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Alejandro Fernández-Ferrero *

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Self-Interacting Extended Scalar Field and Emergent Scale Dynamics in Flat Spacetime

Alejandro Fernández-Ferrero

Basque Country, Bilbao, Spain; alexfdezferr@gmail.com

Abstract

We study a self-interacting scalar field defined in flat spacetime, where the interaction couples the field to its ground state configuration. Using an ansatz for this ground state, the Klein-Gordon equation allows an analytical solution that requires two constraints: one independent of the excitation level and another that depends on it. These constraints organize the accessible configurations and make it possible to describe a process in which the mass evolves until it reaches a stable regime. This leads, at late times, to a residual value whose order of magnitude is similar to that of the lightest neutrinos, although in this framework it is not interpreted as a particle mass but as a geometric remnant of the relaxation process. From these constraints, relations arise between the residual value and the vacuum energy, which in this approach are understood as consequences of the stabilization mechanism itself. In addition, by modeling quantum transitions between adjacent levels of the system, we obtain an effective expansion rate for the scale parameter associated with the field, whose magnitude is compatible with late-time cosmological expansion. The model is presented as an effective framework in flat spacetime, and its scope and limitations are discussed explicitly.

Keywords: self-interacting scalar field; emergent geometry; quantum constraints; vacuum energy; mass relaxation mechanism; cosmological scale dynamics; Minkowski spacetime; dark energy and dark matter unification

1. Introduction

The relationship between spacetime structure and the dynamics of quantum fields [1] remains one of the fundamental problems in theoretical physics. Since the development of general relativity and quantum processes, many works have explored frameworks in which the internal properties of fields can influence the geometry of the universe [2,3]. In this context, models based on the Klein-Gordon equation have played an important role, especially when self - interaction terms are included, since they generate nontrivial dynamics beyond the free-field regime [4–6].

The present work studies a class of quantum fields related to extended formulations used in theoretical frameworks and quantum cosmology [7], whose structure allows the introduction of scalable geometric parameters in flat spacetime. From another viewpoint, several recent proposals examine the possible emergence of geometric properties from extended fields. One line considers the idea of quantum compositeness of gravity, where spacetime is treated as a condensate of gravitons whose saturation controls the vacuum energy [8]. Another line explores quantum gravity scenarios in which dark energy is not a fixed parameter but a dynamical quantity whose scale decreases due to the structure of the vacuum [9]. Both perspectives suggest that certain geometric variables may arise from a more fundamental field substrate, establishing relations between energy density and horizon scale that motivate the analysis presented here.

The central idea of this study is that the accessible configurations of the field χ, defined under specific constraints, do not correspond to conventional energy eigenstates but induce dynamical fluctuations that may be interpreted as effective geometric expansions. This approach makes it possible to obtain analytical relations connecting extreme scales of the universe, from the Planck length as a minimal limit to the radius of the observable universe. To develop this idea, we start from

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a Lagrangian with a self-interaction term of the form $\lambda\chi^4$ treated in a specific regime. The choice $\lambda > 0$ is not arbitrary: in this case, the associated potential is stable and tends to favor expansive behavior [10,11] and an increase in the effective mass parameter [12,13]. In contrast, for $\lambda < 0$ the potential becomes unstable and leads to contractive behavior incompatible with consistent physical scenarios [14,15]. From the Lagrangian with $\lambda > 0$ we derive the Klein-Gordon equation and discuss the assumptions required to obtain an analytical solution. Within this framework, we propose an ansatz χ_0' for the ground state that allows the self-interacting dynamics to be approximated by a term of the form $\lambda\chi_0'^2\chi^2$.

A relevant feature of the model is the emergence of a hierarchy of mass scales, ranging from values of order the Planck mass to scales close to those of the lightest neutrinos, which provides a way to estimate the vacuum energy density in this setting [16]. This hierarchy follows from an analytical relation between extreme spatial scales, showing how the internal structure of the field and its interactions can manifest at macroscopic levels [17,18]. These results indicate that quantum dynamics may play a meaningful role even at cosmological scales [19]. Within this framework, the transition between these scales is described by a relaxation process that leads to a stable value of the mass parameter, a mechanism that is examined in detail in later sections.

Furthermore, the regime $\lambda > 0$ can be associated with expansive behavior compatible with the observed acceleration of the universe, generally attributed to dark energy [20,21], providing an effective framework for describing large-scale evolution. In our analysis, the positive self-interaction of the field in its ground state allows us to define an expansion rate directly from the dynamical solution, with values consistent with classical estimates [22]. In this context, quantum constraints appear that impose discrete values on the product of the spacetime scale parameter and the effective mass parameter, which allows certain geometric features to be interpreted as emerging from the internal dynamics of the system [23].

In summary, this work presents a theoretical framework in which the positive self-interaction of a quantum scalar field ensures stability and introduces natural mechanisms for spacetime expansion and the generation of a mass spectrum. The formulation relies on explicit assumptions and analytical solutions that allow us to explore connections between quantum behavior and large-scale dynamics. It should be emphasized that the analysis is carried out entirely in flat spacetime, and the results must be interpreted within this effective regime, with the expectation of extending these ideas to more realistic scenarios in future work.

Throughout this work, we adopt the standard relativistic notation for spacetime coordinates, denoted by $x^\mu = (x^0, x^1, x^2, x^3)$, along with the metric signature $(+,-,-,-)$. We employ natural units where $\hbar = c = 1$ and the relation between covariant and contravariant indices follows the usual conventions. Occasionally, $x^i = (x^1, x^2, x^3)$ is used as a common index for spatial coordinates or their transforms.

2. Self-Interaction Framework and Analytic Construction

We consider a scalar quantum field χ whose small fluctuations around a ground state profile χ_0 drive the system through self-interaction processes. If χ_0 depends on the spacetime coordinates x^μ with respect to an arbitrary reference point x'^μ, we restrict attention to a neighborhood of size set by a scale parameter δ. Within this region, the detailed structure of χ_0 may be replaced by an approximate profile that retains only its leading spatial and temporal variations, allowing the dynamics to be captured effectively without requiring the exact solution of the full background configuration. This approximation reflects the fact that the dominant self-interaction effects are controlled by the local curvature of χ_0 around x'^μ, rather than its global behavior. Consequently, χ_0 is represented by a simplified ansatz that is sufficiently accurate in the δ-neighborhood while enabling analytical progress in the subsequent derivation.

$$(x^\mu - x'^\mu)^2 < \delta^2 \;\Rightarrow\; \chi_0(x^\mu - x'^\mu) \approx \chi_0'(x^\mu - x'^\mu), \qquad (2.1)$$

where χ_0' denotes an approximation to the ground state profile that will be used to obtain analytical solutions. By introducing the appropriate scaling parameters into the description, one of our aims is to identify quantum constraints on those scales. These constraints will, in turn, define the set of accessible configurations and suggest limitations on the effective spacetime structure, allowing a connection to emerge between χ and the background geometry in which it is defined.

Within this setting we introduce a ground-state ansatz χ_0' tailored to enable an analytical treatment of the Klein-Gordon equation that follows from the Lagrangian specified below. Physically, χ_0' is intended to capture the leading interaction between the ground state and its spacetime environment, accurate enough to support a controlled estimate of vacuum contributions in spatially extended configurations.

