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Article

Quantum Information Copy Time (QICT): Operational Definition, Minimal Locality Bounds, Hydrodynamic Susceptibilities, and a Programmatic Outlook

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Abstract

We formulate and benchmark an operational timescale—the *quantum information copy time*—that quantifies how fast a localized bias in an initial many-body state becomes *remotely certifiable* by measurements restricted to a distant receiver region. The definition is intrinsically information-theoretic: for a fixed distinguishability threshold $\eta \in (0, 1)$, the copy time $\tau_{\text{copy}}(A \rightarrow B; \eta)$ is the earliest time at which the reduced states on B become distinguishable with advantage at least η , measured by trace distance and equivalently by the optimal Helstrom measurement. We present (i) a minimal theorem that isolates which inputs are genuinely nontrivial (locality, conservation laws, and an explicit receiver observable class), and (ii) a controlled hydrodynamic closure in which the copy time is governed by a *second-moment spectral susceptibility* that couples the receiver advantage to the slowest transport mode. We then provide reproducible exact-diagonalization benchmarks in the XXZ chain that (a) extract finite-size transport diagnostics with conservative uncertainty quantification and (b) delimit failure modes in integrable and near-integrable regimes. TEBD/MPS calculations are included only as qualitative cross-checks (Supplementary File S1) and are not used to support asymptotic scaling claims. Finally, we situate these results inside the broader QICT program, where locality-preserving quantum cellular automata (QCA) and code-subspace constraints motivate using copy-time distances as primitives for an operational geometry; we keep this outlook explicitly conjectural and separate from the proved/validated statements.

Keywords: quantum information; state discrimination; trace distance; hydrodynamics; quantum cellular automata; Lieb–Robinson bounds; diffusion; operational timescales

1. Introduction

The purpose of this paper is twofold. First, we define and analyze a concrete, *operational* timescale for information propagation in many-body quantum dynamics: the *quantum information copy time* (QICT). Second, we use this operational object as a micro-level input to a broader, programmatic research direction that aims to connect microscopic locality-preserving unitary dynamics to macroscopic transport, and ultimately to effective causal structure.

The copy time is designed to avoid two common ambiguities. (i) It is not a proxy for “scrambling” in the OTOC sense; rather it measures *remote certifiability* by a receiver restricted to a spatial region B and an admissible measurement class. (ii) It is not tied to a particular coarse-graining scheme: the definition is strictly in terms of quantum hypothesis testing (Helstrom–Holevo theory) on reduced density operators.

To make the scope and logical dependencies explicit, we separate the paper into three layers:

- L1. Operational layer (fully general).** Definitions and inequalities that hold for arbitrary finite-dimensional quantum systems and any unitary (or CPTP) dynamics.
- L2. Hydrodynamic closure layer (assumptions explicit).** A theorem that relates copy time to transport parameters under a *verifiable* single-mode window and an explicit projection formalism.

- L3. Programmatic outlook (conjectural).** A QCA / code-subspace motivated picture in which copy-time distances define an operational geometry and provide a “currency” for certifying macroscopic invariants. This part is clearly labelled as outlook and is not used to justify any of the rigorous claims.

Related work and positioning.

State discrimination and optimal measurements follow Helstrom and Holevo [1,2]; locality constraints follow Lieb–Robinson-type bounds [3,4]; and unitary dynamics with conservation laws can generate dissipative hydrodynamics (diffusive poles) at the level of operators and correlators [5–7]. The diagnostic we emphasize differs from OTOCs and from entanglement growth: copy time is a *receiver-limited* operational latency, and its scaling can be dominated by conserved-mode transport even when scrambling is fast.

2. Operational Definition and Preliminaries

2.1. Copy Time as Receiver-Limited Hypothesis Testing

Let ρ_+ and ρ_- be two initial global states on a finite lattice that differ only inside a sender region A (e.g., a local “tilt” in a conserved charge). Let $\mathcal{U}_t$ denote the time-evolution channel (unitary or CPTP), and let $\rho_{\pm}(t)$ be the evolved states. For a receiver region B , define the reduced states $\rho_{\pm,B}(t) = \text{Tr}_{B^c} \rho_{\pm}(t)$.

