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Posted Date: 15 May 2023

doi: 10.20944/preprints201711.0022.v7

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The Geometrization of Maxwell's Equations and the Emergence of Gravity

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Abstract: Coupling the Maxwell tensor to the Riemann-Christoffel curvature tensor is shown to lead to a geometricized theory of electrodynamics. While this geometricized theory leads directly to the classical Maxwell equations, it also extends their interpretation by giving charge density and mass density, and the four-velocity that describes their motion geometric definitions. These geometric definitions are reminiscent of General Relativity's interpretation of mass in terms of the scalar curvature R and hint at the emergence of gravity. The gravitational theory that does emerge is shown to be equivalent to Einstein's General Relativity augmented by an energy-momentum tensor term that mimics the properties of dark matter and/or dark energy. In summary, the proposed geometrization of the Maxwell tensor puts both electromagnetic and gravitational phenomena on an equal footing with both being tied to the curvature of space-time. Using specific solutions to the proposed theory, the unification brought to electromagnetic and gravitational phenomena, as well as the relationship of those solutions to the corresponding solutions of the classical Maxwell and Einstein field equations are compared.

Keywords: Maxwell's equations; General Relativity; unification; dark matter; dark energy; electromagnetic radiation; gravitational radiation; antimatter; antigravity; quantization; superluminal transport

1. Introduction

Electromagnetic and gravitational fields have long range interactions characterized by speed of light propagation; similarities that suggest these fields should be coupled together at the classical physics level. Although this coupling or unification is a well-worn problem with many potential solutions having been proposed, it is fair to say that there is still no generally accepted classical field theory that can explain both electromagnetism and gravitation in a coupled or unified framework [1]. Today, the descriptions of these fields are generally understood to be distinct and independent, with electromagnetic fields described by Maxwell's equations and gravitational fields described by Einstein's equation of General Relativity. The focus of this manuscript is the assessment of a recently proposed coupling between the Maxwell tensor and the Reimann-Christoffel curvature tensor that leads to a geometricized version of Maxwell's equations from which gravity then emerges.

Assuming the geometry of nature is Riemannian, the classical Maxwell equations will be shown to be a consequence of, [2]

$$F_{\mu\nu;\kappa} = a^{\lambda} R_{\lambda\kappa\mu\nu}\,, \qquad (1)$$

where $F_{\mu\nu}$ is the Maxwell tensor, $R_{\lambda\kappa\mu\nu}$ is the Riemann-Christoffel (R-C) curvature tensor, and a^{λ} is a four-vector related to the familiar vector potential A^{λ} of classical electromagnetism. Including the conserved energy-momentum tensor for matter and electromagnetic fields,

$$\left(\rho_m u^{\mu} u^{\nu} + F^{\mu}{}_{\lambda} F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma}\right)_{;\nu} = 0\,, \qquad (2)$$

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where ρ_m is the scalar mass density, u^λ is its four-velocity, and $g_{\mu\nu}$ the metric tensor, all the equations of classical electromagnetism will be shown to be a consequence of Equations (1) and (2).

Beyond the succinct framework Equation (1) provides for the classical Maxwell equations, its coupling of the R-C tensor to the Maxwell tensor hints at the emergence of gravitational effects. While the gravitational fields due to this coupling are not identical to those predicted by Einstein's equation of General Relativity, they will be shown to be consistent with Einstein's equation of General Relativity augmented by an energy-momentum tensor that mimics the properties of dark matter and/or dark energy.

The goal of this manuscript is to show through an axiomatic development that a continuous field theory with Equations (1) and (2) as axioms encompasses electromagnetic and gravitational phenomena in a unified manner with both tied to the curvature of space-time. After developing the theory and discussing its various aspects, three specific solutions to Equations (1) and (2) are reviewed in Appendices B through D: The first solution is spherically symmetric and represents the asymptotic electric and gravitational fields of a non-rotating, charged particle. The second solution is radiative with two distinct sub solutions, one with electromagnetic radiation in the presence of gravitational radiation and the other with standalone gravitational radiation. This solution in particular illustrates the unification brought to electromagnetic and gravitational phenomena by Equation (1). The third solution is for a maximally symmetric 3-dimensional subspace, for example, representing an isotropic and homogeneous universe. The purpose in developing these solutions is twofold: First, to provide a comparison of solutions to Equations (1) and (2) with those corresponding to the classical Maxwell and Einstein Field Equations (M&EFEs), and second to demonstrate that the solutions to Equations (1) and (2) go further than the classical M&EFEs by uniting electromagnetic and gravitational phenomena.

Throughout the manuscript, geometric units will be used with a metric tensor having signature [+, +, +, -] in which spatial indices run from 1 to 3 and 4 is the time index. The notation within uses commas before tensor indices to indicate ordinary derivatives and semicolons before tensor indices to indicate covariant derivatives. For the definitions of the R-C curvature tensor and the Ricci tensor, the conventions used by Weinberg [3] are followed.

2. Consequences of the Field Equations (1) and (2)

In this section I give a short derivation of the classical equations of electromagnetism in the framework of Equations (1) and (2). The point in going through this purely formal development is to show that Maxwell's equations are derivative only to Equation (1) and the algebraic properties of the R-C tensor with appropriate definitions for the charge density ρ_c and its four-velocity u^ν. After developing the complete set of equations for classical electromagnetism from Equations (1) and (2), I then go on to describe the emergence of gravity that is forced by them.

2.1. The equations of electromagnetism

Maxwell's homogeneous equation and the gauge invariance of $F_{\mu\nu}$

Identifying $F_{\mu\nu}$ in Equation (1) with the Maxwell tensor appears to be an attractive starting point in attempting to geometricize Maxwell's equations due to the following two indicial algebraic properties of the R-C tensor,

$$R_{\lambda\kappa\mu\nu} = -R_{\lambda\kappa\nu\mu}, \tag{3}$$

and

$$R_{\lambda\kappa\mu\nu} + R_{\lambda\nu\kappa\mu} + R_{\lambda\mu\nu\kappa} = 0. \tag{4}$$

Contracting Equation (3) with a^λ demonstrates that $F_{\mu\nu}$ in Equation (1) satisfies,

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$$F_{\mu\nu;\kappa} = -F_{\nu\mu;\kappa}, \tag{5}$$

which forces $F_{\mu\nu}$ to be antisymmetric,

$$F_{\mu\nu} = -F_{\nu\mu}. \tag{6}$$

Contracting Equation (4) with a^λ demonstrates that the antisymmetrized covariant derivative of $F_{\mu\nu}$ vanishes, which is a statement of Maxwell's homogeneous equation,

$$F_{\mu\nu;\kappa} + F_{\nu\kappa;\mu} + F_{\kappa\mu;\nu} = 0 \quad \rightarrow \quad F_{\mu\nu,\kappa} + F_{\nu\kappa,\mu} + F_{\kappa\mu,\nu} = 0. \tag{7}$$

The change from covariant to ordinary derivatives in the last step of (7) is justified by the antisymmetry of $F_{\mu\nu}$.

Having established the antisymmetry of $F_{\mu\nu}$ in (6), and the vanishing of its antisymmetrized derivative in (7), the converse to Poincaré's lemma states that $F_{\mu\nu}$ can itself be expressed as the antisymmetrized derivative of a vector function,

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}, \tag{8}$$

where A_μ is the classical electromagnetic vector potential. Equation (8) then identifies $F_{\mu\nu}$ as gauge invariant, being unaffected when an arbitrary gradient field $\partial_\mu \varphi$ is added to A_μ,

$$A_\mu \rightarrow A_\mu + \partial_\mu \varphi. \tag{9}$$

Maxwell's inhomogeneous equation and the definitions of charge density and four-velocity

Next, Maxwell's inhomogeneous equation and the definitions for the charge density ρ_c and the four-velocity u^λ forced by Equation (1) are derived. Contracting the μ and κ indices in Equation (1) gives,

$$F^{\mu\nu}{}_{;\mu} = a^\lambda R_{\lambda\mu}{}^{\mu\nu} = -a^\lambda R_\lambda{}^\nu, \tag{10}$$

where $R_\lambda{}^\nu$ is the Ricci tensor. Defining the charge current density $\rho_c u^\nu$ by,

$$\rho_c u^\nu \equiv a^\lambda R_\lambda{}^\nu, \tag{11}$$

and then substituting $\rho_c u^\nu$ for $a^\lambda R_\lambda{}^\nu$ in (10) gives Maxwell's inhomogeneous equation,

$$F^{\mu\nu}{}_{;\mu} = -\rho_c u^\nu. \tag{12}$$

To establish the definitions that Equation (11) imposes on the charge density ρ_c and the four-velocity u^ν the following identity valid for any non-null four-vector W^μ is used,

$$W^\mu = \sqrt{|W^\rho W_\rho|}\,\frac{W^\mu}{\sqrt{|W^\sigma W_\sigma|}}. \tag{13}$$

With the aid of (13), any W^μ satisfying $W^\mu W_\mu \neq 0$ can be recast as the product of a scalar density ρ and a four-velocity u^μ,

$$W^\mu = \rho u^\mu, \tag{14}$$

where the scalar density is defined by,

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$$\rho \equiv \pm\sqrt{\left|W^{\rho}W_{\rho}\right|}\,, \tag{15}$$

and the four-velocity by,

$$u^{\mu} \equiv \pm\frac{W^{\mu}}{\sqrt{\left|W^{\sigma}W_{\sigma}\right|}}\,. \tag{16}$$

Note that Equation (16) leads to different normalizations for the four-velocity u^{μ} depending on whether W^{μ} is time-like ($W^{\mu}W_{\mu} < 0$) or space-like ($W^{\mu}W_{\mu} > 0$),

$$u^{\mu}u_{\mu} = \begin{cases} -1 \text{ if } W^{\mu}W_{\mu} < 0 \\ +1 \text{ if } W^{\mu}W_{\mu} > 0 \end{cases}\,. \tag{17}$$

Time-like W^{μ} correspond to subluminal u^{μ} while space-like W^{μ} correspond to superluminal u^{μ}. Identifying W^{ν} with $a^{\lambda}R_{\lambda}^{\ \nu}$ in Equation (11) and using Equations (15) and (16) now gives,

$$\rho_c \equiv \pm\sqrt{\left|a^{\rho}R_{\rho}^{\ \kappa}a^{\sigma}R_{\sigma\kappa}\right|}\,, \tag{18}$$

and,

$$u^{\nu} \equiv \pm\frac{a^{\lambda}R_{\lambda}^{\ \nu}}{\sqrt{\left|a^{\rho}R_{\rho}^{\ \kappa}a^{\sigma}R_{\sigma\kappa}\right|}}\,. \tag{19}$$

Equations (18) and (19) emphasize the underlying geometric character imposed by Equation (1) on Maxwell's equations, with both the charge density ρ_c and the four-velocity field u^{ν} defined in terms of the Ricci tensor $R_{\lambda}^{\ \nu}$ and the four-vector a^{λ}. This geometrization of ρ_c and u^{ν} hints at the emergence of gravity that will be developed subsequently and is reminiscent of classical General Relativity's geometric interpretation of the mass density ρ_m in terms of the curvature scalar R.

