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Article

Backstepping-Based Nonlinear Control of a Grid-Photovoltaic Hybrid Water Pumping System with Multiphase Induction

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Abstract

We address the control problem of a hybrid photovoltaic-grid water pumping system. This system includes a photovoltaic panel, an AC/DC PWM rectifier connected to an electric grid, a DC/AC PWM inverter paired with a Multiphase Induction Motor MPIM, and a centrifugal water pump. The control objectives are to ensure that the water pump flow rate follows a reference signal, regulate the norm of the rotor flux of the motor to its nominal value, maintain the DC link voltage at a reference value for optimal power point tracking (MPPT), and achieve satisfactory power factor correction (PFC). A nonlinear model of the controlled system is developed, followed by the synthesis of a multiloop nonlinear controller using the backstepping technique. This controller ensures global asymptotic stability in closed-loop operation and achieves all control objective.

Keywords: Multiphase Induction Motor; photovoltaic arrays; MPPT; nonlinear control; backstepping technique; Lyapunov stability

1. Introduction

In Morocco, the Ministry of Energy Transition and Sustainable Development has significantly advanced agricultural energy efficiency by promoting solar-powered water pumping systems. These systems, noted for their low maintenance and operational costs, provide a sustainable alternative to traditional energy sources like butane gas and diesel [1]. The adoption of solar pumping has surged, with more than 10,000 systems installed between 2019 and 2020, increasing the total to over 40,000. This expansion is supported by a reduction in photovoltaic panel costs and the economical price of solar pumping at 0.44 dirhams per cubic meter of water—substantially lower than the costs of conventional energy sources [1]. Advanced control strategies have been developed for induction motors in hybrid grid-photovoltaic systems to further augment the viability of solar energy in remote agricultural applications [2–7]. These approaches utilize advanced methods, including Indirect Double Integral Sliding Mode Control for Maximum Power Point Tracking (MPPT) and fuzzy logic, to optimize dynamic performance, thereby improving both efficiency and reliability [4]. Specifically, systems combining photovoltaic and battery storage are designed to ensure a continuous water supply under varying climatic conditions [3]. Technologies such as Field-Oriented Control and Direct Torque Control for induction motor drives have significantly improved operational efficiency and system robustness, minimizing energy losses and optimizing overall performance [5,6,8]. Grid-connected setups utilize Synchronous Reluctance Motors for improved power quality and continuous supply, even during grid power outages [9]. Additionally, the strategic integration of Voltage Source Inverters with existing motor windings facilitates efficient power exchanges between the solar power systems and the grid, minimizing the need for additional power electronics and enhancing overall energy management [10]. This study seeks to devise a control strategy that concurrently governs the AC/DC Rectifier and the 'DC/AC Inverter-Multiphase Induction Motor

Pump' system. The strategy is distinguished by its multi-loop design, featuring a current loop to ensure unit power factor operation in the coupling between the power supply network and the AC/DC rectifier. Additionally, a separate loop is employed to regulate the rectifier's output voltage, ensuring maximum power extraction from the photovoltaic generator, regardless of variations in solar radiation [11]. Moreover, a dual-variable loop is developed to monitor variations in water flow rate and ensure the rotor flux norm remains at its nominal level. The control loops leverage Lyapunov and backstepping techniques to achieve stability and precision [12].

The paper is organized as follows: Section 2 presents the system modeling; Section 3 focuses on the controller design; Section 4 assesses the controller's tracking performance using numerical simulations. The paper concludes with Section 5, which provides final remarks and references.

2. System Modeling

The system configuration depicted in Figure 1 consists of two main components: an AC/DC rectifier connected to a photovoltaic generator (PVG), and a DC/AC inverter that drives a multiphase induction motor (MPIM), which in turn operates a centrifugal water pump. The entire system is connected to a single-phase electrical grid, with both the rectifier and inverter employing Pulse Width Modulation (PWM) technology for efficient electrical conversion. The PV generator consists of 30 SM55 photovoltaic modules connected in series.

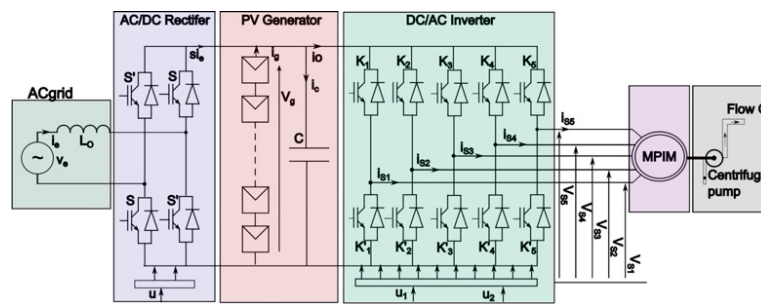


Figure 1. General diagram of the controlled system.

2.1. Photovoltaic Generator Model

Solar cells convert sunlight directly into electrical energy. The equivalent circuit of a photovoltaic (PV) module is illustrated in Figure 2, as described in the established literature [13–16].

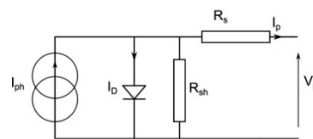


Figure 2. Equivalent circuit of a PV module.