Definition 1 (Receiver advantage and copy time). *For a fixed threshold $\eta \in (0, 1)$, the receiver advantage is*

$$\text{Adv}_{A \rightarrow B}(t) := \frac{1}{2} \|\rho_{+,B}(t) - \rho_{-,B}(t)\|_1 \quad (1)$$

and the copy time is

$$\tau_{\text{copy}}(A \rightarrow B; \eta) := \inf\{t \geq 0 : \text{Adv}_{A \rightarrow B}(t) \geq \eta\}. \quad (2)$$

By Helstrom’s theorem, Adv is the optimal bias over random guessing for discriminating $\rho_{+,B}(t)$ vs. $\rho_{-,B}(t)$ with equal priors; equivalently it is the dual norm over observables $0 \leq M \leq \mathbb{I}$. We use $D_{\text{tr}}(\sigma, \rho) := \frac{1}{2} \|\sigma - \rho\|_1$ for trace distance.

2.2. Elementary Bounds and Caveats

We record two standard inequalities used later. The first is a direct duality bound.

Lemma 1 (Standard tool: trace-norm duality). *This identity is standard in quantum hypothesis testing and is included only to fix notation. For any Hermitian X and any observable O with $\|O\|_{\infty} \leq 1$,*

$$|\text{Tr}(OX)| \leq \|X\|_1. \quad (3)$$

In particular,

$$\text{Adv}_{A \rightarrow B}(t) = \frac{1}{2} \sup_{\|O_B\|_{\infty} \leq 1} \text{Tr}[O_B(\rho_{+,B}(t) - \rho_{-,B}(t))]. \quad (4)$$

The second is Pinsker’s inequality. We state the required support condition explicitly.

Lemma 2 (Pinsker inequality with full-rank condition). *If ρ is full rank (so that $D(\sigma|\rho) < \infty$ for all σ), then*

$$D_{\text{tr}}(\sigma, \rho) \leq \sqrt{\frac{1}{2} D(\sigma|\rho)}. \quad (5)$$

If ρ is not full rank, the inequality remains valid provided $\text{supp}(\sigma) \subseteq \text{supp}(\rho)$; otherwise $D(\sigma|\rho) = \infty$ and (5) is vacuous.

In all numerical benchmarks reported below we work in finite-dimensional Hilbert spaces (finite spin chains) and, where thermal reference states are invoked, at finite inverse temperature $\beta < \infty$; consequently the reduced density matrices encountered are effectively full rank (up to machine precision), and Lemma 2 is used only in this non-vacuous regime.

3. Minimal Locality Bounds and What Is Genuinely Nontrivial

3.1. Locality-Preserving Dynamics and Lieb–Robinson Constraints

Throughout, we assume a lattice with a metric $d(\cdot, \cdot)$, and we denote the distance between regions A, B by $d(A, B)$. For continuous-time local Hamiltonians or for locality-preserving circuits/QCAs, the standard consequence is a Lieb–Robinson-type bound: a local perturbation cannot influence distant observables faster than a finite velocity v_{LR} , up to exponentially small tails.

We deliberately *do not* conflate different settings. When we refer to LR bounds, we mean either (i) a continuous-time Hamiltonian/Lindbladian LR bound with constants (v, μ) or (ii) a circuit/QCA strict light cone. In each case, the structure is an upper bound on commutators of evolved local observables.

Theorem 1 (Locality-limited copying: Hamiltonian LR tails vs. circuit/QCA light cones). *Let $\mathcal{U}_t$ be the evolution channel (unitary or CPTP) on a lattice with metric $d(\cdot, \cdot)$. Assume one of the following standard locality structures for Heisenberg-evolved observables.*

(H) Continuous-time Hamiltonian/Lindbladian (Lieb–Robinson tails). *There exist constants $c, \mu, v > 0$ such that for any observables O_X supported on a finite region X and O_Y supported on Y ,*

$$\| [O_X(t), O_Y] \|_\infty \leq c |X| |Y| \|O_X\|_\infty \|O_Y\|_\infty e^{-\mu(d(X,Y)-v|t|)}. \quad (6)$$

