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Quantization from First Principles: The Structural Origin of $\hbar$ and Spin from Phase Topology

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Abstract

Planck's constant $\hbar$ has long been introduced axiomatically in quantum mechanics, with its numerical value fixed empirically. Yet its *structural necessity* and intrinsic relation to the topology of phase space remain unresolved. In this work we develop a phase-ontological framework—the Ω-model—in which **monodromy** provides the fundamental mechanism of quantization. The structural chain is established as Topology $\longrightarrow$ Monodromy $\longrightarrow$ Quantization Rules $\longrightarrow$ $\hbar$ as universal coherence threshold. Within this perspective, $\hbar$ is not derived as a numerical constant but identified as a universal unit of minimal coherent action that calibrates the model's internal parameters. This formulation yields a unified topological explanation for (i) the inevitability of quantized action and (ii) the dichotomy of bosonic versus fermionic statistics, interpreted as trivial and nontrivial monodromy, respectively. Consistency of the coherence criterion with foundational quantum experiments and even recent high-energy anomalies demonstrates that the Ω-model offers a coherent bridge between topology, phase coherence, and the structural foundations of quantum theory.

Keywords: Planck constant $\hbar$; monodromy; geometric quantization; phase topology; Ω-model; spin; coherence threshold; structural invariants

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1. Introduction

Planck's constant $\hbar$ has remained one of the most fundamental, yet axiomatically introduced, elements of quantum mechanics for more than a century. Since its appearance in the early works of Planck and Bohr, $\hbar$ has served as the defining scale separating the quantum from the classical domain. Its numerical value is known with extraordinary precision, but the *structural necessity* of $\hbar$—why such a constant must exist at all—remains unexplained within the standard framework. Equally unresolved is the simultaneous emergence of two distinct quantum statistics, bosonic and fermionic, and their intrinsic connection to the quantization of action.

Historically, several major approaches have sought to formalize the role of $\hbar$ in physical theory. In the *canonical quantization* program, initiated by Dirac [1,2], $\hbar$ appears axiomatically in the canonical commutation relations,

$$[x, p] = i\hbar,$$

which serve as the bridge between classical Hamiltonian mechanics and quantum theory. In the language of symplectic geometry, $\hbar$ sets the scale for the phase-space cell and thus enforces discreteness. Yet it enters the theory without deeper justification for its universality.

In *geometric quantization*, developed by Kostant [3], Souriau [4], and Woodhouse [5], $\hbar$ arises as the unit of flux required for the integrality condition of the symplectic form, ensuring the existence of a prequantum line bundle. While mathematically elegant, this construction still presupposes $\hbar$ as an external parameter rather than deriving it from the topology of the underlying structure.

The theory of *geometric phases*, initiated by Berry [6] and formalized by Simon [7] and Bohm et al. [8], revealed the holonomic and topological aspects of quantum evolution. Yet, even in this framework, $\hbar$ appears as a background scale, not as a structural necessity. A similar limitation applies to path integral formulations [9], where $\hbar$ enters only as the weight factor in the Feynman amplitude.

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Beyond these formalisms, emergent paradigms suggest that fundamental constants and even spacetime itself may arise from collective or topological phenomena. Sakharov's induced gravity paradigm interprets General Relativity as an effective description emerging from vacuum fluctuations [10]. Topological models of particles as solitons or defects were pioneered by Skyrme [11] and 't Hooft [12], establishing the idea that stability can be of topological origin. More recent approaches extend these ideas: for example, emergent gravity from stochastic flows in topological quantum field theory [13], or effective actions for domain wall dynamics [14].

Despite the depth and elegance of these perspectives, no existing framework offers a unified principle simultaneously explaining: (i) the necessity of quantization, (ii) the dichotomy of bosonic and fermionic statistics, and (iii) the structural role of $\hbar$.

In this work, we propose such a unifying mechanism within the Ω-*model*: a phase-ontological framework in which these features emerge as consequences of the *monodromy* of a universal phase field,

$$\Psi(x) = \rho(x)e^{i\Theta(x)}.$$

Unlike the quantum-mechanical wavefunction, which represents states relative to a fixed spacetime, here Ψ is introduced as a fundamental ontological field: the universal substrate from which spacetime, particles, and interactions themselves emerge. Its phase $\Theta(x)$ provides the structural degree of freedom, while its amplitude $\rho(x)$ measures the density of distinction. The term "monodromy" here refers to the way in which the phase Θ accumulates when Ψ is transported around a non-contractible loop in configuration space—capturing the topological essence of quantization.

