing and dispersion by analyzing molecular interactions.

Application to H₂ and O₂ Nanobubbles

The Chapman-Enskog theory is applied to estimate the diffusion rates of H₂ and O₂ in water by modifying the diffusion coefficient calculations typically used for gas-gas interactions to gas-liquid interactions. The Chapman-Enskog equation provides the diffusion coefficient D , which is essential

to calculate the diffusion rate. After substituting these values into the Chapman-Enskog equation, the calculated diffusion coefficient D for hydrogen in water becomes approximately: $D_{H_2} \approx 2.72 \cdot 10^{-17} m^2 s^{-1}$. In the case of oxygen diffusion, the same process is followed, but with different molecular parameters. The calculated diffusion coefficient D for oxygen in water: $D_{O_2} \approx 1.22 \cdot 10^{-17} m^2 s^{-1}$.

Once the diffusion coefficient is calculated, the diffusion flux J is determined using Fick's law. For hydrogen, $J_{H_2} = 3.25 \cdot 10^{-7} mol \cdot m^{-2} s^{-1}$ and for oxygen, $J_{O_2} = 7.28 \cdot 10^{-8} mol \cdot m^{-2} s^{-1}$. Finally, the diffusion rates R are found by multiplying the diffusion flux by the surface area of the bubble. For hydrogen, $R_{H_2} \approx 1.33 \cdot 10^{-20} mol \cdot s^{-1}$ and for oxygen, $R_{H_2} \approx 2.97 \cdot 10^{-2} mol \cdot s^{-1}$.

The observed diffusion rates are comparatively sluggish, chiefly attributable to the elevated pressure and the interactions occurring between the gaseous phase (in a nanobubble state) and the surrounding water molecules. It can be inferred that the diffusion rates are sufficiently minimal such that the nanobubbles encapsulating H_2 and O_2 exhibit stability, signifying that they will not expeditiously release their gaseous constituents.

II.2. Arrhenius equation for Stability

The Arrhenius equation is key in illustrating chemical kinetics and molecular stability, especially within elaborate systems like nanobubbles. This equation unequivocally connects the rate of a chemical reaction to temperature and activation energy, establishing a foundational framework for comprehending the impact of thermal conditions on molecular interactions and stability. In the realm of nanobubbles, the Arrhenius equation decisively clarifies the dynamics of chemical reactions and stability across varying conditions.

Arrhenius Equation in Chemical Kinetics

The Arrhenius equation is a fundamental tool in chemical kinetics, providing a quantitative relationship between reaction rates and temperature. It is expressed as $k = A \cdot \exp(-E_a/RT)$, where k is the rate constant, A is the pre-exponential factor, E_a is the activation energy, R is the gas constant, and T is the temperature in Kelvin (Alam, 2023). This equation is particularly useful in modeling reactions in gaseous phases, where it aligns with the Boltzmann distribution and equilibrium constants (Brebeneel & Berbente, 2023).

Nanobubbles face stability challenges due to their small size and high internal pressure. They can remain stable under specific conditions like high gas density and certain temperatures (Hewage & Meegoda, 2022). Stability depends on Laplace pressure and force balance at the bubble interface. Imbalance in these forces causes instability (Mi et al., 2023). The Arrhenius equation illustrates how temperature affects nanobubble stability and reaction rates, with higher temperatures potentially destabilizing them (Hong et al., 2022).

The Arrhenius equation has limitations in systems where reaction rates do not consistently rise with temperature. This is seen in complex reactions with specific optimal temperature ranges (Peleg et al., 2012). Super-Arrhenius or sub-Arrhenius behaviors occur at low temperatures, necessitating alternative models for accurate kinetics (Carvalho-Silva et al., 2016).

Molecular dynamics simulations reveal that porosity and temperature impact transport properties like viscosity and diffusivity in nanobubbles. These properties align with the Arrhenius equation, where higher temperatures lower viscosity and increase diffusivity (Hong et al., 2022). The simulations also emphasize that initial gas concentration and density are crucial for nanobubble stability, with higher densities enhancing stability (Hewage & Meegoda, 2022). Even with its drawbacks, the Arrhenius equation is crucial for grasping how temperature influences reaction rates and stability.

Modified Arrhenius Approach

The modified Arrhenius equation accounts for both the pressure and temperature effects on the diffusion process, as well as the activation volume ΔV of gasses like H_2 and O_2 inside nanobubbles.

This is an extension of the Chapman-Enskog theory that incorporates activation energy and pressure, especially at high pressures. The original form of the Arrhenius-like equation for the diffusion coefficient is:

$$R = R^0 \exp\left(\frac{-\Delta H}{k_B T}\right)$$

Where R is the generalized diffusion/rate coefficient, R^0 the characteristic diffusion coefficient/rate, ΔH the activation enthalpy of diffusion, k_B the Boltzmann constant, and T the temperature in Kelvin.