In the development leading up to Maxwell's inhomogeneous Equation (12), I have not imposed the usual restriction on the four-velocity u^{λ} that it be subluminal. I drop this requirement because I am attempting to develop a theory that flows from Equation (1) axiomatically, and there is nothing *a priori* that requires that $a^{\lambda}R_{\lambda}^{\ \nu}$ be time-like. I will comment on this interesting possibility of superluminal velocities later in the manuscript, but for the analysis that follows I will focus on the more familiar case of subluminal velocities.

The conservation of charge

The conservation of charge follows immediately from Maxwell's inhomogeneous Equation (12) and the antisymmetry of $F_{\mu\nu}$. Taking the covariant divergence of Maxwell's inhomogeneous Equation (12),

$$F^{\mu\nu}{}_{;\mu;\nu} = -\left(\rho_c u^{\nu}\right)_{;\nu}\,, \tag{20}$$

and noting $F^{\mu\nu}{}_{;\mu;\nu} \equiv 0$, which is an identity for any antisymmetric tensor $F^{\mu\nu}$, establishes the conservation of charge,

$$\left(\rho_c u^{\nu}\right)_{;\nu} = 0\,. \tag{21}$$

The Lorentz force law and the conservation of mass

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Continuing along the lines of Equation (1) which was empirically chosen to reproduce the classical Maxwell's equations, the conserved energy-momentum tensor given in Equation (2) is now used to derive the Lorentz force law and the conservation of mass equation. To start I distribute the covariant derivative in Equation (2) which gives,

$$\left(\rho_m u^\nu\right)_{;\nu} u^\mu + \rho_m u^\mu{}_{;\nu} u^\nu + F^{\mu\lambda} F^\nu{}_{\lambda;\nu} + F^{\mu\lambda}{}_{;\nu} F^\nu{}_\lambda - \frac{1}{2} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma;\nu} = 0 . \quad (22)$$

With some substitutions and rearrangements using the already derived Maxwell's homogeneous Equation (7) and inhomogeneous Equation (12), equation (22) can be re written as,

$$\left(\rho_m u^\nu\right)_{;\nu} u^\mu + \rho_m u^\mu{}_{;\nu} u^\nu - \rho_c F^\mu{}_\lambda u^\lambda = 0 . \quad (23)$$

Contracting (23) with u_μ, the 2nd and 3rd terms on the LHS are zeroed due to the normalization of u_μ (17) and the antisymmetry of $F_{\mu\nu}$ (6), respectively, leaving

$$\left(\rho_m u^\nu\right)_{;\nu} = 0 , \quad (24)$$

which is the conservation of mass equation. Using [24] to zero out the conservation of mass term in [23] then leaves the Lorentz force law,

$$\rho_m \frac{Du^\mu}{D\tau} = \rho_c F^\mu{}_\lambda u^\lambda , \quad (25)$$

where $\frac{Du^\mu}{D\tau} \equiv u^\mu{}_{;\nu} u^\nu$.

Analogous to the definitions developed for ρ_c and u^ν in Equations (18) and (19), respectively, Equation (25) is now used to define the mass density ρ_m in terms of the fields a^λ, $R_\lambda{}^\nu$, and $F_{\mu\nu}$. Solving Equation (25) for ρ_m and then substituting for ρ_c and u^ν using their definitions gives,

$$\rho_m = \frac{F^\mu{}_\lambda \rho_c u^\lambda}{u^\mu{}_{;\nu} u^\nu} = \frac{F^\mu{}_\lambda a^\sigma R_\sigma{}^\lambda}{\left(\frac{a^\rho R_\rho{}^\mu}{\sqrt{\left|a^\gamma R_\gamma{}^\kappa a^\delta R_{\delta\kappa}\right|}}\right)_{;\nu}\left(\frac{a^\alpha R_\alpha{}^\nu}{\sqrt{\left|a^\beta R_\beta{}^\eta a^\zeta R_{\zeta\eta}\right|}}\right)} \quad (\text{no sum on } \mu) . \quad (26)$$

The classical Lagrangian derivation of Maxwell's equations vs $F_{\mu\nu;\kappa} = a^\lambda R_{\lambda\kappa\mu\nu}$

Except for the geometricized definitions of ρ_m, ρ_c and u^λ given by (26), (18) and (19), respectively, the equations of electromagnetism derived using Equations (1) and (2) are identical to those arrived at using the conventional Lagrangian based classical derivation. In the classical derivation, Maxwell's homogeneous equation must be taken as an axiom so that the vector potential A_μ can be introduced. This introduction of A_μ is necessary so that a scalar Lagrangian consisting of a matter term, a free electromagnetic field term and a matter-field interaction term, can be defined, with the matter-field interaction term written in terms of A_μ. Maxwell's inhomogeneous equation and the Lorentz force law are then derived using a stationary-action calculation in which A_μ and the spatial positions of masses and charges are treated as the dynamic variables that are varied [4]. Finally, an energy-momentum tensor, the same one as in Equation (2), is defined in the classical derivation as the coefficient of the variation of the Lagrangian with respect to $g_{\mu\nu}$, and then shown to be conserved due to the general covariance of the scalar action. Except for the definitions of ρ_m,

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ρ_c and u^λ mentioned above, the equations of electromagnetism are the same regardless of the derivation used, and it is only the starting point axioms that are different. In the conventional Lagrangian based classical derivation, a specific Lagrangian and Maxwell's homogenous equation are taken as the starting point axioms, while in the derivation followed here, Equations (1) and (2) are taken as the starting point axioms.

Relationship of a^λ to the classical electromagnetic vector potential A^λ

In the development followed here, Equation (1) and the vector field a^λ that appears in it are the only truly new pieces of physics that have been introduced. However, it turns out that a^λ is not entirely new, being related to the vector potential A^λ of classical electromagnetism. To see this, take the covariant derivative of both sides of (8),

$$F_{\mu\nu;\kappa} = A_{\nu;\mu;\kappa} - A_{\mu;\nu;\kappa} . \tag{27}$$

and compare it to Equation (1) rewritten as,

$$F_{\mu\nu;\kappa} = a^\lambda R_{\lambda\kappa\mu\nu} = -a_{\kappa;\mu;\nu} + a_{\kappa;\nu;\mu} , \tag{28}$$

where the RHS of (28) follows from the commutation property of covariant derivatives. Equating the RHS's of Equations (27) and (28) gives,

$$-a_{\kappa;\mu;\nu} + a_{\kappa;\nu;\mu} = A_{\nu;\mu;\kappa} - A_{\mu;\nu;\kappa} , \tag{29}$$

establishing a connection between the field a^λ and the vector potential A^λ of classical electromagnetism.

In summary, the theory of electromagnetism based on Equations (1) and (2) does not alter the traditional equations of classical electromagnetism although their derivations have differing axiomatic starting points. In the theory being proposed here, Maxwell's equations are derivative only to Equation (1). Then using Maxwell's equations and the conserved energy-momentum tensor (2), the Lorentz force law and the conservation of mass are derived. Although the derived equations are identical to those of classical electromagnetism, adopting Equation (1) as the starting point does introduce conceptual changes to electromagnetic theory that go beyond the classical interpretation. Notably, the charge density ρ_c and mass density ρ_m, and the four-velocity u^μ that describes their motion are no longer externally inserted fields as they are in the classical physics picture, but instead are determined by the a^λ, $g_{\mu\nu}$ and $F_{\mu\nu}$. These dependencies intermingle electromagnetic and gravitational phenomena in a fundamentally new way. In subsequent sections, the consequences of Equations (1) and (2) will be developed further using specific solutions to show that electromagnetic and gravitational phenomena are effectively described in a unified manner, with both being tied to nonzero space-time curvatures.

2.2. *A theory of gravitation*

The preceding discussion established that the equations of classical electromagnetism follow directly from Equations (1) and (2). Additionally, due to the coupling of the R-C tensor to the Maxwell tensor in Equation (1), some form of gravity can be expected to emerge in the solutions of Equations (1) and (2). This naturally leads to the following question: Will this emergent gravity be equivalent to Einstein's General Relativity,

$$G^{\mu\nu} = -8\pi T^{\mu\nu} , \tag{30}$$

where $G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$ is the Einstein tensor? As shown in Appendix B, using the specific example of a spherically symmetric, non-rotating, charged particle, the Reissner-Nordström metric is an exact solution of Equations (1) and (2), thus establishing that the emergent gravity in the

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proposed theory and classical General Relativity (30) support the same gravitational metric field in the case of spherical symmetry. However, one must go further to determine if Einstein's field equation is a derivable consequence of Equations (1) and (2). To investigate this issue, consider the conserved energy-momentum tensor $T^{\mu\nu}$ given in Equation (2). An immediate consequence of $G^{\mu\nu}$ being independently conserved by the Bianchi identity is that for any constant α, one can define a tensor field $\Lambda^{\mu\nu}$ by,

$$\Lambda^{\mu\nu} \equiv G^{\mu\nu} - \alpha T^{\mu\nu}, \tag{31}$$

that is both symmetric,

$$\Lambda^{\mu\nu} = \Lambda^{\nu\mu}, \tag{32}$$

and conserved,

$$\Lambda^{\mu\nu}{}_{;\nu} = 0 . \tag{33}$$

The value of the constant α in (31) is completely arbitrary and without physical significance because $\Lambda^{\mu\nu}$ as defined can absorb any change in α such that (31) remains satisfied. Taking advantage of this arbitrariness and setting the value of the constant $\alpha = -8\pi$ then gives with a slight rearrangement of (31),

$$G^{\mu\nu} = -8\pi\, T^{\mu\nu} + \Lambda^{\mu\nu}, \tag{34}$$

which is recognized as Einstein's equation of General Relativity (30) augmented on its RHS by the term $\Lambda^{\mu\nu}$. The $\Lambda^{\mu\nu}$ term in (34) exhibits the properties of an energy-momentum tensor appropriate for dark matter and/or dark energy, *viz.*, it is a conserved and symmetric tensor field, it is a source of gravitational fields in addition to energy-momentum tensor $T^{\mu\nu}$ for normal matter and normal energy, and it has no interaction signature beyond the gravitational fields it sources.

At this point it is important to recognize that (34) is a trivial result with no physical significance in the theory being proposed here. This follows because any solution of the Equations (1) and (2) must necessarily be a solution of (34) for some choice $\Lambda^{\mu\nu}$. In fact, the validity of (34) rests only on the existence of a conserved energy-momentum tensor and the properties of the R-C tensor, and so will be true in any physical theory having a conserved energy-momentum tensor. However, the interesting point in the context of the proposed theory is that the value of $\Lambda^{\mu\nu}$ can be calculated from solutions of equations (1) and (2) without postulating the existence of dark matter and/or dark energy.

