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Combined GravElectroMagnetic Forces in a Reissner-Nordström Black-Hole Binary: The Emitted GEM Waves, and the Appearance of the Relativistic Maximum Tension Force and Planck's Constant in the Classical Domain

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Abstract: Two Reissner-Nordström (RN) black holes in a binary, each with mass M and charge Q, are subject to a conservative action-reaction force pair that includes four components ($F_{M\to M}$, $F_{M\to Q}$, $F_{Q\to M}$, and $F_{Q\to Q}$). We combine these four components to obtain the magnitude of the net gravelectromagnetic (GEM) force F as a function of the binary separation R. The resultant force $F(R)$ is especially simple, owing to the conservative nature of the components. The separation R decreases in time, as the binary emits GEM waves. We use an approximation for the dimensionless amplitude h of these waves and the derived force $F(R)$ to develop a formula for h in terms of multiple ratios of the fundamental lengths of the problem: the distance to the binary r, the binary separation R, the Schwarzschild radius R_S, and the RN charge radius R_Q. For $R_Q = 0$, the results reduce to the amplitude of the conventional gravitational radiation produced by the $F_{M\to M}$ force component. In the course of these calculations, we have encountered two entirely unexpected properties of the combined GEM field: Newton's action-reaction principle is not valid for oppositely-charged RN black holes, and the general relativistic maximum force ($F_{\max} = F_P/4$, where F_P is the Planck force) has naturally appeared as the upper limit of classical GEM forces as well. Furthermore, we have obtained some remarkable new relations between universal constants of nature that express Planck's constant h, the Planck mass M_P, and the Compton radius r_c in terms of classical (non-quantum mechanical) constants (electron mass m_e and Avogadro's number N_A). In the same context, the gravitational coupling constant $\alpha_g \propto 1/N_A{}^2$, the fine-structure constant $\alpha \propto 1/N_A{}^2$ as well, and the weak coupling constant $\alpha_w = \sqrt{\alpha} \propto 1/N_A$.

Keywords: black holes; cosmology; gravitation; gravitational waves; Planck system of units

1. Introduction and Motivation

1.1. Conservative Cross-Forces Due to Both Mass and Charge

Conservative forces due to the gravitational field and the electrostatic field have always been treated as independent actions of charged massive sources applied to neighboring objects that carry mass and charge. Despite the obvious fact that Newton's gravitational law and Coulomb's law have the exact same constitution, the two types of resulting forces between charged particles and objects have always been considered separately. Physicists contemplate that the identical behavior of these long-range forces (both falling off as $1/R^2$ with distance R) is due to some unified force of nature, but in the absence of a unification theory, no thoughtful attempts have been made to at least partially unify these two conservative forces.

In recent work, we set out to explore this problem in the classical (nonrelativistic) context. First, we considered the two fundamental force laws, and we attempted to write them in the same form [1]. The result was a new effective gravitational constant $G_\star = 4\pi\varepsilon_0 G$, in which Newton's G also carries an imprint of the vacuum, the vacuum permittivity ε_0; and a new understanding of the source $\sqrt{G_\star}M$ of the gravitational field, where M is the mass of the object that generates the field. The results also pointed clearly to a determination of Newton's G to 10 significant digits,[1] much more precise than the current experimental measurements [2–4].

Next, we considered the cross-forces between masses and charges in a combined conservative gravelectromagnetic (GEM) field [5]. We use this particular term to differentiate our combined GEM field from previous unrelated attempts that have used similar names (e.g., Ref. [6]).

The main idea in Ref. [5] stems from Newton's gravitational law and Coulomb's law: in a unified conservative GEM field, the mass source attracts positive properties of objects entering the field and being subjected to forcing, whereas mass repels negative properties of entering objects, such as negatively-charged objects. Similarly, a positively-charged electrostatic source repels masses as well as positive charges, but the same source attracts negative charges in accordance with Coulomb's law.

The rationale behind these actions of the GEM field relies on the presence of two sources, mass and charge. It appears to be natural behavior for one source (the mass) to attract like (positive) properties of another object, whereas for the other source (the charge) to repel like properties of another object. We have listed the various attractive and repulsive components of the GEM forces in Table 1 below, where the two GEM field sources (mass and charge) are represented by $(+M, \pm Q)$ and the corresponding properties of objects entering the GEM fields and being subjected to forcing by these sources are represented by $(+m, \pm q)$.

Furthermore, there must be a justification for the appearance of two interdependent properties in the combined GEM field. In our view, the dual source is justified by the two ubiquitous force laws: one field source (mass) attracts like fields (masses), whereas the other field source (charge) repels like fields (charges of the same sign); naturally, the opposite occurs for unlike fields subjected to forcing. But once these types of interactions are established, the centuries-old view that they occur independently is biased, to say the least (unless one is willing to concede to the preposterous notion that force fields have "free will" and selective choices available to them).

In an unbiased interaction between fields, when a mass source sees a negative charge, it should act on it based on the property by which this charge is identified (the negative sign) and repel it; similarly, when a positive charge source sees a mass, it should repel it because this is its unbiased nature, to repel like fields. In what follows, we use consistently the sign conventions listed in Table 1, and along the way, we find some support for our assumptions and our sign choices from some well-known stationary relativistic black-hole binary equilibrium states (see part (2) in Section 4).

Table 1. Attractive $(+)$ and repulsive $(-)$ force components. Source $-Q$ (row 2) behaves as source $+M$ (row 1) and oppositely to source $+Q$ (row 3). The reaction forces obey the same scheme after interchanging the labels $M \rightleftharpoons m$ and $Q \rightleftharpoons q$, where $\pm q$ and $+m$ then are the field sources. There is no need for an equivalence principle of masses in this scheme; Newton's constant and Coulomb's constant are carried by the field sources in both the action and the reaction forces.

		Subjected to Force		
		$+m$	$-q$	$+q$
Sources	$+M$	+	−	+
	$-Q$	+	−	+
	$+Q$	−	+	−

1.2. Resultant GEM Forces and Emitted GEM or Gravitational Waves

In this work, we apply the GEM forces studied Ref. [5] to a binary consisting of two orbiting Reissner-Nordström (RN) black holes, a system that is expected to be spiraling in toward a merger due to the emission of GEM waves. For simplicity, we assume that the black holes have the same mass and absolute value of charge, but the two charges can have the same or opposite signs. In the calculations of the resultant forces, we denote initially the sources of the GEM field with capital letters M and $\pm Q$ and the properties of the of the other object subjected to forcing by lower-case letters m and $\pm q$. This distinction is necessary (Table 1): in the combined GEM field, there is no need for a principle of equivalence of masses or charges [1,5], but we do need to know which properties serve as sources of forces, so that we can affix the universal constants G (Newton's constant) and K (Coulomb's constant) to these sources.

For each object feeling the forces due to the field of the other black hole, we combine four conservative force components (with their proper signs), viz.

$$\underset{\text{(Newton's law)}}{F_{M\to m}}\,,\qquad \underset{\text{(Cross-forces)}}{F_{M\to\pm q}\,,\quad F_{\pm Q\to m}}\,,\quad \text{and}\qquad \underset{\text{(Coulomb's law)}}{F_{\pm Q\to\pm q}}\,,$$

to obtain the resultant force F. The detailed calculations are summarized in Section 2.1 below.