At high pressures, ΔH can be approximated as $p\Delta V$, where p is the pressure, and ΔV is the activation volume. Hence, the equation is modified to:

$$R = R^0 \exp\left(\frac{-p\Delta V}{k_B T}\right)$$

To account for the gas-liquid interface tension coefficient γ , the Young-Laplace equation is applied, which gives $p \approx \frac{2\gamma}{r}$, where r is the bubble radius. Substituting this into the equation results in the following modified equation:

$$R = R^0 \exp\left(\frac{-2\gamma\Delta}{k_B T r}\right) \text{ [equation 2]}$$

This equation indicates that the diffusion coefficient is influenced by temperature, pressure, surface tension, and bubble radius. The activation volume ΔV signifies molecular interactions of the gasses and impacts diffusion rates in nanobubbles. The values of ΔV for H_2 and O_2 differ due to their molecular sizes and water interactions, yet the approach remains the same. The modified Arrhenius equation includes activation volume and pressure effects. For both gasses, R is calculated based on ΔV , γ , and r . The diffusion rates demonstrate how internal pressure in nanobubbles affects their stability and gas release, stemming from surface tension.

Theoretical Implications for Bubble Stability

The stability of nanobubbles is greatly influenced by several factors, including pressure, temperature, and molecular interactions. Pressure plays a critical role in determining the stability of bubbles, especially nanobubbles, as governed by the *Young-Laplace equation*:

$$p_{bubble} = p_0 + \frac{2\gamma}{r}$$

The parameters p_{bubble} , p_0 , γ , and r denote the internal and external pressures, surface tension, and bubble radius, respectively. Smaller bubbles experience increased internal pressure due to the $2\gamma/r$ term, leading to gas dissolution and potential destabilization. In environments with elevated external pressure, the internal pressure further increases, enhancing gas dissolution rates. The modified Arrhenius equation incorporates ΔV , where higher pressure amplifies its effect on diffusion rates. Consequently, gasses under increased pressure diffuse more rapidly from the bubble, resulting in decreased bubble longevity. Therefore, elevated internal and external pressures generally facilitate gas diffusion and diminish bubble stability.

Temperature influences bubble stability through its effect on gas kinetic energy and liquid surface tension. Elevated temperature raises the kinetic energy of gas molecules, enhancing gas diffusion into the liquid and promoting bubble shrinkage. The Arrhenius equation's term $\frac{-\Delta H}{k_B T}$ reveals that higher temperatures lower the activation enthalpy ΔH for diffusion, thus increasing diffusion rates. Additionally, increased temperature decreases the liquid's surface tension. Although lower surface tension reduces internal pressure, seemingly aiding stability, it also compromises the bubble interface's ability to retain gas, thus accelerating gas dissolution. At high temperatures, molecular interactions may alter, further destabilizing the bubble. Hence, elevated temperatures generally enhance gas diffusion and reduce surface tension, both contributing to diminished bubble stability.

The interactions between gas molecules and water significantly influence bubble stability. Different gasses exhibit varying solubilities in water, affecting their diffusion rates. Oxygen's higher solubility compared to hydrogen results in faster diffusion from the bubble at constant temperature and pressure. Collision diameter and activation volume indicate the molecular size and interaction

between gas and water molecules. Larger molecules or those with stronger interactions with water, such as oxygen, face greater diffusion resistance than smaller molecules like hydrogen. In the diffusion of H_2 and O_2 , hydrogen's lower molecular mass and smaller collision diameter yield lower diffusion resistance, enabling faster diffusion compared to oxygen. Conversely, the stronger molecular interactions of oxygen with water lead to slower diffusion, promoting slightly greater stability for O_2 -containing bubbles than those with H_2 . At the nanoscale, surface interactions between gas molecules and liquid may stabilize nanobubbles. Certain nanobubbles maintain stability due to a protective layer of surface-active molecules or specific interfacial interactions that impede gas diffusion. Molecular interactions dictate the ease of gas dissolution in water, with stronger interactions typically slowing diffusion and enhancing bubble stability. Conversely, gasses with weaker interactions, exemplified by H_2 , diffuse more rapidly, resulting in quicker bubble destabilization.

In summary, 1) increased pressure enhances gas dissolution but diminishes bubble stability; 2) elevated temperature boosts molecular motion and reduces surface tension, thus accelerating gas diffusion and destabilizing bubbles; and 3) molecular interactions between gas and liquid affect diffusion rates, with weakly interacting gasses (like H_2) being less stable than those with strong interactions (like O_2). In nanobubbles, these factors collectively govern bubble stability and lifespan. Nanobubbles may demonstrate unique stability characteristics due to the intricate interplay of internal pressure, molecular interactions, and surface tension.

III. Calculations and Results

The calculations here are based on an experiment where we generated HHO nanobubbles. The mean diameter is 110nm and the concentration is 50 million nanobubbles/mL.

III.1. Diffusion Coefficients of H_2 and O_2

Important Parameters

Molar mass. Hydrogen: $M_A = 2.016 \cdot 10^{-3} \text{ kg/mol}$, water: $M_B = 18.015 \cdot 10^{-3} \text{ kg/mol}$, and oxygen: $M_A = 32 \cdot 10^{-3} \text{ kg/mol}$. Temperature. $T = 298.15 \text{ K}$ (room temperature). Pressure. Hydrogen: $p = 16.69 \text{ atm} = 1.69 \cdot 10^6 \text{ Pa}$ and oxygen: $p = 8.32 \text{ atm} = 8.43 \cdot 10^5 \text{ Pa}$. Collision diameter. Hydrogen: $\sigma = 2.827 \cdot 10^{-10} \text{ m}$ and oxygen: $\sigma = 3.467 \cdot 10^{-10} \text{ m}$. **Boltzmann constant:** $k_B = 1.38 \cdot 10^{-23} \text{ J/K}$. Collision integral. Hydrogen: $\Omega_D \approx 1$ and oxygen: $\Omega_D \approx 1.17$. Activation volume. Hydrogen: $\Delta V = 2.59 \cdot 10^{-27} \text{ m}^3$ and oxygen: $\Delta V = 5.559 \cdot 10^{-27} \text{ m}^3$. These properties are applied in the Chapman-Enskog equation and the Arrhenius-like models to calculate the diffusion rates and stability of H_2 and O_2 gasses in nanobubbles.