Formally, the structural sequence can be expressed as topology $\longrightarrow$ monodromy $\longrightarrow$ quantization rules $\longrightarrow$ $\hbar$ as coherence threshold.

Bosonic and fermionic statistics appear naturally as manifestations of trivial and nontrivial monodromy, respectively. This observation foreshadows a deeper relation to Spin^c structures, which provide the natural topological framework for half-integer quantization.

In this sense, $\hbar$ is not merely a numerical parameter but a *structural invariant*, comparable in universality to π: just as π encodes the geometry of circles independent of measurement, $\hbar$ encodes the minimal action required to distinguish coherent from incoherent phase configurations. The coherence condition

$$\delta S \geq \hbar,$$

thus plays a role analogous to—but more fundamental than—the Heisenberg uncertainty principle: whereas the latter constrains simultaneous measurements, the former establishes the very boundary of physical distinguishability.

Our approach thereby reinterprets Sakharov's paradigm: the same phase field whose topological configurations generate particles also induces spacetime and interactions, with the entire structure governed by a universal coherence principle. This principle, quantified by the inequality above, will be derived and justified in Section 2.

2. The Ω-Model: A Phase-Ontological Framework

The structural origin of quantization can only be understood within a suitable ontological framework. The Ω-model is founded on a principle of radical emergence: physical reality is not composed of fundamental particles or fields in a pre-existing spacetime, but is instead a manifestation of a single, universal entity — the *phase field*. This idea continues the line of thought initiated by Sakharov's induced gravity [10], Skyrme's solitonic models [11], and 't Hooft's topological defects [12], while also resonating with renormalization group approaches of Wilson [15] and Polchinski [16]. The present framework seeks to unify these insights into a single coherent postulate.

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2.1. The Primordial Substrate: The Universal Phase Field

Definition 1 (Configuration Space of the Universal Phase Field). *The universal phase field is a map*

$$\Psi : M_4 \longrightarrow \mathbb{C}, \qquad \Psi(x) = \rho(x)\, e^{i\Theta(x)},$$

where $\rho(x) \geq 0$ *and* $\Theta(x) \in S^1$. *The set of all such configurations defines the configuration space*

$$\mathscr{C}_\Omega = \{\Psi \mid \rho(x) \geq 0,\ \Theta(x) \in S^1,\ x \in M_4\}.$$

The ontological roles of the components are distinct:

- **Phase** $\Theta(x)$: encodes relations and structure. Its gradients $\partial_\mu \Theta$ seed emergent geometry and dynamics.
- **Amplitude** $\rho(x)$: represents the *density of distinction*, i.e. the capacity of a region to sustain coherent, distinguishable structure.

Remark 1 (Physical Interpretation of $\rho(x)$). *Operationally,* $\rho(x)$ *may be interpreted as proportional to the effective energy density of the phase field. Regions with* $\rho(x) = 0$ *correspond to a "void state," devoid of distinguishable events, while* $\rho(x) > 0$ *indicates the presence of a substrate capable of supporting physical reality.*

2.2. The Criterion of Distinguishability: The Ω-Postulate

Definition 2 (Ω-Postulate of Distinguishability). *A configuration* $\Psi \in \mathscr{C}_\Omega$ *is* physically real *if and only if its action variation satisfies the coherence threshold*

$$\delta S \ \geq \ \hbar. \tag{1}$$

Here δS denotes the minimal difference in action between two infinitesimally close configurations of Ψ.