In this context, we derive and analyze in depth several special cases of interest (Section 2). Furthermore, we use the resultant force F in each case to also obtain an estimate of the expected amplitude h [7] of the emitted GEM waves (Section 3). We find four interesting cases of GEM or gravitational wave emission from binaries which contain RN and/or Schwarzschild black holes (Section 4).

1.3. Outline

The main body of the paper is organized as follows:

- ➢ In Section 2, we describe the four components of the GEM force, and we combine them to derive the resultant forces on to the RN black holes of the binary system.
- ➢ In Section 3, we use the resultant forces to estimate and compare the amplitudes of the GEM waves emitted from each black hole, including the purely gravitational waves emitted from a pair of Schwarzschild black holes and the GEM waves emitted from a Schwarzschild-RN pair.
- ➢ In Section 4, we discuss four possible types of GEM waves emitted from a pair of RN and/or Schwarzschild black holes, depending on the signs and the magnitudes of their charges.
- ➢ In Section 5, we summarize our conclusions and three unexpected properties which emerged from this investigation of the combined GEM field.

2. The Combined Conservative Forces Due to Both Mass and Charge

2.1. The Components of the Conservative Force Field

We consider two RN back holes separated by distance R. One of them has mass M and carries a net charge Q (the sources); the other one has mass m and charge q (subjected to forcing). Although we suppose that $m = M$ and $|q| = |Q|$, the particular choice of distinct symbols helps in distinguishing the two objects. The force field of each black hole is characterized by two sources (mass and charge) and two coupling constants (Newton's gravitational constant G and Coulomb's constant $K = (4\pi\varepsilon_0)^{-1}$ often written in terms of the vacuum permittivity ε_0 [4]).

We intend to calculate the net radial force exerted by the GEM field (M, Q) onto the other black hole (m, q) and the net reaction force vice versa. To this end, we must keep track of the attractive and repulsive components of the force. We adopt the convention that the magnitudes of attractive forces are positive and of repulsive components are negative. We also adopt the usual convention that the gravitational source only attracts masses, whereas the electromagnetic source repels/attracts charges with the same/opposite sign, respectively. We expand these force laws to include the cross-forces $M \rightleftharpoons q$ and $Q \rightleftharpoons m$ as well [5]. So, a mass source attracts/repels a positive/negative charge, respectively; and a positive/negative charge source repels/attracts a mass, respectively. This convention is dictated by the cross-force laws that have the forms $F_{M\to q} \propto Mq/R^2$ and $F_{Q\to m} \propto Qm/R^2$ [5], where a negative charge introduces a negative sign into the cross-force component. The sign conventions of the force components are summarized in Table 1.

Accounting explicitly for the signs of the force components in Table 1, and letting $m = M$ and $q = Q$ in the resulting equations, we can write the net forces in the various cases as follows:

1. Source $(M, +Q) \to (m, +q)$: $F = F_{M\to m} + F_{M\to +q} - F_{+Q\to m} - F_{+Q\to +q}$, viz.

$$F = \frac{GM^2}{R^2} + \frac{\sqrt{GK}\,MQ}{R^2} - \frac{\sqrt{GK}\,QM}{R^2} - \frac{KQ^2}{R^2}\,. \tag{1}$$

2. Source $(M, -Q) \to (m, -q)$: $F = F_{M\to m} - F_{M\to -q} + F_{-Q\to m} - F_{-Q\to -q}$, viz.

$$F = \frac{GM^2}{R^2} - \frac{\sqrt{GK}\,MQ}{R^2} + \frac{\sqrt{GK}\,QM}{R^2} - \frac{KQ^2}{R^2}\,. \tag{2}$$

3. Source $(M, +Q) \to (m, -q)$: $F = F_{M\to m} - F_{M\to -q} - F_{+Q\to m} + F_{+Q\to -q}$, viz.

$$F = \frac{GM^2}{R^2} - \frac{\sqrt{GK}\,MQ}{R^2} - \frac{\sqrt{GK}\,QM}{R^2} + \frac{KQ^2}{R^2}\,. \tag{3}$$

4. Source $(M, -Q) \to (m, +q)$: $F = F_{M\to m} + F_{M\to +q} + F_{-Q\to m} + F_{-Q\to +q}$, viz.

$$F = \frac{GM^2}{R^2} + \frac{\sqrt{GK}\,MQ}{R^2} + \frac{\sqrt{GK}\,QM}{R^2} + \frac{KQ^2}{R^2}\,. \tag{4}$$

2.2. Combining the Four Force Components

Cases 1 and 2 above can be consolidated into one conventional expression that combines Newton's gravitational law and Coulomb's law: the cross-forces cancel out, the gravitational component is attractive as always, and the Coulomb component is repulsive since the charges carry the same sign. Thus, for $Qq > 0$, we find that the two field sources are decoupled, and the net force is

$$F = \frac{GM^2 - KQ^2}{R^2} = \frac{G_\star M^2 - Q^2}{4\pi\varepsilon_0 R^2}\,. \tag{5}$$

In the last step, we have introduced the effective gravitational constant $G_\star \equiv 4\pi\varepsilon_0 G = G/K$ in place of Newton's G [1]. This form is important in that it shows how the two decoupled sources of the conservative GEM field ($\sqrt{G_\star}\,M$ and Q, as they were delineated in the classical expressions of Gauss's law [1]) combine to produce the net GEM force in cases 1 and 2, where $Qq > 0$.

Cases 3 and 4 above may also be combined into one compact expression, although the cross-forces carry opposite signs. Thus, for $Qq < 0$, we find that the net force is

$$F = \frac{\left(\sqrt{G}\,M \pm \sqrt{K}\,Q\right)^2}{R^2} = \frac{\left(\sqrt{G_\star}\,M \pm Q\right)^2}{4\pi\varepsilon_0 R^2}\,, \tag{6}$$

where the sign in the parentheses must be chosen opposite to the sign of the source charge Q. This convention also holds for the reaction forces, in which case we find that Newton's third law of action-reaction is violated in cases 3 and 4, where $Qq < 0$. The imbalance between the two oppositely-directed forces causes a small continuous movement of the center of mass of the binary system, but the effect is too weak to be observed directly. But this GEM property makes a difference in the amplitudes of the emitted GEM waves (see Section 3.2 below).

2.3. Reduced Net Forces

A RN black hole is characterized by two independent length scales corresponding to its physical properties, the field sources M and Q [8,9]: the Schwarzschild radius

$$R_S = \frac{2GM}{c^2}\,, \tag{7}$$

and the charge radius R_Q, which we define here by the equation

$$R_Q = \frac{2\sqrt{GK}\,|Q|}{c^2}\,, \tag{8}$$

where c is the speed of light. We have included a factor of 2 in equation (8) for reasons of convenience in comparisons of the two radii; for instance, the range of acceptable values of the charge radius is simply $0 \le R_Q \le R_S$. But this definition of R_Q modifies the components of the RN metric [10], viz.

$$g_{tt} = \frac{-1}{g_{rr}} = 1 - \frac{R_S}{r} + \left(\frac{R_Q}{2r}\right)^2,$$

where r is the distance from the center of the RN black hole.

