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Article

Problems in Modeling Three-Phase Three-Wire Circuits in the Case of Non-Sinusoidal Periodic Waveforms and Unbalanced Load

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Abstract: Asymmetry in the supply voltage in three-phase circuits disrupts the flow of currents. This worsens the efficiency of the distribution system and increases the problems in determining the mathematical model of the energy system. Among many power theories, the most accurate is the Currents' Physical Components (CPC) power theory, which tries to justify the physical essence of each component. Such knowledge can be used to improve efficiency and reduce transmission losses in the power system. The article discusses the method of mathematical decomposition of current components in the case of a three-wire line connecting an asymmetric power source with of linear time-invariant (LTI) loads. Special cases where irregularities appear in the results of calculations according to the CPC theory has been discussed. The method is illustrated with a numerical examples.

Keywords: Currents' Physical Components (CPC); power definitions; power theory

1. Introduction

Contemporary problems in the energy sector mainly focus on improving the efficiency of energy systems and improving energy quality [1–4]. These activities in the area of power engineering include, among others, monitoring the flow of current and actions to reduce transmission losses [5–7]. In this respect, developing methods for describing the mathematical model of the power system and how to determine the values of individual elements is crucial. Knowing the mathematical model of this system will make it possible to select the topology and parameters of the compensators [9–16].

In measurement practice, we most often only have data in the form of measured electrical quantities on the line connecting the load with the power source. This article focuses on the three-wire connection (**Error! Reference source not found.**).

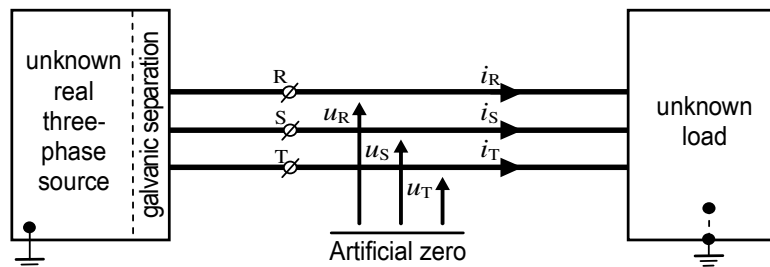


Figure 1. Three-wire connection of the load with the source.

Let a linear time-invariant (LTI) load is supplied by a three-wire line with non-sinusoidal and asymmetrical voltage. The assumption of the LTI load is crucial here, because in the case of a receiver generating harmonics [17–26], identifying all the parameters of the nonlinear load requires a more complicated measurement procedure [27–30].

There are several power theories that allow the description of an electric circuit in terms of energy. The most important are two theories: the first is the IEEE Standard 1459 [31,32], which is

treated as a standard for measurements in electric circuits, and the second is the CPC theory [33–43], which precisely recognizes the individual current components and allows for determining the parameters of compensators.

There is a great conflict in the scientific community between several factions recognizing particular theories. When looking objectively at known power theories, it is necessary to emphasize the advantages of each.

The IEEE Standard 1459, due to its simplicity and simplification, is a good and sufficient way to describe the energy in an electric circuit for billing purposes. Due to the way in which components dependent on physical phenomena are determined, CPC theory is an excellent tool for compensation purposes, design purposes, detailed mathematical analyses and considerations on improving the efficiency of a power system.

The remarks on the CPC theory concerning the incorrectness of the considerations may be considered outdated over time, as work on the detailed description is still in progress and the results obtained bring us ever closer to honoring a universal power theory that does not arouse any controversy. Thanks to the way in which analyses are conducted in this theory, individual components of the current are added to the general mathematical description—analogously to puzzles creating a unified graphic image. If at present day the CPC theory has not taken into account some electrical interaction in the circuit, one can be sure that in the future an additional component will appear which will be a new puzzle in the picture of power theory.

This article focuses on the CPC theory. The first part indicates imprecise descriptions of this theory and proposes corrections supported by computational examples. Next, an analysis was performed for an example measurement scenario.

2. Shift of Vectors by 90°

The voltage measurement [44–50] in a three-wire system is made with respect to an artificial zero (**Error! Reference source not found.**). This circuit is designed in such a way that the three-phase voltage vector does not have a zero symmetrical component. This is possible when the three measuring channels of the voltmeters have the same internal impedances Z_v . In practice, the Z_v value is many times greater than the source and receiver impedances in the tested system, i.e., the Z_v value does not affect the obtained measurements. Current is measured non-invasively using current transformers in the circuit powering a three-phase receiver. This means that the internal impedances Z_A of current meters are many times smaller than the source and receiver impedances. By measuring the instantaneous values of voltages and currents and using the FFT transform, amplitudes and phase shifts are obtained for each harmonic separately. The condition is that the current and voltage waveforms are periodic. When making measurements in a real measuring system, we only have access to the line connecting the source with the receiver (**Error! Reference source not found.**). The source and receiver are unknown. As a result of measuring the instantaneous values of current and voltage and performing the FFT transformation, three-phase vectors are obtained in the form of:

$$\begin{aligned} \mathbf{u}_n &= \begin{bmatrix} u_{R,n} \\ u_{S,n} \\ u_{T,n} \end{bmatrix} = \sqrt{2} \Re \left[\begin{bmatrix} \underline{U}_{R,n} \\ \underline{U}_{S,n} \\ \underline{U}_{T,n} \end{bmatrix} e^{jn\omega_1 t} \right] = \sqrt{2} \Re \{ \underline{\mathbf{U}}_n e^{jn\omega_1 t} \}, & \mathbf{u} = \sum_n \mathbf{u}_n, \\ \mathbf{i}_n &= \begin{bmatrix} i_{R,n} \\ i_{S,n} \\ i_{T,n} \end{bmatrix} = \sqrt{2} \Re \left[\begin{bmatrix} \underline{I}_{R,n} \\ \underline{I}_{S,n} \\ \underline{I}_{T,n} \end{bmatrix} e^{jn\omega_1 t} \right] = \sqrt{2} \Re \{ \underline{\mathbf{I}}_n e^{jn\omega_1 t} \}, & \mathbf{i} = \sum_n \mathbf{i}_n. \end{aligned} \quad (1)$$

When adopting the definitions of these vectors in accordance with the CPC theory, it should be borne in mind that these vectors are shifted by 90° with respect to the elementary theory, because in this case the instantaneous values are based on the cosine function (2) and not the sine function. For example, for voltage it would be:

$$\mathbf{u} = U_0 + \sqrt{2} \sum_{n=1} |\underline{\mathbf{U}}_n| \cdot \cos(n\omega_1 t + \arg\{\underline{\mathbf{U}}_n\}). \quad (2)$$

It is known that the Fourier series coefficients a_n and b_n (3) determined in the period T are:

$$a_n = \frac{2}{T} \int_0^T f(t) \cos(n\omega t) dt, \quad b_n = \frac{2}{T} \int_0^T f(t) \sin(n\omega t) dt, \quad (3)$$

so in CPC theory these coefficients should be swapped:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (b_n \cos(n\omega t) + a_n \sin(n\omega t)). \quad (4)$$

3. Pitfalls in Determining the rms value of Voltage

The active component of current was proposed at the beginning of the 20th century by Prof. Fryze [51]. It makes physical sense for single- and three-phase circuits. As is known, the value of this component is related to the value of active power and the rms value of the voltage. For example, for single-phase circuits, the active component of the current is described by the relations:

$$\|i_a\| = \|u\| \cdot G_e = \frac{P}{\|u\|}, \quad i_a = G_e u = G_e U_0 + \sqrt{2} \Re \left\{ \sum_{n=1}^N G_e \underline{U}_n e^{jn\omega t} \right\}. \quad (5)$$

It can be seen in (5) that the value of the i_a component and the value of the power P depend on the equivalent conductance G_e . Therefore, G_e is a key parameter in determining the active component, and the value of this parameter is determined from the relationship:

$$G_e = P / \|u\|^2. \quad (6)$$

For single-phase systems, the rms value of the voltage is equal:

$$\|u\| = \sqrt{U_0^2 + \sum_{n=1}^N |\underline{U}_n|^2}. \quad (7)$$

Let us note that the rms value of voltage (7) is influenced by all components of this voltage that create the instantaneous value $u(t)$ (e.g., for a three-phase system it is expressed in formula (2)).

Let us assume a situation in which the k th voltage component (where $k \in N$) does not cause a change in the value of the power P in the system. In this situation, the value of the equivalent conductance G_e will not be determined correctly.

Example 1. Let us assume a single-phase receiver constructed from a series connection of a resistor $R = 1 \Omega$ and a capacitor C , which for a pulsation of $\omega = 1$ rad/s has a reactance $X_C = 1 \Omega$. This circuit is powered by voltage $u = 1 + \sqrt{2}[\cos(\omega t) + \cos(3\omega t)]$ V.

According to (7) the rms value of the voltage is: $\|u\| = \sqrt{U_0^2 + U_1^2 + U_3^2} = \sqrt{3}$ V. Active power is the power lost in the resistance R , where for the n th component it is equal: $P_n = |\underline{I}_n|^2 R$, while the current is equal $\underline{I}_n = \frac{U_n}{R - j \frac{X_C}{n\omega_1}}$.

The current values are obtained: $\underline{I} = [0; 0.5 + j0.5; 0.9 + j0.3]$ A, and powers $P = [0; 0.5; 0.9]$ W. The total active power is equal $P = \sum_n P_n = 1.4$ W, i.e., the equivalent conductance (6) is equal $G_e = 0.47$ S.

In a series RC connection, the voltage component U_0 has no energetic effect and does not affect the value of the current flowing through these elements. The direct current component is equal to $I_0 = 0$ and does not depend on the value of R . The correct rms voltage value must be defined as a parameter that causes some effect in the circuit, so in this example it should be determined from the components U_1 and U_3 .

The correct definition of rms voltage is:

$$\|u\| = \sqrt{\sum_{n \in N} U_n^2}, \quad (8)$$

where: N is the set of harmonics for which there is a plexus between the current I_N and the voltage U_N components, i.e., in this example $N = \{1, 3\}$.