In summary, gravitation emerges as a manifestation of the geometricized theory of electromagnetism based on Equations (1) and (2). Specifically, it is the coupling of the derivatives of the Maxwell tensor to the R-C tensor in (1) that brings gravitation into the picture. Importantly, the gravitational theory that emerges does not obey the classical General Relativity field Equation (30), although any solution of Equations (1) and (2) must necessarily be a solution of Equation (34) for some choice of $\Lambda^{\mu\nu}$. While viewing gravitation as a manifestation of electromagnetism and vice versa is not new [5–9], the specific approach being followed here with Equation (1) is new.

2.3. Symmetries of Equations (1) and (2)

Table 1 lists the six fields that have been used in the development of the theory that flows from Equations (1) and (2). Based on these developments, the fields fall into two categories: The fundamental fields a_λ, $g_{\mu\nu}$ and $F_{\mu\nu}$ that Equations (1) and (2) solve for, and the remaining fields ρ_c, u^λ and ρ_m which are defined in terms of the fundamental fields by Equations (18), (19), and (26), respectively.

 doi:10.20944/preprints201711.0022.v7

Table 1. Fields.

Field	Description
a_λ	Four-vector coupling gravitation and electromagnetism – fundamental field
$g_{\mu\nu}$	Metric tensor – fundamental field
$F_{\mu\nu}$	Maxwell tensor – fundamental field
ρ_c	Charge density scalar field – defined by Equation (18)
u^λ	Four-velocity vector field – defined by Equation (19)
ρ_m	Mass density scalar field – defined by Equation (26)

To facilitate identifying the global symmetries of Equations (1) and (2), these equations are collected here along with Equation (11) which gives the definition of the charge current density,

$$F_{\mu\nu;\kappa} = a^\lambda R_{\lambda\kappa\mu\nu} \tag{1}$$

$$\left(\rho_m u^\mu u^\nu + F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma}\right)_{;\nu} = 0 \tag{2}$$

$$a^\lambda R_\lambda{}^\nu = \rho_c u^\nu \tag{11}$$

These three equations succinctly illustrate the three global symmetries imposed on the fields listed in Table 1. The first of these global symmetries corresponds to charge-conjugation,

$$\begin{pmatrix} u^\lambda \\ a^\lambda \\ F^{\mu\nu} \\ g_{\mu\nu} \\ \rho_c \\ \rho_m \end{pmatrix} \to \begin{pmatrix} u^\lambda \\ -a^\lambda \\ -F^{\mu\nu} \\ g_{\mu\nu} \\ -\rho_c \\ \rho_m \end{pmatrix}, \tag{35}$$

the second corresponds to a matter-antimatter transformation as will be discussed in Section 3.4,

$$\begin{pmatrix} u^\lambda \\ a^\lambda \\ F^{\mu\nu} \\ g_{\mu\nu} \\ \rho_c \\ \rho_m \end{pmatrix} \to \begin{pmatrix} -u^\lambda \\ -a^\lambda \\ -F^{\mu\nu} \\ g_{\mu\nu} \\ \rho_c \\ \rho_m \end{pmatrix}, \tag{36}$$

and the third to the product of the first two,

$$\begin{pmatrix} u^\lambda \\ a^\lambda \\ F^{\mu\nu} \\ g_{\mu\nu} \\ \rho_c \\ \rho_m \end{pmatrix} \rightarrow \begin{pmatrix} -u^\lambda \\ a^\lambda \\ F^{\mu\nu} \\ g_{\mu\nu} \\ -\rho_c \\ \rho_m \end{pmatrix}. \tag{37}$$

All three transformations leave Equations (1), (2) and (11) unchanged. Adding the identity transformation to these symmetries forms the Klein-4 group, with the product of any two of the symmetries (35) through (37) giving the remaining symmetry.

Note that among the fields of the theory only $g_{\mu\nu}$ and ρ_m are unchanged by the symmetry transformations, a fact that will be useful for defining self-consistency equations that lead to a mechanism for quantizing the mass, charge, and angular momentum of particle-like solutions in section 3.6. Finally, in addition to the proposed theory's general covariance and global symmetries of Equations (35) through (37), it also exhibits the electromagnetic gauge invariance of classical electromagnetism as detailed in Equations (8) and (9).

3. Discussion

In this section the general features of the theory that flow from Equations (1) and (2) are discussed and compared to the predictions of the classical M&EFEs. To keep the discussion is this section contained, specific solutions to the Equations (1) and (2) which are referenced in this section are collected and presented in Appendices B–D.

3.1. The classical Maxwell equations from $F_{\mu\nu;\kappa} = a^\lambda R_{\lambda\kappa\mu\nu}$

Equation (1) was chosen to reproduce the classical Maxwell equations, but then goes further by defining the charge density ρ_c and the four-velocity u^λ in terms of the Ricci tensor $R_\lambda^{\ \nu}$ and the vector field a^λ. Equation (2), the conserved energy-momentum tensor, then enables the mass density ρ_m to be similarly defined in terms of the fields a_λ, $R_\lambda^{\ \nu}$ and $F_{\mu\nu}$. It is in this sense that a_λ, $g_{\mu\nu}$ and $F_{\mu\nu}$ are the fundamental fields of the theory while ρ_m, ρ_c and u^λ are dependent fields that are defined in terms of the fundamental fields by Equations (26), (18) and (19), respectively. Of the fundamental fields, both the metric tensor $g_{\mu\nu}$ and the Maxwell tensor $F_{\mu\nu}$ are already familiar to Classical Physics, leaving only a^λ which is related to classical electromagnetism's vector potential A^λ by Equation (29) as new.

One of the unusual features of the forgoing development is the explicit dependence of Equation (1) on both the four-vector a^λ and the R-C tensor $R_{\lambda\kappa\mu\nu}$, while on the other hand the classical Maxwell equations which are derived from (1) contain neither a^λ or $R_{\lambda\kappa\mu\nu}$ explicitly,

$$\text{Eq. (1)} \quad \rightarrow \quad \text{Classical Maxwell Eqs.}$$

$$F_{\mu\nu;\kappa} = a^\lambda R_{\lambda\kappa\mu\nu} \quad \rightarrow \quad \begin{cases} F_{\mu\nu,\kappa} + F_{\nu\kappa,\mu} + F_{\kappa\mu,\nu} = 0 \\ F^{\mu\nu}{}_{;\mu} = -\rho_c u^\nu \end{cases}$$

This circumstance conspires to give the classical Maxwell equations the appearance of being valid in flat space-time and motivates their classical interpretation in terms of a conserved charge

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density source term ρ_c and four-velocity u^λ that also appear to exist in flat space-time. However, in the view of Equation (1), ρ_c and u^λ are confined to regions of non-vanishing curvature by Equations (18) and (19), respectively. Thus, the classical Maxwell equations and their interpretation in flat space-time are at best an approximation to the geometricized Maxwell's Equation (1).

3.2. Dark matter and dark energy

With General Relativity as the foundation of gravitational physics today, dark matter and dark energy have been postulated to exist because of the many galactic and cosmological scale observations that cannot be understood using General Relativity with normal matter and normal energy alone. For example, some of the large-scale gravitational features of galaxies and galactic clusters dating back to Zwicky's observations in the 1930's has been explained using dark matter [10], and the acceleration of the universe discovered in the 1990's explained using dark energy [11].

One of the vexing problems facing dark matter and dark energy has been an inability to directly detect them except through their gravitational interaction with normal matter and/or normal energy. However, in Equation (34), which all solutions to Equations (1) and (2) must satisfy, these are exactly the properties of the $\Lambda^{\mu\nu}$ term, it being a source of gravitational fields in addition to the energy momentum tensor for normal matter and normal energy but beyond that it has no interactions.

In the context of Equations (1) and (2), $\Lambda^{\mu\nu}$ can be calculated directly. As already discussed, any solution to Equations (1) and (2) determines the fundamental fields a_λ, $g_{\mu\nu}$ and $F_{\mu\nu}$. Knowing a_λ, $g_{\mu\nu}$ and $F_{\mu\nu}$ then enables the determination of both of $G^{\mu\nu}$ from the definition of the Einstein tensor, and $T^{\mu\nu}$ from its definition given in Equation (2),

$$T^{\mu\nu} \equiv \rho_m u^\mu u^\nu + F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma}, \tag{38}$$

where the definitions for ρ_m and u^ν are given by (26) and (19), respectively. Knowing $G^{\mu\nu}$ and $T^{\mu\nu}$, $\Lambda^{\mu\nu}$ can then be calculated using Equation (34). As an example, the spherically symmetric, particle-like solution to Equations (1) and (2) given in Appendix B outlines a direct calculation of $\Lambda^{\mu\nu}$ in terms of normal matter and normal energy.

With questions today regarding the validity of classical General Relativity beyond the confines of our own solar system [12], and the inability to directly detect dark matter and dark energy, the possible interpretation of the $\Lambda^{\mu\nu}$ term in (34) using only normal matter and normal energy is an enticing feature of Equations (1) and (2). However, it must be acknowledged that one of the challenging tasks facing the theory based on Equations (1) and (2), and one well beyond the analysis presented in this manuscript, is that of finding additional solutions that could be interpreted as agreeing with the rapidly developing observational understanding of galactic and cosmological structures. See for example Appendix D, where a solution based on the Friedmann–Lemaître–Robertson–Walker metric is developed.

3.3. The unification of gravitational and electromagnetic radiation

One of the successes of Equation (1) is the existence of the radiative solutions presented in Appendix C. These solutions describe both electromagnetic and gravitational radiation in the weak field limit, with both phenomena being unified as undulations of the underlying metric field $g_{\mu\nu}$. Two distinct classes of solutions are developed, one in which there is coupled gravitational and electromagnetic radiation, and the other in which there is gravitational radiation but no electromagnetic radiation. Interestingly, in the solution with the coupled gravitational and electromagnetic radiation, the gravitational radiation solution is identical to the weak field gravitational wave of classical General Relativity. Furthermore, because both gravitational and

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electromagnetic radiation fields are due to undulations of the metric field $g_{\mu\nu}$ in the solution of Equations (1) and (2), their propagation speeds are predicted to be identical. This prediction has recently been refined experimentally with observations made during the binary neutron star merger in NGC 4993, 130 million light years from Earth [13]. The nearly simultaneous detection, within 2 seconds of each other, of gravity waves [14] and a burst of gamma rays [15] from this event experimentally constrain the propagation speed of electromagnetic and gravitational radiation to be the same to better than 1 part in 10^{15}.