Calculation Steps

The Chapman-Enskog equation for the diffusion coefficient D of a gas in water (adapted for liquid interactions) is shown before (equation 1). Substituting the parameters above into the Chapman-Enskog equation:

$$D_{H_2} = \frac{3}{16} \cdot \frac{\sqrt{\left(\frac{1}{2.016 \cdot 10^{-3} \text{ kg/mol}} + \frac{1}{18.015 \cdot 10^{-3} \text{ kg/mol}}\right)} \cdot \sqrt{(1.38 \cdot 10^{-23} \text{ J} \cdot \text{K}^{-1} \cdot 298.15 \text{ K})^3}}{1.13 \cdot 10^6 \text{ Pa} \cdot (2.827 \cdot 10^{-10} \text{ m})^2 \cdot 1}$$

$$D_{H_2} = 2.72 \cdot 10^{-17} \text{ m}^2 \text{ s}^{-1}$$

And

$$D_{O_2} = \frac{3}{16} \cdot \frac{\sqrt{\left(\frac{1}{32 \cdot 10^{-3} \text{ kg/mol}} + \frac{1}{18.015 \cdot 10^{-3} \text{ kg/mol}}\right)} \cdot \sqrt{(1.38 \cdot 10^{-23} \text{ J} \cdot \text{K}^{-1} \cdot 298.15 \text{ K})^3}}{8.43 \cdot 10^5 \text{ Pa} \cdot (3.467 \cdot 10^{-10} \text{ m})^2 \cdot 1.17}$$

$$D_{O_2} = 1.22 \cdot 10^{-17} \text{ m}^2 \text{ s}^{-1}$$

Using the *Chapman-Enskog theory*, the estimated diffusion coefficient of hydrogen gas at 16.7 atm and 298.15 K in a bubble of 55 nm radius inside water is approximately $2.72 \cdot 10^{-17} \text{ m}^2 \text{ s}^{-1}$. This result indicates that the diffusion of hydrogen gas in water under these conditions is quite slow, mainly due

to the high pressure and the interaction between hydrogen and the water molecules. Meanwhile for oxygen, the estimated diffusion coefficient at 8.32 atm and 298.15 K is approximately $1.22 \cdot 10^{-17} m^2 s^{-1}$.

The estimated diffusion coefficient D of oxygen gas at 8.32 atm and 298.15 K in a bubble of 55 nm radius inside water is approximately $1.22 \cdot 10^{-17} m^2 s^{-1}$. Given $r = 55 nm$, the surface area of a sphere is:

$$A = 4\pi r^2 = 4\pi \cdot (55 \cdot 10^{-9} m)^2 = 3.8 \cdot 10^{-1} m^2$$

The diffusion flux J of H_2 is calculated by

$$J_{H_2} = D \frac{dC}{dr} = 2.72 \cdot 10^{-17} m^2 s^{-1} \cdot \frac{682 mol \cdot m^{-3}}{5.7 \cdot 10^{-8} m} = 3.25 \cdot 10^{-7} mol \cdot m^{-2} s^{-1}$$

Finally, the diffusion rate R of H_2 :

$$R_{H_2} = J_{H_2} \times A = 3.25 \cdot 10^{-7} mol \cdot m^{-2} s^{-1} \times 4.08 \cdot 10^{-14} m^2 = 1.33 \cdot 10^{-20} mol \cdot s^{-1}$$

Similar approach to O_2 yields $J_{O_2} = 7.28 \cdot 10^{-8} mol \cdot m^{-2} s^{-1}$ and $R_{O_2} = 2.97 \cdot 10^{-21} mol \cdot s^{-1}$.

Nonetheless, we know that the zeta potential prevents the nanobubble from shrinking as quickly due to repulsive forces, leading to a more sustained surface area for gas diffusion. The effective radius of the bubble stays larger for longer, maintaining a consistent rate of gas release. Thus, the diffusion rate J can be modified to reflect the influence of zeta potential as follows:

$$J = \frac{D}{r_{eff}} \Delta C$$

Where D is the diffusion coefficient of the gas, r_{eff} is the effective radius of the nanobubble, which remains larger for a longer time due to the stabilizing effect of zeta potential, and ΔC is the concentration difference between the inside of the nanobubble and the surrounding water. Since,

$$\Delta P = \frac{2\gamma}{r_{eff}} \Rightarrow r_{eff} = \frac{2\gamma}{\Delta P}$$

For HHO nanobubble,

$$r_{eff} = \frac{2 \cdot 0.072}{(2.53 - 0.10) \cdot 10^6} = \frac{1.44 \cdot 10^{-1}}{2.43 \cdot 10^6} = 5.93 \cdot 10^{-8} m$$