The voltage value determined from (8) is equal $\|u\| = \sqrt{2}$ V, this means that the equivalent conductance (6) is equal $G_e = 0.7$ S. By determining the rms value of the active current component

according to (5) and using (7), we obtain $\|i_a\| = 1.4/\sqrt{3} = 0.81$ A, and after using (8), the value of this current is $\|i_a\| = 1.4/\sqrt{2} = 0.99$ A.

A similar problem can be seen in three-phase circuits. In three-phase circuits, the definition of the rms value of the three-phase voltage is equal:

$$\|u\| = \sqrt{\|u_R\|^2 + \|u_S\|^2 + \|u_T\|^2}. \quad (9)$$

The rms values of the L-phase voltages, where $L = \{R, S, T\}$ are also determined from (8). If there are components of the voltage vector that do not participate in the current flow, the equivalent conductance G_e and the active component of the three-phase current $\|i_a\|$ will be determined incorrectly.

4. Notes on the Three-phase Current Components in Time-domain

In three-phase systems, when there is no symmetry in the power source, symmetrical voltage components with: positive U^p , negative U^n and zero U^z sequences are used to build mathematical relationships [52–55]. They are determined from (11), i.e., using three-phase symmetrical unit vectors of the: positive 1^p , negative 1^n and zero 1^z sequences, defined as:

$$1^p = \begin{bmatrix} 1 \\ \alpha^2 \\ \alpha \end{bmatrix}, \quad 1^n = \begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \end{bmatrix}, \quad 1^z = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \quad (10)$$

where: α is the rotation operator which is equal $\alpha = e^{j120^\circ} = e^{j\frac{2\pi}{3}}$.

$$\begin{bmatrix} \underline{U}^z \\ \underline{U}^p \\ \underline{U}^n \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_R \\ \underline{U}_S \\ \underline{U}_T \end{bmatrix}. \quad (11)$$

From this point on, mathematical operations are performed individually on the crms values $\underline{U}^p$, $\underline{U}^n$, $\underline{U}^z$ as for a symmetric source. The transformation (11) is performed for each harmonic separately.

4.1. Definition of instantaneous values of three-phase symmetrical voltage for multiple harmonics

In three-phase unbalanced periodic and non-sinusoidal voltage waveforms for the n th harmonic, there is no symmetrical voltage component of zero sequence, then the following is true:

$$\underline{U}_{R,n} + \underline{U}_{S,n} + \underline{U}_{T,n} = 0, \quad (12)$$

In the case of symmetry, for each n th harmonic, the phase shift is equal α or α^2 . This can be written as an equation:

$$\begin{aligned} u_L &= \sqrt{2} \sum_{n=1} |\underline{U}_{L,n}| \cdot \cos[n(\omega t + s_L + \varphi_n)] = \sqrt{2} \sum_{n=1} |\underline{U}_{L,n}| \cdot \cos(n\omega t + \arg\{\underline{U}_{L,n}\}), \\ u_L &= \sqrt{2} \sum_{n=1} \Re\{\underline{U}_{L,n} \cdot e^{jn\omega t}\}, \end{aligned} \quad (13)$$

where: L – phase symbol, $L = \{R, S, T\}$,

s_L – phase shift equal to $s_L = \left[0, \frac{-2\pi}{3}, \frac{2\pi}{3}\right]$, meaning: $s_R = 0$, $s_S = \arg\{\alpha^2\}$, $s_T = \arg\{\alpha\}$,

φ_n – phase shift between current and voltage for the n th harmonic.

In (13) it can be seen that the argument of the complex number $\underline{U}_{L,n}$ contains: the rotation vector α , the phase shift angle φ_n and the n th harmonic order:

$$\arg\{\underline{U}_{L,n}\} = n(s_L + \varphi_n). \quad (14)$$

Example 2. For the instantaneous voltage value in R phase equal $u_R = \sqrt{2} \cdot \Re\{230e^{j\omega t} + 20e^{j5(\omega t+2)} + 5e^{j7\omega t}\}$ V can be written in the time domain: $u_R = \sqrt{2} \cdot \{230\cos(\omega t) + 20\cos(5(\omega t+2)) + 5\cos(7\omega t)\}$ V. According to (13) in the case of symmetry, the voltages in the remaining phases are equal:

$$u_s = \sqrt{2} \cdot \left\{ 230 \cos\left(\omega t - \frac{2\pi}{3}\right) + 20 \cos\left(5\left(\omega t - \frac{2\pi}{3} + 2\right)\right) + 5 \cos\left(7\left(\omega t - \frac{2\pi}{3}\right)\right) \right\} \text{ V},$$

$$u_T = \sqrt{2} \cdot \left\{ 230 \cos\left(\omega t + \frac{2\pi}{3}\right) + 20 \cos\left(5\left(\omega t + \frac{2\pi}{3} + 2\right)\right) + 5 \cos\left(7\left(\omega t + \frac{2\pi}{3}\right)\right) \right\} \text{ V}$$

The graphical waveforms of these voltages are presented in **Error! Reference source not found..**

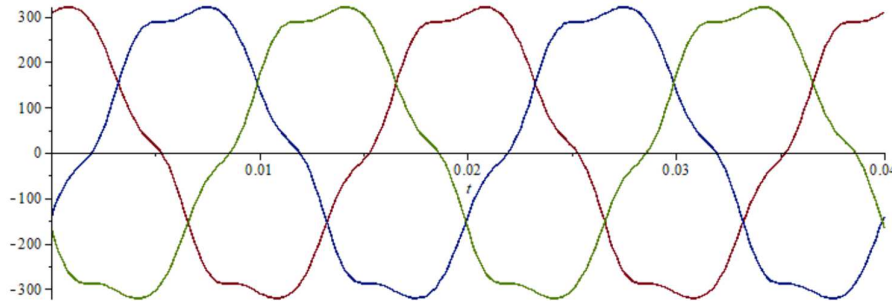


Figure 2. Symmetrical non-sinusoidal voltage waveforms from example 2.