2.1. *Lagrangian Formulation and Choice of Interaction*

We start from a scalar potential containing both a mass term and a self-interaction term,

$$V(\chi) = \frac{1}{2}m^2\chi^2 + \frac{1}{4!}\lambda\chi^4.$$

In the vicinity of the ground state configuration $\chi \to \chi_0$, we treat the self-interaction effectively as quartic and, at larger departures, we regard it as approaching a semiclassical background potential characterized by the ground state itself. This leads us to recast the potential in a form that encodes the feedback between the background profile and fluctuations:

$$V(\chi) = \frac{1}{2}m^2\chi^*\chi + \frac{1}{2}\lambda\chi_0^2(\chi^*\chi). \tag{2.2}$$

Although the interaction term is written in the Lagrangian as $\chi_0^2(\chi^*\chi)$, it is important to emphasize that only the field $\chi(x)$ requires complex conjugation in order to guarantee the reality of the Lagrangian in physical spacetime. The profile $\chi_0(x)$ acts as a background configuration and remains strictly real at all stages of the analysis, including within the transformed space introduced later on. Accordingly, the Lagrangian reads

$$\mathcal{L} = \frac{1}{2}\partial_\mu\chi^*\partial^\mu\chi - \frac{1}{2}m^2\chi^*\chi - \frac{1}{2}\lambda\chi_0^2(\chi^*\chi). \tag{2.3}$$

This formulation features a quadratic coupling $\chi_0^2(\chi^*\chi)$ that represents the interaction between the ground state profile and the field fluctuations. It provides a convenient way to track the backreaction between the vacuum configuration and its spacetime background in an extended field, linking the properties of the effective potential to the intrinsic structure of the ground state. The analysis below demonstrates how this formulation yields the self-interacting Klein-Gordon equation, supports the proposed ground-state ansatz, and establishes the quantum constraints that delimit the accessible configurations.

2.2. *Ground-State Ansatz and Quantum Constraints*

Starting from Lagrangian (2.3), the Klein-Gordon equation for a self-interacting potential is derived

$$\partial_\mu\partial^\mu\chi + m^2\chi + \lambda\chi_0^2\chi = 0, \tag{2.4}$$

At this stage, the field χ depends on the spacetime coordinates x^μ. In order to enable an analytical solution of the differential equation, we replace the ground state of the field, denoted by χ_0, with an ansatz χ_0', thus we can write

$$\partial_\mu\partial^\mu\chi + m^2\chi + \lambda\chi_0'^2\chi = 0, \tag{2.5}$$

The proposed form of the approximation function is as follows

$$\chi_0' = v_0\left[1 - \frac{1}{2}\frac{(x^0 - x'^0)^2}{\delta_t^2} - \frac{1}{2}\frac{(x^i - x'^i)(x^i - x'^i)}{\delta_s^2}\right], \tag{2.6}$$

obviously, it depends on the spacetime x-coordinates. Furthermore, the distances of these coordinates from an arbitrary fixed point x' are introduced scaled by two parameters, δ_t and δ_s, distinguishing, a priori, between temporal and spatial scaling, that is, introducing a different factor

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for each. Among the spatial coordinates, the same factor is used for all spatial components x^i to preserve homogeneity. In addition, a parameter v_0 is introduced to represent the amplitude of the ground state. On the other hand, the negative sign appearing in the ansatz is required to ensure that the ground state χ_0, not the approximation χ'_0, is well-behaved at infinity for the coordinate transformation that is proposed to be carried out in the next step. With all these particularities, we now perform a change of coordinates, moving from the physical spacetime to the dimensionless auxiliary ξ-space, defined below

$$\xi^0 = \frac{x^0 - x'^0}{\delta_t}\ ;\ \xi^i = \frac{x^i - x'^i}{\delta_s}\ \text{, under which } \chi(x) = \tilde{\chi}(\xi) \tag{2.7}$$

Under this transformation, the field is written as $\chi(x) = \tilde{\chi}(\xi)$ and taking into account that $|\xi|^2 = (\xi^0)^2 + \xi^i\xi^i = (\xi^0)^2 + (\xi^1)^2 + (\xi^2)^2 + (\xi^3)^2$, the ansatz takes the form

$$\tilde{\chi}'_0 = v_0\left(1 - \frac{1}{2}|\xi|^2\right), \tag{2.8}$$

in accordance with condition (2.1), the approximation $|\xi|^2 < 1$ is applied only to the ansatz, ensuring that its quadratic form remains valid within the local region where the auxiliary ξ-coordinates are defined, hence

$$\tilde{\chi}'^2_0 \approx v_0^2(1 - |\xi|^2), \tag{2.9}$$

substituting this into the Klein-Gordon equation

$$\frac{1}{\delta_t^2}\partial^2_{\xi^0}\tilde{\chi} - \frac{1}{\delta_s^2}\nabla^2_\xi\tilde{\chi} + m^2\tilde{\chi} + \lambda v_0^2(1 - |\xi|^2)\tilde{\chi} = 0. \tag{2.10}$$

A normalizable ground state solution is given by

$$\tilde{\chi}_0 = v_0 \mathrm{e}^{-\frac{1}{2}|\xi|^2}, \tag{2.11}$$

indeed, once the ansatz has been proposed in the indicated form, a decaying exponential with amplitude v_0 is precisely what one would expect for the ground state. This means that, within the level of approximation adopted in (2.1), we obtain a particular solution consistent with that requirement. To check this, we can simply approximate the decaying exponential to first order in a polynomial expansion and verify that it reproduces the ansatz, meaning that $\tilde{\chi}_0 \to \tilde{\chi}'_0$ when $|\xi|^2 < 1$. Moreover, this function is well-behaved and bounded, tending to vanish outside the limits where the approximation is no longer valid. What remains now is to determine under which constraints it satisfies the differential equation. To this purpose, the function is differentiated and substituted directly into the equation to identify the conditions under which the terms cancel and the equation is satisfied. In consequence, starting with the transformed time coordinate

$$\partial_{\xi^0}\tilde{\chi}_0 = -\xi^0\tilde{\chi}_0 \Rightarrow \partial^2_{\xi^0}\tilde{\chi}_0 = -\tilde{\chi}_0 + (\xi^0)^2\tilde{\chi}_0,$$

and correspondingly with the transformed spatial coordinates, taking into account their homogeneous character

$$\partial_{\xi^i}\tilde{\chi}_0 = -\xi^i\tilde{\chi}_0 \Rightarrow \nabla^2_\xi\tilde{\chi}_0 = -3\tilde{\chi}_0 + \xi^i\xi^i\tilde{\chi}_0,$$

Thus, by substituting both differential expressions, we arrive at the following relation

$$\frac{1}{\delta_t^2}(-\tilde{\chi}_0 + (\xi^0)^2\tilde{\chi}_0) - \frac{1}{\delta_s^2}(-3\tilde{\chi}_0 + \xi^i\xi^i\tilde{\chi}_0) + m^2\tilde{\chi}_0 + \lambda v_0^2\tilde{\chi}_0 - \lambda v_0^2|\xi|^2\tilde{\chi}_0 = 0,$$

on the one hand, we see that it must satisfy

$$\frac{1}{\delta_t^2} = \lambda v_0^2, \text{and } -\frac{1}{\delta_s^2} = \lambda v_0^2, \tag{2.12}$$

and on the other hand, we also see that it must satisfy

$$-\frac{1}{\delta_t^2} + \frac{3}{\delta_s^2} + m^2 + \lambda v_0^2 = 0, \tag{2.13}$$

where the coefficients –1 and +3 stem from the second time derivative and the diagonal terms of the Laplacian evaluated over the ground state solution. Moreover, as a consequence of these constraints, the temporal and spatial scale parameters are uniquely fixed to the values

$$\delta_t = \delta, \text{and } \delta_s = i\delta, \tag{2.14}$$

The appearance of the imaginary unit in the spatial scale parameter $\delta_s = \mathrm{i}\delta$ is not a consequence of the sign convention of the metric itself, but of the Wick-type rotation implicitly performed when transforming Minkowski space into an effectively Euclidean ξ-space. Depending on whether the rotation is applied to the temporal or spatial sector, the factor i would appear in δ_t or δ_s respectively; the choice made here is purely operational and does not affect the physical content of the field. Moreover, this causes both relations that appear in (2.12) to become a single one $\lambda\delta^2 v_0^2 = 1$, and then (2.13) simplifies in this particular case to