The Debye length κ^{-1} measures the thickness of the electrical double layer in a solution. It is influenced by the ionic strength of the solution and the permittivity of the solvent. The Debye length is calculated using the formula:

$$\kappa^{-1} = \sqrt{\frac{\epsilon_0 \epsilon_r k_B T}{2 N_A e^2 I}}$$

Where κ^{-1} is the Debye length (in meters), ϵ_0 is the vacuum permittivity ($8.854 \cdot 10^{-12} F/m$), ϵ_r is the relative permittivity (or dielectric constant) of the medium (for water at 25°C, $\epsilon_r \approx 78.5$), e is the elementary charge ($1.602 \cdot 10^{-19} C$), and I is the ionic strength of the solution (in mol/L). For pure water, the ionic strength is extremely low, $I_{H_2O} = 10^{-7} mol/L$, and for normal saline, $I_{NS} = 0.154 mol/L$

Substituting these values into the Debye length formula:

$$\kappa_{H_2O}^{-1} = \sqrt{\frac{(8.854 \cdot 10^{-12}) \cdot 78.5 (1.38 \cdot 10^{-23}) \cdot 298.15}{2(6.022 \cdot 10^{23})(1.602 \cdot 10^{-19})^2 \cdot 10^{-7}}} \approx 9.58 \cdot 10^{-7}$$

After simplifying the units and performing the calculation, the **Debye length** for pure water is $\kappa_{H_2O}^{-1} \approx 958 nm$ (relatively long). Again, substituting these values into the Debye length formula for normal saline:

$$\kappa_{NS}^{-1} = \sqrt{\frac{(8.854 \cdot 10^{-12}) \cdot 78.5 (1.38 \cdot 10^{-23}) \cdot 298.15}{2(6.022 \cdot 10^{23})(1.602 \cdot 10^{-19})^2 \cdot 0.154}} \approx 0.79 \cdot 10^{-9}$$

After simplifying, the **Debye length** for saline is $\kappa_{NS}^{-1} \approx 0.79 nm$. This shorter Debye length reflects the fact that saline has a high ionic strength, leading to much stronger screening of electrostatic forces.

Modifying earlier equation to incorporate κ^{-1} to estimate the r_{eff} more accurately,

$$J = \frac{D}{r_0 + \kappa^{-1}} \Delta C$$

Values of Henry's Law constant at room temperature (298 K) are: $k_{H_2} = 7.8 \cdot 10^{-4} \text{ mol} \cdot \text{m}^{-3} \text{ Pa}^{-1}$ and $k_{O_2} = 1.3 \cdot 10^{-3} \text{ mol} \cdot \text{m}^{-3} \text{ Pa}^{-1}$. Substituting those values into $C = k_H P$, the concentrations of H_2 and O_2 inside the nanobubbles are $C_{H_2}^i = 1.98 \text{ mol} \cdot \text{m}^{-3}$ and $C_{O_2}^i = 3.29 \text{ mol} \cdot \text{m}^{-3}$. The concentration of H_2 and O_2 in water (outside the nanobubble) at 1 atm and room temperatures are $C_{H_2}^o = 0.079 \text{ mol} \cdot \text{m}^{-3}$ and $C_{O_2}^o = 0.132 \text{ mol} \cdot \text{m}^{-3}$.

Thus, the J of hydrogen and oxygen in pure water are,

$$J_{H_2}^{H_2O} = \frac{D}{r_0 + \kappa_{H_2O}^{-1}} \Delta C = \frac{2.72 \cdot 10^{-17}}{(55 + 958) \cdot 10^{-9}} (1.98 - 0.079) = 5.10 \cdot 10^{-11} \text{ mol} \cdot \text{m}^{-2} \text{ s}^{-1}$$

$$J_{O_2}^{H_2O} = \frac{D}{r_0 + \kappa_{H_2O}^{-1}} \Delta C = \frac{1.22 \cdot 10^{-17}}{(55 + 958) \cdot 10^{-9}} (3.29 - 0.132) = 3.80 \cdot 10^{-11} \text{ mol} \cdot \text{m}^{-2} \text{ s}^{-1}$$

Substituting the values obtained before, for H_2 and O_2 ,

$$Q(t)_{H_2}^{H_2O} = \int_0^t 5.10 \cdot 10^{-11} \text{ mol} \cdot \text{m}^{-2} \text{ s}^{-1} \times 3.8 \cdot 10^{-14} \text{ m}^2 dt = 1.938 \cdot 10^{-24} (t) \text{ mol}$$

and

$$Q(t)_{O_2}^{H_2O} = \int_0^t 3.80 \cdot 10^{-11} \text{ mol} \cdot \text{m}^{-2} \text{ s}^{-1} \times 3.8 \cdot 10^{-14} \text{ m}^2 dt = 1.444 \cdot 10^{-24} (t) \text{ mol}$$

From this, we can estimate how long the gas will dissipate from the nanobubble

$$\text{Dissipation}_{H_2}^{H_2O} = 4.75 \cdot 10^{-19} \text{ mol} \div 1.938 \cdot 10^{-24} \text{ mol} \cdot \text{s}^{-1} \approx 68 \text{ hours}$$

and

$$\text{Dissipation}_{O_2}^{H_2O} = 2.37 \cdot 10^{-19} \text{ mol} \div 1.444 \cdot 10^{-24} \text{ mol} \cdot \text{s}^{-1} \approx 46 \text{ hours}$$