In (13), the separation of the n th harmonic order and phase shifts: s_L, φ_n simplifies the mathematical analysis of three-phase waveforms.

4.2. Using of symmetrical components in the case of many harmonics

In the case of sinusoidal waveforms, the three-phase asymmetrical source is replaced by the sum of three symmetrical sources, which from (11) for phase R as the reference phase, create the equation:

$$\underline{U} = \underline{U}^z + \underline{U}^p + \underline{U}^n = \mathbf{1}^z \underline{U}^z + \mathbf{1}^p \underline{U}^p + \mathbf{1}^n \underline{U}^n. \quad (15)$$

where: $\underline{U} = [\underline{U}_R \quad \underline{U}_S \quad \underline{U}_T]^T$.

Similarly to (15), in articles [50] for non-sinusoidal periodic waveforms for the n th harmonic the following relation is used:

$$\underline{U}_n = \underline{U}_n^z + \underline{U}_n^p + \underline{U}_n^n = \mathbf{1}^z \cdot \underline{U}_n^z + \mathbf{1}^p \cdot \underline{U}_n^p + \mathbf{1}^n \cdot \underline{U}_n^n. \quad (16)$$

So far, in the case of periodic non-sinusoidal waveforms, the values of symmetrical components have been determined in the same way as for sinusoidal circuits – i.e., using the transformation (11). For waveforms consisting of many harmonics, the formula (14) is valid. It follows that the turnover factor α is multiplied n times. Therefore, equation (11) for the n th harmonic should look like this:

$$\begin{bmatrix} \underline{U}_n^z \\ \underline{U}_n^p \\ \underline{U}_n^n \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^n & \alpha^{2n} \\ 1 & \alpha^{2n} & \alpha^n \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_{R,n} \\ \underline{U}_{S,n} \\ \underline{U}_{T,n} \end{bmatrix}, \quad (17)$$

while the inverse transformation is described by the relationship:

$$\begin{bmatrix} \underline{U}_{R,n} \\ \underline{U}_{S,n} \\ \underline{U}_{T,n} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^{2n} & \alpha^n \\ 1 & \alpha^n & \alpha^{2n} \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_n^z \\ \underline{U}_n^p \\ \underline{U}_n^n \end{bmatrix}. \quad (18)$$

The new vectors $\mathbf{1}_n^p, \mathbf{1}_n^n$, arise after raising vectors (10) to the n th power:

$$\mathbf{1}_n^p \stackrel{\text{df}}{=} (\mathbf{1}^p)^n = \begin{bmatrix} 1 \\ \alpha^{2 \cdot n} \\ \alpha^n \end{bmatrix}, \quad \mathbf{1}_n^n \stackrel{\text{df}}{=} (\mathbf{1}^n)^n = \begin{bmatrix} 1 \\ \alpha^n \\ \alpha^{2 \cdot n} \end{bmatrix}. \quad (19)$$

The coefficients (19) was called multiplied three-phase unit vectors.

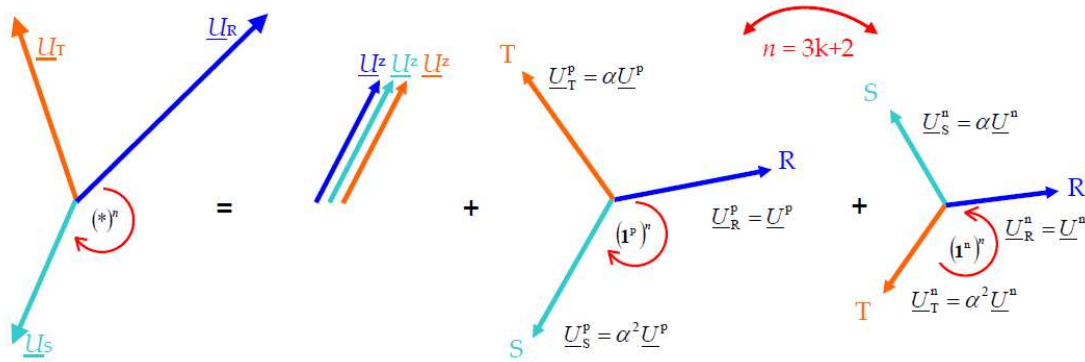


Figure 3. Graphical interpretation of multiplied three-phase unit vectors, where k is a natural number $k \in \{N\}$.