3.4. The emergence of antimatter and its behavior in electromagnetic and gravitational fields

One of the interesting features of Equations (1) and (2) is that the properties of antimatter emerge naturally in their solutions. Traditionally, these properties emerge in quantum mechanical treatments but here emerge in the context of a classical continuous field theory due to the global symmetry (36) of Equations (1) and (2), with every matter containing solution having a corresponding antimatter solution generated by the symmetry transformation (36). This transformation is physically equivalent to the view that a particle's antiparticle is the particle moving backwards through time [16]. Said another way, the time-like component of the four-velocity is positive for matter and negative for antimatter,

$$u^4 \begin{cases} >0 \ \text{for matter} \\ <0 \ \text{for antimatter} \end{cases}. \tag{39}$$

Building on the distinction between matter and antimatter, their behavior in external electromagnetic and gravitational fields is briefly reviewed here. To see more rigorously that antimatter can be viewed as matter moving backwards through time, consider the four-velocity associated with a fixed quantity of charge and mass density,

$$u^\lambda = \frac{dx^\lambda}{d\tau}. \tag{40}$$

Under the matter-antimatter transformation (36), $u^\lambda \to -u^\lambda$, or equivalently $d\tau \to -d\tau$. In a locally inertial coordinate system, this motivates the following expression for the four-velocity in terms of the coordinate time,

$$u^\lambda = \frac{dx^\lambda}{d\tau} = s_{m-a}\gamma \frac{dx^\lambda}{dt} = s_{m-a}\gamma \begin{pmatrix} \vec{v} \\ 1 \end{pmatrix}, \tag{41}$$

where $\vec{v}$ is the ordinary 3-space velocity of the charge and mass density, $\gamma = 1/\sqrt{1-\vec{v}^2}$, and s_{m-a} is a matter-antimatter parameter defined by,

$$s_{m-a} = \begin{cases} +1 \ \text{for matter} \\ -1 \ \text{for antimatter} \end{cases}. \tag{42}$$

Equation (41) establishes that corresponding matter and antimatter solutions travel in opposite coordinate-time directions relative to each other, while Equation (36), the matter-antimatter symmetry transformation requires ρ_c does not change sign under such transformations. To see that this is consistent with the usual view in which antiparticles have the opposite charge of their corresponding particles, I use (41) to illustrate the behavior of a region containing charged matter or antimatter in an external electromagnetic field. Consider a region with an external electromagnetic field defined by,

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_z & -B_y & E_x \\ -B_z & 0 & B_x & E_y \\ B_y & -B_x & 0 & E_z \\ -E_x & -E_y & -E_z & 0 \end{pmatrix}, \tag{43}$$

in a locally inertial coordinate system. Starting with the Lorentz force law (25), and then expanding and rearranging slightly leads to the following development,

$$\begin{gathered} \rho_p \frac{Du^\mu}{D\tau} = \rho_c F^\mu{}_\lambda u^\lambda \\ \downarrow \\ \rho_p s_{m-a} \gamma \frac{du^\mu}{dt} = \rho_c F^\mu{}_\lambda u^\lambda \\ \downarrow \\ \rho_p s_{m-a} \gamma \frac{d}{dt} \begin{pmatrix} s_{m-a} \gamma \vec{v} \\ s_{m-a} \gamma \end{pmatrix} = \rho_c \begin{pmatrix} 0 & B_z & -B_y & E_x \\ -B_z & 0 & B_x & E_y \\ B_y & -B_x & 0 & E_z \\ E_x & E_y & E_z & 0 \end{pmatrix} \begin{pmatrix} s_{m-a} \gamma v_x \\ s_{m-a} \gamma v_y \\ s_{m-a} \gamma v_z \\ s_{m-a} \gamma \end{pmatrix} \\ \downarrow \\ \rho_p \frac{d}{dt} \begin{pmatrix} \gamma \vec{v} \\ \gamma \end{pmatrix} = s_{m-a} \rho_c \begin{pmatrix} \vec{E} + \vec{v} \times \vec{B} \\ \vec{v} \cdot \vec{E} \end{pmatrix} \end{gathered} \tag{44}$$

which on the last line above ends up at the conventional form of the Lorentz force law except for the extra factor of s_{m-a} on the RHS. This factor of s_{m-a} gives the product $s_{m-a} \rho_c$ the appearance that antimatter charge density has the opposite sign of matter charge density when interacting with an external electromagnetic field.

Next, I investigate the behavior of antimatter in an external gravitational field. There is no question about the gravitational fields generated by matter and antimatter, they are identical under the matter-antimatter symmetry (36), as $g_{\mu\nu}$ is unchanged by that transformation. To understand whether antimatter is attracted or repelled by an external gravitational field, I again go to the Lorentz force law (25) but this time assume there is no electromagnetic field present, just a gravitational field given by a Schwarzschild metric generated by a central mass $m > 0$ that is composed of either matter or antimatter. I explicitly call out $m > 0$ because I am endeavoring to develop a physical theory that flows axiomatically from (1) and (2), and at this point in the development there is nothing to preclude the existence of negative mass density $\rho_m < 0$, a consideration I will return to in section 3.5. Placing a test particle having mass m_{test} composed of either matter or antimatter a distance r from the center of the gravitational field and assuming the test particle is initially at rest, the trajectory of the test particle is that of a geodesic given by the following development,

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$$\begin{gathered} m_{test}\frac{Du^{\mu}}{D\tau}=0 \\ \downarrow \\ s_{m-a}\,\gamma\frac{du^{\mu}}{dt}=-\Gamma^{\mu}{}_{\nu\rho}u^{\nu}u^{\rho} \\ \downarrow \\ s_{m-a}\,\gamma\frac{d}{dt}\left(s_{m-a}\gamma\frac{d}{dt}\begin{pmatrix} r\\ \theta\\ \phi\\ t\end{pmatrix}\right)=-\Gamma^{\mu}{}_{\nu\rho}u^{\nu}u^{\rho}\approx-\Gamma^{\mu}{}_{44}u^{4}u^{4}=-\begin{pmatrix}\left(1-\frac{2m}{r}\right)\frac{m}{r^{2}}\\ 0\\ 0\\ 0\end{pmatrix}\left(\frac{s_{m-a}}{\sqrt{1-\frac{2m}{r}}}\right)^{2}=-\begin{pmatrix}\frac{m}{r^{2}}\\ 0\\ 0\\ 0\end{pmatrix}s_{m-a}{}^{2} \end{gathered} \tag{45}$$

where $s_{m-a}=\pm 1$ references whether the test particle is composed of matter or antimatter as defined by (42). In the last line of (45), I have approximated the RHS using the initial at rest value of the test particle's four-velocity $u^{\mu}=\left(0,0,0,s_{m-a}/\sqrt{1-2m/r}\right)$, and additionally used the fact that the only nonzero $\Gamma^{\mu}{}_{44}$ in a Schwarzschild metric is $\Gamma^{1}{}_{44}=\left(1-\frac{2m}{r}\right)m/r^{2}$. Simplifying the LHS of the last line in (45) by noting that initially $\gamma=1$ then gives,

$$\frac{d^{2}r}{dt^{2}}\approx-\frac{m}{r^{2}}, \tag{46}$$

which is independent of s_{m-a}, and so demonstrates that the proposed theory predicts both matter and antimatter test particles will be attracted by the source of the gravitational field, and this regardless of whether the source of the gravitational field is matter or antimatter. The result that the test particle is attracted toward the source of the gravitational field is also independent of whether the test particle's mass, m_{test}, is positive or negative, this because the geodesic trajectory (46) is independent of m_{test}.

3.5. Possibility of negative mass solutions and antigravity

As already noted, there appears to be nothing in Equations (1) and (2) that precludes the possibility of negative mass density $\rho_m<0$. The existence of negative mass density is equivalent to the existence of antigravity because negative mass density generates gravitational fields that are repulsive, *viz.*, Equation (46) with $m<0$. However, logical inconsistencies are introduced if negative mass density were to exist. As just shown, Equation (46) with $m>0$ predicts a test particle at some distance from the origin will feel an attractive gravitational force regardless of whether the test particle is comprised of matter or antimatter and regardless of whether its mass is positive or negative. Now consider Equation (46) with the central mass $m<0$. Using the same argument as in the previous section, the test particle in this case will feel a repulsive gravitational force regardless of whether it is comprised of matter or antimatter and regardless of whether its mass is positive or negative. These two situations directly contradict each other. For example, in the first case the negative mass test particle is gravitationally attracted toward the positive mass particle located at the origin, but in the second case the positive mass test particle is gravitationally repelled by the negative mass particle located at the origin. This contradiction makes Equations (1) and (2) logically inconsistent if negative mass density were to exist. The only way to avoid this logical contradiction is to require that mass density be non-negative always. This condition, that mass density ρ_m be non-

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negative always is also consistent with the global symmetry transformations (35) through (37) where it was noted that the field ρ_m does not change sign under any of the symmetry transformations.

It is interesting to note that the existence of negative mass in the context of classical General Relativity has been extensively studied [17,18] and invoked, particularly when trying to find stable particle-like solutions using the conventional Einstein field equations [19–21]. However, in the context of the present theory based on Equations (1) and (2) the existence of negative mass density leads to a logical contradiction that can only be resolved by requiring mass density be non-negative always, i.e., $\rho_m \geq 0$.

3.6. Conjecture for quantizing the charge and mass of particle-like solutions

When considering solutions for the asymptotic electromagnetic and gravitational fields of charged particles, the metric tensor $g_{\mu\nu}$ is expressed parametrically in terms of the particle's mass and charge. For example, the solution to Equations (1) and (2) presented in Appendix B and summarized in (63) represents the asymptotic electric and gravitational fields of a spherically symmetric, non-rotating, charged particle with $g_{\mu\nu}$ given by the Reissner-Nordström metric,

$$g_{\mu\nu} = \begin{pmatrix} \dfrac{1}{\left(1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}\right)} & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\mathrm{Sin}^2(\theta) & 0 \\ 0 & 0 & 0 & -\left(1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}\right) \end{pmatrix}, \qquad (47)$$

where q and m represent the particle's total charge and total mass, respectively. Because the mass density ρ_m and charge density ρ_c are specified as part of any solution to Equations (1) and (2), a self-consistency condition exists in which the particle's total charge q and total mass m must agree with the spatially integrated charge and mass density, respectively. For the charge, this amounts to requiring,

$$q = \int \rho_c u^4 \sqrt{\gamma_{sp}}\, d^3x\,, \qquad (48)$$

where γ_{sp} is the determinant of the spatial metric defined by [22],

$$\gamma_{sp\,ij} = g_{ij} - \frac{g_{i4}\, g_{j4}}{g_{44}}, \qquad (49)$$

and *i* and *j* run over the spatial dimensions 1, 2 and 3. For the mass, the analogous self-consistency condition is,

$$m = \int \rho_m \left|u^4\right| \sqrt{\gamma_{sp}}\, d^3x\,. \qquad (50)$$

The reason for the absolute value of u^4 in the mass self-consistency condition (50) but not in the charge self-consistency condition (48) are the global symmetries (35) through (37) exhibited by the theory's Equations (1) and (2), and the requirement that the self-consistency conditions exhibit those same symmetries. The conjecture being put forth here is that (48) and (50) represent self-consistency constraints on the charge and the mass, respectively, that any particle-like solution to Equations (1) and (2) must satisfy if the solutions are to be physically realizable.