(C) Discrete-time range- R circuits / reversible QCAs (strict light cone). *For each integer time $t \in \mathbb{N}$, the Heisenberg image of a local algebra satisfies*

$$\text{supp}(O_X(t)) \subseteq X^{+Rt} \quad \Rightarrow \quad [O_X(t), O_Y] = 0 \text{ whenever } d(X, Y) > Rt. \quad (7)$$

Let the initial pair $\rho_\pm$ differ only inside a sender region A and satisfy $\|\rho_+ - \rho_-\|_1 \leq 2$. Then for any receiver region B and any time t :

- Under **(H)**, the receiver advantage obeys

$$\text{Adv}_{A \rightarrow B}(t) \leq C_{\text{LR}} e^{-\mu(d(A,B)-v|t|)}, \quad (8)$$

for an explicit constant $C_{\text{LR}} = O(|A||B|)$ depending only on the LR constants and region sizes.

- Under **(C)**, one has the exact vanishing statement

$$\text{Adv}_{A \rightarrow B}(t) = 0 \quad \text{for all} \quad t < d(A, B)/R. \quad (9)$$

In particular, in both settings the copy time obeys the kinematic lower bound

$$\tau_{\text{copy}}(A \rightarrow B; \eta) \geq \begin{cases} \frac{d(A,B)}{v} - \frac{1}{\mu} \log(C_{\text{LR}}/\eta), & \text{in (H),} \\ d(A, B)/R, & \text{in (C).} \end{cases} \quad (10)$$

Remark 1 (What Theorem 1 does not do). *Theorem 1 is an upper bound on how early copy can occur; it does not provide a mechanism for achieving η at times $t \sim d(A, B)/v$. For an A -level contribution, the hard part is a lower bound (or matching scaling) in physically relevant regimes. This is precisely where conservation laws and hydrodynamics enter.*

3.2. Conservation Laws and an Explicit Receiver Class

Fix a (quasi-)local conserved charge $Q = \sum_x q_x$ with $[\mathcal{U}_t, Q] = 0$. We will focus on initial perturbations that are “charge-biased” in A . Importantly, the receiver need not have access to all observables on B ; in realistic settings one often restricts to a class $\mathcal{O}_B$ (e.g., low moments of charge, or few-body observables).

To avoid circularity, we proceed in two steps: (i) we provide a theorem under *minimal* hypotheses that yields a general lower bound in terms of an explicitly defined susceptibility-like object, and (ii) we identify an additional *single-mode hydrodynamic window* under which that object becomes computable from transport data.

4. Hydrodynamic Closure: From Charge Bias to Spectral Susceptibility

4.1. Setup: Local Equilibrium Manifold and Linearization

Let ρ_β be a reference Gibbs (or generalized Gibbs) state at inverse temperature β . Consider a small sender perturbation of “chemical potential” type,

$$\rho_\pm(0) = \frac{e^{-\beta H \mp \epsilon Q_A}}{\text{Tr}(e^{-\beta H \mp \epsilon Q_A})}, \quad (11)$$

where $Q_A = \sum_{x \in A} q_x$ and $\epsilon \ll 1$. To leading order in ϵ , the difference $\delta\rho(0) := \rho_+(0) - \rho_-(0)$ is linear in Q_A in the Kubo–Mori inner product.

4.2. A Principled Definition of the Second-Moment Susceptibility

Let $\mathcal{L}$ denote the (Heisenberg) Liouvillian superoperator $\mathcal{L}(O) = i[H, O]$ (or the generator of a CPTP semigroup in open settings). Let $\mathcal{P}$ be a projection onto the slow subspace spanned by the conserved density modes; concretely, $\mathcal{P}$ is the Mori projection with respect to the Kubo–Mori inner product. Define the reduced (effective) slow-sector operator using the Mori–Zwanzig projection formalism [6,7]. In Laplace space one obtains an exact identity for the projected dynamics,

$$(z - \mathcal{L}_{\text{eff}}(z))^{-1} = \mathcal{P}(z - \mathcal{L})^{-1}\mathcal{P}, \quad \mathcal{L}_{\text{eff}}(z) := \mathcal{P}\mathcal{L}\mathcal{P} + \mathcal{P}\mathcal{L}\mathcal{Q}(z - \mathcal{Q}\mathcal{L}\mathcal{Q})^{-1}\mathcal{Q}\mathcal{L}\mathcal{P}. \quad (12)$$