Example 3. In the three-wire line from **Error! Reference source not found.**, where the relationship $\underline{U}_{R,n} + \underline{U}_{S,n} + \underline{U}_{T,n} = 0$ is valid, the following voltage values were measured:

$$\begin{aligned} u_R &= \sqrt{2} \cdot \{300 \sin(\omega t) + 30 \sin(5\omega t) + 3 \sin(9\omega t)\} \text{ V}, \\ u_S &= \sqrt{2} \cdot \{250 \sin(\omega t + s_s) + 25 \sin(5(\omega t + s_s)) + 2 \sin(9(\omega t + s_s))\} \text{ V}, \\ u_T &= \sqrt{2} \cdot \{50\sqrt{3} \sin(\omega t + 128.95^\circ) + 5\sqrt{3} \sin(5\omega t - 128.95^\circ) + 5 \sin(9\omega t + 180^\circ)\} \text{ V}. \end{aligned} \quad (20)$$

The symmetric components (16) determined from (11), give the solution in the form:

$$\begin{aligned} \underline{U}_n^z &= \frac{1}{3} (\underline{U}_{R,n} + \underline{U}_{S,n} + \underline{U}_{T,n}), \\ \underline{U}_n^p &= \frac{1}{3} (\underline{U}_{R,n} + \alpha \underline{U}_{S,n} + \alpha^2 \underline{U}_{T,n}), \\ \underline{U}_n^n &= \frac{1}{3} (\underline{U}_{R,n} + \alpha^2 \underline{U}_{S,n} + \alpha \underline{U}_{T,n}). \end{aligned} \quad (21)$$

After substituting the numerical data, we get:

$$\begin{bmatrix} \underline{U}_1^p \\ \underline{U}_5^p \\ \underline{U}_9^p \end{bmatrix} = \begin{bmatrix} 275 + j \frac{25\sqrt{3}}{3} \\ \frac{5}{2} + j \frac{5\sqrt{3}}{6} \\ \frac{3}{2} + j \frac{7\sqrt{3}}{6} \end{bmatrix}, \quad \begin{bmatrix} \underline{U}_1^n \\ \underline{U}_5^n \\ \underline{U}_9^n \end{bmatrix} = \begin{bmatrix} 25 - j \frac{25\sqrt{3}}{3} \\ \frac{55}{2} - j \frac{5\sqrt{3}}{6} \\ \frac{3}{2} - j \frac{7\sqrt{3}}{6} \end{bmatrix}, \quad \begin{bmatrix} \underline{U}_1^z \\ \underline{U}_5^z \\ \underline{U}_9^z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (22)$$

The same components determined from (18), give the solution:

$$\begin{aligned} \underline{U}_n^z &= \frac{1}{3} (\underline{U}_{R,n} + \underline{U}_{S,n} + \underline{U}_{T,n}), \\ \underline{U}_n^p &= \frac{1}{3} (\underline{U}_{R,n} + \alpha^n \underline{U}_{S,n} + \alpha^{2n} \underline{U}_{T,n}), \\ \underline{U}_n^n &= \frac{1}{3} (\underline{U}_{R,n} + \alpha^{2n} \underline{U}_{S,n} + \alpha^n \underline{U}_{T,n}), \end{aligned} \quad (23)$$

which in this example gives the values:

$$\begin{bmatrix} \underline{U}_1^p \\ \underline{U}_5^p \\ \underline{U}_9^p \end{bmatrix} = \begin{bmatrix} 275 + j \frac{25\sqrt{3}}{3} \\ \frac{55}{2} - j \frac{5\sqrt{3}}{6} \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \underline{U}_1^n \\ \underline{U}_5^n \\ \underline{U}_9^n \end{bmatrix} = \begin{bmatrix} 25 - j \frac{25\sqrt{3}}{3} \\ \frac{5}{2} + j \frac{5\sqrt{3}}{6} \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \underline{U}_1^z \\ \underline{U}_5^z \\ \underline{U}_9^z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (24)$$

Comparing solutions (22) and (24) it should be noted that differences appear when $n > 1$. In practice, when $n = 3k + 2$ (where k is a natural number $k \in \{N_+\}$) the positive $\underline{U}_n^p$ and negative $\underline{U}_n^n$ symmetric components are swapped (**Error! Reference source not found.**).

The appearance of non-zero $\underline{U}_9^p$ and $\underline{U}_9^n$ components in equation (22) also indicates an incorrect transformation. When equation $\underline{U}_{R,n} + \underline{U}_{S,n} + \underline{U}_{T,n} = 0$ is satisfied, the appearance of zero-order harmonics (for $n = 3k$) is unnatural. In functions (20), in real systems, the amplitude of the 9th harmonic cannot be different from zero. This translates into zero values of a symmetrical component of the zero sequence $\underline{U}_n^z$ for any n th harmonic.

5. Revised CPC Theory Definitions

The introduction of a correction to the decomposition into symmetrical components for periodic non-sinusoidal waveforms results in a different mathematical notation for electrical quantities for three-phase systems.