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Finally, when considering metrics that include nonzero angular momentum, as for example would be required for particles having an intrinsic magnetic field, the same approach used here to quantize the particle's mass and charge could be used to quantize its angular momentum. Traditionally the quantization of mass, charge and angular momentum are introduced in quantum mechanical treatments but here are conjectured within the framework of a classical continuous field-theoretic description of nature and are another example of how the proposed theory differs from the classical M&EFEs.

3.7. Possibility of superluminal transport if $a^\lambda R_\lambda{}^\nu$ is space-like

Having chosen the form of Equations (1) and (2), all subsequent results presented in this manuscript have been mathematically derivative to them. As an example, after the definitions of the charge density ρ_c and the four-velocity u^λ were developed in Equations (18) and (19), respectively, Maxwell's inhomogeneous Equation (12) was shown to follow from Equation (1). Noteworthy in the definition for ρ_c is that in addition to its motion being described in terms of subluminal transport, the development naturally includes the case of superluminal transport. Because I am attempting to develop the theory that flows axiomatically from Equations (1) and (2), and because there is nothing *a priori* that precludes the possibility of $a^\lambda R_\lambda{}^\nu$ being space-like which corresponds to superluminal transport by Equation (19), I have carried this as a possibility, although one that must be regarded as speculative at this point because the specific solutions investigated within have not exhibited it. Although not pursued further here, the possibility of superluminal transport in the context of a classical field theory may be an interesting and timely avenue of investigation as recent research has suggested the possible existence of nonlocal correlations stronger than those predicted by quantum theory [23].

4. Conclusion

The choice of the Equation (1) as the foundation of the geometricized theory of electromagnetism developed within was driven by the desire to preserve as much as possible the physics embodied in the classical Maxwell equations. Using a vector field a^λ that is related to the familiar vector potential A^λ of classical electromagnetism, Equation (1) couples the Maxwell tensor to the Riemann-Christoffel curvature tensor and reproduces the classical Maxwell equations in their entirety. However, the interpretation of the Maxwell equations derived from Equation (1) goes further than their classical interpretation, for example, giving the charge density ρ_c, the mass density ρ_m, and the four-velocity u^λ a geometric underpinning with definitions in terms of a^λ, $g_{\mu\nu}$, and $F_{\mu\nu}$. It is this geometric underpinning that ties electromagnetism to gravitation. Although the gravity emerging from (1) and (2) is different from that described by General Relativity, it is consistent with Einstein's field equations of General Relativity augmented by a symmetric and conserved tensor field that in the context of General Relativity exhibits the properties of an energy-momentum tensor appropriate for dark matter and/or dark energy. However, in the context of Equations (1) and (2), this augmenting field is determined by conventional matter and energy alone.

Using the specific solutions to Equations (1) and (2) that are covered in the appendices, the unification brought to electromagnetic and gravitational phenomena is developed. Also discussed are the unique features/interpretations of Equations (1) and (2) that set them apart from the classical M&EFEs. These distinguishing features include the emergence of antimatter and its behavior in electromagnetic and gravitational fields, the emergence of dark matter and dark energy mimicking terms in the context of General Relativity, an underlying unification of electromagnetic and gravitational radiation, and the impossibility of negative mass solutions that would generate repulsive gravitational fields or antigravity. Although not yet based on specific solutions to the proposed theory, a method for quantizing the charge, mass, and angular momentum of particle-like solutions, as well as the possibility of superluminal transport when $a^{\lambda}R_{\lambda}^{\ \nu}$ is space-like, are conjectured.

The work presented here was reported in a preliminary form in references [2] and [24]. The same coupling between the Maxwell tensor and the R-C tensor given in Equation (1) was first reported in those references, although in a somewhat modified form. The discussion of systems of first order partial differential equations and the existence of solutions to such systems was also given in reference [2] but is included in Appendix A to keep the mathematical description of the proposed theory self-contained. New to this manuscript is the discussion of the global symmetries of Equations (1) and (2), and based on those global symmetries the interpretation of the particle-like solution has been advanced as has the discussion of self-consistency conditions in section 3.6. The discussion of Einstein's equation of General Relativity augmented by a term that can mimic the properties of dark matter and/or dark energy is also new to this manuscript, as is the discussion of the solution based on the FLRW metric. The present manuscript also corrects an error in the weak field analysis of reference [2], leading to an expanded discussion of electromagnetic radiation and its underlying gravitational radiation. The discussion of the impossibility of both negative mass solutions and antigravity is new. The speculation on superluminal transport if $a^{\lambda}R_{\lambda}^{\ \nu}$ is space-like is also new. Finally, the appendix containing the analysis of the Cauchy initial value problem as it relates to Equations (1) and (2) is new and included to replace an incorrect discussion of the logical consistency of the field equations that was given in reference [2].

Acknowledgements: For his penetrating insights and suggestions offered during the early phases of this work [24]. I would like to express my appreciation and thanks to David Eimerl. For many useful conversations and critical comments during the preparation of this manuscript I would like to thank my colleagues at Lawrence Livermore National Laboratory: Steve Libby, Michael Schneider, Mark Rotter, Ed Hartouni, Charles Boley and Alexander Rubenchik. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

Appendix A. Solutions to Equations (1) and (2)

Equation (1) represents a mixed system of first order partial differential equations for $F_{\mu\nu}$ and illustrates one of the mathematical complexities that must be addressed when attempting to find solutions to Equations (1) and (2) [25]. Specifically, mixed systems of first order partial differential equations must satisfy integrability conditions if solutions are to exist [26]. One statement of these integrability conditions is given by,

$$F_{\mu\nu;\kappa;\lambda} - F_{\mu\nu;\lambda;\kappa} = -F_{\mu\sigma}R^{\sigma}{}_{\nu\kappa\lambda} - F_{\sigma\nu}R^{\sigma}{}_{\mu\kappa\lambda}, \tag{51}$$

which can be derived using the commutation relations for covariant derivatives. Using (1) to substitute for $F_{\mu\nu;\kappa}$ in (51) gives,

$$\left(a^{\rho}R_{\rho\kappa\mu\nu}\right)_{;\lambda} - \left(a^{\rho}R_{\rho\lambda\mu\nu}\right)_{;k} = -F_{\mu\sigma}R^{\sigma}{}_{\nu k\lambda} - F_{\sigma\nu}R^{\sigma}{}_{\mu\kappa\lambda}, \tag{52}$$

which can be interpreted as conditions that are automatically satisfied by any solution consisting of expressions for $g_{\mu\nu}$, a^{λ} and $F_{\mu\nu}$ that satisfy (1). With (52) as integrability conditions that must be satisfied by any solution of (1), the question that naturally arises is this: Are these integrability conditions so restrictive that perhaps no solution to the proposed theory exists? Although this view could be construed as making the proposed field theory uninteresting because perhaps no solutions exist, it will be shown that solutions that are consistent with known solutions of the classical M&EFEs can be found. Additionally, Equation (52), which is linear in $F_{\mu\nu}$ is often useful in developing solutions to Equations (1) and (2), an approach that will be used in the solution to be found in Appendix B. Finally, to further elucidate questions regarding solutions of the proposed theory, an outline showing how the field equations can be solved numerically is given in Appendix E, where an analysis is presented of Equations (1) and (2) in terms of a Cauchy initial value problem.

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In the following three appendices solutions to Equations (1) and (2) are presented. The first solution is spherically symmetric, representing the asymptotic electric and gravitational fields of a non-rotating, charged particle. The second solution is radiative with two distinct sub solutions, one with electromagnetic radiation in the presence of gravitational radiation and the other with standalone gravitational radiation. The third solution has a maximally symmetric 3-dimensional subspace, for example, representing an isotropic and homogeneous universe. The purpose in developing these solutions is twofold: First, to provide a comparison of solutions to Equations (1) and (2) with the corresponding solutions to the classical M&EFEs, and second to demonstrate that the solutions to Equations (1) and (2) go further than the classical M&EFEs by uniting electromagnetic and gravitational phenomena.

Appendix B. Spherically symmetric solution

Here a solution representing the asymptotic fields of a non-rotating, spherically symmetric, charged mass is investigated. It is demonstrated that the Reissner-Nordström metric with an appropriate choice for $F_{\mu\nu}, a^{\lambda}, u^{\lambda}, \rho_c$ and ρ_m satisfies Equations (1) and (2). Although the presentation in this section is purely formal, it is included here for several reasons. First, if the theory could not describe the asymptotic electric and gravitational fields of a charged particle it would be of no interest on physical grounds. Second, and as already discussed, Equation (1) requires the solution of a mixed system of first order partial differential equations, a system that may be so restrictive that no solutions exist, and so an outline of at least one methodology to a solution is warranted.

To proceed, I draw on a solution for a spherically symmetric charged particle that was previously derived [27]. Starting with the Reissner-Nordström metric [28],

$$g_{\mu\nu} = \begin{pmatrix} \dfrac{1}{\left(1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}\right)} & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\mathrm{Sin}^2(\theta) & 0 \\ 0 & 0 & 0 & -\left(1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}\right) \end{pmatrix}, \tag{53}$$

and the Ricci tensor that follows from it,

$$R_{\lambda}{}^{\nu} = \frac{q^2}{r^4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{54}$$

I investigate a trial solution in which a^{λ} is constant,

$$a^{\lambda} = (0,0,0,c_1), \tag{55}$$

where the value of the constant c_1 is yet to determined. One of the physical motivations for (55) is that it satisfies the conservation of charge Equation (21), a necessary requirement for any solution. Next, using the definition of charge current density from Equation (11), and the expressions for $R^{\lambda}{}_{\nu}$ and a^{λ} from (54) and (55), respectively, gives,

$$\rho_c u^{\nu} = a^{\lambda} R_{\lambda}{}^{\nu} = \left(0,0,0,c_1\frac{q^2}{r^4}\right). \tag{56}$$

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Using the definitions for the charge density ρ_c and the four-velocity u^λ from Equations (18) and (19), respectively, then gives,

$$\rho_c = \pm|c_1|\frac{q^2}{r^4}\sqrt{1-\frac{2m}{r}+\frac{q^2}{r^2}}\,, \tag{57}$$

and

$$u^\lambda = \left(0,0,0,\pm\frac{c_1}{|c_1|}\frac{1}{\sqrt{1-\frac{2m}{r}+\frac{q^2}{r^2}}}\right). \tag{58}$$

The next step is to satisfy Equation (1) by solving for $F_{\mu\nu}$. Rather than tackle this head on by directly trying to find a solution to the mixed system of first order partial differential equations that is (1), I instead solve the integrability Equations (52), which are linear in $F_{\mu\nu}$ for $F_{\mu\nu}$. Proceeding in this manner, all the integrability equations are satisfied for $F_{\mu\nu}$ given by,

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_\phi & -B_\theta & E_r \\ -B_\phi & 0 & B_r & E_\theta \\ B_\theta & -B_r & 0 & E_\phi \\ -E_r & -E_\theta & -E_\phi & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \frac{(mr-q^2)}{r^3}c_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{(mr-q^2)}{r^3}c_1 & 0 & 0 & 0 \end{pmatrix}. \tag{59}$$

By direct substitution it is easily verified that $F_{\mu\nu}$ as given in (59) is a solution of (1) [29]. Choosing the value of the undetermined constant to be,

$$c_1 = q/m\,, \tag{60}$$

then gives an electric field that agrees with the Coulomb field for a point charge to leading order in $1/r$.