Remark 2 (Action Functional). *For* $\Psi \in \mathscr{C}_\Omega$*, the effective action is given by*

$$S[\Psi] = \int_{M_4} \mathcal{L}[\Psi, \partial_\mu \Psi]\, d^4x, \qquad \mathcal{L} = \partial_\mu \Psi^* \partial^\mu \Psi - V(\Psi^* \Psi).$$

At this stage, the precise microscopic form of $V(\Psi^*\Psi)$ *remains unspecified; its role is to ensure stability of localized configurations, analogous to solitonic models [11].*

Remark 3 (Ontological Status of the Threshold). *Unlike in canonical quantization, where* $\hbar$ *is inserted axiomatically [1,2], here the inequality*

$$\delta S \geq \hbar$$

is not a quantization rule but an ontological filter of reality. *It establishes* $\hbar$ *as a structural invariant, analogous to* π*, marking the boundary between distinguishable and indistinguishable phase configurations.*

2.3. Emergent Geometry and Matter

The coherence threshold induces a structural cascade:

$$\Psi \quad \longrightarrow \quad \delta S \geq \hbar \quad \longrightarrow \quad \text{Physical Reality.}$$

Definition 3 (Emergent Metric). *The spacetime metric arises as the statistical correlation of phase gradients:*

$$g_{\mu\nu}(x) \ \sim \ C\, \langle \partial_\mu \Theta(x)\, \partial_\nu \Theta(x) \rangle,$$

where C is a proportionality factor conjectured to encode Newton's constant G, linking the Ω-model to induced gravity in the spirit of Sakharov [10].

Remark 4 (On Averaging). *The averaging $\langle\cdot\rangle$ denotes coarse-graining over regions larger than the coherence length of the phase field. Nontrivial curvature arises when $\partial_\mu\Theta$ forms vortex-like or monopole-like configurations, consistent with topological defect models [11,12].*

2.4. Examples of Emergent Structures

Example 1 (Emergent Temporal Metric). *For $\Theta(t,\mathbf{x}) = \omega t$, $\rho = \rho_0 > 0$, one finds*

$$g_{\mu\nu} = \mathrm{diag}(C\omega^2, 0, 0, 0),$$

corresponding to a purely temporal metric.

Example 2 (Emergent Lorentzian Metric). *For $\Theta(t,\mathbf{x}) = \omega t - kx$, one obtains*

$$g_{\mu\nu} = C\,\mathrm{diag}(\omega^2, -k^2, 0, 0),$$

a Lorentzian signature $(+,-)$ in $1+1$ dimensions.

2.5. Illustrative Example: Harmonic Oscillator Quantization

Example 3 (Harmonic Oscillator). *For the classical harmonic oscillator*

$$H = \frac{p^2}{2m} + \tfrac{1}{2}m\omega^2 q^2,$$

the action over a closed trajectory is

$$S_\gamma = \oint_\gamma p\,dq = \frac{2\pi E}{\omega}.$$

Trivial monodromy enforces

$$S_\gamma = 2\pi n\hbar, \quad n \in \mathbb{Z},$$

yielding quantized energies

$$E_n = n\hbar\omega.$$

This reproduces the integer-spaced spectrum; the zero-point shift $E_0 = \frac{1}{2}\hbar\omega$ requires additional refinements. Unlike canonical quantization, where this rule is postulated, here it arises as a consequence of monodromy of the phase field.

2.6. Outlook: Spin and Spin^c Structures

Nontrivial monodromy corresponds to half-integer quantization, naturally linked to Spin^c structures on emergent manifolds. This provides a geometric mechanism for fermionic statistics and the Pauli principle, to be developed in Section 3. A canonical example is the rigid quantum rotor, where half-integer spin states correspond precisely to nontrivial monodromy.

Lemma 1 (Quantization from Monodromy). *Let $\Psi \in \mathscr{C}_\Omega$ be a configuration of the universal phase field. If Ψ is transported adiabatically along a closed, non-contractible loop γ in configuration space, then the requirement of single-valued physical observables enforces*

$$S_\gamma \;=\; 2\pi n\hbar, \qquad n \in \tfrac{1}{2}\mathbb{Z}.$$

Sketch of Proof. The holonomy of the phase Θ around γ defines a monodromy element of $U(1)$. Consistency of the prequantum bundle requires this holonomy to be trivial in the observable sector. For trivial spin structure, this yields integer quantization ($n \in \mathbb{Z}$); for nontrivial spin structure, half-

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integer quantization arises ($n \in \frac{1}{2}\mathbb{Z}$). Thus quantization emerges directly from topological constraints on Ψ rather than being externally imposed. $\square$

3. The Topological Origin of Quantization Rules

The Ω-model asserts that quantization rules are not arbitrary axioms but inevitable consequences of topology. The central mechanism is *monodromy*: the change of phase that a wavefunction acquires when transported along a non-contractible closed loop in configuration space. In intuitive terms, monodromy encodes how the phase of a quantum state "remembers" the topology of its path [6,7].