The ideal performance of a solar array, often represented in the current-voltage (I_p - V_p) characteristics where the series resistance (R_s) is assumed zero and the shunt resistance (R_{sh}) infinite, is described by:

$$I_p = I_{ph} - I_o \{ \exp(A V_p) - 1 \} \quad (1)$$

where

$$A = \frac{q}{\gamma K T}$$

$$I_{ph} = [I_{SCR} + K_I(T - T_r)] \frac{\lambda}{1000}$$

$$I_o = I_{or} \left[\frac{T}{T_r} \right]^3 \exp \left[\frac{q E_{G0}}{\gamma K} \left(\frac{1}{T_r} - \frac{1}{T} \right) \right]$$

Here, I_{ph} represents the photocurrent generated under specific radiation conditions; I_o is the diode’s reverse saturation current; I_{or} is the saturation current at the reference temperature; I_{or} denotes the short-circuit current under standard conditions (298.15 K and 1 kW/m²); K_I is the temperature coefficient at I_{scr} ; λ denotes solar irradiance; E_{GO} is the band gap energy of silicon; γ is the diode ideality factor; T_r is the reference temperature; T is the actual cell temperature; k is Boltzmann’s constant; and q is the electron charge. Detailed formulas for I_{ph} and I_o are provided in sources such as [15], highlighting their dependency on T and λ . The photovoltaic generator (PVG) is composed of multiple strings of PV modules connected in series and parallel configurations to achieve the required voltage and current levels. This setup exhibits nonlinear current-voltage (I_g - V_g) characteristics, ideally expressed as:

$$I_g = I_{phg} - I_o \{ \exp (A_g V_g) - 1 \} \tag{2}$$

In this equation, I_g denotes the current flowing through the PVG, while V_g represents the voltage across the PVG. The constant $A_g = A/N_s$ adjusts A to account for the number of series-connected cells, while $I_{og} = N_p I_o$ and $I_{phg} = N_p I_{ph}$ scale the saturation current and photocurrent, respectively, based on N_p , the number of parallel paths within the PVG. The specific PV module used in this study is the SM55, which consists of 36 mono-crystalline cells connected in series. The electrical specifications for this array are outlined in Table 1.

Table 1. Electrical specifications of the SM55 solar module.

Maximum Power	P_m (W)	55
Short circuit current	I_{SCR} (A)	3.45
Open circuit voltage	V_{oc} (V)	21.7
Voltage at max power point	V_m (V)	17.4
Current at max power point	I_m (A)	3.15
Temperature Coefficient I_{sc}	K_I (A/K)	$4 \cdot 10^{-4}$

2.2. Modeling AC/DC Rectifier

Connected to the power network, the H-bridge converter configuration consists of four IGBTs (refer to Figure 1). The behavior of this subsystem is described by Kirchhoff’s laws, resulting in the following differential equations:

$$\frac{di_e}{dt} = \frac{v_e}{L_o} - \frac{sV_g}{L_o} \tag{3}$$

$$\frac{dV_g}{dt} = \frac{si_e}{2C} + \frac{I_g}{2C} - \frac{i_o}{2C} \tag{4}$$

Here, i_e is the current through the inductor L_o , V_g is the voltage across the output capacitor $2C$ from the PV array, and I_g is the generated current from the photovoltaic (PV) generator. The variable v_e , defined as $\sqrt{2}E\cos(\omega_e t)$, represents the sinusoidal grid voltage, where E and ω_e are the amplitude and angular frequency of the grid voltage, respectively. The variable i_o denotes the current flowing into the inverter, while s is the switching state of the H-bridge, which can toggle between -1 and 1:

$$s = \begin{cases} 1 & \text{if } S \text{ On and } S' \text{ Off} \\ -1 & \text{if } S \text{ Off and } S' \text{ On} \end{cases} \tag{5}$$

Although this model effectively describes the physical dynamics of the rectifier, its switch-based nature makes it unsuitable for control design in traditional frameworks that require continuous control inputs. Thus, for the purposes of control, an average model is used (3-4) as follows [17]:

$$\frac{dx_1}{dt} = \frac{v_e}{L_o} - \frac{ux_2}{L_o} \tag{6}$$

$$\frac{dx_2}{dt} = \frac{ux_1}{2C} + \frac{I_g}{2C} - \frac{i_o}{2C} \tag{7}$$

The variables x_1, x_2 , and u represent the averaged values of i_e , V_g , and s over switching periods, respectively.

2.3. Modeling “DC/AC Inverter – MPIM” Association

The dynamic model of a Multiphase Induction Motor (MPIM) in the stator-oriented (α, β) coordinates is given by [18–20]:

$$\frac{d\Omega}{dt} = -\frac{f}{J}\Omega + \frac{M_{sr}}{L_r} \frac{p}{J} (i_{s\beta}\phi_{r\alpha} - i_{s\alpha}\phi_{r\beta}) - \frac{T_L}{J} \quad (8a)$$

$$\frac{di_{s\alpha}}{dt} = \frac{R_r}{L_r} \frac{M_{sr}\phi_{r\alpha}}{\sigma L_s L_r} + p \frac{M_{sr}\phi_{r\beta}\Omega}{\sigma L_s L_r} - \gamma i_{s\alpha} + \frac{v_{s\alpha}}{\sigma L_s} \quad (8b)$$

$$\frac{di_{s\beta}}{dt} = \frac{R_r}{L_r} \frac{M_{sr}\phi_{r\beta}}{\sigma L_s L_r} - p \frac{M_{sr}\phi_{r\alpha}\Omega}{\sigma L_s L_r} - \gamma i_{s\beta} + \frac{v_{s\beta}}{\sigma L_s} \quad (8c)$$

$$\frac{d\phi_{r\alpha}}{dt} = -\frac{R_r}{L_r} \phi_{r\alpha} + \frac{R_r}{L_r} M_{sr} i_{s\alpha} - p\Omega \phi_{r\beta} \quad (8d)$$

$$\frac{d\phi_{r\beta}}{dt} = -\frac{R_r}{L_r} \phi_{r\beta} + \frac{R_r}{L_r} M_{sr} i_{s\beta} + p\Omega \phi_{r\alpha} \quad (8e)$$