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_\phi & -B_\theta & E_r \\ -B_\phi & 0 & B_r & E_\theta \\ B_\theta & -B_r & 0 & E_\phi \\ -E_r & -E_\theta & -E_\phi & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \frac{q}{r^2}-\frac{q^3/m}{r^3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{q}{r^2}+\frac{q^3/m}{r^3} & 0 & 0 & 0 \end{pmatrix} \tag{61}$$

Finally, the mass density ρ_m is found using Equation (26),

$$\rho_m = \frac{q^4}{m^2r^4}\left(1-\frac{2m}{r}+\frac{q^2}{r^2}\right). \tag{62}$$

To summarize, the following expressions for $g_{\mu\nu}$, $F_{\mu\nu}$, a^λ, u^λ, ρ_c and ρ_m are an exact solution to Equations (1) and (2):

$$g_{\mu\nu}=\begin{pmatrix} \dfrac{1}{\left(1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}\right)} & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\mathrm{Sin}^2(\theta) & 0 \\ 0 & 0 & 0 & -\left(1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}\right) \end{pmatrix}$$

$$F_{\mu\nu}=\begin{pmatrix} 0 & B_\phi & -B_\theta & E_r \\ -B_\phi & 0 & B_r & E_\theta \\ B_\theta & -B_r & 0 & E_\phi \\ -E_r & -E_\theta & -E_\phi & 0 \end{pmatrix}=s_{c-c}s_{m-a}\begin{pmatrix} 0 & 0 & 0 & \dfrac{q}{r^2}-\dfrac{q^3/m}{r^3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\dfrac{q}{r^2}+\dfrac{q^3/m}{r^3} & 0 & 0 & 0 \end{pmatrix} \tag{63}$$

$$u^\lambda = s_{m-a}\left(0,0,0,\frac{1}{\sqrt{1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}}}\right)$$

$$a^\lambda = s_{c-c}s_{m-a}\left(0,0,0,\frac{q}{m}\right)$$

$$\rho_c = s_{c-c}\frac{q^3}{m}\frac{\sqrt{1-\dfrac{2m}{r}+\dfrac{q^2}{r^2}}}{r^4}$$

$$\rho_m = \frac{q^4}{m^2}\frac{(1-\dfrac{2m}{r}+\dfrac{q^2}{r^2})}{r^4}$$

In (63), the multiplicative parameters s_{c-c} and s_{m-a} in the equations for $F_{\mu\nu}$, u^λ, a^λ and ρ_c take on the values ±1 and correspond to the global charge-conjugation symmetry transformation (35) and the global matter-antimatter symmetry transformation (36), respectively, which was discussed in section 3.4. Except for the possibility of both matter and antimatter solutions, the physical interpretation of solution (63) is almost identical to that of the classical M&EFEs, i.e., a non-rotating, spherically symmetric particle having charge q and mass m. The metric tensor which is identical to the Reissner-Nordström metric establishes that the theory based on Equations (1) and (2), and Einstein's General Relativity predict the same gravitational fields in this case. However, solution (63) does differ from the classical picture in several ways. For example, the mass and charge are not localized; with both ρ_m and ρ_c having a spatial extent that fall off as $1/r^4$. Also, the radial electric field,

$$E_r=\frac{q}{r^2}-\frac{q^3/m}{r^3}=\frac{q}{r^2}\left(1-\frac{q^2/m}{r}\right), \tag{64}$$

while agreeing with the Coulomb field q/r^2 to leading order in $1/r$ does have a higher order term. This next term depends on both the charge and mass of the particle. Taking an electron as an example, its electric field as given by (64) would be,

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$$E_r = \frac{q_e}{r^2}\left(1 - \frac{q_e^{\,2}/m_e}{r}\right) = \frac{q_e}{r^2}\left(1 - \frac{r_e}{r}\right), \tag{65}$$

where $r_e = q_e^{\,2}/m_e \sim 2.8x10^{-15}m$ and is recognized as the classical radius of the electron.

The gravitational field predicted by the solution investigated here agrees with the corresponding solution to Einstein's General Relativity (30), with both described by the Reissner-Nordström metric. However, it is important to note that the classical General Relativity field Equations (30) are not satisfied using the energy-momentum tensor of Equation (2),

$$T^{\mu\nu} = \rho_m u^\mu u^\nu + F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma}, \tag{66}$$

calculated using the values for ρ_m, u^ν and $F_{\mu\nu}$ given in (63). However, Einstein's equation of General Relativity augmented by the $\Lambda^{\mu\nu}$ term on its RHS in equation (34) is trivially satisfied. For completeness, the values of $G^{\mu\nu}$, $T^{\mu\nu}$ and $\Lambda^{\mu\nu}$ that go with solution (63) are given here:

$$G^{\mu\nu} = \begin{pmatrix} \frac{q^2}{r^4}\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right) & 0 & 0 & 0 \\ 0 & -\frac{q^2}{r^6} & 0 & 0 \\ 0 & 0 & -\frac{q^2 \csc^2(\theta)}{r^6} & 0 \\ 0 & 0 & 0 & -\frac{q^2}{r^4}\frac{1}{\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)} \end{pmatrix}$$

$$T^{\mu\nu} = \begin{pmatrix} -\frac{q^2\left(q^2 - mr\right)^2\left(q^2 + r(-2m + r)\right)}{2m^2 r^8} & 0 & 0 & 0 \\ 0 & \frac{q^2\left(q^2 - mr\right)^2}{2m^2 r^8} & 0 & 0 \\ 0 & 0 & \frac{q^2\left(q^2 - mr\right)^2 \csc^2(\theta)}{2m^2 r^8} & 0 \\ 0 & 0 & 0 & \frac{3q^6 + m^2 q^2 r^2 + 2q^4 r(-3m + r)}{2m^2 r^4\left(q^2 + r(-2m + r)\right)} \end{pmatrix} \tag{67}$$

$$\Lambda^{\mu\nu} = G^{\mu\nu} + 8\pi T^{\mu\nu}$$

In the context of classical General Relativity's Equation (30), the interpretation of $\Lambda^{\mu\nu}$ is that of the energy-momentum tensor for dark matter and/or dark energy which serves as a source term for gravitational fields in addition to $T^{\mu\nu}$. However, in the context of the proposed theory, $\Lambda^{\mu\nu}$ depends only on the existence of normal matter and normal energy and is a consequence of Equations (1) and (2), again emphasizing that theory of gravitation emerging from (1) and (2) differs from that of classical General Relativity.

For the spherically symmetric solution investigated here, the conjectured charge and mass self-consistency conditions discussed in section 3.6 and given by Equations (48) and (50), respectively, both diverge, and so are not satisfied by the solution given in (63). The upshot of this observation is, that while the solution given in (63) represents a formal mathematical solution of Equations (1) and (2) that describe the asymptotic gravitational and electrical fields of a particle-like solution, it cannot

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represent a physically allowed solution. The possibility of finding solutions that satisfy both Equations (1) and (2), and the charge and mass boundary conditions (48) and (50) remains an open question at this point. However, interesting possibilities exist beyond the spherically symmetric solution based on the Reissner-Nordström metric investigated here. For example, the modified Reissner-Nordström and modified Kerr-Newman metrics developed by S.M. Blinder [30] give finite values for the RHS of both (48) and (50).

Appendix C. Radiative solutions in the weak field limit

Electromagnetic radiation

Working in the weak field limit, here expressions are derived for a propagating electromagnetic plane wave in terms of the vector field a^{λ} and the metric tensor $g_{\mu\nu}$. This example establishes a fundamental relationship between electromagnetic and gravitational radiation imposed by Equation (1), predicting that both are manifestations of wave propagation of the underlying metric $g_{\mu\nu}$. To begin, consider an electromagnetic plane wave having frequency ω, propagating in the +*z*-direction and polarized in the *x*-direction. The classical Maxwell tensor for this field is given by,

$$F_{\mu\nu} = \begin{pmatrix} 0 & 0 & -B_y & E_x \\ 0 & 0 & 0 & 0 \\ B_y & 0 & 0 & 0 \\ -E_x & 0 & 0 & 0 \end{pmatrix} e^{i\omega(t-z)} . \tag{68}$$

where E_x and B_y are the constant field amplitudes of the electromagnetic wave. Next, assume a near-Minkowski weak field metric given by,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} e^{i\omega(t-z)} . \tag{69}$$

where $h_{\mu\nu}$ are complex constants satisfying $|h_{\mu\nu}| \ll 1$, $\eta_{\mu\nu} = diag[1,1,1,-1]$, and the vector field a^{λ} is assumed to be constant and given by,

$$a^{\lambda} = \left(a^1, a^2, a^3, a^4\right) . \tag{70}$$

I proceed by substituting for $F_{\mu\nu}$ using (68), $g_{\mu\nu}$ using (69), and a^{λ} using (70) into Equation (1), and then only retaining terms to first order in the fields $h_{\mu\nu}$ and $F_{\mu\nu}$, both of which are assumed to be small and of the same order [31]. Doing this leads to a set of 8 independent linear equations for the 16 unknown constants: $h_{\mu\nu}$, a^{λ}, E_x and B_y. Solving these 8 independent equations gives 8 field components E_x, B_y, h_{13}, h_{22}, h_{23}, h_{34}, a^2 and a^3 in terms of 8 free constants a^1, a^4, h_{11}, h_{12}, h_{14}, h_{24}, h_{33}, and h_{44} gives,

$$\begin{aligned} E_x &= i\,\omega \frac{\left(h_{11}{}^2 + h_{12}{}^2\right)}{2\,h_{11}} a^1 \\ B_y &= E_x = i\,\omega \frac{\left(h_{11}{}^2 + h_{12}{}^2\right)}{2\,h_{11}} a^1 \end{aligned} , \tag{71}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & -h_{14} & h_{14} \\ h_{12} & -h_{11} & -h_{24} & h_{24} \\ -h_{14} & -h_{24} & h_{33} & -\frac{1}{2}(h_{33}+h_{44}) \\ h_{14} & h_{24} & -\frac{1}{2}(h_{33}+h_{44}) & h_{44} \end{pmatrix} e^{i\omega(t-z)} \quad (72)$$

and

$$a^{\lambda} = \left(a^{1}, a^{1}\frac{h_{12}}{h_{11}}, a^{4}, a^{4} \right). \quad (73)$$