3.1. Monodromy as a Homomorphism

Formally, monodromy is defined as a group homomorphism

$$\text{Mon} : \pi_1(P, x_0) \longrightarrow U(1), \tag{2}$$

where P is the configuration space and $\pi_1(P, x_0)$ its fundamental group. The image lies in the abelian group $U(1)$, reflecting the freedom of wavefunctions to acquire a phase. For systems in $3+1$ dimensions, physical consistency reduces this to the discrete subgroup $\{\pm 1\}$, yielding the familiar dichotomy of bosons and fermions. In lower dimensions, notably $2+1$, the full $U(1)$ image becomes possible, leading to anyonic statistics [17–19].

Remark 5 (Physical meaning of loops γ)**.** *The abstract loop $\gamma \in \pi_1(P, x_0)$ corresponds physically to processes such as a 360° rotation of a particle, or the exchange of two identical particles. The topological class of γ determines the allowed statistics.*

3.2. Two Fundamental Scenarios of Monodromy

3.2.1. Scenario 1: Trivial Monodromy (Bosons)

For a loop γ in P homotopic to the identity, the monodromy is trivial:

$$\psi(\Theta + 2\pi) = +\psi(\Theta). \tag{3}$$

Consistency of the bundle holonomy for observable sections of $\mathcal{E} = \mathcal{L} \otimes \mathcal{S}$ requires

$$S_\gamma \;=\; \oint_\gamma p\,dq \;=\; 2\pi n\hbar, \qquad n \in \mathbb{Z}, \tag{4}$$

which yields integer quantization of the action. This is the bosonic case.

3.2.2. Scenario 2: Nontrivial Monodromy (Fermions)

For a loop γ representing a 2π rotation in a space requiring the spin double cover, one finds

$$\psi(\Theta + 2\pi) = -\psi(\Theta), \qquad \psi(\Theta + 4\pi) = \psi(\Theta). \tag{5}$$

The corresponding action along γ,

$$S_\gamma \;=\; \oint_\gamma p\,dq,$$

must satisfy

$$\frac{S_\gamma}{\hbar} = (2n+1)\pi, \qquad n \in \mathbb{Z}, \tag{6}$$

equivalently

$$S_\gamma = 2\pi\left(n + \tfrac{1}{2}\right)\hbar. \tag{7}$$

This condition follows from $\exp(-iS_\gamma/\hbar) = -1$ and encodes the geometric origin of fermionic spin.

3.3. Theorem: Quantization from Monodromy

Let $\Psi \in \mathscr{C}_\Omega$ be a configuration of the universal phase field. If Ψ is transported adiabatically along a non-contractible loop $\gamma \in \pi_1(P, x_0)$, then the consistency of observable sections of the composite bundle $\mathcal{E} = \mathcal{L} \otimes \mathcal{S}$ enforces the quantization rule

$$S_\gamma = 2\pi n\hbar, \qquad n \in \tfrac{1}{2}\mathbb{Z}. \tag{8}$$

Sketch of Proof. The holonomy of Ψ along γ defines a $U(1)$ monodromy element. For trivial spin structure, holonomy triviality gives $n \in \mathbb{Z}$ (bosons). For nontrivial spin structure, the spin bundle $\mathcal{S}$ contributes a factor -1, yielding $n \in \frac{1}{2}\mathbb{Z}$ (fermions). Thus quantization follows directly from the topological constraints on Ψ, without postulates. □

3.4. Connection to Phase-Space Cells

The action quantization condition

$$S_\gamma = \oint_\gamma p\, dq = 2\pi n\hbar$$

is equivalent to the existence of a minimal "cell of action" of size $\sim \hbar$. This directly recovers the familiar discreteness of phase space and the uncertainty principle as topological consequences of monodromy.