In normal saline, the J of hydrogen and oxygen are,

$$J_{H_2}^{NS} = \frac{D}{r_0 + \kappa_{NS}^{-1}} \Delta C = \frac{2.72 \cdot 10^{-17}}{(55 + 0.79) \cdot 10^{-9}} (1.98 - 0.079) = 9.27 \cdot 10^{-10} \text{ mol} \cdot \text{m}^{-2} \text{ s}^{-1}$$

$$J_{O_2}^{NS} = \frac{D}{r_0 + \kappa_{NS}^{-1}} \Delta C = \frac{1.22 \cdot 10^{-17}}{(55 + 0.79) \cdot 10^{-9}} (3.29 - 0.132) = 6.91 \cdot 10^{-10} \text{ mol} \cdot \text{m}^{-2} \text{ s}^{-1}$$

Substituting the values obtained before, for H_2 and O_2 ,

$$Q(t)_{H_2}^{NS} = \int_0^t 9.27 \cdot 10^{-10} \text{ mol} \cdot \text{m}^{-2} \text{ s}^{-1} \times 3.8 \cdot 10^{-14} \text{ m}^2 dt = 3.523 \cdot 10^{-23} (t) \text{ mol}$$

and

$$Q(t)_{O_2}^{NS} = \int_0^t 6.91 \cdot 10^{-10} \text{ mol} \cdot \text{m}^{-2} \text{ s}^{-1} \times 3.8 \cdot 10^{-14} \text{ m}^2 dt = 2.626 \cdot 10^{-23} (t) \text{ mol}$$

From this, we can estimate how long the gas will dissipate from the nanobubble

$$\text{Dissipation}_{H_2}^{NS} = 4.75 \cdot 10^{-19} \text{ mol} \div 3.523 \cdot 10^{-23} \text{ mol} \cdot \text{s}^{-1} \approx 3.75 \text{ hours}$$

and

$$\text{Dissipation}_{O_2}^{NS} = 2.37 \cdot 10^{-19} \text{ mol} \div 2.626 \cdot 10^{-23} \text{ mol} \cdot \text{s}^{-1} \approx 2.5 \text{ hours}$$

Therefore, the dissipation of H_2 and O_2 from an HHO nanobubble in water is between 46-68 hours and in normal saline is between 2.5-3.75 hours.

III.2. Activation Volume and Diffusion Rates

Calculation of Activation Volume

The activation volume ΔV represents the volume change associated with molecular movement, particularly in diffusion processes, and is influenced by the internal and external pressures of the nanobubble. The modified Arrhenius equation for the diffusion rate R is given as equation 2 above. We'll use the following formula to calculate the activation volume:

$$\Delta V \approx \frac{k_B T}{P_{\text{internal}} - P_{\text{external}}} \approx \frac{1.38 \cdot 10^{-23} \times 298.15}{(1.69 - 0.10) \cdot 10^6} \approx 2.59 \cdot 10^{-27}$$

Thus, the H_2 activation volume (ΔV_{H_2}) is $2.59 \cdot 10^{-27} \text{ m}^3$. To convert to volume per mole, multiply by Avogadro's number, where $\Delta V_{H_2/\text{mol}}$ is $1.56 \cdot 10^{-3} \text{ m}^3/\text{mol}$.

Meanwhile for O_2 :

$$\Delta V \approx \frac{k_B T}{P_{\text{internal}} - P_{\text{external}}} \approx \frac{1.38 \cdot 10^{-23} \times 298.15}{(0.843 - 0.101) \cdot 10^6} \approx 5.55 \cdot 10^{-27}$$

Thus, the O₂ activation volume (ΔV_{O_2}) is $5.55 \cdot 10^{-27} m^3$, and the volume per mole, $\Delta V_{O_2/mol}$ is $3.34 \cdot 10^{-3} m^3/mol$.

These activation volumes are crucial in determining how pressure affects the diffusion rates of these gasses in nanobubbles. Hydrogen, with a smaller activation volume, diffuses faster, leading to less stability compared to oxygen, which has a larger activation volume, making oxygen nanobubbles more stable under the same conditions.

Diffusion Rate Analysis

The initial diffusion rate: $R_{H_2}^0 = 1.33 \cdot 10^{-2} mol/s$ and $R_{O_2}^0 = 2.97 \cdot 10^{-21} mol/s$. Referring back to equation 2, for H₂ - substitute the values into the exponent term:

$$Exponent_{H_2} = \frac{-2 \times 0.072 \times 1.56 \cdot 10^{-3}}{1.38 \cdot 10^{-2} \times 298.15 \times 5.7 \cdot 10^{-8}} = \frac{-2.25 \cdot 10^{-7}}{2.35 \cdot 10^{-30}} = -9.57 \cdot 10^{22}$$

And as for O₂ - the exponent term:

$$Exponent_{O_2} = \frac{-2 \times 0.072 \times 3.34 \cdot 10^{-3}}{1.38 \cdot 10^{-23} \times 298.15 \times 5.7 \cdot 10^{-8}} = \frac{-4.81 \cdot 10^{-7}}{2.35 \cdot 10^{-30}} = -2.35 \cdot 10^{23}$$

The exponent term for H₂ and O₂ are extremely large and negative, $exp(-9.57 \cdot 10^{22})$ and $exp(-2.35 \cdot 10^{23})$, respectively. The new diffusion rate R_{H_2} and R_{O_2} are hence *both* ≈ 0 or *negligible*. With the modified diffusion rate approaching zero, the hydrogen or oxygen gas will no longer diffuse out of the nanobubble. This indicates *extreme stability*, meaning the nanobubble can persist for long durations without the gas escaping.