This model incorporates key parameters such as the rotor and stator resistances (R_r , R_s), inductances (L_r , L_s), mutual inductance (M_{sr}), number of pole pairs (p), system inertia (J), and friction coefficient (f). The stator voltages $v_{s\alpha}$ and $v_{s\beta}$ can be independently controlled, linking them directly to the respective control actions:

$$v_{s\alpha} = x_2 u_1, \quad v_{s\beta} = x_2 u_2, \quad i_o = (u_1 \bar{i}_{s\alpha} + u_2 \bar{i}_{s\beta}) \quad (9)$$

This setup allows for precise control over the motor's behavior, leveraging modern PWM techniques to adjust the motor's operation dynamically. The variables u_1 and u_2 are averaged representations of the (α, β) components of the multiphase duty ratio system signals (k_1, k_2, k_3, k_4, k_5). These control signals are derived by applying the Park transformation to the PWM signals and averaging over the PWM cycles. The variable i_o denotes the input current to the inverter, and the binary signals k_1, k_2, k_3, k_4, k_5 are defined by the following switching logic:

$$k_i = \begin{cases} 1 & \text{if } K_i \text{ is on and } K'_i \text{ is off,} \\ 0 & \text{if } K_i \text{ is off and } K'_i \text{ is on,} \end{cases} \quad i = 1, 2, 3, 4, 5 \quad (10)$$

Additionally, we define the averaged state variables for the control system:

$$x_4 = \bar{\Omega}, \quad x_5 = \bar{i}_{s\alpha}, \quad x_6 = \bar{i}_{s\beta}, \quad x_7 = \bar{\phi}_{r\alpha}, \quad x_8 = \bar{\phi}_{r\beta} \quad (11)$$

where the overline notation, $\bar{\cdot}$, indicates averaging over PWM periods.

For modeling purposes, the averaged dynamic system is represented by the following set of differential equations:

$$\frac{dx_3}{dt} = -\frac{f}{J} x_3 + p \frac{M_{sr}}{L_r} \frac{1}{J} (x_5 x_6 - x_4 x_7) - \frac{T_L}{J} \quad (12a)$$

$$\frac{dx_4}{dt} = \frac{R_r}{L_r} \frac{M_{sr} x_6}{\sigma L_s L_r} + p \frac{M_{sr} x_7 x_3}{\sigma L_s L_r} - \gamma x_4 + \frac{x_2 u_1}{\sigma L_s} \quad (12b)$$

$$\frac{dx_5}{dt} = \frac{R_r}{L_r} \frac{M_{sr} x_7}{\sigma L_s L_r} - p \frac{M_{sr} x_6 x_3}{\sigma L_s L_r} - \gamma x_5 + \frac{x_2 u_2}{\sigma L_s} \quad (12c)$$

$$\frac{dx_6}{dt} = -\frac{R_r}{L_r} x_6 + \frac{R_r}{L_r} M_{sr} x_4 - p x_3 x_7 \quad (12d)$$

$$\frac{dx_7}{dt} = -\frac{R_r}{L_r} x_7 + \frac{R_r}{L_r} M_{sr} x_5 + p x_3 x_6 \quad (12e)$$

2.4. Modeling the Centrifugal Pump

Centrifugal pumps, commonly used in photovoltaic energy applications, apply a load torque:

$$T_L = G\Omega^2 \quad (13)$$

Correspondingly, the flow rate Q of the pump varies proportionally with the rotor speed:

$$Q = \kappa\Omega \quad (14)$$

where G and κ are constants specific to the centrifugal pump's characteristics.

2.5. System Model

The state-space equations previously derived are consolidated into a comprehensive global state-space model representing the entire system, including the Multiphase Induction Motor, Centrifugal Pump, and the associated DC/AC Inverter:

$$\frac{dx_1}{dt} = \frac{v_e}{L_o} - \frac{ux_2}{L_o} \quad (15a)$$

$$\frac{dx_2}{dt} = \frac{ux_1}{2C} + \frac{I_g}{2C} - \frac{i_o}{2C} \quad (15b)$$

$$\frac{dx_3}{dt} = -\frac{f}{J}x_3 + p\frac{M_{sr}}{L_r}\frac{1}{J}(x_5x_6 - x_4x_7) - \frac{G}{J}x_3^2 \quad (15c)$$

$$\frac{dx_4}{dt} = \frac{R_r}{L_r}\frac{M_{sr}x_6}{\sigma L_s L_r} + p\frac{M_{sr}x_7x_3}{\sigma L_s L_r} - \gamma x_4 + \frac{x_2u_1}{\sigma L_s} \quad (15d)$$

$$\frac{dx_5}{dt} = \frac{R_r}{L_r}\frac{M_{sr}x_7}{\sigma L_s L_r} - p\frac{M_{sr}x_6x_3}{\sigma L_s L_r} - \gamma x_5 + \frac{x_2u_2}{\sigma L_s} \quad (15e)$$

$$\frac{dx_6}{dt} = -\frac{R_r}{L_r}x_6 + \frac{R_r}{L_r}M_{sr}x_4 - px_3x_7 \quad (15f)$$

$$\frac{dx_7}{dt} = -\frac{R_r}{L_r}x_7 + \frac{R_r}{L_r}M_{sr}x_5 + px_3x_6 \quad (15g)$$