This solution illustrates several ways in which the new theory departs from the classical physics view of electromagnetic radiation. Of most significance, the undulations in the electromagnetic field are due to undulations in the underlying metric field $g_{\mu\nu}$ given in (72). This result also underscores that the existence of electromagnetic radiation is forbidden in strictly flat space-time. An interesting aspect of this solution is that while the electromagnetic field necessitates the presence of an underlying gravitational radiation field, the underlying gravitational field is not completely defined by the electromagnetic field. The supporting gravitational radiation has 6 undetermined constants $(h_{11}, h_{12}, h_{14}, h_{24}, h_{33}, h_{44})$ with the only restriction being $|h_{\mu\nu}| \ll 1$ and $h_{11} \neq 0$ as required by (71). Further insight into the physical content of the metric (72) is evident after making the infinitesimal coordinate transformation from $x^{\mu} \to x'^{\mu}$ given by,

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \to \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} x - \frac{i}{\omega} h_{14}\, e^{i\omega(t-z)} \\ y - \frac{i}{\omega} h_{24}\, e^{i\omega(t-z)} \\ z + \frac{i}{2\omega} h_{33}\, e^{i\omega(t-z)} \\ t + \frac{i}{2\omega} h_{44}\, e^{i\omega(t-z)} \end{pmatrix}, \quad (74)$$

and only retaining terms to first order in the *h's*. Doing this, the metric (72) is transformed to,

$$g'_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & 0 & 0 \\ h_{12} & -h_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i\omega(t-z)}, \quad (75)$$

while E'_x and B'_y, the transformed electric and magnetic field amplitudes, respectively, are identical to E_x and B_y given in (71). Note, only the h_{11} and h_{12} components of the metric (75) have an absolute physical significance and $h_{22} = -h_{11}$ which makes the gravitational plane wave solution (75) identical to the gravitational plane wave solution of the classical Einstein field equations [32,33].

Gravitational radiation

The forgoing analysis demonstrated the necessity of having an underlying gravitational wave to support the presence of an electromagnetic wave, but the converse is not true and gravitational radiation can exist independent of electromagnetic radiation. The following analysis demonstrates

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this by solving for the structure of gravitational radiation in the absence of electromagnetic radiation. Following the same weak field formalism for the unknown fields $h_{\mu\nu}$ given in (69), but this time zeroing out E_x and B_y in (68), leads to the following solutions for $g_{\mu\nu}$ and a^λ,

$$g_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & -h_{14} & h_{14} \\ h_{12} & \frac{h_{12}{}^2}{h_{11}} & -h_{24} & h_{24} \\ -h_{14} & -h_{24} & h_{33} & -\frac{h_{33}+h_{44}}{2} \\ h_{14} & h_{24} & -\frac{h_{33}+h_{44}}{2} & h_{44} \end{pmatrix} e^{i\omega(t-z)} \tag{76}$$

and,

$$a^\lambda = \left(a^1, -a^1 \frac{h_{11}}{h_{12}}, a^4, a^4 \right). \tag{77}$$

Both $g_{\mu\nu}$ given by (76) and a^λ given by (77) are modified from their solutions in the presence of an electromagnetic wave as given by (72) and (73), respectively. Performing a transformation to the same primed coordinate system as given in (74), here gives the metric field,

$$g'_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} h_{11} & h_{12} & 0 & 0 \\ h_{12} & \frac{h_{12}{}^2}{h_{11}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i\omega(t-z)}, \tag{78}$$

illustrating again that only the h_{11} and h_{12} components have an absolute physical significance. Of particular note is the change in the value of the h_{22} component depending on whether the gravitational wave supports an electromagnetic wave as in (75) or is standalone as in (78).

In this section, Equation (1) has been shown to have a weak field electromagnetic plane wave solution identical to that of the classical Maxwell equations. Additionally, the gravitational radiation solution that underpins this electromagnetic plane wave is identical to the weak field gravitational wave solution of classical General Relativity. The solutions of Equation (1) are again seen to be consistent with those of the classical M&EFEs but to go further by providing an underlying unification between electromagnetic and gravitational phenomena.

Appendix D. Solution with a maximally symmetric 3-dimensional subspace

Next, I consider the time-dependent Friedmann–Lemaître–Robertson–Walker (FLRW) metric,

$$g_{\mu\nu} = \begin{pmatrix} \frac{R_s{}^2(t)}{1-kr^2} & 0 & 0 & 0 \\ 0 & R_s{}^2(t)r^2 & 0 & 0 \\ 0 & 0 & R_s{}^2(t)r^2\,\mathrm{Sin}^2(\theta) & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{79}$$

where k equals +1, 0 or -1 depending on whether the spatial curvature is positive, zero or negative, respectively, and $R_s(t)$ is a time-dependent scale factor. Just as in the case of General Relativity where the FLRW metric is a cosmological solution representing a homogeneous and isotropic universe it is the same for Equation (1) with an appropriate choice for the time development of $R_s(t)$. To derive the time dependence of $R_s(t)$, I note the 3-dimensional spatial subspace of (79) is maximally symmetric and so any tensor fields that inhabit that subspace must also be maximally symmetric [34]. Specifically, this restricts the form of a^{μ} to be,

$$a^{\mu} = \left(0,0,0,a^{4}(t)\right), \tag{80}$$

and forces the antisymmetric Maxwell tensor to vanish,

$$F_{\mu\nu} = 0. \tag{81}$$

Because $F_{\mu\nu}$ vanishes so must $F_{\mu\nu;\kappa}$,

$$F_{\mu\nu;\kappa} = 0, \tag{82}$$

which on substitution in (1) forces,

$$a^{\lambda} R_{\lambda\kappa\mu\nu} = 0. \tag{83}$$

This in turn forces,

$$a^{\lambda} R_{\lambda}^{\ \nu} = 0, \tag{84}$$

which gives $\rho_c = 0$ by Equation (18). Substituting a^{μ} given by (80), and the FLRW metric given by (79) into (83) then leads to the following set of equations to be satisfied,

$$\begin{aligned} a^4(t)R_{4114} &= a^4(t)\left(\frac{R_s(t)}{k\,r^2-1}\frac{d^2R_s(t)}{dt^2}\right)=0 \\ a^4(t)R_{4224} &= a^4(t)\left(-r^2\,R_s(t)\frac{d^2R_s(t)}{dt^2}\right)=0 \\ a^4(t)R_{4334} &= a^4(t)\ \left(-r^2\,R_s(t)\ \mathrm{Sin}^2(\theta)\frac{d^2R_s(t)}{dt^2}\right)=0 \end{aligned} \tag{85}$$

with all other components of (83) not listed in (85) being trivially satisfied, *i.e.*, $0=0$. The nontrivial components given in equations (85) are all satisfied if,

$$\frac{d^2R_s(t)}{dt^2} = 0, \tag{86}$$

or,

$$R_s(t) = R_{s0} + v_s\,t, \tag{87}$$

where R_{s0} is the scale factor at time *t=0*, and v_s is its constant rate of change.

Summarizing, the predictions of the new theory for a homogeneous and isotropic solution are:

1. It must be charge neutral, $\rho_c = 0$.
2. The scale factor $R_s(t)$ changes linearly with time.
3. The spatial curvature of the solution can be positive, negative or 0.

The second prediction above regarding the linear time dependence of the scale factor $R_s(t)$ differs from the predictions of the Friedmann models of General Relativity and again emphasizes that the theory of gravitation emerging here differs from that described by classical General

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Relativity. In fact, Equation (87) giving the time rate of change of the scale factor $R_s(t)$ depends only on Equation (1), the geometricized version of Maxwell's equations.

Appendix E. The Cauchy problem applied to Equations (1) an (2)

One of the unusual features of Equations (1) and (2) is the lack of any explicit derivatives of the vector field a^{λ}, a situation which raises questions about the time dependent development of a^{λ}. To further elucidate this and other questions regarding solutions of Equations (1) and (2), and to outline how they can be solved numerically, they are here analyzed in terms of a Cauchy initial value problem.

Given initial conditions for the fields in Table 1 at all spatial locations, a procedure is outlined that propagates those fields in time. To begin, assume $g_{\mu\nu}, F_{\mu\nu}, u^{\lambda}, \rho_c, \rho_m$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ are known at all spatial coordinates at some initial coordinate time t_0. Note that the initial value of field a^{λ} is not required, rather it will be solved for using Equation (1) as described below. Also note that in addition to $g_{\mu\nu}$ the initial values of $\frac{\partial g_{\mu\nu}}{\partial t}$ must be specified because Equation (1) contains the R-C tensor and so is second order in the time derivatives of $g_{\mu\nu}$, a situation analogous to classical General Relativity. The goal of the Cauchy method as it applies here is to start with specified initial conditions for $g_{\mu\nu}, F_{\mu\nu}, u^{\lambda}, \rho_c, \rho_m$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ at t_0, and then using the Equations (1) and (2) solve for $a^{\lambda}, R_{\lambda\kappa\mu\nu}, \frac{\partial F_{\mu\nu}}{\partial t}, \frac{\partial u^{\lambda}}{\partial t}, \frac{\partial \rho_m}{\partial t}, \frac{\partial \rho_c}{\partial t}$ and $\frac{\partial^2 g_{\mu\nu}}{\partial t^2}$ at t_0. Armed with these values at t_0, it is straight forward to propagate the fields $g_{\mu\nu}, F_{\mu\nu}, u^{\lambda}, \rho_c, \rho_m$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ from their initial conditions at t_0 to $t_0 + dt$ and then solve for $a^{\lambda}, R_{\lambda\kappa\mu\nu}, \frac{\partial F_{\mu\nu}}{\partial t}, \frac{\partial u^{\lambda}}{\partial t}, \frac{\partial \rho_m}{\partial t}, \frac{\partial \rho_c}{\partial t}$ and $\frac{\partial^2 g_{\mu\nu}}{\partial t^2}$ at $t_0 + dt$ using the same procedure that was used to find them at t_0. Repeating this procedure, values for the fields in Table 1 can then be found at all times. One additional requirement on the field values specified by initial conditions is that they must be self-consistent with the Equations (1) and (2), i.e., the specified initial conditions must be consistent with a solution existing to Equations (1) and (2).