$$\delta^2 m^2 = 3.$$

Up to this point, we have analyzed the particular solution. There is no doubt that the proposed equation and its ground state solution are mathematically very similar to the standard approach used to solve the quantum harmonic oscillator [26]. Hence, we propose a general solution consisting of the Hermite polynomial $H_n(\xi) = H_n(\xi^0)H_n(\xi^1)H_n(\xi^2)H_n(\xi^3)$ multiplied by the decaying exponential of the particular solution, namely

$$\tilde{\chi}_n = \sum_n C_n H_n(\xi) \mathrm{e}^{-\frac{1}{2}|\xi|^2}, \tag{2.15}$$

consequently, the amplitude for the ground state, $n = 0$, must be $C_0 = v_0$. This general solution has the usual form associated with separation of variables. In this regard, a parameter α^2 could have been introduced to redefine the mass m^2 that appears in the Lagrangian, that is, $m'^2 = m^2 + \alpha^2$. However, in the case under study, such a redefinition of the mass is not considered necessary since it does not contribute anything significant to the results; therefore, the analysis proceeds with the original value in the Lagrangian. On the other hand, due to the structure of the general solution involving Hermite polynomials, the field $\chi(x)$ may acquire complex components after the inverse transformation $\xi = \xi(x)$, since this mapping introduces an imaginary factor. Nevertheless, this does not affect the analytical treatment in the transformed space, where $\tilde{\chi}(\xi)$ is consistently handled as a real field. The explicit presence of the product $\chi^*\chi$ in the physical Lagrangian ensures hermiticity without imposing restrictions on the quantum number n or on the parity of the excitation levels, thereby preserving the coherence of the entire construction. Moreover, following the same consistency conditions as for the ground state, we obtain a quantum constraint that does not depend on the excitation level of the field

$$\lambda\delta^2 v_0^2 = 1, \tag{2.16}$$

and by induction, in the general case, we also obtain the following quantum constraint, which does depend on the excitation level

$$\delta^2 m^2 = 2\left(4n + \frac{3}{2}\right), \tag{2.17}$$

where $n = n_0 = n_1 = n_2 = n_3$, that is, a simplified way of writing it by assuming homogeneity in the spacetime coordinates, although its use in general does not necessarily require imposing that condition. Therefore, we refer to these algebraic consistency relations as quantum constraints because they arise from the discrete structure of the Hermite solutions.

Using the transformation relations (2.7) and the quantum constraints imposed on the scaling parameters δ_t and δ_s in (2.14), we can relate a distance in the physical spacetime to its corresponding counterpart in the auxiliary ξ-space as follows

$$\frac{1}{\delta^2}(x_\mu - x'_\mu)(x^\mu - x'^\mu) = |\xi|^2, \tag{2.18}$$

The left-hand side of this expression, except for the multiplicative scale factor δ, is purely geometric. In fact, it corresponds to the relativistic interval s^2, therefore the transformation shown below has a geometric character, in the sense that it relates the Lorentz invariant spacetime interval s^2 to the dimensionless Euclidean distance $|\xi|^2$ in the ξ-space, this is

$$s^2 = \delta^2|\xi|^2, \tag{2.19}$$

This relation is controlled by the parameter δ, which effectively rescales the spacetime interval. Since the auxiliary ξ-coordinates are obtained from the spacetime x-coordinates in a manner analogous to a Wick rotation, according to (2.14), in that context, the proposed geometric relation is more consistent as a whole than the individual transformations in (2.7). On the other hand, we can specify that the spacetime interval scaled by δ ultimately depends on the state $\tilde{\chi}_n$ in which the field is found, through the quantum constraints obtained. This means that the dilation of the interval results from a possible transition between states or from a particular field configuration. This does not correspond to a modification of the Minkowski metric, but to an emergent rescaling in the transformed ξ-space. In other words, it can be interpreted as certain geometric features of the spacetime background emerging from the field itself.

2.3. Time-Independent Regime and Extended Configuration Space

In this particular case, and in order to isolate the purely spatial behaviour of the system, we work directly in the transformed ξ-space, meaning that we assume from the outset a field $\tilde{\chi}(\xi)$ that depends explicitly on the ξ-coordinates. This implies starting from a transformed Lagrangian written in the following form, which serves as the basis for the subsequent derivation

$$\tilde{\mathcal{L}}(\xi) = \frac{1}{2\delta_t^2}\left(\partial_{\xi^0}\tilde{\chi}\right)^2 - \frac{1}{2\delta_s^2}\left(\nabla_\xi\tilde{\chi}\right)^2 - \frac{1}{2}m^2\tilde{\chi}^2 - \frac{1}{2}\lambda\tilde{\chi}_0^2\tilde{\chi}^2, \tag{2.20}$$

it is the Lagrangian in (2.3) but rewritten using the transformations in (2.7). On the other hand, it must be considered that the action $S(x)$ transforms as $\tilde{S}(\xi) = \int_{-\infty}^{\infty}\delta_t\delta_s^3 d^4\xi\ \tilde{\mathcal{L}}(\xi)$. Now, we assume that the scaling parameters satisfy the condition $\delta_t \gg \delta_s$. As a consequence, the transformed time coordinate $\xi^0 \ll \xi^i$, this condition effectively suppresses the time-dependence of the field in the transformed description, making $\tilde{\chi}$ independent of ξ^0 in a controlled approximation, then, $\tilde{\chi}(\xi^0,\xi^1,\xi^2,\xi^3) \to \tilde{\chi}(\xi^1,\xi^2,\xi^3) = \tilde{\chi}(\xi^i)$. From a physical point of view, this amounts to assuming an implicit time τ that is fully extended and evolves much more slowly than t, with the relation $\tau = t/\delta_t$; this auxiliary time does not play a dynamical role, but merely parametrizes a slow background evolution. The essential point is that the field then depends only on the spatial components, and the spacetime interval s^2 becomes a spatial interval, yielding a proper distance L^2. The temporal coordinate τ is effectively left aside, behaving like an absolute time independent of the chosen reference frame, in this way, we avoid any undefined or ambiguous behavior when determining the time evolution of the system parameters. Under these conditions, we can write

$$\tilde{\mathcal{L}}(\xi) = -\frac{1}{2\delta_s^2}\left(\nabla_\xi\tilde{\chi}\right)^2 - \frac{1}{2}m^2\tilde{\chi}^2 - \frac{1}{2}\lambda\tilde{\chi}_0^2\tilde{\chi}^2, \tag{2.21}$$

and consequently, in this case, the action transforms as $\tilde{S}(\xi) = \int_{-\infty}^{\infty}\delta_s^3 d^3\xi\ \tilde{\mathcal{L}}(\xi)$, which is fully equivalent to the physical action, with the measure adjusted only by the scaling factors introduced in the coordinate transformation. From this point on, the steps to derive the Klein-Gordon equation and obtain a general solution are essentially the same as in the previous section, with only minor differences. For example, in this case $|\xi|^2 = \xi^i\xi^i = (\xi^1)^2 + (\xi^2)^2 + (\xi^3)^2$, where it is preferable to use the notation $\xi^i\xi^i$ instead of $|\xi|^2$ to make clear that only spatial components are being summed. As in the previous subsection, the ansatz $\tilde{\chi}_0'$ is intended to capture the leading behaviour of the background configuration in the region where the transformed expansion remains valid, we have

$$\tilde{\chi}_0'^2 \approx v_0^2(1 - \xi^i\xi^i), \tag{2.22}$$

and by substituting this into the Klein-Gordon equation derived from the proposed Lagrangian, we obtain

$$-\frac{1}{\delta_s^2}\nabla_\xi^2\tilde{\chi} + m^2\tilde{\chi} + \lambda v_0^2(1 - \xi^i\xi^i)\tilde{\chi} = 0, \tag{2.23}$$