3. Controller Design

3.1. Control Targets

The system aims to achieve the following primary control Targets:

(1) Optimization of DC Link Voltage: The DC link voltage V_g must precisely follow a state-dependent reference voltage $V_{gref}=F(P)$, where P represents the output power of the PVG. This function F is designed to ensure that $V_g=V_{gref}$ aligns with the operating point of the PVG at its maximum power.

(2) Power Factor Correction Requirement (PFC): The output current i_e from the rectifier must be sinusoidal and phase-aligned with the AC supply voltage v_e .

(3) Regulation of Water Flow Rate: The water flow rate Q should follow a predefined reference signal Q^{ref} .

(4) Regulation of Rotor Flux: The norm of the rotor flux $\Phi_r = \sqrt{x_6^2 + x_7^2}$ should be controlled to match a reference value Φ_{ref} , ideally its nominal value.

These controllers are developed using the backstepping control [12].

3.2. Optimization of DC Link Voltage

The voltage-reference optimizer is based on the power-voltage relationship (P - V_g) characteristics, which do not require the measurement of solar radiation λ . This design strategy is adapted to the typical operating conditions of the photovoltaic generator at $45^\circ C$, a common temperature under which the generator frequently operates. The objective here is to develop a voltage-reference optimizer that meets the Maximum Power Point Tracking criteria, dynamically calculating the optimal voltage V_{gopt} to guarantee maximum power extraction and efficient transmission to the multiphase induction motor via the DC/AC inverter.

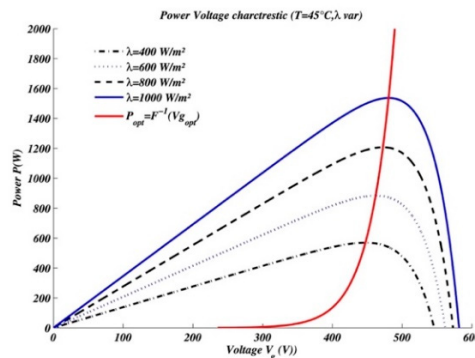


Figure 3. Power-voltage (P - V) characteristics of the PVG, under constant temperature and varying radiation.

The voltage-reference optimizer is formulated as a polynomial function $V_{g_{opt}}=F(P_{opt})$, as detailed in [11]:

$$F(P) = h_n P^n + h_{n-1} P^{n-1} + \dots + h_1 P + h_0 \quad (16)$$

In this formula, the coefficients h_i represent the specific characteristics of the photovoltaic generator system (PVG).

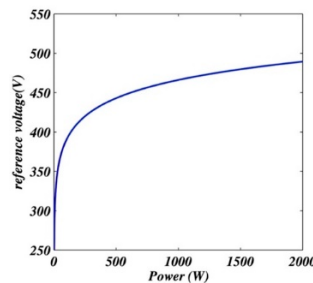


Figure 4. Optimal power-voltage characteristic obtained from polynomial interpolation for the SM55 module series.

3.3. Power Factor Correction (PFC)

The objective of Power Factor Correction (PFC) is to keep the grid current sinusoidal and in phase with the AC supply voltage. A control regulator fine-tunes the current x_1 to align with the reference signal x_1^* , defined as:

$$x_1^* = kv_e \quad (17)$$

where k is a real parameter influencing the direction of power flow. Introducing the current tracking error:

$$z_1 = x_1 - x_1^* \quad (18)$$

Given the dynamics in (6), the error evolves according to:

$$\dot{z}_1 = \frac{1}{L_o} v_e - \frac{1}{L_o} u x_2 - \dot{x}_1^* \quad (19)$$

A stabilizing control strategy is developed using a quadratic Lyapunov function $V_1 = 0.5z_1^2$, ensuring that the time derivative $\dot{V}_1$ becomes negative definite when the control input is:

$$u = \frac{L_o}{x_2} \left(c_1 z_1 + \frac{v_e}{L_o} - \dot{x}_1^* \right) \quad (20)$$

where c_1 is a positive design parameter. The stability and effectiveness of this control strategy are outlined in the subsequent proposition, demonstrating straightforward proof from the conditions defined.

Proposition 1. Given the system described by the current equation (6) and the control law (20), with a positively set c_1 , and provided the reference $x_1^* = kv_e$ and its first derivative are available, the following properties hold:

1. The current loop governed by $\dot{z}_1 = -c_1 z_1$ exhibits global exponential stability, implying that z_1 diminishes exponentially irrespective of initial conditions.
2. Additionally, if k converges to a finite value, then the PFC requirement is asymptotically met on average, meaning the average input current x_1 exponentially approaches the reference kv_e over time.