3.5. Generalizations: Anyons and Beyond

In two-dimensional systems ($2+1$ spacetime dimensions), the fundamental group $\pi_1(P)$ can support representations into the full $U(1)$, not just $\{\pm 1\}$. This yields a continuum of possible statistics (anyons), as first proposed by Wilczek [17,18] and later applied to topological quantum computation by Kitaev [19]. In contrast, in $3+1$ dimensions the representation is restricted to $\{\pm 1\}$, explaining why only bosons and fermions occur in ordinary quantum field theory. The Ω-model thus naturally situates both conventional and exotic statistics within a unified monodromy framework.

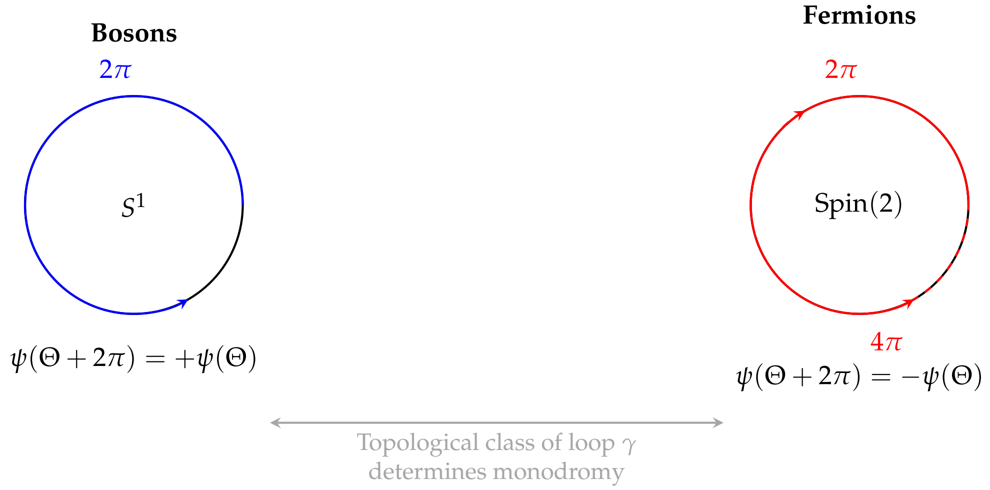

Figure 1. Two topological scenarios for the global phase Θ in the Ω-model. **Left:** Trivial monodromy ($\psi \to +\psi$) for S^1, leading to integer quantization (bosons). **Right:** Nontrivial monodromy ($\psi \to -\psi$ for 2π loop, identity for 4π) for Spin(2), yielding half-integer quantization (fermions).

4. Rigorous Derivation via Geometric Quantization

The intuitive picture presented in Section 3, where quantization emerges from the monodromy of the phase field, can be made mathematically rigorous within the framework of geometric quantization. This formalism, developed by Kostant [3], Souriau [4], Woodhouse [5], Atiyah [20], and Kirillov [21], provides a canonical procedure for constructing quantum theory from a classical symplectic manifold.

In the Ω-model, the universal phase field Ψ naturally induces a symplectic structure on configuration space, making geometric quantization the canonical language to formalize the monodromy principle and the coherence threshold $\delta S \geq \hbar$.

Our derivation proceeds in five structured steps: construction of bundles, definition of states, consistency condition, evaluation of holonomies, and synthesis of quantization rules.

4.1. Step 1: Constructing the Geometric Bundles

Let (P, ω) denote the symplectic phase space.

The Prequantum Line Bundle $\mathcal{L}$:

The quantum phase is represented by a section of a Hermitian line bundle $\mathcal{L}$ with unitary connection ∇, whose curvature satisfies the Kostant–Souriau condition

$$F_\nabla = -\frac{i}{\hbar}\,\omega. \tag{9}$$

Existence of such a bundle requires the Dirac integrality condition:

$$\frac{[\omega]}{2\pi\hbar} \in H^2(P, \mathbb{Z}),$$

which is equivalent to the quantization of symplectic flux through 2-cycles. Physically, this ensures that action fluxes are compatible with the Ω-postulate of coherence; otherwise, configurations are inconsistent.

The Spin^c Bundle $\mathcal{S}$:

In full generality, the relevant topological structure is a Spin^c bundle [22], of which the spin bundle is a special representative. Its holonomy takes values in $\{\pm 1\}$, encoding trivial or nontrivial spin structures.