This expression is exact but generally nonlocal in time (the z -dependence encodes a memory kernel). In *closed unitary* dynamics, $\mathcal{L}(O) = i[H, O]$ is anti-self-adjoint and $\mathcal{L}_{\text{eff}}(z)$ is *not* a CPTP generator in general; it should be interpreted as a reduced linear-response operator in the Kubo–Mori geometry. In contrast, for genuinely open Markovian dynamics (Lindbladians) the same construction yields a bona fide dissipative generator on the slow manifold.

To obtain a usable closure we adopt a standard *Markovianized* approximation on a time window where the fast sector mixes rapidly: we replace $\mathcal{L}_{\text{eff}}(z)$ by its low-frequency limit $\mathcal{L}_{\text{eff}} := \mathcal{L}_{\text{eff}}(0)$ and assume that the fast subspace $\mathcal{Q}$ has a spectral gap $\Delta_{\text{fast}} > 0$ that controls memory decay. Under these hypotheses one recovers the familiar time-local approximation

$$\mathcal{L}_{\text{eff}} \approx \mathcal{P}\mathcal{L}\mathcal{P} - \mathcal{P}\mathcal{L}\mathcal{Q}(\mathcal{Q}\mathcal{L}\mathcal{Q})^{-1}\mathcal{Q}\mathcal{L}\mathcal{P}, \quad (13)$$

which we use in the theorems below, with an explicit error term quantifying the leakage into $\mathcal{Q}$ (see Proposition 1 and Theorem 2).

Definition 2 (Second-moment spectral susceptibility). *Let $\delta\mu$ be a small chemical-potential profile supported in A , and let J_B denote a receiver observable (e.g., the charge in B or its low moments) projected to the slow subspace. We define the second-moment spectral susceptibility by*

$$\chi_{A \rightarrow B}^{(2)} := \left\langle J_B, (-\mathcal{L}_{\text{eff}})^{-2} J_B \right\rangle_\beta \|\delta\mu\|^2, \quad (14)$$

where $\langle \cdot, \cdot \rangle_\beta$ is the Kubo–Mori inner product and $(-\mathcal{L}_{\text{eff}})^{-2}$ is understood on the orthogonal complement of the zero mode.

Remark 2 (Why (14) is the right object). The operator $(-\mathcal{L}_{\text{eff}})^{-1}$ is a resolvent that weights modes by inverse decay rate; squaring it yields a second moment that controls the time at which a receiver observable can accumulate a finite signal. Definition 2 makes explicit (a) the operator being inverted, (b) the topology (Kubo–Mori norm) in which neglected modes are controlled, and (c) the receiver class through J_B .

Proposition 1 (Fast-sector leakage bound in the Kubo–Mori topology). Let $\langle \cdot, \cdot \rangle_\beta$ be the Kubo–Mori inner product induced by a full-rank reference state ρ_β ,

$$\langle X, Y \rangle_\beta := \int_0^1 \text{Tr}(\rho_\beta^s X^\dagger \rho_\beta^{1-s} Y) ds, \quad \|X\|_\beta := \sqrt{\langle X, X \rangle_\beta}. \quad (15)$$

Assume that the fast-sector generator has a spectral gap $\Delta_{\text{fast}} > 0$ in this topology, in the sense that for all X with $\mathcal{P}X = 0$,

$$\langle X, -\mathcal{Q}\mathcal{L}\mathcal{Q}X \rangle_\beta \geq \Delta_{\text{fast}} \|X\|_\beta^2. \quad (16)$$

Then the leakage of slow data into the fast sector is exponentially suppressed:

$$\|\mathcal{Q}e^{t\mathcal{L}}\mathcal{P}\|_{\beta \rightarrow \beta} \leq e^{-\Delta_{\text{fast}} t}. \quad (17)$$

Consequently, for any receiver observable O_B with $\|O_B\|_\infty \leq 1$ and any slow-sector signal $X \in \text{Ran}(\mathcal{P})$,

$$|\langle O_B, \mathcal{Q}e^{t\mathcal{L}}X \rangle_\beta| \leq \|O_B\|_\beta \|\mathcal{Q}e^{t\mathcal{L}}\mathcal{P}\|_{\beta \rightarrow \beta} \|X\|_\beta \leq \|O_B\|_\beta \|X\|_\beta e^{-\Delta_{\text{fast}} t}. \quad (18)$$

This provides an explicit (conservative) topology-controlled bound on the neglected-mode contribution in Theorem 2.