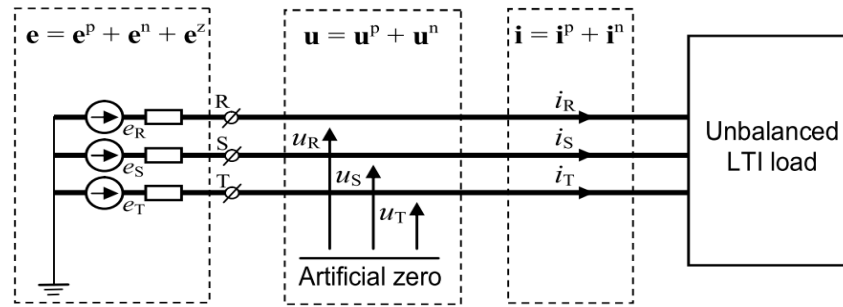


Figure 4. Decomposition into three-phase symmetrical components.

The vector of the supply voltage $\mathbf{u}$ as referenced to an artificial zero of the circuit (**Error! Reference source not found.**) can be decomposed into symmetrical components of the positive and the negative sequence:

$$\mathbf{u} = \sum_n \mathbf{u}_n = \sqrt{2} \Re \left\{ \sum_{n=1} \left(\mathbf{1}^p \underline{U}_n^p + \mathbf{1}^n \underline{U}_n^n \right) \cdot e^{jn\omega t} \right\} = \mathbf{u}^p + \mathbf{u}^n. \quad (25)$$

The symbols $\underline{U}_n^p$ and $\underline{U}_n^n$ in (25) denote the crms values of the symmetrical components of the n th order supply voltage harmonic with positive and negative sequence, respectively. They are equal

$$\begin{bmatrix} \underline{U}_n^p \\ \underline{U}_n^n \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \alpha^n & \alpha^{2n} \\ 1 & \alpha^{2n} & \alpha^n \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_{R,n} \\ \underline{U}_{S,n} \\ \underline{U}_{T,n} \end{bmatrix}. \quad (26)$$

The three-phase rms value of the supply voltage, equal to the root of the sum of the squares of the effective values of the line voltages, is:

$$\|\mathbf{u}\| = \sqrt{\sum_{L=R,S,T} \|u_L\|^2}, \text{ where } \|u_L\| = \sqrt{\sum_{n=1} |\underline{U}_{L,n}|^2}. \quad (27)$$

The load in **Error! Reference source not found.** is equivalent with respect to active power P to a balanced resistive load of conductance G_b :

$$G_b = \frac{P}{\|\mathbf{u}\|^2}. \quad (28)$$

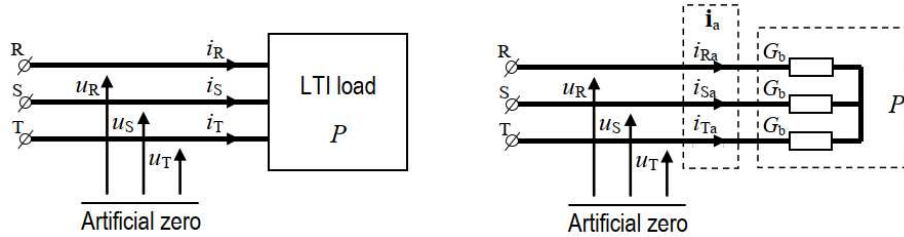


Figure 5. Three-phase load and balanced resistive load equivalent in terms of active power P .

The current flowing through such an equivalent load is called the active current and is equal:

$$\mathbf{i}_a = \begin{bmatrix} i_{Ra} \\ i_{Sa} \\ i_{Ta} \end{bmatrix} = G_b \mathbf{u} = \sqrt{2} \Re \left\{ \sum_{n=1} G_b \left(\underbrace{\mathbf{1}^p U_n^p}_{\underline{\mathbf{U}}_n^p} + \underbrace{\mathbf{1}^n U_n^n}_{\underline{\mathbf{U}}_n^n} \right) \cdot e^{jn\omega t} \right\}, \quad \|\mathbf{i}_a\| = G_b \cdot \|\mathbf{u}\|. \quad (29)$$

As a result of calculating equation (29), we obtain the same form as in the previous articles that did not take into account the multiplied vector in (26), because the $\underline{\mathbf{U}}^p$ and $\underline{\mathbf{U}}^n$ components are swapped—which does not affect the result in the case of their sum. However, to improve readability, the elementary division between these components should be maintained.

The load for each harmonic has active and reactive powers. For the n th order harmonic, they are equal:

$$P_n = \Re \{ \underline{\mathbf{U}}_n^T \cdot \underline{\mathbf{I}}_n^* \} = \Re \{ \underline{U}_{R,n} I_{R,n}^* + \underline{U}_{S,n} I_{S,n}^* + \underline{U}_{T,n} I_{T,n}^* \}, \quad Q_n = \Im \{ \underline{\mathbf{U}}_n^T \cdot \underline{\mathbf{I}}_n^* \}. \quad (30)$$

When for the n th order harmonic the load is unbalanced, it is useful to define the balanced admittance $\underline{Y}_b$. This admittance is already symmetrical and depends on the voltage $\mathbf{u}_n$, the active powers P_n and reactive powers Q_n , and is equal:

$$\underline{Y}_{b,n} = G_{b,n} + jB_{b,n} = \frac{P_n - jQ_n}{\|\mathbf{u}_n\|^2} = \frac{\underline{C}_n^*}{\sum_{L=R,S,T} |\underline{U}_{L,n}|^2}. \quad (31)$$