It is observed that a ground state solution to this equation can likewise be obtained through a decaying exponential with amplitude v_0, of the form

$$\tilde{\chi}_0 = v_0 \mathrm{e}^{-\frac{1}{2}\xi^i\xi^i}, \tag{2.24}$$

following the same procedure as in the previous section, we obtain the same quantum constraint $\lambda\delta^2 v_0^2 = 1$, with $\delta_s = \mathrm{i}\delta$, as well as a slightly different one that depends on the excitation level, precisely like

$$\delta^2 m^2 = 2(3n + 1), \tag{2.25}$$

with $n = n_1 = n_2 = n_3$, corresponding to the three spatial dimensions in the homogeneous case. A general solution for the field can also be written as

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$$\tilde{\chi}_n = \sum_n C_n H_n(\xi) e^{-\frac{1}{2}\xi^i\xi^i}, \tag{2.26}$$

with $H_n(\xi) = H_n(\xi^1)H_n(\xi^2)H_n(\xi^3)$, where $H_n(\xi^i)$ denotes the Hermite polynomials for each individual coordinate component. Likewise, in this case, the amplitude C_0 of the ground state must coincide with the amplitude v_0 defined for the ansatz. To clarify, the appearance of the factor i in δ_s reflects the Wick-type structure of the transformation and does not imply any complex behaviour of $\tilde{\chi}(\xi)$, which remains strictly real in this space.

On the other hand, following equation (2.18), a distance in physical space relates to its counterpart in the transformed ξ-space through

$$-\frac{1}{\delta^2}[(x^i - x'^i)(x^i - x'^i)] = \xi^i\xi^i, \tag{2.27}$$

taking the definition of the relativistic interval s into account, together with the fact that $\xi^0 \ll \xi^i$, and recalling that the sign convention $(+,-,-,-)$ is being used, a physical distance L can be defined such that $L^2 = -s^2(\xi^0 \ll \xi^i)$. This is because, with this convention, the spatial interval is negative, so a minus sign is introduced in its definition to make it positive. In this way, we can write

$$L^2 = \delta^2\, \xi^i\xi^i. \tag{2.28}$$

This geometric transformation is similar to (2.19), which was derived in a general form including the temporal coordinate, but in this case, where an extended time interval has been assumed, only a spatial interval is being evaluated. This equivalence between the physical distance and the spatial interval means that the distance L can be also regarded as a spatial geometric invariant.

The general solution (2.26) obtained for the field and the methodology developed around it is similar to that used in the quantum harmonic oscillator, so its properties can be applied here as well. This allows us to define differential operators $\hat{a}(\xi)$ and $\hat{a}^\dagger(\xi)$ that induce excitations, namely

$$\hat{a}(\xi) = \frac{1}{\sqrt{2}}(\xi + \partial\xi)\,;\ \hat{a}^\dagger(\xi) = \frac{1}{\sqrt{2}}(\xi - \partial\xi). \tag{2.29}$$

These operators retain the same algebraic structure as in the standard harmonic oscillator formulation, since the differential representation in ξ-space is fully real. Furthermore, these are not occupation operators in the sense of creating or destroying particles in a given mode; in an extended field such as the one considered here, that cannot be defined in such a direct and simple way. Rather, $\hat{a}(\xi)$ acts as an operator that removes excitations, while $\hat{a}^\dagger(\xi)$ acts as an operator that generates excitations. With this in mind, the following relations that describe them are

$$\hat{a}(\xi)|n\rangle = \sqrt{n}|n-1\rangle\,;\ \hat{a}^\dagger(\xi)|n\rangle = \sqrt{n+1}|n+1\rangle. \tag{2.30}$$

In this setting, the number operator $\hat{N}$ is defined as

$$\hat{N} = \hat{a}^\dagger(\xi)\hat{a}(\xi), \tag{2.31}$$

although these operators mirror those of the usual creation-annihilation algebra, their role here is purely to classify spatial excitations of the extended field $\tilde{\chi}$." Then, $\hat{N}$ satisfies

$$\hat{N}|n\rangle = n|n\rangle. \tag{2.32}$$

Applied to a given state, this operator yields the number of excitations; this property will be used to determine, in the large-n limit, the evolution of the field through its emergent parameters.

3. Results and Interpretation

In this section we evaluate the physical implications of the quantum constraints derived in Section 2, focusing on the effective mass, the emergent spectrum, the vacuum energy density and the dynamical behaviour of the scale parameter.

3.1. Effective Mass and Level Dependence

To compute a physically measurable effective mass, we can rewrite the Lagrangian (2.21) grouped in the following way

$$\tilde{\mathcal{L}}(\xi) = \frac{1}{2\delta^2}\left(\nabla_\xi\tilde{\chi}\right)^2 - \frac{1}{2}(m^2 + \lambda\tilde{\chi}_0^2)\tilde{\chi}^2, \tag{3.1}$$

it is important to stress that the effective mass defined here does not arise from radiative corrections in the usual quantum field theoretic sense, but from the coupling between $\tilde{\chi}$ and its background configuration $\tilde{\chi}_0$. This allows us to identify the following term, m^2_{eff}, which is habitually referred to as the effective mass in this context, then

$$m^2_{\text{eff}} = m^2 + \lambda\langle\tilde{\chi}^2_0\rangle, \tag{3.2}$$

we see that it is the sum of the bare quadratic mass term m^2 and a term proportional to the square of the field's ground state $\tilde{\chi}^2_0$. Naturally, this latter term depends on the transformed ξ-coordinates, so it is necessary to perform some kind of averaging in that transformed space in order for the effective mass to be well defined. In this regard, the average $\langle\tilde{\chi}_0\rangle$ is evaluated over the transformed Euclidean space, where $\tilde{\chi}_0$ is strictly real and normalizable as follows

$$\langle\tilde{\chi}_0\rangle = \int_{-\infty}^{\infty} \delta_s^3 d^3\xi \; \mathrm{e}^{-\frac{1}{2}\xi^i\xi^i} = \delta_s^3(2\pi)^{3/2}, \tag{3.3}$$

this would act as a reference volume that will be used to consistently normalize the averages throughout the analysis. Then, with this definition in place, we can proceed to carry out the following calculation

$$\langle\tilde{\chi}^2_0\rangle = \frac{1}{\delta_s^3(2\pi)^{3/2}}\int_{-\infty}^{\infty} \delta_s^3 d^3\xi \; \tilde{\chi}^2_0 = \frac{1}{(2\pi)^{3/2}} v_0^2 \int_{-\infty}^{\infty} d^3\xi \; \mathrm{e}^{-\xi^i\xi^i} = \frac{1}{2\sqrt{2}} v_0^2, \tag{3.4}$$

substituting this value into the effective mass gives

$$m^2_{\text{eff}} = m^2 + \frac{1}{2\sqrt{2}}\lambda v_0^2, \tag{3.5}$$

now, using the quantum constraint $\lambda\delta^2 v_0^2 = 1$ together with (2.25), we have

$$m^2_{\text{eff}} = \left(1 + \frac{1}{4\sqrt{2}(3n+1)}\right)m^2, \tag{3.6}$$

that is, the effective mass is determined directly by the bare mass, modified by a factor that depends on the field excitation level n. This means that we can examine how the effective mass changes from $n = 0$ up to $n \to \infty$. For example, for $n = 0$, we obtain

$$m_{\text{eff}} = \left(1 + \frac{1}{4\sqrt{2}}\right)^{1/2} m \; \approx 1.08\, m, \tag{3.7}$$

for $n \to \infty$, its value is

$$m_{\text{eff}} = m. \tag{3.8}$$

Consequently, the effective mass varies only marginally over such a wide range of values, from $n = 0$ to $n \to \infty$. This mild dependence on the excitation number is a consequence of the large hierarchy induced by the quantum constraint $\lambda\delta^2 v_0^2 = 1$. To a good approximation, its value can therefore be taken to be essentially equal to the bare mass in the original Lagrangian.