3.4. Regulation of DC Link Voltage

The objective is to formulate a control law for adjusting the ratio k in (17) to regulate the inverter input voltage $x_2 = \bar{V}_g$ to a target reference value $V_{g_{ref}}$. Establishing the relationship between the control input k and the output voltage x_2 is critical, as highlighted in the following proposition.

Proposition 2. Under the assumptions previously established in Proposition 1, the power rectifier described by (6-7) and controlled via (20) exhibits the following behavior:

1. The output voltage x_2 adjusts in response to changes in k according to:

$$2C \frac{dx_2}{dt} = \frac{1}{x_2} (kv_e^2 + z_1 v_e) + I_g - i_o \quad (21)$$

2. The squared voltage $y = x_2^2$ responds to adjustments in k as follows:

$$\frac{dy}{dt} = \frac{1}{C} (kv_e^2 + z_1 v_e) + \frac{1}{C} (I_g x_2 - i_o x_2) \quad (22)$$

Given that the reference for the squared voltage is $y_{ref} = V_{gref}^2$, the tracking error $z_2 = y - y_{ref}$ evolves according to:

$$\dot{z}_2 = \frac{E^2}{C} (k + k \cos(2\omega_e t)) + \frac{\sqrt{2}E}{C} z_1 \cos(\omega_e t) + \frac{I_g x_2 - i_o x_2}{C} - \dot{y}_{ref} \quad (23)$$

This equation includes periodic terms due to $v_e = \sqrt{2}E \cos(\omega_e t)$, where $v_e^2 = E^2(1 + \cos(2\omega_e t))$. To stabilize the system described by (23), a quadratic Lyapunov function is used:

$$V_2 = 0.5 z_2^2 \quad (24)$$

By setting the control law to neutralize periodic fluctuations and ensure a negative definite $\dot{V}_2$, we simplify:

$$\begin{aligned} kE^2(1 + \cos(2\omega_e t)) + \sqrt{2}E z_1 \cos(\omega_e t) \\ = C c_2 z_2 + I_g x_2 - i_o x_2 - C \dot{y}_{ref} \end{aligned} \quad (25)$$

Considering practical implementation and avoiding excessive control chattering, a filtered version of the control solution is proposed:

$$k + dk = d \frac{C}{E^2} \left(c_2 z_2 + \frac{I_g x_2 - i_o x_2}{C} - \dot{y}_{ref} \right) \quad (26)$$

This design uses regulator parameters d and c_2 , both positive constants. The complete dynamics and control effectiveness are summarized in the following proposition:

Proposition 3. With the AC/DC rectifier system defined by (67) and incorporating control laws (20) and (26), the closedloop dynamics in (z_1, z_2, k) coordinates are governed by:

$$\dot{Z} = AZ + g(x) \left(\frac{I_g x_2 - i_o x_2}{C} - \dot{y}_{ref} \right) + f(Z, t) \quad (27)$$

where:

$$Z = \begin{bmatrix} z_1 \\ z_2 \\ k \end{bmatrix}, A = \begin{bmatrix} -c_1 & 0 & 0 \\ 0 & 0 & -\frac{E^2}{C} \\ 0 & \frac{dC}{E^2} c_2 & -d \end{bmatrix}, g(x) = \begin{bmatrix} 0 \\ 1 \\ \frac{dC}{E^2} \end{bmatrix} \quad (28)$$

$$f(Z, t) = \begin{bmatrix} 0 \\ -\frac{E^2}{C} k \cos(2\omega_e t) - \frac{\sqrt{2}E}{C} z_1 \cos(\omega_e t) \\ 0 \end{bmatrix} \quad (29)$$

3.5. Design of Water Flow Rate and Rotor Flux Norm Regulator

The control of water flow rate and flux norm poses a significant challenge, involving the entire system composed of the induction machine and centrifugal pump, as detailed in equations (15c - 15g) and (14). The reference for the water flow rate, Q_{ref} , is set as a bounded and differentiable function of time, with both its first and second derivatives being well-defined and limited. This ensures that Q_{ref} can be efficiently managed through second-order linear filters. The flux norm reference, Q_{ref} , is established at its nominal value.

The controller is designed in two stages using the tuning functions backstepping adaptive method [12]. Initially, we define the tracking errors:

$$z_3 = Q_{ref} - Q = Q_{ref} - \kappa x_3 \quad (30a)$$

$$z_4 = \Phi_{ref}^2 - \Phi_f^2 = \Phi_{ref}^2 - (x_6^2 + x_7^2) \quad (30b)$$

Step 1. We employ the state equations (15d- 15g) to describe the dynamics of errors z_3 and z_4 :

$$\dot{z}_3 = \dot{Q}_{\text{ref}} + \kappa \left(\frac{fx_3}{J} - p \frac{M_{\text{sr}}}{JL_r} (x_5x_6 - x_4x_7) + \frac{G}{J} x_3^2 \right) \quad (31a)$$

$$\dot{z}_4 = 2\Phi_{\text{ref}}\dot{\Phi}_{\text{ref}} - \frac{2R_r M_{\text{sr}}}{L_r} (x_4x_6 + x_5x_7) + \frac{2R_r \Phi_r^2}{L_r} \quad (31b)$$