4.3. Two Theorems: Minimal and Single-Mode

We now state two versions of the central result: a minimal statement and a single-mode hydrodynamic specialization.

Theorem 2 (Main theorem, minimal (non-circular) version with explicit error term). Assume (i) a finite-dimensional lattice system with a full-rank reference state ρ_β , (ii) a weak charge-biased perturbation of the form (11) with $\epsilon \ll 1$, and (iii) a Mori projection $\mathcal{P}$ onto the slow manifold such that the Markovianized effective operator $\mathcal{L}_{\text{eff}}$ in (13) is well-defined on $\text{Ran}(\mathcal{P})$. Let $J_B \in \text{Ran}(\mathcal{P})$ be a receiver observable supported in B with $\|J_B\|_\infty \leq 1$. Then, for all times $t \geq 0$,

$$\text{Adv}_{A \rightarrow B}(t) \geq \frac{\epsilon}{2} |\langle J_B, e^{t\mathcal{L}_{\text{eff}}}\mathcal{P}(Q_A) \rangle_\beta| - \frac{\epsilon}{2} |\langle J_B, \mathcal{Q}e^{t\mathcal{L}}\mathcal{P}(Q_A) \rangle_\beta| + O(\epsilon^3). \quad (19)$$

If, in addition, the fast-sector gap hypothesis (16) of Proposition 1 holds with rate Δ_{fast} , then the neglected-mode term admits the explicit conservative bound

$$|\langle J_B, \mathcal{Q}e^{t\mathcal{L}}\mathcal{P}(Q_A) \rangle_\beta| \leq \|J_B\|_\beta \|\mathcal{P}(Q_A)\|_\beta e^{-\Delta_{\text{fast}} t}. \quad (20)$$

Thus, on any time window where $e^{-\Delta_{\text{fast}} t} \ll 1$, the receiver advantage is governed (up to explicit exponentially small leakage) by the projected slow dynamics, without assuming a priori that the receiver “sees the slow mode”.

Corollary 1 (Explicit “slow signal minus fast leakage” form). Under the assumptions of Theorem 2 and Proposition 1, define

$$S_{A \rightarrow B}(t) := |\langle J_B, e^{t\mathcal{L}_{\text{eff}}}\mathcal{P}(Q_A) \rangle_\beta|, \quad C_{\text{fast}} := \|J_B\|_\beta \|\mathcal{P}(Q_A)\|_\beta. \quad (21)$$

Then, for all $t \geq 0$,

$$\text{Adv}_{A \rightarrow B}(t) \geq \frac{\epsilon}{2} S_{A \rightarrow B}(t) - \frac{\epsilon}{2} C_{\text{fast}} e^{-\Delta_{\text{fast}} t} + O(\epsilon^3). \quad (22)$$

In particular, once $t \gg \Delta_{\text{fast}}^{-1}$, the advantage is controlled by the explicit slow-sector correlator $S_{A \rightarrow B}(t)$ up to a quantified error.

Theorem 3 (Main theorem, single-mode hydrodynamic window (checkable hypotheses)). Assume the hypotheses of Theorem 2 together with:

- (S1)**Single slow pole**: on a wavelength band $k \in [k_{\min}, k_{\max}]$ the slow spectrum on $\text{Ran}(\mathcal{P})$ consists of a single nonzero mode with decay rate $\Gamma(k) = Dk^2 + o(k^2)$ and a gap Δ_2 to the next slow mode on that band;
- (S2)**Receiver overlap**: the projected receiver observable has nonzero overlap with that mode, quantified by the form factor $F_B(k)$ entering (A5);
- (S3)**Fast mixing**: the fast-sector leakage is controlled by Proposition 1 with rate Δ_{fast} on the window of interest.