Both hydrogen and oxygen nanobubbles exhibit extremely high stability with their modified diffusion rates approaching zero. The large negative exponent in the Arrhenius-like equation, resulting from the significant pressure inside the nanobubbles and the activation volume, effectively prevents the diffusion of these gasses. This suggests that nanobubbles containing either gas will remain intact and not dissipate quickly, making them ideally stable for extended use.

IV. Discussion

The comparison of evanescence and permanence of HHO nanobubbles using the Chapman-Enskog and Arrhenius approaches provides a comprehensive insight into the mechanisms governing gas diffusion and nanobubble stability.

The Chapman-Enskog theory decisively centers on gas diffusion rates, particularly emphasizing the molecular interactions within a gas and between gas and liquid. This theory is rigorously applied to calculate the diffusion coefficients of gasses in water within nanobubbles. In this method, diffusion coefficients for gasses are unequivocally derived using factors such as the molar mass of the gasses, the temperature and pressure of the gas in the nanobubbles, and the collision diameter of gas molecules. The Chapman-Enskog equation for the diffusion of gasses in water definitively yields $D_{hydrogen} = 2.72 \cdot 10^{-1} m^2/s$ and $D_{oxygen} = 1.22 \cdot 10^{-17} m^2/s$. These values emphatically underscore the slow diffusion of both gasses due to the high pressure inside the nanobubbles and the molecular interactions with water. Hydrogen diffuses marginally faster than oxygen; however, both gasses exhibit low diffusion rates, which firmly contribute to the nanobubbles' stability over time. The critical point in the Chapman-Enskog approach is the unequivocal relationship between molecular size, pressure, and temperature, illustrating how gasses escape from nanobubbles at a sluggish rate. This directly impacts the evanescence of the nanobubbles. The slow diffusion unequivocally enables the nanobubbles to endure for hours to days, contingent on the surrounding medium (46-68 hours in water, 2.5-3.75 hours in saline).

The Arrhenius approach is assertively utilized to investigate the stability and longevity of the nanobubbles based on activation energy and internal pressure. This approach decisively calculates the modified diffusion rate by considering the zeta potential and the activation volume of the gas, which effectively prevents rapid diffusion. For nanobubbles, the internal and external pressures critically influence the activation volume. The activation volumes determined for hydrogen and oxygen are $\Delta V_{hydrogen} = 1.56 \cdot 10^{-3} m^3/mol$ and is $\Delta V_{oxygen} = 3.34 \cdot 10^{-3} m^3/mol$. These values are emphatically employed in the Arrhenius-like model to demonstrate how high internal pressure and

substantial activation volumes lead to exceedingly slow diffusion rates. As the exponent becomes exceedingly large and negative, the diffusion rate approaches zero, signifying long-term stability or permanence of the nanobubble. This stability is markedly more pronounced in the Arrhenius approach, indicating that, under specific conditions, nanobubbles can undoubtedly persist without significant gas loss for extended durations.

The Chapman-Enskog theory delivers a definitive estimate of gas diffusion rates grounded in molecular properties and interactions, asserting that although the diffusion of gasses from nanobubbles is slow, it undeniably occurs at measurable rates. The Arrhenius approach, in stark contrast, underscores the almost negligible diffusion rate once the stabilizing effects of zeta potential and pressure are factored in. It firmly asserts that nanobubbles can maintain stability for significantly longer durations than what the Chapman-Enskog theory anticipates. From the Chapman-Enskog perspective, nanobubbles possess a constrained lifespan, with gas diffusion leading to their inevitable dissipation. The gas loss rate is indeed slow, permitting stability for hours to days, yet it is clear that nanobubbles are not indefinitely stable. The Arrhenius approach, conversely, stresses permanence. The substantial negative exponent in the Arrhenius equation creates a situation where the nanobubble's diffusion rate approaches zero, signifying that the gas is effectively "trapped" within the bubble for prolonged periods, culminating in remarkable stability and negligible gas loss. The Chapman-Enskog theory asserts that medium conditions (such as the difference between water and saline) significantly influence the stability of nanobubbles. In saline, for example, the elevated ionic strength accelerates dissipation, whereas in pure water, nanobubbles exhibit enhanced stability. The Arrhenius approach further incorporates internal pressures and activation volumes, emphasizing that nanobubbles with increased internal pressure will maintain stability for much longer due to the near-total suppression of gas diffusion.

Both theories elucidate different dimensions of the evanescence and permanence of HHO nanobubbles. Chapman-Enskog concentrates on molecular diffusion and demonstrates gradual dissipation over hours to days. Arrhenius highlights the stability of nanobubbles and posits that under specific conditions, they can remain intact for extremely long durations due to minimal diffusion. Thus, the Chapman-Enskog approach addresses the slow, yet inevitable, evanescence of nanobubbles, while the Arrhenius model unequivocally proposes the potential for permanence under optimal conditions.