3.2. Hierarchy of Scales and Mass Stabilization

According to the analysis in the previous section, assuming an effective mass equal to the bare mass, we can estimate the value of the effective mass m_{eff_1}, for the case $n = 0$, taking the scale parameter of the ground state to have a lower bound equal to the Planck length, that is, $\delta = \ell_{\text{p}}$. In this way, using (2.25) directly, we can write

$$m_{\text{eff}_1} = m = \frac{\sqrt{2}}{\delta} = \frac{\sqrt{2}}{\ell_{\text{p}}} = \sqrt{2}\, m_{\text{p}}, \tag{3.9}$$

this yields a value of the order of the Planck mass m_{p}, which can be regarded as the expected value for a supposedly primordial field state in the case $n = 0$.

Another interesting case is to assume that the mass evolves from this initial primordial state to values with $n \gg 0$. We can represent this evolution by increasing both terms in relation (2.25). On the left-hand side we assume small, continuous increments, making it normally differentiable; this is justified because the variables involved are expected to approach a continuous spectrum for very large values of n. The term on the right-hand side, which depends only on n, simply increases by one unit, thus

$$\Delta(\delta^2 m^2) = 2[(3(n+1)+1) - (3n+1)], \tag{3.10}$$

working it out, we obtain

$$\Delta(\delta^2 m^2) = 2\delta m^2 \Delta\delta + 2\delta^2 m \Delta m = 6. \tag{3.11}$$

The identity $\Delta(\delta^2 m^2) = 6$ encapsulates the entire relaxation mechanism: it is the algebraic bridge between the discrete quantum structure of the field and the emergent large-scale behaviour. Once the stabilization condition $\Delta m = 0$ is imposed, this relation fixes the residual mass purely from geometric information. This condition represents a natural stabilization threshold for the effective mass, beyond which the internal dynamics no longer modify m. In other words, a transition occurs in which the scale parameter δ becomes dependent only on n, and under these conditions the mass is

$$m(\delta) = \sqrt{3}(\delta\Delta\delta)^{-1/2}. \tag{3.12}$$

This relation reveals that the stabilized mass is determined solely by the ratio between the global scale δ and the minimal increment $\Delta\delta$, linking microscopic and macroscopic scales within a single algebraic identity. Thus, we can set δ to a value equal to the radius of the observable universe, that is, $\delta = R = 4.4 \times 10^{26}$ m. For the increment of δ, and to remain consistent with the first case, we take it to be equal to the Planck length, $\Delta\delta = \ell_{\mathrm{p}}$. This choice corresponds to an initial minimal increment determined by the Planck scale and a final configuration encompassing the observable Universe. Substituting these values into the previous expression yields a result of

$$m_{\mathrm{eff}_2} = m = \sqrt{3}(\delta\Delta\delta)^{-1/2} = \sqrt{3}\left(R\ell_{\mathrm{p}}\right)^{-1/2} = 4.04 \times 10^{-3}\ \mathrm{eV}. \tag{3.13}$$

The value obtained for m_{eff_2} is of the same order as the mass of the lightest neutrinos, the smallest particle mass observed in the known universe [27,28]. This numerical coincidence does not rely on phenomenological inputs, but follows directly from the internal structure of the model. This value is obtained through a relaxation process starting from an initial state [29]. Once a stabilization condition is imposed, we suppose a priori the process ends at the moment when the underlying geometry becomes fully developed [30], which in this case occurs at $\delta = R$. In this way, a mass spectrum is generated from a primordial state $n = 0$, with an initial effective mass $m_{\mathrm{eff}_1} = \sqrt{2}\, m_{\mathrm{p}}$, to a stabilized state $n \gg 0$ with $m_{\mathrm{eff}_2} = 4.04 \times 10^{-3}$ eV.

Now, by substituting the mass value obtained in (3.12) into the quantum constraint (2.25), we obtain

$$\delta = 2\ell_{\mathrm{p}} n, \tag{3.14}$$

here, δ takes the form of a distance with discrete values, and its smallest possible value is directly tied to the Planck length. This direct dependence of δ on the discrete quantum number n follows from the stabilization condition $\Delta m = 0$; that is, it arises once the mass relaxation process has concluded. In this manner, a scenario emerges in which the field state $\tilde{\chi}_n$ acquires the ability to generate a spatial length element through δ and the dilation relation (2.28) associated with the underlying geometry in which the field is defined. At the same time, both n and δ appear as emergent quantum parameters of the field under the imposed quantum constraints. This suggests a form of self-consistency between the field and the geometric properties of the background on which it is established. What does appear clear is that, according to (3.14), the field excitations described by n have the capacity to assign length elements as a statistical property and, therefore, to modify the geometry of a given region in that way. It is true that the quantum number associated with n becomes very large once the mass has relaxed. This implies that the distances attributed to δ behave in a predictable classical regime, approaching ordered, quasi-continuous values. It is worth noting that although δ has the dimensional form of a distance, its essential role as an emergent parameter is that of a scale factor. This means that a length constructed from spatial coordinates does not necessarily have to be discrete or share the structure of δ. It is not the spatial coordinates themselves that are directly constrained, but rather the emergent parameters that define them. Consequently, we can determine the value of n from which stabilization occurs

$$n = \frac{1}{2}\frac{R}{\ell_{\mathrm{p}}} \approx 10^{61}, \tag{3.15}$$

obviously, much greater than zero [26]. Such large values of n justify treating the spectrum as effectively continuous in the dynamical analysis of Section 3.4. Likewise, under the same conditions, the ground state amplitude is computed as

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$$v_0 = \frac{1}{\sqrt{\lambda}} \frac{1}{2\ell_{\mathrm{p}} n}, \tag{3.16}$$

to this end, we have used the quantum constraint (2.16) together with the dependence of δ on n established here. This provides another relation showing that, in the stabilization regime, the ground state depends directly on the excited states as a consequence of the self-interaction processes encoded in the original Lagrangian.

3.3. Vacuum Energy and Scaling Relations

In general, the quantum vacuum is difficult to define precisely, and this becomes even more problematic for extended fields. The reason is that, in such systems, the canonical Fock space formalism cannot be rigorously applied. In most of these cases, one cannot unambiguously define particle creation and annihilation operators, and thus cannot assign well-defined occupation numbers to the field modes. In fact, identifying those modes with precision is itself ambiguous, and even the notion of localized individual particles becomes diffuse and ill defined.

As a central hypothesis, we assume that the vacuum of the field $\tilde{\chi}$ coincides with its ground state, identified with $n = 0$. This is the basic assumption we adopt to compute the vacuum energy density, interpreting n not as an occupation number, which cannot be rigorously established for extended fields, but as the index that labels the different excitation levels allowed by the field's own solutions. Since increasing values of n correspond to higher excitation states, it is reasonable to associate the level $n = 0$ with a state without real excitations, in the operational sense required by the notion of vacuum. Furthermore, the level $n = 0$ corresponds to the field configuration (2.24), characterized by the amplitude v_0 and a decaying exponential whose argument is $\xi^i \xi^i$, a spatial geometric invariant according to (2.28). This implies that the vacuum state, in this sense, is the same for all observers related by the relevant spatial symmetries, thus eliminating ambiguity in its definition [31].

On the other hand, the vacuum must be associated with the lowest energy state of the system, or more precisely, with the lowest energy state compatible with the quantum constraints imposed on the field. If we minimize the potential $V(\tilde{\chi}_0)$ defined in (2.2), that is, if we solve $V'(\tilde{\chi}_0) = 0$ in a neighborhood of $\tilde{\chi}_0$ and impose the constraints (2.16) and (2.25) for $n = 0$, we obtain a value of $\tilde{\chi}^2_{\min} = -v_0^2$. In some treatments this relation appears and leads to a negative vacuum energy density, which is not compatible with an expanding system in the standard cosmological model. However, the fact that the minimum energy is controlled directly by the amplitude of the ground state reinforces the conclusion that v_0 carries the essential information about the vacuum in our case.