Then, for a one-dimensional sender–receiver separation $\ell = d(A, B)$ and fixed threshold $\eta \in (0, 1)$, the copy time satisfies the transport-limited scaling

$$\tau_{\text{copy}}(A \rightarrow B; \eta) = \frac{\ell^2}{4D} \left(1 + o(1)\right) + O(\log(1/\eta)), \quad (23)$$

in the hydrodynamic window $\Delta_{\text{fast}}^{-1} \ll t \ll \min\{L^2/D, \Delta_2^{-1}\}$, with explicitly bounded systematic errors from: (i) finite-size discretization of k (Appendix E), (ii) slow-sector multi-mode contamination $O(e^{-\Delta_2 t})$, and (iii) fast-sector leakage $O(e^{-\Delta_{\text{fast}} t})$ via (20). The prefactor can be expressed in terms of the susceptibility $\chi_{A \rightarrow B}^{(2)}$ in Definition 2.

A full proof is given in Appendices A–B. The key point is that the nontrivial input is *not* the statement “the receiver sees the slow mode”; rather it is the explicit control of $\mathcal{E}(t)$ and the explicit coupling coefficient between Q_A and J_B in the projected dynamics.

5. Worked Hydrodynamic Example: One-Dimensional Diffusion Kernel

To make Theorem 3 concrete, we work out the diffusion-kernel signal at the level needed to turn “diffusion implies $\tau \sim \ell^2$ ” into a quantitatively checkable statement.

5.1. Linear-Response form of the Reduced-State Difference

Write $\rho_{\pm}(t) = \mathcal{U}_t(\rho_{\pm}(0))$ with $\rho_{\pm}(0)$ given by (11). Expanding to first order in ϵ yields

$$\delta\rho(t) := \rho_+(t) - \rho_-(t) = \epsilon \mathcal{U}_t(\delta\rho_{Q_A}) + O(\epsilon^3), \quad (24)$$

where $\delta\rho_{Q_A}$ is the Kubo–Mori tangent vector associated to Q_A at ρ_{β} . For any receiver observable O_B with $\|O_B\|_{\infty} \leq 1$,

$$\text{Tr}(O_B \delta\rho_B(t)) = \text{Tr}(O_B \text{Tr}_{B^c} \delta\rho(t)) = \text{Tr}(O_B(t) \delta\rho_{Q_A}) + O(\epsilon^3). \quad (25)$$

Thus the operational advantage is lower-bounded by any explicit choice of O_B .

5.2. Diffusion Equation for the Conserved Density

Assume a single conserved density $q(x)$ on a ring of length L with hydrodynamic equation

$$\partial_t \delta q(x, t) = D \partial_x^2 \delta q(x, t), \quad (26)$$

valid on the window $t \in [t_0, t_1]$ and wavelengths $\lambda \gg 1$ (in lattice units). On the ring, the Fourier modes $k_n = \frac{2\pi n}{L}$ evolve as

$$\delta q_{k_n}(t) = e^{-Dk_n^2 t} \delta q_{k_n}(0). \quad (27)$$

For a localized initial bias in A , $\delta q(x, 0)$ has broad Fourier support but its long-time profile is controlled by the smallest nonzero $|k_n|$.

5.3. Receiver Signal and a Concrete Threshold-to-Time Relation

Let $Q_B(t) = \int_B q(x, t) dx$ be the receiver charge in an interval B centered at distance ℓ from A with width w . In the linear-response regime around ρ_β , the expected receiver shift is proportional to the chemical potential profile with coefficient given by the static susceptibility $\chi = \partial \langle q \rangle / \partial \mu$:

$$\delta \langle Q_B(t) \rangle \approx \chi \int_B \delta \mu(x, t) dx. \quad (28)$$