The insights derived from the study of HHO nanobubbles through both the Chapman-Enskog and Arrhenius frameworks exert a substantial influence on the comprehensive understanding of nanobubble chemistry and gas diffusion mechanisms. The nanobubbles can display exceptional stability, far surpassing previous assumptions regarding gas bubbles in liquid. Conventional perceptions of gas bubbles in water indicated that they dissipate relatively quickly due to the high diffusion rates of gasses such as hydrogen and oxygen. Nonetheless, the findings decisively reveal that the zeta potential and internal pressure are critical factors in stabilizing nanobubbles. The zeta potential generates repulsive electrostatic forces that unequivocally prevent the collapse of nanobubbles, while internal pressure, in concert with the bubble's activation volume, dramatically curtails gas diffusion. The Arrhenius approach forecasts near-zero diffusion rates, clearly indicating that under optimal conditions, nanobubbles may maintain stability for extended periods or even indefinitely. This fundamentally challenges prior assumptions that nanobubbles are transient, thereby unlocking new avenues for their application in long-term scenarios where sustained gas retention is essential.

Nanobubble research enhances understanding of gas diffusion in confined spaces. The Chapman-Enskog and Arrhenius models demonstrate unique nanoscale gas diffusion behaviors. Traditional diffusion theory relies on molecular size, pressure, and temperature. Chapman-Enskog confirms that smaller molecules diffuse faster, but both gasses face slow diffusion due to nanobubble pressure. The Arrhenius model shows activation volume can trap gasses in nanobubbles. In nanobubbles, diffusion rates near zero defy traditional macroscopic diffusion laws. The research clarifies surface chemistry at the nanobubble interface. Gas-liquid interactions at the nanoscale are complex. Nanobubbles' higher surface area-to-volume ratio enhances gas-liquid interactions. Ionic

environments significantly affect nanobubble stability. Nanobubble persistence under varying conditions underscores the electric double layer's role. Manipulating ionic strength could control nanobubble longevity for various applications.

In biomedical fields, nanobubbles serve as drug delivery vehicles. Near-zero diffusion rates allow gas-loaded nanobubbles to retain therapeutic gasses for longer. Tunable nanobubble stability aids targeted drug delivery. Nanobubbles can release payloads based on environmental changes, ensuring precise control. Extreme stability in certain conditions could transform multiple industries. In industrial applications, nanobubbles enhance chemical reactions by prolonging gas presence. In nanomedicine, they optimize ultrasound imaging and gas transport. In environmental science, nanobubbles improve aeration, oil recovery, and carbon sequestration effectiveness. The Chapman-Enskog and Arrhenius models are essential for understanding gas diffusion in nanobubbles and predicting stability. These models are flawed due to their idealized assumptions. Real systems are complex, influenced by molecular interactions and environmental factors. While these models provide insights, they require refinement and validation for practical applications.

V. Conclusion

The study of HHO nanobubbles reveals vital insights into gas diffusion and stability. The Chapman-Enskog theory explains the slow gas diffusion from nanobubbles due to high pressure and interactions with water. This slow process leads to a limited lifespan of nanobubbles, with gas dissipating over hours or days. In contrast, the Arrhenius approach indicates that under ideal conditions, nanobubbles can achieve long-term stability with minimal gas diffusion. This stability is influenced by factors like zeta potential and activation volume, which help retain gas within the nanobubbles. While useful, these theoretical models have constraints. They depend on simplifications that may not accurately reflect real-world complexities. Thus, more experimental validation and refinement are essential for a deeper understanding of nanobubble behavior.

In summary, the Chapman-Enskog theory highlights the fleeting nature of nanobubbles, while the Arrhenius approach opens doors to potential permanence, presenting thrilling opportunities for long-lasting gas stability applications.

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References

1. Abad, E. and Garzó, V., 2023. Diffusion of intruders in granular suspensions: Enskog theory and random walk interpretation. *Physical review. E*, 108(2-1), pp.024903-024903.
2. Brebenel, M. and Berbente, C., 2023. A new approach for mathematical modeling of chemical kinetics. *Technical Sciences*, 8(1), pp.43-52.
3. Carvalho-Silva, V.H., Coutinho, N.D. and Aquilanti, V., 2016, December. Description of deviations from Arrhenius behavior in chemical kinetics and materials science. In *AIP conference proceedings* (Vol. 1790, No. 1). AIP Publishing.
4. Cho, C.H., Shin, H.J., Singh, B., Kim, K. and Park, M.H., 2023. Assessment of sub-200-nm nanobubbles with ultra-high stability in water. *Applied Water Science*, 13(7), p.149.
5. Dou, R., 2023. Deriving the Arrhenius Equation and the Pre-Exponential Factor.
6. Foudas, A.W., Kosheleva, R.I., Favvas, E.P., Kostoglou, M., Mitropoulos, A.C. and Kyzas, G.Z., 2023. Fundamentals and applications of nanobubbles: A review. *Chemical Engineering Research and Design*, 189, pp.64-86.
7. Gao, Z., Wu, W., Sun, W. and Wang, B., 2021. Understanding the stabilization of a bulk nanobubble: A molecular dynamics analysis. *Langmuir*, 37(38), pp.11281-11291.
8. Gómez González, R., Abad, E., Bravo Yuste, S. and Garzó, V., 2023. Diffusion of intruders in granular suspensions: Enskog theory and random walk interpretation. *Physical Review E*, 108(2), p.024903.
9. Hewage, S.A. and Meegoda, J.N., 2022. Molecular dynamics simulation of bulk nanobubbles. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, 650, p.129565.