With this in mind, we propose a general relation for computing the vacuum energy density ρ_{vac}, which contains two terms: a kinetic contribution $E_c(\tilde{\chi}_0)$ and a potential contribution $E_c(\tilde{\chi}_0)$. Both are averaged in the ground state over the entire spatial volume. The corresponding expression is

$$\rho_{vac} = \langle E_c(\tilde{\chi}_0) \rangle + \langle V(\tilde{\chi}_0) \rangle, \tag{3.17}$$

computing the kinetic energy term derived from the Lagrangian (3.1), with $\delta = \ell_{\mathrm{p}}$, as

$$E_c(\tilde{\chi}_0) = \frac{1}{2\ell_{\mathrm{p}}^2} \left(\nabla_\xi \tilde{\chi}_0\right)^2, \tag{3.18}$$

given the homogeneity of the spatial coordinate terms, we have

$$\langle E_c(\tilde{\chi}_0) \rangle = \frac{1}{2\ell_{\mathrm{p}}^2} \langle \left(\nabla_\xi \tilde{\chi}_0\right)^2 \rangle = \frac{3}{2\ell_{\mathrm{p}}^2} \langle \left(\partial_{\xi^i} \tilde{\chi}_0\right)^2 \rangle,$$

differentiating the ground state

$$\partial_{\xi^i} \tilde{\chi}_0 = -\xi^i \tilde{\chi}_0,$$

computing the average kinetic energy contribution, taking into account that it is averaged over the entire spatial volume according to (3.3), we obtain

$$\langle E_c(\tilde{\chi}_0) \rangle = \frac{3}{2\ell_{\mathrm{p}}^2} \langle (-\xi^i \tilde{\chi}_0)^2 \rangle = \frac{3}{2\ell_{\mathrm{p}}^2} \langle (\xi^i)^2 \tilde{\chi}_0^2 \rangle = \frac{3}{2\ell_{\mathrm{p}}^2 \delta_s^3 (2\pi)^{3/2}} v_0^2 \int_{-\infty}^{\infty} \delta_s^3 d^3\xi \, (\xi^i)^2 \mathrm{e}^{-\xi^i \xi^i},$$

this kinetic contribution is computed entirely in the ξ-space, where $\tilde{\chi}_0$ is real and the Euclidean structure ensures normalizability, the result is

$$\langle E_c(\tilde{\chi}_0)\rangle = \frac{3}{8\sqrt{2}}\frac{v_0^2}{\ell_{\mathrm{p}}^2}\,. \tag{3.19}$$

In exactly the same way, we can proceed with the next potential term

$$V(\tilde{\chi}_0) = \frac{1}{2}m^2\tilde{\chi}_0^2 + \frac{1}{2}\lambda\tilde{\chi}_0^4, \tag{3.20}$$

computing the average of the potential, taking into account that the first term has already been evaluated in (3.4)

$$\langle V(\tilde{\chi}_0)\rangle = \frac{1}{2}m^2\langle\tilde{\chi}_0^2\rangle + \frac{1}{2}\lambda\langle\tilde{\chi}_0^4\rangle = \frac{1}{2}m^2\left(\frac{1}{2\sqrt{2}}v_0^2\right) + \frac{1}{2\delta_s^3(2\pi)^{3/2}}\lambda v_0^4\int_{-\infty}^{\infty}\delta_s^3 d^3\xi\; \mathrm{e}^{-2\,\xi^i\xi^i},$$

by operating, simplifying and substituting terms, we obtain

$$\langle V(\tilde{\chi}_0)\rangle = \frac{1}{4\sqrt{2}}m^2v_0^2 + \frac{1}{16}\,\lambda v_0^4 = \frac{8+\sqrt{2}}{16\sqrt{2}}\frac{v_0^2}{\ell_{\mathrm{p}}^2}\,, \tag{3.21}$$

in which the potential term must be evaluated consistently with the quantum constraints derived in Section 2.3,

$$m_{(n=0)}^2 = \frac{2}{\ell_{\mathrm{p}}^2} \text{ and } \lambda_{(\delta=\ell_p)} = \frac{1}{\ell_{\mathrm{p}}^2 v_0^2}.$$

After these calculations, inserting the computed values of $\langle E_c(\tilde{\chi}_0)\rangle$ and $\langle V(\tilde{\chi}_0)\rangle$ into the starting expression (3.17) yields the result

$$\rho_{\mathrm{vac}} = \frac{14+\sqrt{2}}{16\sqrt{2}}\frac{v_0^2}{\ell_{\mathrm{p}}^2} \approx \frac{2}{3}\frac{v_0^2}{\ell_{\mathrm{p}}^2}\,, \tag{3.22}$$

we finally obtain a value for the vacuum energy density that depends exclusively on the amplitude v_0 of the ground state. This dependence reflects the essential role of the ground state amplitude in regulating the vacuum structure. However, it must be interpreted carefully because it is a dynamical one. The reason is that v_0 depends directly on n for $n \gg 0$, and therefore on the quantum state of the field and, consequently, on its evolution, as will be shown in the next section.

Choosing the numerical factor of ρ_{vac} by $2/3$, and taking into account the quantum constraint $\lambda\delta^2 v_0^2 = 1$, the vacuum energy density can be written as

$$\rho_{\mathrm{vac}} = \frac{2}{3}\frac{1}{\lambda\delta^2\ell_{\mathrm{p}}^2}\,, \tag{3.23}$$

this is a relation in which the vacuum energy density $\rho_{\mathrm{vac}}(\delta)$ is expressed in terms of a specific value of the model's main scale, implying that different densities can be obtained for different states of the scale. Then, given the scale of δ under consideration, for $\delta = \ell_{\mathrm{p}}$, a vacuum energy density for a contracted state $\rho_{\mathrm{vac}}^{\mathrm{c}}$, which represents the maximum energy density achievable within the model, can be defined as

$$\rho_{\mathrm{vac}}^{\mathrm{c}} = \frac{2}{3}\frac{1}{\lambda\ell_{\mathrm{p}}^4}\,, \tag{3.24}$$

at the opposite side of the scale, for $\delta = R$, a vacuum energy density for an extended state $\rho_{\mathrm{vac}}^{\mathrm{e}}$ [32,33], can likewise be defined as

$$\rho_{\mathrm{vac}}^{\mathrm{e}} = \frac{2}{3}\frac{1}{\lambda R^2\ell_{\mathrm{p}}^2}\,. \tag{3.25}$$

Given these two densities, which basically reflect a dynamic transition between two different scales, we can propose a ratio that matches the well-known hierarchy between the naive QFT vacuum density and the observed vacuum energy scale, ensuring the overall consistency of the model. This leads to the following expression

$$\frac{\rho_{\mathrm{vac}}^{\mathrm{c}}}{\rho_{\mathrm{vac}}^{\mathrm{e}}} = \frac{R^2}{\ell_{\mathrm{p}}^2} \sim 10^{122}, \tag{3.26}$$

a little over 120 orders of magnitude. This value reflects the well-known enormous gap between the energy density assigned to a primordial contracted state and that assigned to a fully expanded state; furthermore, within our framework, it points to a possible transition of the field $\tilde{\chi}$ from an $n = 0$ state to states with $n \gg 0$.