4.2. Step 2: Defining the Physical State

The full state space is the tensor product

$$\mathcal{E} := \mathcal{L} \otimes \mathcal{S} \longrightarrow P, \tag{10}$$

so that the abstract universal field $\Psi \in \mathscr{C}_\Omega$ of the Ω-model is realized concretely as a section of $\mathcal{E}$.

4.3. Step 3: Consistency Condition

Physical observables must be globally unambiguous. This imposes the requirement

$$\mathrm{Hol}_{\mathcal{E}}(\gamma) = 1, \qquad \forall \gamma \in \pi_1(P), \tag{11}$$

ensuring that observable sections remain consistent after parallel transport around any closed loop γ.

4.4. Step 4: Holonomies of the Component Bundles

Lemma 1 (Prequantum Holonomy).

For $\gamma = \partial D$, the holonomy of $\mathcal{L}$ is

$$\mathrm{Hol}_{\mathcal{L}}(\gamma) = \exp\!\left(-\frac{i}{\hbar}\int_D \omega\right) = \exp\!\left(-\frac{iS_\gamma}{\hbar}\right), \tag{12}$$

where $S_\gamma = \oint_\gamma p\,dq$ is the classical action along γ, equivalently the symplectic flux of ω through D.

Lemma 2 (Spin^c Holonomy).

The holonomy of $\mathcal{S}$ is a topological sign:

$$\mathrm{Hol}_{\mathcal{S}}(\gamma) = \chi_{\mathcal{S}}(\gamma) \in \{+1, -1\}. \tag{13}$$

4.5. Step 5: Synthesis and Quantization Rules

Theorem (Quantization Rule from Bundle Consistency).

For any closed loop $\gamma \in \pi_1(P)$, consistency of observable sections of $\mathcal{E}$ enforces

$$\mathrm{Hol}_{\mathcal{E}}(\gamma) = \exp\left(-\frac{iS_\gamma}{\hbar}\right) \cdot \chi_{\mathcal{S}}(\gamma) = 1. \tag{14}$$

Sketch of Proof. If $\chi_{\mathcal{S}}(\gamma) = +1$, then $\exp(-iS_\gamma/\hbar) = 1$, giving

$$S_\gamma = 2\pi n\hbar, \quad n \in \mathbb{Z}.$$

If $\chi_{\mathcal{S}}(\gamma) = -1$, then $\exp(-iS_\gamma/\hbar) = -1$, giving

$$S_\gamma = (2n+1)\pi\hbar = 2\pi\left(n + \tfrac{1}{2}\right)\hbar, \quad n \in \mathbb{Z}.$$

Thus the general solution is compactly written as

$$S_\gamma = 2\pi k\hbar, \qquad k \in \tfrac{1}{2}\mathbb{Z},$$

with $k \in \mathbb{Z}$ (bosons) or $k \in \mathbb{Z} + \frac{1}{2}$ (fermions). □

Remark 6 (Ω-model Interpretation). *The loop action S_γ is the integrated version of the local threshold $\delta S \geq \hbar$, ensuring global consistency across cycles. Hence the rigorous holonomy condition of geometric quantization is precisely the global manifestation of the Ω-model's coherence postulate.*

Remark 7 (Anyons and 2+1D Systems). *In $3+1$ dimensions, $\chi_{\mathcal{S}}$ is restricted to $\{\pm 1\}$, yielding bosons and fermions. In contrast, in $2+1$ dimensions the fundamental group of the configuration space is the braid group, $\pi_1(P) \cong B_n$, which admits continuous representations into $U(1)$. This leads to fractional statistics and anyons [17–19]. Thus the Ω-model framework naturally generalizes to topological phases beyond bosons and fermions.*

In summary, geometric quantization formalizes the monodromy principle of the Ω-model and confirms that Planck's constant $\hbar$ emerges as a structural invariant of bundle consistency — a universal threshold of coherence — rather than an externally imposed parameter.

5. Discussion: The Status of $\hbar$ and Emergent Spin

The rigorous derivation in Section 4 establishes that quantization rules arise as unavoidable consequences of phase-space topology and bundle consistency. This has profound implications for two cornerstones of quantum mechanics: the meaning of Planck's constant $\hbar$ and the origin of spin.