Next, by substituting equations (7) and (8) into equations (5) and (6), we find for the net forces that

$$F = \frac{c^4}{4G}\left(\frac{{R_S}^2 - {R_Q}^2}{R^2}\right) \qquad (\text{Cases 1 and 2},\;\; Qq > 0)\,, \tag{9}$$

and

$$F = \frac{c^4}{4G}\left(\frac{R_S \pm R_Q}{R}\right)^2 \qquad \left(\text{Cases 3 and 4},\;\; Qq < 0,\;\; \text{with } {}^{+}_{-} \text{ if source } {}^{Q<0}_{Q>0}\right). \tag{10}$$

The leading factor of $c^4/(4G)$ admits multiple interpretations. On one hand, the particular combination of units is the Planck force [5,11], viz.

$$F_P \equiv \frac{c^4}{G} = 1.210\,307\,23 \times 10^{44}\ \text{N}\,, \tag{11}$$

which introduces the correct dimensions of [force] in the right-hand sides of equations (9) and (10).

On the other hand, the factor of 4 reduces the Planck force to the value of maximum tension force F_{max} established in general relativity [12–15], viz.

$$F_{\text{max}} = \frac{c^4}{4G} = 3.025\,768\,08 \times 10^{43}\ \text{N}\,. \tag{12}$$

The significance of the maximum tension force in the present work is that F_{max} has indeed appeared in our classical analysis of GEM forces. Up until now, it was commonly believed that a maximum force could not be obtained in classical Newtonian gravity [13], although it was found in some modified theories of gravity applied to black-hole binaries (e.g., Moffat, Brans-Dicke, and pure Lovelock theories [15]).

The 'absence' of F_{max} from Newtonian mechanics was previously attributed in part to the absence of Planck's constant h in the definition (11) of the Planck force F_P [13]. We see now that this conjecture is not correct: as soon as we considered all force components of the combined GEM field, force F_{max}

appeared naturally in the leading factors of the conservative net forces (9) and (10). In these equations, we expect that $R_S + R_Q < R$ (even when the horizons of the black holes touch), which then implies that the net forces F in binaries containing Schwarzschild and/or RN black holes also obey the condition $F < F_{\rm max}$.

In the following sections, we use the symbol h to represent both Planck's constant and the amplitude of emitted GEM waves. This choice follows standard notation for each quantity and should not lead to confusion, as the intended meaning is always clear from the context in which h appears.

2.4. Gauss's Law for GEM Fields

From an inspection of the details of the above equations (1)-(6), we can formulate Gauss's law for the combined GEM field: For a spherical surface S of radius R and normal area element $dS_k = \hat{n}_k dS$, enclosing a finite charged mass distribution (M, Q), Gauss's law for the GEM field components E_{ij} takes the matrix form

$$\text{Fluxes } \mathcal{F}_{ij} \ := \ \oiint_S (E_{ij} \cdot \hat{n}_k)\, dS \ = \ \begin{pmatrix} \mathcal{F}_{Mm} & \mathcal{F}_{Mq} \\ \mathcal{F}_{Qm} & \mathcal{F}_{Qq} \end{pmatrix} \ = \ \frac{1}{\varepsilon_0} \begin{pmatrix} G_\star M & \sqrt{G_\star} M \\ -\sqrt{G_\star} Q & -Q \end{pmatrix}, \tag{13}$$

where the signs were chosen according to the adopted convention (+ for attraction, − for repulsion); charge Q carries an intrinsic sign; mass $M > 0$; and summation over the indices i, j gives the total flux $\mathcal{F}$ of the conservative GEM field, viz. $\mathcal{F} = \sum_{ij} \mathcal{F}_{ij}$.

Integration over the spherical surface S to determine E_{ij} from the source matrix $\mathcal{F}_{ij}$, followed by multiplication of the forced vector $\binom{m}{q}$ by $E_{ij} = \mathcal{F}_{ij}/(4\pi R^2)$ from the left, results in equations (5) and (6) above. We note that q in vector $\binom{m}{q}$ also carries an intrinsic sign, which allows for coverage of the various force components without using ± symbols. But then, the resultant force equations become too compact and harder to follow. As a case in point, the various force components derived from the above fluxes $\mathcal{F}_{ij}$ in compact form are

$$\begin{pmatrix} F_{M\to m,q} \\ F_{Q\to m,q} \end{pmatrix} \ = \ \frac{1}{R^2} \begin{pmatrix} GM & \sqrt{GK} M \\ -\sqrt{GK} Q & -KQ \end{pmatrix} \begin{pmatrix} m \\ q \end{pmatrix}, \tag{14}$$

where G is Newton's constant and $K = (4\pi\varepsilon_0)^{-1}$ is Coulomb's constant; with SI values precise to 10 significant digits [5] of

$$G \ = \ 6.674\,015\,081 \times 10^{-11} \ \ \mathrm{m^3\,kg^{-1}\,s^{-2}}, \tag{15}$$

and

$$K \ = \ 8.987\,551\,786 \times 10^{+9} \ \ \mathrm{m^3\,kg\,s^{-2}\,C^{-2}}, \tag{16}$$

respectively.

3. The Emitted Gravitational Waves

3.1. Conventional Estimates

In linearized gravity, the typical amplitude h of the perturbation h_{ab} applied to the flat spacetime metric η_{ab} (i.e., the 'metric perturbation' h_{ab} with $h \equiv ||h_{ab}|| \ll 1$) is estimated to be (equation (4.38) in Ref. [7]) of the order of

$$h \ = \ \frac{G}{c^4} \frac{d^2}{dt^2} \left(\frac{I_{\rm RN}}{r} \right) \ = \ \frac{1}{F_{\rm P}} \frac{d^2}{dt^2} \left(\frac{I_{\rm RN}}{r} \right), \tag{17}$$

where t represents time, $I_{\rm RN} = MR^2$ is the moment of inertia of each orbiting RN black hole, and r is the distance of the binary system from the observer. At this early stage, we notice the appearance of the Planck force (not $F_{\rm max}$ yet) in the denominator of the right-hand side.

We assume that the mass of the black hole M and the distance r do not vary in time, in which case equation (17) takes the forms

$$h = \frac{1}{F_P}\left(\frac{M}{r}\right)\left[\frac{d^2}{dt^2}\left(R^2\right)\right] = \frac{2}{F_P}\left(\frac{R}{r}\right)\left[M\frac{d^2R}{dt^2} + \frac{M}{R}\left(\frac{dR}{dt}\right)^2\right] = \frac{2}{F_P}\left(\frac{R}{r}\right)\left[F + \frac{MV^2}{R}\right], \tag{18}$$

where the separation $R = R(t)$ is a function of time and $V(t) = dR/dt$ is the radial velocity.