Alternatively, $\rho_{\mathrm{vac}}^{\mathrm{c}}$ is expressed solely as a function of the mass using (3.9)

$$\rho_{\mathrm{vac}}^{\mathrm{c}} = \frac{2}{3}\frac{m_{\mathrm{p}}^4}{\lambda}\,, \tag{3.27}$$

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the same can be done for $\rho^{\mathrm{e}}_{\mathrm{vac}}$, but in this case using (3.13) is also obtained

$$\rho^{\mathrm{e}}_{\mathrm{vac}} = \frac{2}{27}\frac{m^4}{\lambda}\,. \tag{3.28}$$

Moreover, if $\rho^{\mathrm{e}}_{\mathrm{vac}}$ is identified with the energy density ρ_Λ associated with the cosmological constant Λ, which is entirely natural in this extended field framework, the value of λ can be estimated by converting everything to the same units as

$$\rho^{\mathrm{e}}_{\mathrm{vac}} = \rho_\Lambda \Rightarrow \frac{2}{3}\frac{\hbar c}{\lambda R^2 \ell_{\mathrm{p}}^2} = \frac{\Lambda c^4}{8\pi G}\,,$$

thus, the result we obtain is

$$\lambda = \frac{2}{3}\frac{8\pi}{\Lambda R^2} \approx 0.8\,, \tag{3.29}$$

a very reasonable value for λ, close to unity, without invoking large orders of magnitude to justify density adjustments. With this adjusted value of self-interaction coupling constant, we can return to (3.28) that express a numerical relation between the vacuum energy density derived from the cosmological constant and the mass m^4, except for a small numerical coefficient. This expression is often described in the specialized literature as a coincidence when m is assumed to correspond to a neutrino mass [16]. However, it is not a coincidence, it is the result of a numerical equivalence derived directly from the fundamental relation (3.22). This equivalence stems from the quantum constraints used to compute $\rho^{\mathrm{e}}_{\mathrm{vac}}$; in particular, from $\lambda\delta^2 v_0^2 = 1$ that does not depend on the state of the field and, specifically, from the stabilization condition $\Delta m = 0$. Consequently, this yields a residual mass value that, as previously shown, is remarkably close to that of light neutrinos.

Finally, assuming a surface $S = 4\pi\delta^2$ that encloses the vacuum energy density ρ_{vac} as a bounding surface, and taking into account the quantum constraint $\lambda\delta^2 v_0^2 = 1$, and that the Planck length can be written in terms of the gravitational constant G, we combine these quantities to obtain the following interesting expression

$$S\rho_{\mathrm{vac}} = \frac{8\pi c^4}{3\lambda G}\,, \tag{3.30}$$

it is noteworthy that this geometric relation emerges without invoking gravitational dynamics, since the model remains strictly in flat spacetime. Moreover, the relation has been written explicitly in SI units to show that it does not depend on $\hbar$. This means that the computed vacuum energy density ρ_{vac}, in particular, and more generally its relation to the enclosing surface, exhibit a behavior that is independent of the quantum scale. Thus, this supports that the fundamental physical expression of ρ_{vac} is given by relation (3.22), or alternatively by this other relation (3.23); conversely, the vacuum energy density does not arise as fundamental form from the expressions proportional to m^4, which in this case should be understood only as a singular numerical equivalence. On the other hand, the product $S\rho_{\mathrm{vac}}$ is constant, indicating a possible coupling between them [34–37]. To justify this, we can refer to the self-interaction term $\tilde{\chi}_0^2\tilde{\chi}^2$ in the original Lagrangian, which describes an interaction between ground state and other states of the field. In this framework, if we relate the vacuum energy density ρ_{vac} with $\tilde{\chi}_0$, through v_0, it is reasonable to associate the surface S with the dynamics of the $\tilde{\chi}$ field. This implies that a specific state of the field, or any combination of states and its possible transitions, may be related to its surroundings giving rise to an evolution of the surface S.

Continuing in this direction, it is useful to further manipulate the previous expression. According to the interpretation given, we may define a vacuum energy E_0 contained within a volume V, such that the vacuum energy density can be written as $\rho_{\mathrm{vac}} = \Delta E_0/\Delta V$. Moreover, we may assume that the volume element is defined by a surface S and an infinitesimal length $\Delta\ell$ perpendicular to it, this is, $\Delta V = S\Delta\ell$. Substituting these relations into (3.30), we can then write

$$\Delta E_0 = \frac{8\pi c^4}{3\lambda G}\Delta\ell\,. \tag{3.31}$$

In this way, we obtain a direct linear relation between a portion of vacuum energy and a geometric length element. More importantly, the numerical factor connecting them contains the Einstein tension c^4/G, together with the self-interaction constant λ. This suggests the possibility of a self-conversion process between vacuum energy and geometric elements of the underlying geometry.

3.4. Dynamical Scale Evolution and Emergent Expansion

Certainly, the geometric transformation (2.28) shares some similarities with the flat FLRW metric. This suggests, as has been hinted in previous sections, the need to identify a dynamical law for the scale parameter δ that accounts for the evolution of the field $\tilde{\chi}$, allowing it to transition in an orderly way between its states. For example, given that metric, consider measuring the spatial distance between two coordinate points (x_1, y_1, z_1) and (x_2, y_2, z_2). It is also assumed that the measurement is performed simultaneously at both points, that is, $t_1 = t_2$. If the flat FLRW metric can be written as

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2),$$

then, in this particular case, the resulting physical length L is

$$L^2 = a^2(t) L^2_{\text{comoving}},$$

that is, the length L expands in time through a scale factor $a(t)$ multiplying a fixed comoving length L_{comoving}. The same calculation can be carried out by applying (2.27) to points 1 and 2, yielding an observable physical distance L equivalent to the previous case. This analogy does not imply that δ replaces the cosmological scale factor, but rather that it induces an emergent dilation compatible with FLRW-type expansion. In this sense, we can write

$$L^2 = \left(x_2^i - x_1^i\right)\left(x_2^i - x_1^i\right) = \delta^2\, \xi^i \xi^i,$$

similarly, this distance can be dilated by a parameter δ that rescales the transformed dimensionless Euclidean distance $\xi^i \xi^i$.

Thus, when a physical length L is evaluated in both frameworks, the parameters $a(t)$ and δ play the same role in the corresponding processes of expansion and dilation. These factors appear in formally different mathematical expressions: an FLRW metric on a flat background in the case of $a(t)$, obtained by solving the General Relativity equations for a homogeneous universe; and, in the case of δ, a geometric transformation used to describe a homogeneous extended $\tilde{\chi}$ field in space. However, despite the methodological differences in their origin, their interpretation coincides. This opens the possibility of defining, at least in principle, a time dependence for the scale parameter δ, allowing a static and scalable mathematical transformation to give rise to an active and dynamical scenario, which can be interpreted as an expansion of the observable physical space [38,39]. In this sense, the extended $\tilde{\chi}$ field may be said to exhibit an expansive dynamics compatible with FLRW. It must be emphasized that the FLRW metric and the geometric transformation (2.28) arise from entirely different physical considerations, yet both converge in proposing the same physical mechanism of expansion of their respective spacetime backgrounds. To this end, let us evolve both sides of equality (3.14) in time τ, that is

$$\dot{\delta} = 2\ell_{\text{p}} \frac{\Delta n}{\Delta \tau} = 2\ell_{\text{p}} \frac{1}{\Delta \tau}, \tag{3.32}$$

Assuming, as we are in the regime $n \gg 0$, that the transitions between the states of the field $\tilde{\chi}_n$ occur slowly and in an orderly manner over a time τ, that is, under an adiabatic regime and in the absence of any external perturbative mechanisms, then $1/\Delta\tau$ may reasonably be taken as a proportional approximation to the transition probability between states n and $n+1$. In turn, that probability is taken to be the square of the corresponding transition amplitude, namely

$$\frac{1}{\Delta \tau} \propto P(n \to n+1) = \left|\langle n+1|\hat{N}(\varepsilon)|n\rangle\right|^2, \tag{3.33}$$

the transition operator used is $\hat{N}(\varepsilon)$, this is the number operator $\hat{N}$ defined in (2.31), but with the aim of adding a perturbation term ε to destabilize the system and induce transitions between states. Therefore, we write

$$\hat{N}(\varepsilon) = (\hat{a}^\dagger + \varepsilon)(\hat{a} + \varepsilon). \tag{3.34}$$