In (31a) and (31b), the expressions $p\kappa \frac{M_{\text{sr}}}{JL_r} (x_5x_6 - x_7x_4)$ and $\text{ref} 2 \frac{R_r}{L_r} M_{\text{sr}} (x_4x_6 + x_5x_7)$ act as virtual control signals. To stabilize the error system, we set:

$$\mu_1 \stackrel{\text{def}}{=} c_3 z_3 + \dot{Q}_{\text{ref}} + \kappa \frac{G}{J} x_3^2 + \kappa \frac{f}{J} x_3, \quad (32a)$$

$$v_1 \stackrel{\text{def}}{=} c_4 z_4 + 2\Phi_{\text{ref}}\dot{\Phi}_{\text{ref}} + 2 \frac{R_r}{L_r} \Phi_r^2. \quad (32b)$$

where c_3 and c_4 are positive design parameters. Employing a Lyapunov function:

$$V_1 = 0.5(z_3^2 + z_4^2) \quad (33)$$

yields:

$$\dot{V}_1 = -c_3 z_3^2 - c_4 z_4^2 \quad (34)$$

This formulation indicates global asymptotic stability. Introducing new errors for unactualized virtual controls:

$$z_5 = \mu_1 - p\kappa \frac{M_{\text{sr}}}{JL_r} (x_5x_6 - x_7x_4) \quad (35a)$$

$$z_6 = v_1 - 2 \frac{R_r}{L_r} M_{\text{sr}} (x_4x_6 + x_5x_7) \quad (35b)$$

Thus, the modified dynamics of the errors z_3 and z_4 are:

$$\dot{z}_3 = -c_3 z_3 + z_5 \quad (36a)$$

$$\dot{z}_4 = -c_4 z_4 + z_6 \quad (36b)$$

Step 2: The second phase of the design involves selecting the actual control signals, u_1 and u_2 , to ensure that all errors (z_3, z_4, z_5, z_6) converge to zero. We begin by examining how these errors depend on the control signals (u_1, u_2). Starting with z_5 , from (35a) we derive:

$$\dot{z}_5 = \dot{\mu}_1 - p\kappa \frac{M_{\text{sr}}}{JL_r} (\dot{x}_5x_6 + x_5\dot{x}_6 - \dot{x}_4x_7 - x_4\dot{x}_7) \quad (37)$$

which simplifies to:

$$\dot{z}_5 = \mu_2 - p\kappa \frac{M_{\text{sr}}}{JL_r} \frac{x_2}{\sigma L_s} (x_7u_1 - x_6u_2) \quad (38)$$

with μ_2 defined as:

$$\begin{aligned} \mu_2 = & -c_3^2 z_3 + \ddot{Q}_{\text{ref}} - 2\kappa \frac{G^2}{J^2} x_3^3 + p^2 \kappa \frac{M_{\text{sr}}^2}{J\sigma L_s L_r^2} x_3 \Phi_r^2 \\ & + c_3 z_5 + p^2 \kappa \frac{M_{\text{sr}}}{JL_r} x_3 (x_5x_7 + x_4x_6) \\ & + p\kappa \frac{M_{\text{sr}}}{JL_r} \left(\frac{R_r}{L_r} + \gamma \right) (x_6x_5 - x_4x_7) - 3\kappa \frac{fG}{J^2} x_3^2 \end{aligned} \quad (39)$$

For z_6 , we have:

$$\dot{z}_6 = \dot{v}_1 - 2 \frac{R_r}{L_r} M_{\text{sr}} (\dot{x}_4x_6 + x_4\dot{x}_6 + \dot{x}_5x_7 + x_5\dot{x}_7) \quad (40)$$

which simplifies to:

$$\dot{z}_6 = v_2 - 2 \frac{R_r}{L_r} M_{\text{sr}} \frac{x_2}{\sigma L_s} (u_1x_6 + u_2x_7) \quad (41)$$

with v_2 defined as:

$$\begin{aligned} v_2 = & c_4(-c_4 z_4 + z_6) + 2(\dot{\Phi}_{\text{ref}}^2 + \Phi_{\text{ref}}\ddot{\Phi}_{\text{ref}}) \\ & + 2 \frac{R_r}{L_r} M_{\text{sr}} p x_3 (x_4x_7 - x_5x_6) \\ & - \left(4 \left(\frac{R_r}{L_r} \right)^2 + 2 \left(\frac{R_r}{L_r} \right)^2 \frac{M_{\text{sr}}}{\sigma L_s L_r} M_{\text{sr}} \right) \Phi_r^2 \\ & - 2 \left(\frac{R_r}{L_r} \right)^2 M_{\text{sr}}^2 (x_5^2 + x_4^2) \end{aligned} \quad (42)$$

To control this system, we define an augmented Lyapunov function:

$$V_3 = \frac{1}{2}(z_3^2 + z_4^2 + z_5^2 + z_6^2) \quad (43)$$

whose time derivative is:

$$\dot{V}_3 = z_3\dot{z}_3 + z_4\dot{z}_4 + z_5\dot{z}_5 + z_6\dot{z}_6 \quad (44)$$

yielding:

$$\begin{aligned} \dot{V}_3 = & -c_3z_3^2 - c_4z_4^2 - c_5z_5^2 - c_6z_6^2 \\ & + z_6 \left[c_6z_6 + z_4 + v_2 - 2\frac{R_r}{L_r}M_{sr}\frac{x_2}{\sigma L_s}(x_6u_1 + x_7u_2) \right] \end{aligned} \quad (45)$$

In this formulation, c_5 and c_6 are introduced as new positive real design parameters.