5.1. The Status of $\hbar$ as a Structural Invariant

Within the Ω-model, Planck's constant is reinterpreted not as an externally imposed parameter, but as a *structural invariant* of the phase-ontological framework.

The analogy with π is instructive: in Euclidean geometry π is not chosen but necessarily emerges as the ratio of circumference to diameter. Likewise, in geometric quantization $\hbar$ inevitably appears as the scale that ties the symplectic form ω to its integral cohomology class [5,20]. Quantization of action is therefore as unavoidable in a nontrivial phase space as the appearance of π in circular geometry.

This framework does not derive the numerical value of $\hbar$ from pure mathematics. Instead, it explains why a universal quantum of action must exist at all. The experimentally measured value then calibrates hidden parameters (such as the vacuum amplitude ρ_0), and this calibration hints at possible links to measurable cosmological quantities (vacuum energy density, coherence lengths, or even the cosmological constant). Thus, $\hbar$ is revealed as the universal scale of phase coherence, not merely a unit conversion factor.

5.2. Spin as a Topological Phenomenon

Spin emerges in the Ω-model not as an intrinsic label, but as the physical manifestation of nontrivial monodromy $\psi \mapsto -\psi$ of the universal phase field. This reproduces the topological origin of half-integer quantization through Spin^c structures [22,23].[1]

This insight provides a direct geometric explanation of the Pauli Exclusion Principle. The exchange of two identical fermions is topologically equivalent to a 2π rotation, enforcing a global sign flip:

$$\Psi(\text{particle } 1, \text{particle } 2) = -\Psi(\text{particle } 2, \text{particle } 1).$$

Thus the Pauli principle—responsible for atomic structure, stability of matter, and chemistry itself—emerges as a corollary of phase monodromy, rather than an independent axiom. This interpretation resonates with neutron interferometry experiments [24], where 4π-periodicity of spinor states is observed directly.

6. Consistency with Physical Phenomena

A physical theory must not only be mathematically self-consistent, but also compatible with experimental evidence. This section demonstrates that the central postulate of the Ω-model—the coherence threshold

$$\delta S \geq \hbar, \tag{15}$$

which defines the minimal action required for physical distinguishability and stability—is consistently realized across a wide spectrum of quantum phenomena.

6.1. Qualitative Support from Foundational Experiments

Several cornerstone experiments in quantum physics, traditionally interpreted within the standard framework, acquire a direct and natural explanation within the Ω-model.

Aharonov–Bohm Effect:

An electron's wave function acquires a measurable phase when encircling a magnetic flux, even though it passes only through field-free regions [25–28]. The observable is the accumulated phase $\Theta = \oint A_\mu dx^\mu$, a global quantity sensitive only to the topology of the path. This directly illustrates the Ω-model's phase ontology and confirms that physical effects depend on the action integral rather than on local forces.

Neutron Interferometry:

Experiments by Rauch, Colella, and collaborators [24,29] demonstrated that neutron wave functions require a 4π rotation to return to their initial state. This is direct evidence of nontrivial monodromy $\psi \mapsto -\psi$, which in the Ω-model arises naturally from the topology of the universal phase field. It confirms the topological origin of spin and the necessity of the half-integer quantization rule.

dc-SQUID Flux Quantization:

Superconducting quantum interference devices exploit a macroscopic phase coherence. The quantization of magnetic flux in units of $h/2e$ [30–32] directly reflects the condition $\delta S \geq \hbar$ applied to

[1] The general setting is a Spin^c bundle, of which the spin bundle $\mathcal{S}$ in Section 4 is a representative case.

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a superconducting loop. This is a macroscopic realization of the Ω-postulate, where single-valuedness of the order parameter parallels global holonomy consistency.

Quantum Hall Effect:

The quantization of Hall conductance in integer multiples of e^2/h [33–35] is a canonical example of a topological invariant realized in experiment. Since $h = 2\pi\hbar$, this corresponds to integer filling of phase-space cells of area $\hbar$. Thus, the QHE provides a direct macroscopic manifestation of the Ω-model's coherence threshold.

Quantum Decoherence:

Studies of decoherence show that quantum superpositions lose coherence rapidly when $\delta S < \hbar$ due to environmental coupling [36,37]. This aligns precisely with the Ω-model's postulate that insufficient action cannot sustain physically distinguishable states.