Next we adopt the equality $MV^2/R = F$, and we cast equation (18) in a compact form, viz.

$$h = \left(\frac{R}{r}\right)\left(\frac{4F}{F_P}\right) = \left(\frac{R}{r}\right)\left(\frac{F}{F_{max}}\right). \tag{19}$$

The assumption $MV^2/R = F$ implies the differential equation $RR'' = (R')^2$, where primes denote derivatives with respect to time t. The particular solution that corresponds to orbit decay is $R(t) \propto \exp(-ft)$, where f is the frequency of the emitted gravitational waves. This solution also dictates that the infall speed $V \propto f$ and the radial acceleration $a \propto f^2$.

For the conventional purely gravitational waves, equations (9) and (10) with $R_Q = 0$ reduce to the form $F = F_{max}(R_S/R)^2$. Then, equation (19) produces an estimate of the expected amplitude of purely gravitational waves, viz.

$$h = \frac{{R_S}^2}{Rr}. \tag{20}$$

When the horizons of two orbiting Schwarzschild black holes touch, then $R = 2R_S$ and $h = R_S/(2r)$. As an example, for $M = 5M_\odot$ and $r = 8$ kpc, the wave amplitude is quite strong, viz. $h = 3 \times 10^{-17}$. But for a binary separation of $R = 1$ pc, equation (20) gives $h = 3 \times 10^{-29}$, a signal that is weaker by 12 orders of magnitude.

3.2. The Influence of GEM Forces

By substituting equations (9) and (10) into equation (19), we find that

$$h_Q = \frac{{R_S}^2 - {R_Q}^2}{Rr} \qquad (\text{Cases 1 and 2, } \ Qq > 0)\,, \tag{21}$$

and

$$(h_{Q\times})_\pm = \frac{\left(R_S \pm R_Q\right)^2}{Rr} \qquad \left(\text{Cases 3 and 4, } \ Qq < 0, \ \text{ with } \substack{+\\-} \text{ if source } \substack{Q<0\\Q>0}\right). \tag{22}$$

Equation (21) applies for RN black holes with charges of the same sign. Clearly, the Coulomb forces work toward diminishing the amplitude h_Q.

Equation (22) applies for RN black holes with charges of the opposite sign. Although not immediately obvious, the Coulomb and the cross-forces work toward amplifying the amplitude $h_{Q\times}$. The two options described by the $\pm$ symbol form an action-reaction pair in which the two opposite forces do not have equal magnitudes (Section 2.2). Thus, both forces are present in a RN binary, and one of the black holes (the one that is positively charged) will certainly emit waves of amplitude larger than those from its companion and those from a pure Schwarzschild binary, viz.

$$(h_{Q\times})_+ = \frac{\left(R_S + R_Q\right)^2}{Rr} \qquad (\text{where source } Q < 0 \text{ and forced emitting } q > 0)\,. \tag{23}$$

Evidently, the relative contribution of the cross-forces in the above equations depends on the ratio of length scales R_Q/R_S. Using equations (7) and (8), we determine that

$$\frac{R_Q}{R_S} = \frac{|Q|}{\sqrt{G_\star}M}. \tag{24}$$

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Thus, the ratio R_Q/R_S describes precisely the ratio of strengths of the two sources of the GEM field [1]. This identification could not have been made in the past, when the significance of $G_\star$ was not known, and the source corresponding to the gravitational component of the GEM field was thought to be the mass M or, in some cases stemming from Gauss's law for gravity [1,16], the 'standard gravitational parameter' $\mu = GM$ [17,18].

The ratio of GEM sources is equal to 1 (i.e., $R_S = R_Q$) for a charge-to-mass ratio of

$$\frac{|Q|}{M} = \sqrt{G_\star} = 8.617\,333\,262 \times 10^{-11} \text{ C kg}^{-1} . \tag{25}$$

This numerical value is indeed precise to 10 significant digits (as was determined in Ref. [5]).

3.3. A Schwarzschild-RN Black-Hole Binary

In the special case of a RN black hole paired with a Schwarzschild black hole, the same procedure as above produces the following results:

$$F = F_{\max} \left(\frac{{R_S}^2 \pm R_S R_Q}{R^2} \right) \qquad \left(\text{Cases 5 and 6 with } \begin{smallmatrix}+\\-\end{smallmatrix} \text{ if source } \begin{smallmatrix}Q<0\\Q>0\end{smallmatrix}\right) , \tag{26}$$

and

$$(h_{Q\times})_{\pm} = \frac{R_S(R_S \pm R_Q)}{Rr} \qquad \left(\text{Cases 5 and 6, with } \begin{smallmatrix}+\\-\end{smallmatrix} \text{ if source } \begin{smallmatrix}Q<0\\Q>0\end{smallmatrix}\right) . \tag{27}$$

The reaction forces of the uncharged on to the charged black hole are obtained by switching the $\pm$ signs to $\mp$.

In the above cases 5 and 6, the maximum wave amplitude is obtained for the action force $(M, -Q) \to m$ and the reaction force $m \to (M, +Q)$, viz.

$$(h_{Q\times})_{+} = \frac{R_S(R_S + R_Q)}{Rr} . \tag{28}$$

4. Multiple GEM Emission Possibilities

Equations (21) and (22) open a wide range of possibilities for the emission of GEM waves from black-hole binaries:

(1) *Purely gravitational waves from both components*.—If the charges of both RN black holes are negligible ($R_Q \ll R_S$), then both objects will emit the expected gravitational waves whose amplitude h is estimated by equation (20) above. Of course, wave interference is expected in this case, and the resultant amplitude may be as large as $2h$.

(2) *No gravitational or GEM waves from the binary system*.—Equation (21) represents the result of both the action and the reaction GEM forces in Cases 1 and 2 above; thus, the emission of GEM waves ceases in this case (with $Qq > 0$) when the special condition $R_S = R_Q$ is realized (if it can be realized). Then, the ratio $|Q|/M$ is given by equation (25).

This case is the classical analogue of the famous relativistic Majumdar-Papapetrou stationary equilibrium solution, in which the Newtonian gravitational attraction cancels exactly the Coulomb repulsion between two or more RN black holes carrying the same charge in both sign and magnitude [19–24]. The classical equilibrium is possible because the cross-forces also cancel out only for the case with $Qq > 0$. For this reason, this stationary equilibrium solution provides strong support for our assumptions concerning the actions of the cross-forces, as they were described in Sections 1 and 2 and outlined in Table 1.

(3) *GEM waves only from one component*.—Equation (22) represents the result of both the action and the reaction GEM forces in Cases 3 and 4 above (the $\pm$ signs correspond to the type of force

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exerted on to one and the other oppositely-charged black hole); thus, for $Qq < 0$ and $R_S = R_Q$, one object ($Q < 0$) is not emitting GEM waves, whereas the other object ($q > 0$) is emitting strong GEM waves whose amplitude is estimated to be $4h$, i.e., at least $2\times$ stronger than the interfering gravitational waves in part (1) above (this is despite the absence of interference and amplification of these GEM waves).