The idea is that the operator $\hat{N}$, by itself, cannot induce transitions between different field states. Thus, we have introduced a perturbation whose nature and origin will be clarified later. Under these conditions, the calculation proceeds in the standard way, considering that the states n are orthonormal and satisfy relations (2.30)

$$\langle n+1|\hat{N}(\varepsilon)|n\rangle = \langle n+1|(\hat{a}^\dagger + \varepsilon)(\hat{a} + \varepsilon)|n\rangle = \langle n+1|\hat{N} + \varepsilon(\hat{a}^\dagger + \hat{a}) + \varepsilon^2|n\rangle,$$

then, by operating, all the terms become zero except for

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$$\langle n+1|\hat{N}(\varepsilon)|n\rangle = \varepsilon\langle n+1|\hat{a}^{\dagger}|n\rangle = \varepsilon\sqrt{n+1},$$

finally, $1/\Delta\tau$ can be written by introducing a proportionality constant A as

$$\frac{1}{\Delta\tau} \propto \varepsilon^2 n = A\,\varepsilon^2 n, \tag{3.35}$$

returning to (3.32), substituting the value obtained for $1/\Delta\tau$, and using (3.14) again to eliminate n we obtain

$$\frac{\dot{\delta}}{\delta} = A\varepsilon^2. \tag{3.36}$$

The constants A and ε encapsulate the effective transition rate between neighbouring configurations, and their precise values are determined up to order-unity factors; within this framework, we find that the dynamics of the parameter δ is consistent with the Hubble's law, as is $a(t)$, and thus it is compatible with the dark energy processes involved in the standard model.

It is of interest to examine in more detail the properties of the constants A and ε. For example, we can refer to the definition of the operator $\hat{a}(\varepsilon)$ to determine the meaning of the perturbation ε introduced into the system, namely

$$\hat{a}(\varepsilon) = \hat{a} + \varepsilon = \frac{1}{\sqrt{2}}\left((\xi + \sqrt{2}\varepsilon) + \partial\xi\right) \Rightarrow \xi(\varepsilon) = \xi + \sqrt{2}\varepsilon,$$

that is, ε appears as a dispersion σ_ε of the variable ξ, or equivalently, ε manifests itself as an intrinsic fluctuation of the system. If we compare this dispersion with the corresponding one for the vacuum state, which is exactly a normalized Gaussian function according to (2.24), we find

$$\xi(\varepsilon) = \xi + \sigma_\varepsilon \Rightarrow \sqrt{2}\varepsilon = \sigma_\varepsilon \approx 1 \Rightarrow \varepsilon \approx \frac{1}{\sqrt{2}}. \tag{3.37}$$

This estimate is consistent with treating ε as an intrinsic fluctuation of the Gaussian ground state. As a result, the amplitude of the perturbation ε has an upper bound slightly below unity, and its effect on the expansion dynamics of the system can be interpreted as arising from quantum vacuum fluctuations that propagate through the field via the self-interaction processes described in the Lagrangian. Regarding the value of A, we can assume that it is a scaling factor. Since it has dimensions of s^{-1}, we may interpret it as the frequency associated with one cycle of the system. At the scales we are considering, $n \gg 0$, it is reasonable to expect that such a cycle is of order $A \approx c/R$. With this value, and using the previously determined value of $\varepsilon \approx 1/\sqrt{2}$, we can rewrite (3.36) and obtain

$$\frac{\dot{\delta}}{\delta} \approx 3.4 \times 10^{-19}\,(s^{-1}). \tag{3.38}$$

This value lies within the expected order of magnitude of the Hubble parameter at late times. Taken together, the considerations presented here make it reasonable to say that the spacetime expansion described by $a(t)$ could, in our case, be identified with a field $\tilde{\chi}$ that extends under the influence of δ, sharing the same dynamics as $a(t)$ but governed differently due to a set of quantum constraints. The specific process involved, as described above, arises from fluctuations in the vacuum of the field. Moreover, this is supported by a favorable mathematical structure, namely one with transition probabilities proportional to the excitation number, which drives an ordered extension of the field and thereby dilates the surrounding space. This interpretation could relegate the space coordinates x^i to a passive role, perhaps as mere labels without physical meaning, suggesting instead that parameters emerging from quantum fields with deeper significance may take their place.

In this context, the hypothesis $\Delta m = 0$, in (3.11), which defines the moment when the mass m has relaxed and becomes stabilized, we interpret this as a regime change in which the field enters an expansion phase described by (3.38), a phase that in the standard model is associated with dark energy domain. It seems remarkably coincidental that imposing a mass stabilization condition on a quantum constraint of the field unexpectedly yields an evolutionary mechanism compatible with FLRW. Viewed from this perspective, both phases appear perfectly synchronized, with the underlying processes emerging naturally without forcing any dynamics [40–44]. Consequently, a closer examination of what this actually implies shows that

$$\Delta m = 0 \ \Rightarrow\ \rho \propto \frac{1}{\delta^3} \text{ and } p = 0, \tag{3.39}$$

that is, the density evolves inversely with the volume, freely and without exerting pressure on its surroundings, a notable feature of dark matter once it enters the expansion regime driven by dark energy. This suggests a qualitative link between the stabilization of m and the onset of the expansion driven by δ, pointing to a unified description of dark matter and dark energy within the same field framework. Moreover, since the phase transition entails a significant loss of vacuum energy, the interactions between the vacuum and the rest of the field become less energetic, naturally reducing the activity of the system. In this model, where the vacuum acts as the driver and regulator of the dynamics, this decrease supports conditions compatible with high quantum occupation states and possible coherent configurations. In this context, the residual mass m_{eff_2} obtained in (3.13) may gravitationally condense into bosonic halos that contribute to dark matter. Thus, the formation of cold dark matter and the onset of the expansion driven by dark energy become linked through the mass stabilization condition, an internal requirement of the model and consistent with a unified dark sector framework.

4. Conclusions

In this work we have presented an effective framework based on an extended self-interacting scalar field defined in flat spacetime. The formulation relies on a specific interaction between the field and its ground state configuration, from which a set of quantum constraints emerges. These constraints give rise to discrete parameters that organize the accessible configurations of the system and allow the appearance of geometric features within this effective description.

A central aspect of the analysis is the relaxation mechanism associated with the mass parameter. Under the imposed constraints, the system evolves from an initial regime near the Planck scale to a stable residual value whose magnitude is comparable to the lightest neutrino masses, although in this framework it is interpreted as a geometric remnant rather than a particle mass. This process provides a coherent way of connecting microscopic and macroscopic scales within the same logical structure.

Within this setting we have also evaluated the vacuum energy density associated with the ground state of the field. The analysis shows that this quantity scales with the geometric parameters introduced in the model, and that the relation between residual mass scale and vacuum energy arises naturally from the underlying constraints. Although the framework is effective and limited to flat spacetime, these results offer a controlled way to compare contracted and extended regimes and to relate the self-interaction strength to observational estimates of the cosmological vacuum density.

The scale parameters that emerge from the constraints are not static, but display a dynamical behavior linked to transitions between neighbouring configurations of the field. By modelling these transitions, we obtain an effective expansion rate for the scale parameter, compatible in order of magnitude with that expected for a late-time accelerated regime. This provides an interpretation of how large-scale evolution may arise from quantum processes in an extended field, without implying a direct identification with classical cosmological dynamics. The stabilization of the residual scale also suggests possible connections with dark-sector phenomenology at a qualitative level.

Overall, the model developed here illustrates how self-interaction, ground state structure and quantum constraints can combine to produce effective geometric behavior. The results highlight several conceptual links between internal field dynamics, vacuum properties and large-scale evolution. Although simplified by the assumption of flat spacetime, the framework offers a basis from which more complete or realistic extensions may be explored in future work.

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