In what follows, Greek indices ($\mu, \nu, \kappa, \ \ldots$) take on the usual space-time coordinates 1-4, while Latin indices ($i, j, k, \ \ldots$) are restricted to spatial coordinates, 1-3 only. Since the values of $g_{\mu\nu}$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ are known at all spatial coordinates at time t_0, the values of $\frac{\partial g_{\mu\nu}}{\partial x^i}, \frac{\partial^2 g_{\mu\nu}}{\partial x^i \partial x^j}$ and $\frac{\partial^2 g_{\mu\nu}}{\partial x^i \partial t}$ can be calculated at all spatial coordinates at time t_0. This leaves the ten quantities $\frac{\partial^2 g_{\mu\nu}}{\partial t^2}$ as the only second derivatives of $g_{\mu\nu}$ not known at t_0. To find the values of $\frac{\partial^2 g_{\mu\nu}}{\partial t^2}$ at t_0 proceed as follows. First find the values of the six $\frac{\partial^2 g_{ij}}{\partial t^2}$ at t_0 using a subset of equations from (1), the subset containing only those equations having spatial derivatives of $F_{\mu\nu}$ on the LHS and at most one time-index in each occurrence of the R-C tensor on the RHS. These equations will be used to solve for the values of a^{λ} at time t_0. In all there are 12 such equations out of the 24 that comprise (1), as listed here:

$$
\begin{aligned}
F_{12;1} &= a^{\lambda} R_{\lambda 112} \\
F_{13;1} &= a^{\lambda} R_{\lambda 113} \\
F_{23;1} &= a^{\lambda} R_{\lambda 123} \\
F_{12;2} &= a^{\lambda} R_{\lambda 212} \\
F_{13;2} &= a^{\lambda} R_{\lambda 213} \\
F_{23;2} &= a^{\lambda} R_{\lambda 223} \\
F_{12;3} &= a^{\lambda} R_{\lambda 312} \\
F_{13;3} &= a^{\lambda} R_{\lambda 313} \\
F_{23;3} &= a^{\lambda} R_{\lambda 323} \\
F_{12;4} &= -F_{24;1} - F_{41;2} = a^{\lambda} R_{\lambda 412} \\
F_{13;4} &= -F_{34;1} - F_{41;3} = a^{\lambda} R_{\lambda 413} \\
F_{23;4} &= -F_{34;2} - F_{42;3} = a^{\lambda} R_{\lambda 423}
\end{aligned}
\quad . \tag{88}
$$

The last three equations in (88) use Maxwell's homogeneous Equation (7) to express the time derivative of the Maxwell tensor component on the LHS as the sum of the spatial derivatives of two other Maxwell tensor components. The importance of having only spatial derivatives of the Maxwell tensor components on the LHS of (88) is that they are all known quantities at time t_0, *i.e.*, since all the $F_{\mu\nu}$ are known at time t_0, all $\frac{\partial F_{\mu\nu}}{\partial x^i}$ and $F_{\mu\nu;i}$ can be calculated at time t_0. Equally important, is that the RHS of the 12 equations that comprise (88) contain at most a single time index in each occurrence of their R-C tensor and so are also known at time t_0. To see that this is so I examine the general form of the R-C tensor in a locally inertial coordinate system where all first derivatives of $g_{\mu\nu}$ vanish, *i.e.*,

$$
R_{\lambda\kappa\mu\nu} = \frac{1}{2}\left(\frac{\partial^2 g_{\mu\lambda}}{\partial x^{\nu}\partial x^{\kappa}} - \frac{\partial^2 g_{\mu\kappa}}{\partial x^{\nu}\partial x^{\lambda}} - \frac{\partial^2 g_{\nu\lambda}}{\partial x^{\mu}\partial x^{\kappa}} + \frac{\partial^2 g_{\kappa\nu}}{\partial x^{\mu}\partial x^{\lambda}} \right). \tag{89}
$$

Note, having at most a single time index on the RHS of (89) means that the R-C tensor is made up entirely of terms from $\frac{\partial^2 g_{\mu\nu}}{\partial x^i \partial x^j}$ and $\frac{\partial^2 g_{\mu\nu}}{\partial x^i \partial t}$, all of which are known at time t_0. Examining the set of equations (88), there are 12 equations for 4 unknowns, the unknowns being the components of a^{λ}. These 12 equations can be solved for a^{λ} at time t_0 provided the initial conditions were chosen self-consistently with Equations (1) and (2), i.e., chosen such that a solution to the field equations is indeed possible.

Knowing the value of a^{λ} at time t_0, I now proceed to determine the R-C tensor components with two time indices at time t_0. Going back to the 24 equations that comprise the set of equations (1), here I collect the subset of those equations in which the LHS is known at time t_0, i.e., contains only spatial derivatives of the Maxwell tensor, and the RHS has an R-C tensor component that contains two time indices:

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$$\begin{aligned}
F_{14;1} &= a^{\lambda} R_{\lambda 114} \\
F_{24;1} &= a^{\lambda} R_{\lambda 124} \\
F_{34;1} &= a^{\lambda} R_{\lambda 134} \\
F_{14;2} &= a^{\lambda} R_{\lambda 214} \\
F_{24;2} &= a^{\lambda} R_{\lambda 224} \\
F_{34;2} &= a^{\lambda} R_{\lambda 234} \\
F_{14;3} &= a^{\lambda} R_{\lambda 314} \\
F_{24;3} &= a^{\lambda} R_{\lambda 324} \\
F_{34;3} &= a^{\lambda} R_{\lambda 334}
\end{aligned} \quad . \tag{90}$$

Each of the equations in (90) contains only one unknown, the R-C component having two time indices. In total, there are six such independent R-C tensor components:

$$\begin{aligned}
&R_{1414} \\
&R_{1424} \\
&R_{1434} \\
&R_{2424} \\
&R_{2434} \\
&R_{3434}
\end{aligned} \quad . \tag{91}$$

so the system of nine equations (90) can be algebraically solved for the six unknown R-C components at time t_0. With this I now know the value of all components of the R-C tensor at time t_0. From the t_0 values of the R-C tensor components listed in (91), the values of the six unknown $\frac{\partial^2 g_{ij}}{\partial t^2}$ at t_0 can be found.

There are three remaining equations from the set of equations (1) that have not yet been addressed:

$$\begin{aligned}
F_{14;4} &= a^{\lambda} R_{\lambda 414} \\
F_{24;4} &= a^{\lambda} R_{\lambda 424} \\
F_{34;4} &= a^{\lambda} R_{\lambda 434}
\end{aligned} \quad . \tag{92}$$

These are the equations for which the temporal derivatives of the Maxwell tensor components are not yet known. Because all values of the R-C tensor and a^{λ} are now known at t_0, these three remaining time-differentiated components of the Maxwell tensor can now be solved for directly using (92), giving complete knowledge of $\frac{\partial F_{\mu\nu}}{\partial t}$ at time t_0.

If the values of the four $\frac{\partial^2 g_{\mu 4}}{\partial t^2}$ could be calculated then all $\frac{\partial^2 g_{\mu\nu}}{\partial t^2}$ would be known and all $\frac{\partial g_{\mu\nu}}{\partial t}$ could be propagated from t_0 to $t_0 + dt$. Just as is the case with classical General Relativity, the four $\frac{\partial^2 g_{\mu 4}}{\partial t^2}$ can be determined from the four coordinate conditions that are fixed by the choice of coordinate system [35]. Recapping, at t_0 the following quantities are now known: $g_{\mu\nu}, F_{\mu\nu}, u^{\lambda}, \rho_c, \rho_m$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ are defined by initial conditions. Next: $a^{\lambda}, \frac{\partial^2 g_{\mu\nu}}{\partial x^{\kappa}\partial x^{\lambda}}, R_{\lambda\kappa\mu\nu}$, and $\frac{\partial F_{\mu\nu}}{\partial x^{\lambda}}$ are solved for using those initial conditions, the field equations, and the four coordinate conditions that are fixed by the choice of coordinate system. Still needed to propagate the initial conditions in time from t_0 to t_0+dt are $\frac{\partial u^{\mu}}{\partial t}, \frac{\partial \rho_m}{\partial t}$ and $\frac{\partial \rho_c}{\partial t}$. Using the Lorentz force law (25), the following development,

$$\begin{gathered}\rho_m \frac{Du^{\mu}}{D\tau} = \rho_c u^{\lambda} F^{\mu}{}_{\lambda} \\ \downarrow \\ \rho_m u^{\mu}{}_{;\nu} u^{\nu} = \rho_c u^{\lambda} F^{\mu}{}_{\lambda} \\ \downarrow \\ \rho_m u^{\mu}{}_{;4} u^4 = -\rho_m u^{\mu}{}_{;i} u^i + \rho_c u^{\lambda} F^{\mu}{}_{\lambda} \\ \downarrow \\ \rho_m \left(\frac{\partial u^{\mu}}{\partial t} + \Gamma^{\mu}{}_{4\sigma} u^{\sigma} \right) u^4 = -\rho_m u^{\mu}{}_{;i} u^i + \rho_c u^{\lambda} F^{\mu}{}_{\lambda}\end{gathered} \tag{93}$$

shows on the last line above that $\frac{\partial u^{\mu}}{\partial t}$ can be solved for at t_0 in terms of knowns at t_0. Next, using the conservation of mass Equation (24) and knowing $\frac{\partial u^{\mu}}{\partial t}$ at t_0, the following development,

$$\begin{gathered}\left(\rho_m u^{\nu}\right)_{;\nu} = 0 \\ \downarrow \\ \left(\rho_m u^4\right)_{;4} = -\left(\rho_m u^i\right)_{;i} \\ \downarrow \\ \frac{\partial \rho_m}{\partial t} u^4 = -\rho_m u^4{}_{;4} - \left(\rho_m u^i\right)_{;i}\end{gathered} \tag{94}$$

shows on the last line above that $\frac{\partial \rho_m}{\partial t}$ can be solved for at t_0 in terms of knowns at t_0. Following an analogous development for ρ_c using the charge conservation Equation (20), $\frac{\partial \rho_c}{\partial t}$ can be solved for at t_0 in terms of knowns at t_0. With these, the values of $a^{\lambda}, R_{\lambda\kappa\mu\nu}, \frac{\partial F_{\mu\nu}}{\partial t}, \frac{\partial u^{\lambda}}{\partial t}, \frac{\partial \rho_m}{\partial t}, \frac{\partial \rho_c}{\partial t}$ and $\frac{\partial^2 g_{\mu\nu}}{\partial t^2}$ are all known at t_0 and can be used to propagate the initial conditions

$g_{\mu\nu}, F_{\mu\nu}, u^{\lambda}, \rho_c, \rho_m$ and $\dfrac{\partial g_{\mu\nu}}{\partial t}$ at t_0 to time $t_0 + dt$. Iterating the process, the fields in Table 1 can be determined at all times.

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