(4) *GEM waves from an uncharged component*.—Similarly to part (3), equation (28) represents the result of the action GEM force $(M, -Q) \to m$ in Case 5 above; thus, for RN source $Q < 0$ such that $R_Q = R_S$, the Schwarzschild black hole of mass m $(= M)$ is emitting GEM waves of amplitude $2h$ and the RN black hole is not emitting waves. Therefore, such GEM waves may be hard to distinguish from the interfering gravitational waves of part (1) above.

The stronger GEM waves with amplitudes of $4h$ were obtained in part (3) above. But at this point, it is not known whether the special scaling $R_Q = R_S$ can occur in nature (or, more generally, $R_Q \lesssim R_S$, which makes RN black holes interesting as the two field sources have roughly comparable strengths). For instance, equation (25) implies that, for $R_Q = R_S$, a $5M_\odot$ RN black hole must have a charge excess or deficit of $\sim 10^{16}$ moles of electrons[2,3,4] (corresponding to an ionization fraction of only $\sim 10^{-18}$ since a $5M_\odot$ mass corresponds to 10^{34} moles of hydrogen atoms).

Nevertheless, whether such a large concentration of charge ($|Q| \sim 10^{21}$ C) can be achieved by stellar black holes is a contemporary mystery. For more details on the theoretically expected range of charges in RN black holes and the perceived upper limit of $R_Q = R_S$ for the so-called 'extremal black holes' with naked singularities, see Refs. [25–27], the online summaries [28,29] and the recent research results in Refs. [30,31].

5. Summary, Discussion, and Conclusions

5.1. Summary

In this work, we have calculated the resultant GEM forces on to a RN black-hole binary [25,26]. These forces are generated by a combined conservative field that includes source terms due to both mass and charge (Section 2). For simplicity, we have assumed that the objects carry the same mass and the same and absolute value of charge. We have also included the special case of GEM forces between a Schwarzschild-RN black-hole pair (Section 3.3).

Based on past experience with Newton's gravitational law and Coulomb's law, we have also assumed that the mass source M of one black hole attracts the positive properties m and $+q$ of the other component, but M generates a repulsive cross-force on to a charge $-q$ of the other component. Similarly, a charge source $+Q$ repels $+q$ and m but attracts $-q$ of the other black hole; and a charge source $-Q$ attracts both $+q$ and m. The signs of the various components of the action and the reaction forces are listed in Table 1.

We used the resultant forces to estimate the amplitude of GEM waves emitted by such binaries [7], as the black holes are spiraling in toward one another (Section 3). We described four cases of wave emission depending on the signs and the values of the two charge properties (Section 4). In the critical case of comparable RN source strengths (i.e., $|Q| = \sqrt{G_\star}M$ or $R_Q = R_S$, see equations (7) and (8)), the largest GEM amplitude occurs in the case of two oppositely-charged RN black holes (equation (23)). The wave is emitted from the positively-charged black hole, and its amplitude is $4\times$ that of the purely gravitational waves emitted from a single Schwarzschild black-hole (equation (20)). On the other hand, for $R_Q = R_S$ and $Qq > 0$, no GEM waves are emitted because the black holes are in stationary equilibrium [19–22].

5.2. Unexpected Results

The investigation of the resultant forces from the combined GEM field has revealed two entirely unexpected properties of the combined conservative GEM field:

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(a) The ubiquitous action-reaction principle (Newton's third law of motion) is only valid for a Schwarzschild binary ($Q = q = 0$) and for a RN binary with $Qq > 0$. The magnitudes of the two oppositely-directed forces are not equal in the RN case with $Qq < 0$ (Section 2.2) and in the Schwarzschild-RN case with only one of the two charges being equal to zero (Section 3.3).

(b) The maximum force $F_{\rm max} = F_{\rm P}/4$ (where $F_{\rm P}$ is the Planck force [11]) determined exclusively in general relativistic theories [12–15] and believed to be absent from Newtonian gravity has appeared naturally in the calculations of the resultant GEM forces F (equations (9)-(12)), and it is clear that these forces obey the condition $F < F_{\rm max}$ (Section 2.3).

Furthermore, our calculations in an example given at the bottom of Section 4 concerning the net charge and the total mass of 1 mole of electrons have produced one old well-known result and one new astonishing result (the details are given in Notes 1-4), respectively:

(1) *Faraday's constant* $C_{\rm Far}$.—As expected, the net charge of 1 mole of electrons is equal to $C_{\rm Far} \simeq 96$ kC.

(2) *Planck's constant* h.—Quite unexpectedly, the total mass of 1 mole of electrons is equal to 10 Planck masses. Solved for h, this relation determines Planck's constant from a subset of non-quantum mechanical constants. We repeat it here for the reader's convenience:

$$h = \frac{G m_{\rm e}^2}{c} \left(\frac{N_{\rm A}}{10}\right)^2 , \tag{29}$$

where G is Newton's gravitational constant, c is the speed of light, $N_{\rm A}$ is Avogadro's number, and $m_{\rm e}$ is the mass of the electron. The result is precise to 9 significant digits (Note 4), provided that our adjusted values of G (equation(15)) and $N_{\rm A}$ (Note 3) are adopted; otherwise, using only CODATA values [4], the result is higher by 1%.

5.3. The Two Independent Coupling Constants of the GEM Field

Equation (29) dispels the centuries-old belief that Planck's constant belongs exclusively to quantum mechanics, thus h cannot possibly appear in classical physics. Furthermore, equation (29) reveals yet another example of the same universal constant appearing in different physical contexts, viz.

$$\left(\frac{N_{\rm A}}{10}\right)^2 = \frac{1}{\alpha_{\rm g}} , \tag{30}$$

where $\alpha_{\rm g}$ is the gravitational coupling constant defined by $\alpha_{\rm g} \equiv G{m_{\rm e}}^2/(hc)$. In these contexts (ideal gases and gravity), the constants are dimensionless, thus there is no need for adjustment of units.

On the other hand, the EM coupling constant of the combined GEM field, the fine-structure constant α cannot be tied to $N_{\rm A}$ exclusively, thus it remains an independent entity in the GEM field, unlike the weak coupling constant $\alpha_{\rm w} = \sqrt{\alpha}$ [5] in a combined GEW field (where EW denotes the electroweak field). For the fine-structure constant, we find the relation

$$\alpha = \frac{e^2}{G_\star m_{\rm e}^2} \left(\frac{N_{\rm A}}{10}\right)^{-2} , \tag{31}$$

in which the interrelation between the EM source and the gravitational source is now evident in the ratio $e/(G_\star m_{\rm e})$, although we also see a dependence of α on the charge-to-mass ratio $(e/m_{\rm e})$.

5.4. Ramifications and Conclusions

5.4.1. Classical Constants Independent of h and e

Equation (29) implies that many physical constants have a classical origin because the universal constants that depend on Planck's h or $\hbar$ (presently perceived to have a quantum mechanical nature) can be redefined in terms of well-known classical constants. Running down the list of physical constants given in Ref. [32], we classify the following universal constants [4] (that do not depend on the elementary charge e) as having a classical origin:

(1) Planck's constant h ($\hbar$ is not included—we have argued for its retirement from science [1,33]).
(2) Boltzmann's constant k_B (entropy is not a quantum property).
(3) Stefan-Boltzmann constant $\sigma = {k_B}^4/(h^3c^2)$ (defined here in terms of c, k_B, and h, not $\hbar$).
(4) First radiation constants $c_1 = 2\pi hc^2$ and, for spectral radiance, $c_{1L} = 2hc^2$ per steradian.
(5) Second radiation constant $c_2 = hc/k_B$.
(6) Molar Planck constant $N_A h$.
(7) Quantum of circulation $h/(2m_e)$.
(8) Compton radius $r_c = h/(m_e c)$.

