Article

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Posted Date: 15 January 2024

doi: 10.20944/preprints202401.1126.v1

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MOND, MONG and the Oort Cloud

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Abstract: The definitive negative results of various direct searches for dark matter particles and non-observation of them has increasingly indicated that we should consider alternate models like the Modification of Newtonian Dynamics and Modification of Newtonian Gravity. It is puzzling how the Oort cloud, despite its great distance from the Sun, is gravitationally bound to the Sun's gravity. In this work, we suggest that this could be explained by the modifications to Newtonian dynamics and to gravity.

Keywords: Oort cloud; modified newtonian dynamics; modified newtonian gravity

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1. Introduction

One of the as yet unresolved problems in cosmology is that the universe seems not to be dominated by ordinary baryonic matter. But instead, by a form of non-luminous matter called the dark matter (DM), which is about five times more abundant than baryonic matter [1]. There exists substantial evidence for the existence of dark matter within the vast expanse of the Universe. Major evidence stems from its gravitational effects, which account for the flat rotation curves observed in galaxies [2]. Another compelling evidence arises from the measurements of fluctuations in the cosmic microwave background radiation. The pattern of temperature fluctuations observed in the CMB aligns with the presence of dark matter [3]. Consequently, the cumulative evidence strongly indicates that dark matter constitutes approximately 85% of the total matter in the Universe.

However, over the past few decades, several experiments to detect the elusive DM particles have yielded no positive results so far. PandaX-II dark matter experiment has reported that no DM candidates have been observed [4]. The upgraded XENON1T has set most stringent limits on DM interaction cross-section, and yet the detection of DM remains elusive [5]. XENONnT is the most recent upgrade of the XENON experiments consists of more than eight tones of XENON mass. Running since 2020, the detector has registered just 16 events so far, most of which is attributed to electronic recoils or neutron collisions. Results from XENONnT has further reduced the limit of interaction cross section to $10^{-48}cm^2$ with no conclusive evidence of dark matter [6].

2. Modifications of Newtonian Gravity (MONG)

In view of the negative results from dark matter detection experiments, alternate theories attempt to modify Newton's law of gravitation or Einstein's theory of relativity to account for the observations that necessitate the presence of dark matter. In this context the Vulcan example is sometimes pointed out. The anomaly that was observed in Mercury's orbit was explained by a new theory of gravity (i.e. general relativity (GR)) rather than a missing planet (dark matter?). One of the alternate models is the Modification of Newtonian Dynamics (MOND) which was proposed by Milgrom to account for the flat rotational curves of galaxies [7]. MOND introduces an ad-hoc introduction of a fundamental acceleration $a_0 \approx 10^{-8}cm^{-2}$. Below this acceleration, the Newtonian law gets altered and the gravitational acceleration is modified to,

$$a = \frac{(GMa_0)^{1/2}}{r} \quad (1)$$

This acceleration gives a force that goes as $1/r$, which accounts for a constant rotation velocity given as

$$v_c = (GMa_0)^{1/4} \tag{2}$$

This is independent of r when the gravitational acceleration drops below a_0 [7].

Another theory of MOdified Gravity (MOG) arrives at strikingly similar results by considering two scalar fields and one vector field to Einstein's theory of gravity. The vector field in the theory resembles a Lorentz force where each particle has a charge proportional to its inertial mass [8, 9, 10].

In a previous study [11], we demonstrated that a minimal acceleration occurs organically, correlating with the lowest gravitational field intensity, particularly observed at the peripheries of galaxies and galaxy clusters. This minimum acceleration turns out to be $10^{-8} cm/s^2$, i.e. the same as MOND acceleration. So in MONG, we arrive at this minimal acceleration without any ad hoc assumptions. The minimum acceleration is given by,

$$a_{min} = \frac{GM}{r_{max}^2} \tag{3}$$

Here r_{max} is the maximum size constrain on the gravitationally bound structures.

The flat rotation curves can alternatively be explained by considering Modifications of Newtonian Gravity (MONG) by adding an additional gravitational self-energy density term, $K(\nabla\phi)^2$. This leads to the modified Poisson equation [12],

$$\nabla^2\phi = 4\pi G\rho + K(\nabla\phi)^2 + \Lambda c^2 \tag{4}$$

where, $K \approx \left(\frac{G^2}{c^2}\right)$ is a constant and the gravitational self-energy density is given by $K(\nabla\phi)^2$.

The solution of equation (4) yields:

$$\phi = (Q + K')\ln\frac{r}{r_o} \tag{5}$$

where $Q = 4\pi G\rho_o {r_o}^2$ and $K' \approx \frac{GM}{r_o}$ are constants.

Equation (5) gives a force (per unit mass) of the form,

$$F = \frac{K''}{r} \tag{6}$$

where $K'' = (GMa_{min})^{1/2}$ is also a constant.

Equation (5) implies a logarithmic increase in the gravitational potential at distances corresponding to $a \leq a_{min}$.

3. Oort Cloud

The solar system is dominated by the Sun's mass $M_\odot$. At a distance of $r \approx 10^{17} cm$, the gravitational acceleration drops to a_0. This distance is about a few thousand Astronomical Units (AU). If Planet Nine orbits at distance of over 300 AU, the solar gravitational acceleration could drop to this value. The Oort cloud, postulated to house trillions of comets is believed to be at a distance of 10^5 AU, i.e. $\sim 10^{18}\ cm$. The solar gravitational (Newtonian) acceleration at this distance would be

$$a_{Oort} = \frac{GM_\odot}{r^2} \approx 10^{-10}\ cm/s^2 \tag{4}$$

This is well below a_0.

The orbital velocity due to Sun's field at that distance is,

$$v \approx \left(\frac{GM}{r}\right)^{1/2} \approx 10^4\ cm/s \tag{5}$$

So it is difficult to understand how the Oort cloud is bound to the Sun's gravity with such a low gravitational acceleration and orbital velocity.

However, MOND and MONG would be expected to play a role at such a low acceleration. The MOND acceleration will be,

$$a = \frac{(GM_\odot a_0)^{1/2}}{r} \approx 10^{-9}\ cm/s^2 \tag{6}$$

This is greater than around 10 times the Newtonian value. Irrespective of distance, MOND would imply an orbital velocity of,

$$v = (GMa_0)^{1/4} \approx 0.5\ \times 10^5\ cm/s \tag{7}$$

i.e. $0.5\ km/s$, much higher than the Newtonian value. MONG implies a logarithmic relation for the gravitational potential

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$$\phi = (GMa_0)^{1/2} \ln \frac{r}{r_0} \tag{8}$$

where $r_0 \approx 10^4 AU$ is the distance at which the acceleration becomes a_0.

The gravitational potential is a few times higher than the Newtonian value, and the orbital velocity is a constant at $0.5\ km/s$ (much higher than the Newtonian value). Thus we see how Oort cloud, despite its great distance from the Sun, can be gravitationally bound to the Sun's gravity.

4. Conclusion

Even the Voyager spacecrafts are a very long way off from $r_0 \approx 10^4 AU$ (they are about 150 AU away and unlikely to transmit beyond 2030). So contrary to earlier ideas that the Pioneer anomaly could be a test of MOND, it can be concluded that we have to go much beyond $(10^4 AU)$ before the effects of MOND and MONG becomes noticeable. In the context of modified theories, recent work on a large number of widely separated binary stars (their separation have been obtained by Gaia data), where their relative acceleration drops well below a_0, independently supports a modification of Newtonian dynamics and gravity [13].

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