6.2. Illustrative Consistency Check: The CMS Dimuon Anomaly (As Future Directions)

As a qualitative consistency test, we note that a recent dimuon excess reported by the CMS Collaboration [38] can be interpreted in the Ω-model as a candidate for a higher-order ($n > 1$) coherent phase configuration.

The relevant estimate is based on the action

$$\delta S \sim \Delta E \times \Delta t,$$

where $\Delta E \approx 5.6\,\text{GeV}$ is the energy scale of the excess, and Δt is its characteristic coherence timescale. Although the precise mapping of Δt requires detailed detector-level analysis, the scale of the phenomenon ensures that $\delta S \gg \hbar$, placing it firmly within the coherent regime predicted by the model.

We emphasize that this interpretation is speculative and does not replace dedicated BSM analyses, but it illustrates how collider anomalies can be consistently embedded into the coherence-threshold framework.

Summary

From mesoscopic interference (Aharonov–Bohm, SQUIDs) to fundamental spinor behavior (neutron interferometry) and topological macroscopic invariants (Quantum Hall effect), the Ω-model's criterion $\delta S \geq \hbar$ is borne out across experimental domains. Decoherence phenomena mark the complementary limit $\delta S < \hbar$, while even recent collider anomalies appear consistent with the coherence threshold.

Ultimately, across scales from interferometry to condensed matter and high-energy physics, the Ω-model frames Planck's constant not as an arbitrary parameter but as the *universal threshold of coherence*—a principle unifying quantum phenomena from the microscopic to the cosmic.

7. Limitations and Future Directions

Near-Term Limitations

Constants

The model explains the necessity of $\hbar$ but does not predict its numerical value. $\hbar$ remains an empirical input, though its linkage to vacuum amplitude and coherence length suggests possible pathways toward quantitative predictions.

Gravity

Although geometry and curvature arise from phase gradients, a full derivation of Einstein's equations is not yet achieved. Bridging from quantum topology to macroscopic dynamics will require renormalization group or nonperturbative methods.

Manifold Assumptions

We assume a smooth M_4, but spacetime itself may emerge from pre-geometric combinatorial structures.

Programmatic Outlook

Mass Spectrum of Particles

Elementary particles appear as stable, topological excitations of the universal phase field. A central challenge is to derive their mass ratios (e.g., m_μ / m_e) from topological stability conditions.

Emergent Gravity

Following Jacobson [39], Padmanabhan [40], and more recent work [13,41,42], the Ω-model motivates a program where spacetime curvature and gravity emerge from phase coherence.

Dark Energy and Topological Defects

Vacuum tension and dark energy may reflect coherent background structures. Links to domain walls and decoherence studies [36,37,43] open a perspective on cosmology.

Exotic Statistics and Fractonic Phases

The framework generalizes naturally to anyons [18,19] and fractons [44], showing the universality of the Ω-principle across condensed matter and high-energy physics.

From Ω-I to Ω-II

This work (Ω-*I*) clarifies the structural role of $\hbar$ and spin. The next stage (Ω-*II*) will extend toward emergent gravity, vacuum structure, and cosmology.

In summary, the Ω-model provides a unified framework where $\hbar$ is elevated to the status of a structural constant, spin emerges from topology, and physical reality itself is filtered through the coherence criterion $\delta S \geq \hbar$. This perspective clarifies long-standing quantum principles and charts ambitious new research directions: the derivation of particle masses, the emergence of spacetime geometry, the dynamics of dark energy, and the exploration of exotic statistics.

Ultimately, the Ω-model suggests that *Planck's constant is not an arbitrary parameter but the universal threshold of coherence, testable from quantum interferometry to cosmology*.

8. Conclusion

The Ω-model reframes quantization as a structural consequence of phase topology rather than an imposed axiom. Planck's constant $\hbar$ emerges as a universal threshold of coherence, while spin and the Pauli principle follow naturally from nontrivial monodromy of the universal phase field. This perspective unifies diverse quantum phenomena—from interferometry to condensed matter—under a single coherence criterion. In doing so, the Ω-model positions $\hbar$ not as an arbitrary parameter but as a structural invariant of physical reality, providing both conceptual clarity and concrete pathways for future theoretical and experimental exploration.

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