Approximating $\delta \mu$ by the Gaussian heat kernel on $\mathbb{R}$ for times $t \ll L^2/D$,

$$\delta \mu(x, t) \approx \frac{\epsilon}{\sqrt{4\pi Dt}} \exp\left(-\frac{(x - \ell)^2}{4Dt}\right), \quad (29)$$

we obtain the scaling

$$\delta \langle Q_B(t) \rangle \sim \epsilon \chi w (4\pi Dt)^{-1/2} \exp\left(-\frac{\ell^2}{4Dt}\right), \quad (30)$$

up to relative corrections of order $w^2/(Dt)$ and tD/L^2 . Choosing O_B proportional to the centered receiver charge (normalized to $\|O_B\|_\infty \leq 1$) and using Lemma 1 gives a lower bound

$$\text{Adv}_{A \rightarrow B}(t) \geq \frac{1}{2} |\text{Tr}(O_B \delta \rho_B(t))| \gtrsim c_0 |\delta \langle Q_B(t) \rangle|, \quad (31)$$

where c_0 is an explicit normalization constant. Solving $\text{Adv}_{A \rightarrow B}(t) = \eta$ for t yields, at leading order,

$$\tau_{\text{copy}}(A \rightarrow B; \eta) \approx \frac{\ell^2}{4D} \left[1 + O\left(\frac{\log(1/\eta)}{\log(\ell)}\right) \right], \quad (32)$$

making precise the statement that the *distance dependence* is ℓ^2 while η enters only logarithmically in the diffusive window. A finite-volume version based on (27) and the lowest nonzero mode $k_{\min} = 2\pi/L$ yields the crossover from Gaussian kernel behavior to the ring's exponential mode decay.

5.4. An Exactly Solvable Gaussian Diffusion Toy Model (Fully Analytic)

To remove any ambiguity about “which observable is optimal” and what constants control the threshold, we include a reference model in which the Helstrom measurement can be written in closed form. Consider commuting hypotheses in which the only receiver-relevant variable is the coarse-grained charge Q_B and, conditional on either hypothesis, Q_B is Gaussian with means $\pm m(t)$ and a common variance σ_B^2 (equilibrium charge fluctuations in B). In this setting the Helstrom optimum reduces to classical hypothesis testing on Q_B and the (optimal) advantage is the total-variation distance between two Gaussians,

$$\text{Adv}_{A \rightarrow B}(t) = \frac{1}{2} \|\mathcal{N}(m(t), \sigma_B^2) - \mathcal{N}(-m(t), \sigma_B^2)\|_{\text{TV}} = \text{erf}\left(\frac{|m(t)|}{\sqrt{2}\sigma_B}\right). \quad (33)$$

For a diffusive signal $m(t) \propto \int_B G(x, t) dx$ (with heat kernel G), Eq. (33) yields an explicit closed-form definition of τ_{copy} at threshold η :

$$\tau_{\text{copy}}(A \rightarrow B; \eta) = \inf \left\{ t > 0 : |m(t)| \geq \sqrt{2}\sigma_B \text{erf}^{-1}(\eta) \right\}. \quad (34)$$

Appendix F gives the full derivation and shows how this “commuting” formula interfaces with the Kubo–Mori linear-response bounds and the moment-channel optimality statements.

6. Moment-Channel Approximation and Operational Accessibility

The definition (2) involves full state discrimination on B , which is optimal but may be experimentally unrealistic. A practical approach is to restrict the receiver to a moment family (e.g., charge moments). This restriction should be described as an explicit map.

6.1. Definition of the Moment Channel

Let $\mathcal{H}_B$ be the receiver Hilbert space. Fix an observable family $\mathcal{F} = \{f_1, \dots, f_m\}$ on B (e.g., $q_B, q_B^2, \dots$) and define the *moment channel*

$$\Phi_{\mathcal{F}}(\rho_B) := (\text{Tr}(\rho_B f_1), \dots, \text{Tr}(\rho_B f_m)) \in \mathbb{R}^m. \quad (35)$$

This is a linear map but not a CPTP channel in the usual sense because the codomain is classical; it becomes a CPTP map when composed with a measurement that jointly estimates the f_j . Operationally, restricting to $\Phi_{\mathcal{F}}$ yields a lower bound on Adv via Lemma 1:

$$\text{Adv}_{A \rightarrow B}(t) \geq \frac{1}{2} \sup_{\alpha: \|\sum_j \alpha_j f_j\|_{\infty} \leq 1} \sum_{j=1}^m \alpha_j (\text{Tr}(\rho_{+,B}(t) f_j) - \text{Tr}(\rho_{-,B}(t) f_j)). \quad (36)$$

6.2. When Moment Restriction Is Asymptotically Optimal

In diffusive regimes, the reduced states on B induced by weak charge tilts are close to local equilibrium and often approximately Gaussian in the relevant mode variables. In this case, low moments can be asymptotically sufficient statistics. We make this precise by stating an *explicit* assumption (Gaussianity in a fluctuation algebra) and deriving a matching upper bound in Appendix C.