10. Hong, S.N., Ri, J.H., Mun, S.Y. and Yu, C.J., 2022. Revealing the influence of porosity and temperature on transport properties of nanobubble solution with molecular dynamics simulations. *Journal of Molecular Liquids*, 367, p.120518.
11. Jain, S. and Qiao, L., 2018. Understanding combustion of H₂/O₂ gasses inside nanobubbles generated by water electrolysis using reactive molecular dynamic simulations. *The Journal of Physical Chemistry A*, 122(24), pp.5261-5269.
12. Kuang, Y., Feng, Y., Dong, H., Yang, L. and Zheng, Y., 2022. Evolution process and stabilization mechanism of different gas nanobubbles based on improved statistical analysis. *Nano Select*, 3(6), pp.1091-1101.
13. Kulkarni, A.D., Gulecha, V.S., Dolas, R.T., Zalte, A.G., Deore, S.R., Deore, S.S. and Pande, V.V., 2022. Nanobubbles: Fundamentals and recent drug delivery applications. *Int. J. Health Sci*, 6, pp.1004-1025.
14. Lei, J., Huang, D., Zhao, W., Liu, S. and Yue, Y., 2024. Investigating the stability mechanisms of single bulk Nanobubbles: A molecular dynamics perspective. *International Journal of Heat and Mass Transfer*, 225, p.125407.
15. Loyalka, S.K., Tipton, E.L. and Tompson, R.V., 2007. Chapman–Enskog solutions to arbitrary order in Sonine polynomials I: Simple, rigid-sphere gas. *Physica A: Statistical Mechanics and its Applications*, 379(2), pp.417-435.
16. Mi, S., Hou, C. and Ge, W., 2024. Theoretical study on the stability of nanobubbles and its verification in molecular dynamics simulation. *Particuology*, 87, pp.99-105.
17. Miyazaki, K., Srinivas, G. and Bagchi, B., 2001. The Enskog theory for transport coefficients of simple fluids with continuous potentials. *The Journal of Chemical Physics*, 114(14), pp.6276-6285.
18. Pan, G., He, G., Zhang, M., Zhou, Q., Tyliczszak, T., Tai, R., Guo, J., Bi, L., Wang, L. and Zhang, H., 2016. Nanobubbles at hydrophilic particle–water interfaces. *Langmuir*, 32(43), pp.11133-11137.
19. Peleg, M., Normand, M.D. and Corradini, M.G., 2012. The Arrhenius equation revisited. *Critical reviews in food science and nutrition*, 52(9), pp.830-851.
20. Senthilkumar, G. and Aravind Kumar, J., 2023. Nanobubbles: a promising efficient tool for therapeutic delivery of antibacterial agents for the Staphylococcus aureus infections. *Applied Nanoscience*, 13(9), pp.6177-6190.
21. Sverdrup, K., Kimmerle, S.J. and Berg, P., 2017. Computational investigation of the stability and dissolution of nanobubbles. *Applied Mathematical Modelling*, 49, pp.199-219.
22. Tan, B.H., An, H. and Ohl, C.D., 2019. Stability, dynamics, and tolerance to undersaturation of surface nanobubbles. *Physical Review Letters*, 122(13), p.134502.
23. Tipton, E.L., Tompson, R.V. and Loyalka, S., 2009. Chapman–Enskog solutions to arbitrary order in Sonine polynomials III: Diffusion, thermal diffusion, and thermal conductivity in a binary, rigid-sphere, gas mixture. *European Journal of Mechanics-B/Fluids*, 28(3), pp.353-386.
24. Wang, S., Liu, M. and Dong, Y., 2013. Understanding the stability of surface nanobubbles. *Journal of Physics: Condensed Matter*, 25(18), p.184007.
25. Wang, Y., Zhou, L., Hu, J. and Zhang, L., 2022. Theoretical analysis on the stability of single bulk nanobubble. *Frontiers in Materials*, 9, p.824283.
26. Xu, W., Wang, Y., Huang, Q., Wang, X., Zhou, L., Wang, X., Wen, B., Guan, N., Hu, J., Zhou, X. and Zhang, L., 2023. The generation and stability of bulk nanobubbles by compression-decompression method: The role of dissolved gas. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, 657, p.130488.
27. Yasui, K., Tuziuti, T. and Kanematsu, W., 2018. Mysteries of bulk nanobubbles (ultrafine bubbles); stability and radical formation. *Ultrasonics sonochemistry*, 48, pp.259-266.
28. Zhang, Y., Zhu, X., Wood, J.A. and Lohse, D., 2024. Threshold current density for diffusion-controlled stability of electrolytic surface nanobubbles. *Proceedings of the National Academy of Sciences*, 121(21), p.e2321958121.

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