In definition (31) the symbol “ C ” was used instead of the symbol “ S ”. This is an intentional action by authors [50] to avoid confusing the complex power $P_n + jQ_n$ with the apparent power S_n , which may also contain components other than only active and reactive power. These powers was shown in **Error! Reference source not found.**

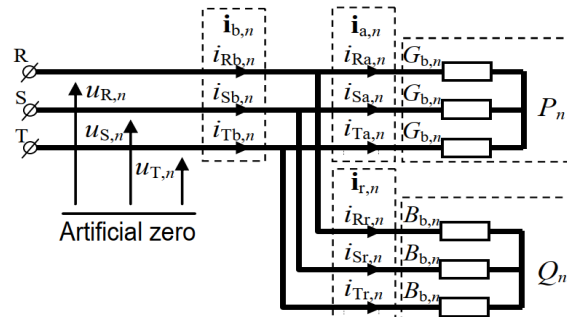


Figure 6. Graphical interpretation of the equivalent admittance for the n th order harmonic.

The supply current of such an equivalent load consists of the active current (32) and the reactive current (33):

$$\mathbf{i}_{a,n} = G_{b,n} \mathbf{u}_n = \sqrt{2} \Re \left\{ G_{b,n} \left(\mathbf{1}^p \underline{U}_n^p + \mathbf{1}^n \underline{U}_n^n \right) \cdot e^{jn\omega t} \right\}, \quad (32)$$

$$\mathbf{i}_{r,n} = \sqrt{2} \Re \left\{ jB_{b,n} \left(\mathbf{1}^p \underline{U}_n^p + \mathbf{1}^n \underline{U}_n^n \right) \cdot e^{jn\omega t} \right\}. \quad (33)$$

The $\mathbf{i}_r$ current component appears in the load current, because of the phase shift of the load current harmonics relative to the supply voltage harmonics. It can be considered as a **reactive current** component on the load, whose value is:

$$\mathbf{i}_r = \sum_n \mathbf{i}_{r,n} = \sqrt{2} \Re \left\{ \sum_{n=1} jB_{b,n} \left(\mathbf{1}^p \underline{U}_n^p + \mathbf{1}^n \underline{U}_n^n \right) \cdot e^{jn\omega t} \right\}. \quad (34)$$

$\underline{Y}_{b,n}$ is the admittance of the equivalent balanced load for the n th order harmonic. In practice, the load may be unbalanced, so the n th order harmonic of the load current $\mathbf{i}_n$ may contain the **unbalanced current** component:

$$\mathbf{i}_{u,n} = \mathbf{i}_n - \mathbf{i}_{b,n} = \mathbf{i}_n - (\mathbf{i}_{a,n} + \mathbf{i}_{r,n}) = \sqrt{2} \Re \left\{ \left[\underline{Y}_n - \underline{Y}_{b,n} \right] \left(\mathbf{1}^p \underline{U}_n^p + \mathbf{1}^n \underline{U}_n^n \right) \cdot e^{jn\omega t} \right\}, \quad \mathbf{i}_u = \sum_{n=1} \mathbf{i}_{u,n}. \quad (35)$$

In the case of the (32) component, the relation holds:

$$\sum_{n=1} \mathbf{i}_{a,n} \neq \mathbf{i}_a = G_b \sum_{n=1} \mathbf{u}_n. \quad (36)$$

The inequality (36) occurs when the conductance $G_{b,n}$ for harmonic frequencies are different from the conductance G_b of the equivalent balanced load. The difference between these components is:

$$\sum_{n=1} \mathbf{i}_{a,n} - \mathbf{i}_a = \sqrt{2} \Re \left\{ \left(G_{b,n} - G_b \right) \cdot \left(\mathbf{1}^p \underline{U}_n^p + \mathbf{1}^n \underline{U}_n^n \right) \cdot e^{jn\omega t} \right\} = \mathbf{i}_s. \quad (37)$$

The current $\mathbf{i}_s$ is called a **scattered current** component because conductances at harmonic frequencies $G_{b,n}$ are usually scattered around the balanced conductance G_b and the current $\mathbf{i}_s$ is the effect of this scattering.

The current of the load was presented as the sum of the specific components:

$$\mathbf{i} = \mathbf{i}_a + \mathbf{i}_s + \mathbf{i}_r + \mathbf{i}_u. \quad (38)$$

Because of association of this load currents components with physical phenomena in the load, these currents are referred to as the Currents' Physical Components (CPC). This does not mean that these currents exist physically, as they are mathematical entities, not physical ones.

Each of these currents is associated with a different physical phenomenon in the load. The active current $\mathbf{i}_a$ is related to the phenomenon of supplying active power P to the load. The scattered current $\mathbf{i}_s$ is related to the phenomenon of changing the equivalent conductance $G_{b,n}$ with the harmonic order n . The reactive current $\mathbf{i}_r$ is related to the phenomenon of phase shift of the current relative to the supply voltage for the n th harmonic. The unbalanced current $\mathbf{i}_u$ is related to the load imbalance for the n th harmonic.

The correction introduced to the definitions presented so far in this article does not change the rms values of these components. Only the equations defined in the time domain change.