The dynamics of the system as described by equation (45) suggest that the control inputs u_1 and u_2 must be carefully selected to nullify the expressions within the curly brackets on the right side of (45). By zeroing these terms and solving the resulting system of linear equations, we obtain the control law:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \delta_0 & \delta_1 \\ \delta_2 & \delta_3 \end{bmatrix}^{-1} \begin{bmatrix} J\mu_2 + Jc_5z_5 + Jz_3 \\ c_6z_6 + v_2 + z_4 \end{bmatrix} \quad (46)$$

where the coefficients are defined as:

$$\begin{aligned} \delta_0 &= 2R_r\frac{x_2}{\sigma L_s}\frac{M_{sr}}{L_r}x_6, & \delta_1 &= 2R_r\frac{x_2}{\sigma L_s}\frac{M_{sr}}{L_r}x_7, \\ \delta_2 &= p\kappa\frac{x_2}{\sigma L_s}\frac{M_{sr}}{L_r}x_7, & \delta_3 &= -p\kappa\frac{x_2}{\sigma L_s}\frac{M_{sr}}{L_r}x_6. \end{aligned} \quad (47)$$

It is notable that the matrix $\begin{bmatrix} \delta_0 & \delta_1 \\ \delta_2 & \delta_3 \end{bmatrix}$ is nonsingular. Indeed, its determinant,

$$D = \delta_0\delta_3 - \delta_1\delta_2 = -2p\kappa R_r \left(\frac{x_2}{\sigma L_r L_s} M_{sr} \right)^2 \Phi_r^2$$

does not vanish in practice due to the machine's inherent magnetic flux. Implementing the control law (46) for u_1 and u_2 leads to:

$$\dot{V}_3 = -c_3z_3^2 - c_4z_4^2 - c_5z_5^2 - c_6z_6^2 \quad (48)$$

demonstrating that this is a negative definite function over the state space defined by (z_3, z_4, z_5, z_6) .

This leads us to the following proposition:

Proposition 4. Consider the closed-loop system comprising the induction machine, modeled by (15c) - (15g), and the nonlinear control laws (36a), (36b), (38), and (41). The system exhibits the following properties:

1. The closed-loop error system, in the coordinates (z_3, z_4, z_5, z_6) , follows the equations:

$$\begin{aligned} \dot{z}_3 &= -c_3z_3 + z_5 & (49a) \\ \dot{z}_4 &= -c_4z_4 + z_6 & (49b) \\ \dot{z}_5 &= -c_5z_5 - z_3 & (49c) \\ \dot{z}_6 &= -c_6z_6 - z_4 & (49d) \end{aligned}$$
2. This linear system is globally asymptotically stable with respect to the Lyapunov function V_3 , ensuring that errors (z_3, z_4, z_5, z_6) , diminish exponentially fast regardless of initial conditions.

4. Results from Simulation

The control strategy outlined in Section 3, which integrates the optimizer (16) along with the control laws (20), (26), and (46), has been thoroughly evaluated through simulation. The specifications for the simulated system are as follows:

- Electric Grid: Single phase, 230V, 50Hz
- AC/DC Rectifier: Inductance $L_o=16\text{mH}$; Capacitance $C=1.6\text{mF}$; Modulation frequency 10 kHz
- Photovoltaic Modules: 30 series-connected SM55 modules, each rated at 55 W, detailed in Table 1.
- Multiphase Induction Motor: Nominal power $P_n= 2.2 \text{ kW}$; Voltage $U_{sn}=400\text{V}$; Current $I_{sn}=5\text{A}$; Frequency $f=50\text{Hz}$; Speed $\Omega_n=1435\text{rpm}$.

The design parameters ($c_1 = 100, c_2 = 10, d = 1000, c_3 = 50, c_4 = 100, c_5 = 200, c_6 = 500$) were selected through trial and error, proving their efficacy in simulations. The experimental setup

was modeled in Matlab/Simulink, utilizing a simulation timestep of $5\mu\text{s}$, necessary to accommodate the 10 kHz switching frequency of the inverter and rectifier.

Radiation variation, depicted in Figure 5, includes scenarios of low ($\lambda=200\text{W}/\text{m}^2$) and high ($\lambda=900\text{W}/\text{m}^2$) radiation. Simulations were performed at a constant temperature of 45°C . The graph shows how the voltage reference dynamically adjusts to changes in solar radiation, demonstrating the responsiveness of the system to environmental changes.

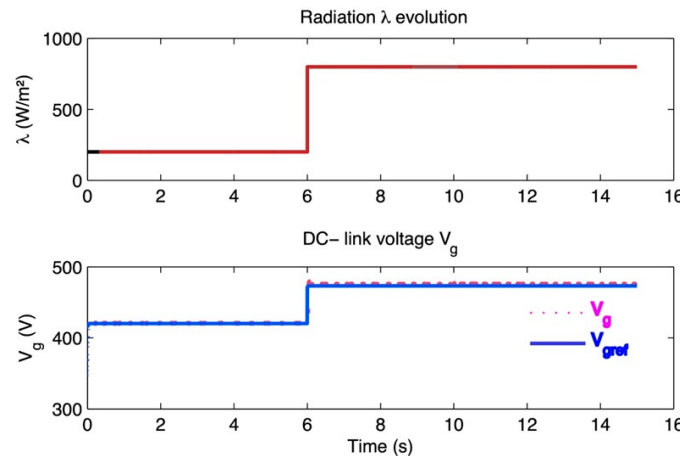


Figure 5. Upper: Radiation λ variation (W/m^2), lower: Photovoltaic voltage V_g (V).