We have not included in this list the vacuum constants ($\epsilon_0, \mu_0, c, Z_0$) or the known classical constants (G, K, N_A, and the molar gas constant $N_A k_B$), or e-dependent constants, such as the conductance quantum $2e^2/h$, the von Klitzing constant h/e^2, the Josephson constant $2e/h$, the magnetic flux quantum $h/(2e)$, the fine-structure constant α, the gravitational coupling constant α_g, the Bohr and nuclear magnetons μ_B and μ_N, the classical electron radius r_e, the Bohr radius r_B, the Thomson cross-section σ_e, the Rydberg constant R_∞, and the Faraday constant C_{Far} [4].

5.4.2. The Compton Radius r_c

In the above list, the appearance of the Compton radius r_c is especially interesting because its classical foundation differentiates this constant from the other two atomic radii, r_e and r_B. In fact, r_c is the geometric mean (G-M) of r_e and r_B, as well as the G-M of all three radii [33]. The Bohr radius and the classical electron radius are then produced by scaling r_c by α^{-1} and α, respectively.

The G-Ms relate the three atomic radii, and r_c is the only radius that does not depend on the charge of the electron. Using equation (29), we can redefine the Compton radius by the classical relation

$$r_c = \frac{Gm_e}{c^2}\left(\frac{N_A}{10}\right)^2 . \tag{32}$$

5.4.3. Unit Conventions Behind the Factor of $(N_A/10)^2$ in Equations (29)-(32)

Equation (30) reveals the origin of the factor of $(N_A/10)^2$: it shows that Avogadro's number determines the gravitational coupling constant, viz. $\alpha_g \propto 1/N_A^2$. But unlike the natural constant N_A, the gravitational coupling constant is a man-made quantity—and the factor of 10^2 comes in to provide the appropriate scaling for the value of our so-defined α_g.

Properly scaled, Avogadro's number also determines the (man-made) Planck mass M_P, viz.

$$M_P = n\, m_e\,, \tag{33}$$

where the 'reduced Avogadro number' is defined by

$$n \equiv N_A/10\,, \tag{34}$$

thus leading to a succinct determination of α_g as well, viz.

$$\alpha_g = 1/n^2\,. \tag{35}$$

In this sequence of events, the reduced Avogadro number n (not Planck's h) is the fundamental natural constant, and it produces our subjective (man-made) constants of α_g (all by itself) and M_P (via the electron mass).[5] In the next step, the classical Planck mass $M_P = n m_e$ determines the remaining physical constants, so that equations (29), (31), and (32) produce the following quantities:

$$h = \frac{G{M_P}^2}{c}, \tag{36}$$

$$\alpha = \frac{Ke^2}{G{M_P}^2}, \tag{37}$$

and

$$r_c = \frac{G{M_P}^2}{m_e c^2}, \tag{38}$$

respectively.

The constants α_g and M_P thus defined by equations (33)-(35) simplify a large number of physical quantities (dimensionless and dimensional), and allow for unequivocal interpretations. We summarize here six examples of general interest:

① The normalized gravitational coupling constant $\beta_g = \alpha_g/\alpha$ [5,33] measures the relative strength of gravitational coupling against the measurable by experiment fine-structure constant; thus, β_g must be considered as a fundamental dimensionless unit in any system of units. From equations (30) and (31), we find that β_g is independent of Avogadro's number, as is indeed seen in its formal definition [5]

$$\beta_g \equiv G_\star \left(\frac{m_e}{e}\right)^2. \tag{39}$$

② The Bekenstein-Hawking formula for the entropy of a black hole of mass M_{BH} [36–38] is $S_{BH} = k_B A/(2L_P)^2$, where A is the area of its event horizon and L_P is the Planck length [11]. For a Schwarzschild black hole, we set its horizon area to $A = 4\pi {R_S}^2$, and we also define the Planck length in terms of h, not $\hbar$ (Table 2); then, the Bekenstein-Hawking formula takes the concise form

$$S_{BH} = 4\pi k_B \left(\frac{M_{BH}}{M_P}\right)^2. \tag{40}$$

The factor of 4π (the imprint of the three-dimensional vacuum [1,33]) has emerged in this equation to denote that S_{BH} is the total entropy enclosed within the volume of the black hole.

③ The Bekenstein bound for the maximum entropy of a body of mass M, radius R, and rest-energy E [39–43] is $S_{max} = k_B(2\pi R)E/(hc)$. Written in this form, the equation gives a misleading signal because $(2\pi R)$ is a two-dimensional quantity; although it reduces to equation (40) for a black hole with $R = R_S$ and $E = c^2 M_{BH}$. The apparent geometric issue is resolved when S_{max} is reformulated in terms of the Planck mass: using equation (36) to eliminate h, we find that

$$S_{max} = 4\pi k_B \left(\frac{R}{R_S}\right)\left(\frac{M}{M_P}\right)\left(\frac{E}{E_P}\right), \tag{41}$$

where $E_P = c^2 M_P$ is the Planck energy. The appearance of the comparative ratio R/R_S points to the fundamental nature of the Schwarzschild radius R_S [41–43] (in contrast to the man-made Planck length L_P), including the natural (i.e., not man-made) factor of 2 that appears in equation (7) above: introducing the ratio R/L_P in equation (41) leads to a simpler formula, viz.

$$S_{max} = 2\pi k_B \left(\frac{R}{L_P}\right)\left(\frac{E}{E_P}\right), \tag{42}$$

which, however, displays the apparent 2π geometric issue previously discussed, arising from the subjective definition of the Planck length.

④ The thermal Hawking temperature of a black hole (also called Hawking-Unruh or Davies-Unruh temperature in related contexts) [44–47] is defined here as $\Theta_{BH} = ha/(k_B c)$, where a denotes acceleration. As usual, this definition is given in terms of h (not $\hbar$), but it is also devoid of a man-made factor of $4\pi^2$.[6] For a Schwarzschild black hole of mass M_{BH} and surface acceleration of $a = GM_{BH}/R_S^2 = F_{max}/M_{BH} = c^4/(4GM_{BH})$ on the horizon, we find a concise formula for Θ_{BH}, viz.

$$\frac{\Theta_{BH}}{\Theta_P} = \frac{1}{4}\left(\frac{M_{BH}}{M_P}\right)^{-1}, \quad (43)$$

where Θ_P is the Planck temperature (Table 2). The factor of 1/4 stems from the maximum force $F_{max} = c^4/(4G)$ (equation (12) above), which is realized on the horizon $R = R_S$ of the Schwarzschild black hole, where the acceleration $a = F_{max}/M_{BH}$.