The situation becomes more complicated when the unbalanced current component is decomposed and the values of the compensator parameters are determined. In the article [36] the unbalanced current component was decomposed into two elements:

$$\mathbf{i}_u = \mathbf{i}_u^p + \mathbf{i}_u^n. \quad (39)$$

The rms values of both components depend on the division into positive and negative sequence components. When equation (16) is taken into account, these components can be represented in the form:

$$\mathbf{i}_u^p = \sqrt{2} \Re \left\{ \sum_{n=1} \left(\underline{Y}_{d,n} \underline{U}_n^p + \underline{Y}_{u,n}^n \underline{U}_n^n \right) \mathbf{1}^p e^{jn\omega t} \right\}, \quad (40)$$

$$\mathbf{i}_u^n = \sqrt{2} \Re \left\{ \sum_{n=1} \left(\underline{Y}_{d,n} \underline{U}_n^n + \underline{Y}_{u,n}^p \underline{U}_n^p \right) \mathbf{1}^n e^{jn\omega t} \right\}, \quad (41)$$

where, according to [36] the voltage asymmetry dependent admittance:

$$\underline{Y}_{d,n} = \underline{Y}_{e,n} - \underline{Y}_{b,n}, \quad (42)$$

the **equivalent admittance**:

$$\underline{Y}_{e,n} = \underline{Y}_{ST,n} + \underline{Y}_{TR,n} + \underline{Y}_{RS,n}, \quad (43)$$

and the **unbalanced admittances** is:

$$\begin{aligned} \underline{Y}_{u,n}^p &= -(\underline{Y}_{ST,n} + \alpha \underline{Y}_{TR,n} + \alpha^2 \underline{Y}_{RS,n}), \\ \underline{Y}_{u,n}^n &= -(\underline{Y}_{ST,n} + \alpha^2 \underline{Y}_{TR,n} + \alpha \underline{Y}_{RS,n}). \end{aligned} \quad (44)$$

The admittances (42), (43), (44), (31) depend on the admittance values of the three-phase load, while the rotation vector α acts in the same direction of rotation—regardless of the harmonic order.

As a result of changing the values of the symmetrical voltage components $\underline{U}_n^p$ and $\underline{U}_n^n$, which should be determined from (26), the values of the current unbalanced components (40) and (41) will change. A different way of defining these variables causes the instantaneous and rms values of both components to change, and the formulas determining the reactive compensator parameters must be re-developed.

6. Conclusions

The computational problems revealed in this article are important when performing mathematical analyses in electrical systems powered by sources with periodic non-sinusoidal waveforms.

1. The 90° shift of vectors discussed in point 2 is important in the case of time-domain notation and comparing calculation results with oscilloscope measurements. To deal with this problem, one can use the relationship (4), or each of the determined numerical values in the time domain can be shifted by a constant angle of -90°.
2. The problem shown in point 3 concerns a special case when harmonics appear that do not participate in the transmission of the energy. An example is a situation when the load has a series capacitance, which, as is known, does not carry a DC component. In such a situation, it remains to use formula (8) only for those harmonics that are related to energetic interactions.
3. The method of notation of three-phase waveforms, discussed in point 4, revealed the need to change the definition of symmetrical components (17) when the instantaneous values are described by periodic non-sinusoidal functions. Determining the symmetric components using multiplied three-phase unit vectors (19) improves the mathematical notation. This observation revealed the need to improve the development of algorithms determining the unbalanced components and parameters of reactive compensators. These issues should be considered in further research.

Symbols

$\mathbf{1}$	three phase symmetrical unit vector
a, b	Fourier series coefficients
α	rotation vector
B_b	balanced susceptance, S
C	capacitance, F
e	electromotive force, emf, V

φ	phase shift
G_b	balanced resistive load of conductance, S
G_e	equivalent conductance, S
$\mathbf{i}$	vector of instantaneous currents in a three-phase system, A
$\underline{\mathbf{I}}$	vector of complex currents in a three-phase system, A
i_R, i_S, i_T	instantaneous values of line currents, A
$\mathbf{i}_a$	active component of the current—three-phase vector, A
$\mathbf{i}_r$	reactive component of the current—three-phase vector, A
$\mathbf{i}_s$	scattered component of the current—three-phase vector, A
$\mathbf{i}_u$	unbalanced component of the current—three-phase vector, A
N	set of harmonics
P	active power, W
Q	reactive power, var
R	resistance, Ω
s_L	phase shift
t	time, s
T	the repetition period of the instantaneous value, rad/s
$\mathbf{u}$	vector of instantaneous voltages in a three-phase system, V
$\underline{\mathbf{U}}$	vector of complex voltages in a three-phase system, V
u_R, u_S, u_T	instantaneous voltage values relative to the virtual star point, V
ω	basic pulsation, rad/s
X	reactance, Ω
$\underline{Y}_b$	balanced admittance, S
$\underline{Y}_d$	voltage asymmetry dependent admittance, S
$\underline{Y}_e$	equivalent admittance, S
$\underline{Y}_u$	unbalanced admittance, S

Subscripts, superscripts

R,S,T,N	phase and neutral wires
n	harmonic number
L	phase number, $L = \{R,S,T\}$
p, n, z	positive, negative, zero sequence

Acronyms

CPC	Currents' Physical Components
crms	complex root mean square

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