The control system drives the machine to operate at variable water flow rates, as illustrated in Figure 6. The rotor flux norm Φ_{ref} , set at its nominal value of 1 Weber, shows successful convergence to the reference values, indicating precise control over both flow rate and rotor flux.

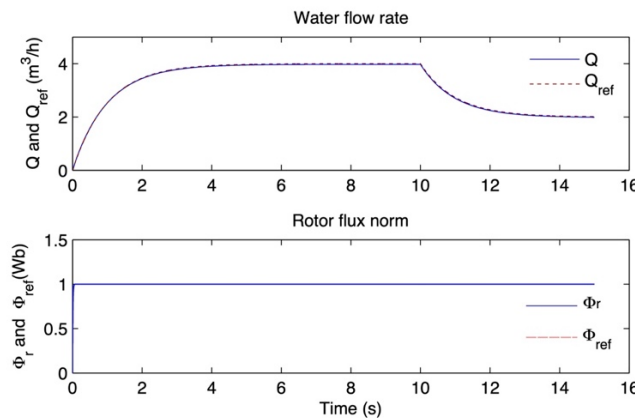


Figure 6. Upper: Water flow rate $Q(\text{m}^3/\text{h})$, lower: Rotor flux norm Φ_{ref} (Wb).

Figure 7 highlights the effectiveness of Power Factor Correction (PFC), demonstrating how the input current synchronizes with the grid voltage while the system adjusts to changes in solar radiation and load demands. Notably, the control ratio stabilizes rapidly after any fluctuations, showcasing the controller's robustness and adaptability. The results confirm that the multiloop controller effectively meets its objectives, delivering satisfactory transient performance and efficiently managing various system states under changing environmental conditions. As shown in Figure 7, the upper graph presents the behavior of the measured input current. The current frequency remains consistent and synchronized with the voltage frequency. Specifically, the current either aligns with or is out of phase with the supply network voltage, which is typical during periods of low solar radiation when the photovoltaic (PV) generator produces insufficient power to meet the demands of

the induction motor and centrifugal pump system. Conversely, during periods of high solar radiation, the PV generator produces excess power, which is then fed back into the grid. This behavior aligns with the requirements for Power Factor Correction (PFC). Additionally, the upper graph in Figure 7 shows that the control ratio stabilizes at a consistent value after transient fluctuations caused by changes in solar radiation and the water flow rate reference. It is important to note that the control ratio becomes negative when the system exports excess power to the grid.

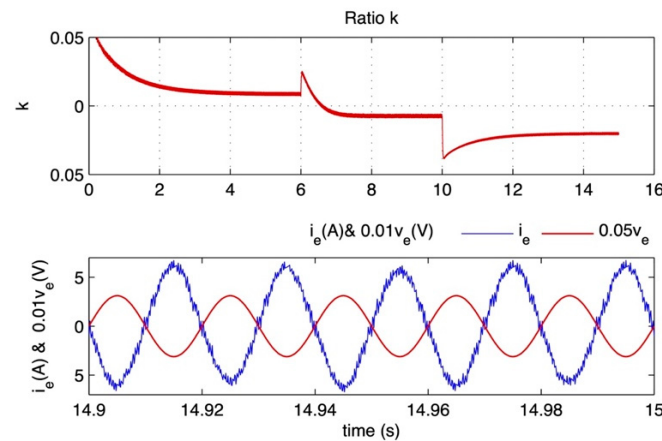


Figure 7. Upper: Ratio k , lower: Zoom on the curves of i_e and $0.01 v_e$ over the time interval [14.9s, 15s].

The closed-loop responses illustrated in Figures 5 and 7 clearly indicate that the multiloop controller successfully achieves all its specified objectives, delivering satisfactory transient performance.

5. Conclusions

This research tackles the challenge of achieving Maximum Power Point Tracking (MPPT) in the grid-connected photovoltaic systems. We propose a novel control strategy employing backstepping design controllers, derived from the system's nonlinear model (15a-15g). This approach integrates the dynamics of the DC/AC inverter with the three-phase grid inductor, thereby eliminating the need for a DC/DC chopper. To adapt to varying operational conditions, particularly fluctuations in solar radiation, a voltage reference optimizer is implemented.

The developed MPPT controller combines this optimizer with voltage reference generation (16) and control laws (46), offering a more efficient solution for identifying the maximum power point of the PV array. This method outperforms traditional techniques like Resistive Control Coefficient (RCC), Incremental Conductance (IncCond), and Perturb and Observe (P&O) by achieving MPPT without oscillations around the maximum power point and avoiding the pitfalls of using power, current, and voltage derivatives, which can lead to divide-by-zero errors.

The robustness and effectiveness of this MPPT controller have been validated through simulations, demonstrating its ability to meet control objectives under diverse environmental conditions.

Abbreviations

The following abbreviations are used in this manuscript:

MPIM	Multiphase Induction Motor
PVG	Photovoltaic Generator
MPPT	Maximum Power Point Tracking
PFC	Power Factor Correction
PWM	Pulse Width Modulation

IncCond	Incremental Conductance
P&O	Perturb and Observe
RCC	Resistive Control Coefficient

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