⑤ A new deeper interpretation of Heisenberg's position-momentum (x, p_x) uncertainty principle [48–50] emerges from equation (36) and the relations listed in Table 3:

Table 2. Original Planck units formulated in terms of $\{G,\, m_e,\, c,\, k_B,\, K\}$ and scaled by $n \equiv N_A/10$ via the Planck mass $M_P = n\, m_e$.

Unit	Symbol	Planck Definition	Reformulation
Mass	M_P	$M_P = \sqrt{h\,c/G}$	$M_P = n\,m_e$
Length	L_P	$L_P = \sqrt{h\,G/c^3}$	$L_P = M_P\,G/c^2$
Time	T_P	$T_P = \sqrt{h\,G/c^5}$	$T_P = M_P\,G/c^3$
Temperature	Θ_P	$\Theta_P = \sqrt{h\,c^5/G}/k_B$	$\Theta_P = M_P\,c^2/k_B$
Force	F_P	$F_P = c^4/G$	$F_P = M_P\,c^2/L_P$
Pressure	P_P	$P_P = c^7/(h\,G^2)$	$P_P = M_P\,c^2/{L_P}^3$
Acceleration	a_P	$a_P = \sqrt{c^7/(h\,G)}$	$a_P = c^2/L_P$

Table 3. EM Planck units formulated in terms of $\{G,\, m_e,\, c,\, k_B,\, K\}$ and some of them scaled by $n \equiv N_A/10$ via the Planck mass $M_P = n\,m_e$. The EM units are simplified considerably by the introduction of the effective gravitational constants $G_\star = G/K = 4\pi\varepsilon_0\, G$ and $G_B = GK/c^2 = G\,\mu_0/(4\pi)$, and the impedance of free space $Z_0/(4\pi) = K/c$.

Unit	Symbol	Planck Definition	Reformulation
Charge	Q_P	$Q_P = \sqrt{h\,c/K}$	$Q_P = M_P\sqrt{G_\star}$
Magnetic Flux	Φ_P	$\Phi_P = \sqrt{K\,h/c}$	$\Phi_P = M_P\sqrt{G_B}$
Voltage	$\mathcal{V}_P$	$\mathcal{V}_P = \sqrt{K\,c^4/G}$	$\mathcal{V}_P = c^2/\sqrt{G_\star}$
Electric Current	$\mathcal{I}_P$	$\mathcal{I}_P = \sqrt{c^6/(GK)}$	$\mathcal{I}_P = c^2/\sqrt{G_B}$
Electric Resistance	$\mathcal{R}_P$	$\mathcal{R}_P = K/c$	$\mathcal{R}_P = Z_0/(4\pi)$
Capacitance	$\mathcal{C}_P$	$\mathcal{C}_P = \sqrt{h\,G/(K^2\,c^3)}$	$\mathcal{C}_P = G_\star\,M_P/c^2$
Inductance	$\mathcal{L}_P$	$\mathcal{L}_P = \sqrt{h\,G\,K^2/c^7}$	$\mathcal{L}_P = G_B\,M_P/c^2$

Note: The reformulated units produce a remarkably simple expression for Planck's constant h; using $M_P = n m_e$ (Table 2), equation (29) is recast as $h = G_\star {M_P}^2\, \mathcal{R}_P$, where the interrelation between gravity and the vacuum constants in formulating the classical form of h becomes apparent (see also Section 5.4.3.⑤ below).

- Written in the standard form $\Delta x\, \Delta p_x \geq \hbar/2$, the inequality is misleading: Dirac's $\hbar$ is a two-dimensional (2-D) constant, whereas the standard deviations $(\Delta x, \Delta p_x)$ are one-dimensional (1-D) uncertainties. This issue about $\hbar$ was exposed and explored in Ref. [33] for the first time. However, there is another issue that has not been discussed until now: the lower bound of the inequality depends only on Planck's h, and there is no justification for such a presumption because h is not a lower limit in nature (very much like the vacuum's impedance Z_0 and MOND's critical acceleration a_0 that are simply thresholds).
- Written in terms of Planck's h, the standard form $\Delta x\, \Delta p_x \geq h/(4\pi)$ shows a 3-D vacuum tag of 4π, a signature that the 1-D motion unfolds within three-dimensional (3-D) space. But,

for as long as h is believed to be a fundamental constant of nature, the question about the physical significance of the lower bound $h/(4\pi)$ cannot be answered.

- The lower bound in Heisenberg's inequality, viz. $h/(4\pi)$, is now easily understood in two different, fully consistent, instructive ways:
  (a) We consider the uncertainty principle written in terms of h as given in the note to Table 3, so that h is derived from classical constants, viz.

$$h \;=\; G_\star M_P{}^2\, \mathcal{R}_P\,. \tag{44}$$

  Here, h is indeed a lower limit; its minimum value comes from the permittivity ϵ_0 embedded in $G_\star$; whereas $\mathcal{R}_P \propto Z_0$ is an imprint of the impedance of free 3-D space, significant for the motions of charged particles in any number of dimensions.
  (b) Reverting to equation (36), viz. $h = GM_P{}^2/c$, we arrive at the same conclusion since $1/c$ is a lower limit in nature and G and $M_P = nm_e$ are natural constants.

  We note however that, in stark contrast to Planck's h, equation (37), viz. $\alpha = e^2/(G_\star M_P{}^2)$, indicates that the fine-structure constant α attains a maximum value due to the vacuum permittivity ϵ_0 embedded in $G_\star$ that appears in the denominator of the comparative ratio.

⑥ The Casimir force per unit area between two parallel conducting plates [51] has occupied many physicists over the past 80 years. Its magnitude was determined by several different methods (e.g., [51–55]), and it was confirmed experimentally to $\lesssim 1\%$ accuracy (e.g., [56–58], and references therein). The Casimir effect was originally thought to be a quantum effect that originates from vacuum energy fluctuations and provides proof that zero-point energies in quantum-field ground states are real. These notions were conclusively refuted [55,59,60], except for the quantum nature of the effect ($\hbar$ is present in the equations). In our times, the Casimir force is believed to be the relativistic analogue of the classical van der Waals force in which retardation effects are taken into account [55,59–64], and it is produced by the matter-EM interaction term in the QED Hamiltonian [59].

Here, we revisit the Casimir effect in light of our results:

- Equations (29), (33), (34), and (36) highlight the classical origin of Planck's constant, thereby dispelling the notion that the nature of the Casimir force lies in quantum mechanics. Thus, this force is the long-known van der Waals force [61,65] corrected to account for the finite speed of light.
- Another issue concerns the appearance of geometric terms in the equations for the Casimir effect. The full treatment of the effect shows π-dependent coefficients introduced by counting the density of states along the surface of the plates, which does not raise any questions. Written in terms of the quantities of Table 2, the Casimir pressure P_c is

$$P_c \;=\; -\,\frac{\pi}{480}\,\frac{GM_P{}^2}{d^4} \;=\; -\,\frac{\pi}{480}\,P_P\left(\frac{L_P}{d}\right)^4, \tag{45}$$

  where d is the distance between the flat, parallel, perfectly conducting plates.
- The simplified 1-D scalar analogue of the effect [54,55] should not contain any geometric terms, which is indeed the case when the 1-D Casimir force F_c is expressed in terms of the quantities of Table 2 (but not when expressed in terms of $\hbar$), viz.

$$F_c \;=\; -\,\frac{1}{48}\,\frac{GM_P{}^2}{d^2} \;=\; -\,\frac{F_{max}}{12}\left(\frac{L_P}{d}\right)^2, \tag{46}$$

where $F_{\rm max}$ is given by equation (12) above.

5.4.4. Reformulated Planck Units

Equation (29) permits a reformulation of the famous Planck system of units [11] in terms of the constants $\{G,\, m_{\rm e},\, c,\, k_{\rm B},\, K\}$, in which the ratio $n \equiv N_{\rm A}/10$ appears as a dimensionless scale factor. The original Planck units are listed in Table 2, and the extended electromagnetic (EM) units are listed in Table 3.

The appearance of the two effective gravitational constants $G_\star$ and G_B and their G-M relation $\sqrt{G_B/G_\star} = Z_0/(4\pi) = \mathcal{R}_{\rm P}$ in the EM Planck units is notable. The G-M, in particular, points to the well-known relation $\mathcal{R}_{\rm P} = \mathcal{V}_{\rm P}/\mathcal{I}_{\rm P}$ (Ohm's law), as well as to its magnetic analogue $\mathcal{R}_{\rm P} = \Phi_{\rm P}/Q_{\rm P}$.

An alternative way of looking at the G-M relation $\sqrt{G_B/G_\star}$ is also deduced from Table 3, viz. $\sqrt{G_B/G_\star} = K/c$. The other G-M of the constants $G_\star$ and G_B is also interesting, as it produces an analogous combination of units with G in place of K, viz. $\sqrt{G_B\, G_\star} = G/c$.

Furthermore, the multiplication of these two G-Ms produces two compelling definitions of the coupling constant $\sqrt{GK}$ of the new cross-forces shown in equations (1)-(4) of Section 2.1, viz.

$$\sqrt{GK} \; = c\sqrt{G_B} \; = K\sqrt{G_\star}\,, \tag{47}$$

where

$$G_B \;\equiv\; \left(\frac{\mu_0}{4\pi}\right) G\,, \tag{48}$$

and $G_\star \equiv (4\pi\epsilon_0)\, G$, respectively.

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Abbreviations

The following abbreviations are used in this manuscript:

CODATA	Committee On Data
EM	ElectroMagnetic
GEM	GravElectroMagnetic
GEW	GravElectroWeak
G-M	Geometric Mean
QED	Quantum ElectroDynamics
RN	Reissner-Nordström
SI	Système International d'unités
1-D, 2-D, etc.	one-dimensional, two-dimensional, etc.

Notes

1 The value of $G = 6.674\,015\,081 \times 10^{-11}$ m^3 kg^{-1} s^{-2} was obtained to much higher precision than in modern experiments [2–4] by observing that the numerical value of the square root of the effective gravitational constant $G_\star = 4\pi\varepsilon_0 G$ in SI units is identical to the numerical value of Boltzmann's constant k_B measured in MeV K^{-1} [4], viz. $\mathcal{N}(\sqrt{G_\star}) = \mathcal{N}(k_B)$, where the function $\mathcal{N}(\cdot)$ denotes that units are set aside [1,5]. Furthermore, when k_B is measured in MJ K^{-1}, then $\mathcal{N}(\sqrt{e^2 G_\star}) = \mathcal{N}(k_B)$, where e is the elementary charge. Such numerical identities are not coincidental, but rather indications that nature uses constants of the same strength in different physical contexts. In Note 2 below, we report another such numerical identity that we discovered in the course of this work.

2 One mole of electrons contains N_A electrons, where $N_A = 6.022\,140\,76 \times 10^{23}$ particles per mole ('Avogadro constant' [4]). Although not directly related to the numerical estimates at the bottom of Section 4, it is natural to also calculate the mass and the charge of one mole of electrons each with mass m_e and charge $-e$. Using CODATA values [4], we determine that $N_A m_e = 10 M_p = 5.455\,628\,31 \times 10^{-7}$ kg, where M_P is the Planck mass defined in terms of Planck's constant h (not $\hbar$) [1,5]; and that $N_A|e| = C_{Far} = 9.648\,533\,21 \times 10^4$ C, where C_{Far} is Faraday's constant for 1 mole of protons [4].

3 The minute discrepancy in the third significant digit of the values in equality $N_A m_e = 10 M_p$ of Note 2 poses a serious problem for the SI system of units: the universal constants h and c used to define the Planck mass as well as Avogadro's number N_A and Faraday's constant have all been defined to be "exact" to 9 significant digits (and we have determined G to 10 significant digits); thus, one of the exact constants must be redefined and its value should be changed slightly. As an example, we choose to rework N_A starting from the above identity, and we find the following updated numerical values: $N_A = 5.989\,020\,20 \times 10^{23}$ particles per mole and $N_A|e| = C_{Far} = 9.595\,468\,23 \times 10^4$ C per mole. Compared to the current CODATA values [4], these updated values are both lower by 0.55%.

4 The identity $N_A m_e = 10 M_p$ is ground-breaking and may amend physics as we know it: substituting $M_P = \sqrt{hc/G}$ and solving for h, we find the astonishing relation

$$h = \frac{G m_e^2}{c} \left(\frac{N_A}{10}\right)^2 ,$$

that determines Planck's constant from well-known classical (non-quantum mechanical) constants. Using current CODATA values for c and m_e [4], as well as the new values of G (Note 1) and N_A (Note 3), this equation determines Planck's constant $h = 6.626\,070\,15 \times 10^{-34}$ J Hz^{-1}. On the other hand, the same equation with CODATA values for all constants on the right-hand side gives a value of h that is higher by 1%.

5 We define the reduced Avogadro number $n := N_A/10$, a subjective (man-made) quantity so as not to upset the metric foundation of physics currently relying on 'our ten fingers' (i.e., powers of 10, as they were conceived and disseminated to future generations by the great Archimedes of Syracuse [34] in his ground-breaking work '*The Sand Reckoner*' [35]), and on the SI system of units for subjective (man-made) modern-day measurements. That would be the case, had we required for α_g and M_P to attain new (albeit natural) values via the properly reformulated *natural definitions* of $\widehat{\alpha}_g := 1/N_A^2$ and $\widehat{M}_P := N_A m_e$, respectively.

6 The original definition of the Hawking temperature $\Theta_{BH} = \hbar a/(2\pi k_B c)$ [44,47] contains a geometric scaling of $\hbar/(2\pi) \propto 1/(4\pi^2)$ that does not make sense because all physical quantities involved are intrinsically three-dimensional in nature, thus they need no geometric imprint of any dimensionality to be inserted by the vacuum. In the original definition of Θ_{BH}, one 2π comes from Dirac's miscue concerning $\hbar$ [33], and the other 2π comes from the treatment of plane waves with angular frequency of $\omega = 2\pi f$ [44,47]. When these two 2π terms are discarded, then equation (43) is derived, as was done in part ④ of Section 5.4.3.

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