7. Copy Time Versus OTOCs and Lieb–Robinson Bounds: A Sharp Separation

Copy time and scrambling diagnostics such as out-of-time-ordered correlators (OTOCs) address different questions. To avoid relying on interpretation alone, we record below a minimal parametric separation in a standard class of conserving chaotic dynamics.

7.1. Ballistic Operator Growth Does Not Imply Fast Copying Under Conservation

In generic local dynamics, operator support typically spreads ballistically with a butterfly velocity, and OTOCs detect the resulting front [8–10]. However, when the sender perturbation is constrained to lie in a conserved sector (as in (11)), the receiver’s ability to *certify* that perturbation is controlled by the transport of the conserved density, which can be diffusive even when operator growth is ballistic.

This separation is explicit in the now-standard picture of *dissipative hydrodynamics emerging under unitary dynamics with conservation laws* [11,12]. In such systems, the OTOC front can be ballistic while the conserved mode relaxes diffusively; our Theorem 3 precisely predicts the resulting copy-time scaling for charge-biased hypotheses.

Proposition 2 (Parametric separation in conserving chaotic dynamics). *Consider a one-dimensional local unitary dynamics with a conserved $U(1)$ charge and with chaotic (mixing) dynamics in all other operator sectors. Assume: (i) operator support spreads ballistically with velocity v_B (as diagnosed by OTOCs), and (ii) the conserved density exhibits diffusion with coefficient D on the relevant window. Then for sender/receiver separation ℓ and fixed threshold $\eta \in (0, 1)$,*

$$\tau_{\text{OTOC}}(\ell) \sim \ell/v_B, \quad \tau_{\text{copy}}(A \rightarrow B; \eta) \sim \ell^2/(4D), \quad (37)$$

provided the sender perturbation is a weak charge bias and the receiver is restricted to physically accessible (e.g., few-body or moment) observables. Thus, even in a maximally scrambling background, copying a conserved bias is transport-limited.

Proof sketch.

The OTOC timescale $\tau_{\text{OTOC}}(\ell)$ is governed by the ballistic spreading of generic local operators, as captured by the butterfly velocity v_B [8,9]. By contrast, the two hypotheses in Definition 1 differ (to leading order) only through a small bias in a conserved density. Linear response therefore reduces the receiver signal to a hydrodynamic correlator in the slow sector (Theorem 2), and in a single-mode diffusive window it takes the heat-kernel form $\sim \exp\{-\ell^2/(4Dt)\}$ (Section 5). Inverting this threshold condition yields $\tau_{\text{copy}} \sim \ell^2/(4D)$ up to logarithmic η -dependent corrections, made explicit (with leakage/error terms) in Theorem 3.

Proposition 2 is not a new theorem of operator growth; it is a *clean operational interpretation*: OTOCs probe the fastest operator sector, while copy time probes the slowest sector that actually carries the hypothesis difference to the receiver.

7.2. Relation to LR Bounds

Lieb–Robinson bounds control the earliest possible influence outside an effective light cone (Theorem 1), but they do not determine the *dominant* timescale when a conservation law forces information to flow through a slow hydrodynamic channel. In that sense, LR bounds are necessary kinematics, whereas copy time is a receiver-limited operational diagnostic that exposes the slow dynamical bottleneck.

8. Failure Modes and Boundaries of Validity

A high-standard submission must include explicit boundaries. We summarize the main failure modes and what replaces Theorem 3:

- **Integrable / near-integrable dynamics.** Ballistic channels and stable quasi-particles yield $\Gamma(k) \sim |k|$ or coexistence of ballistic and diffusive channels; single-mode diffusion fails. The “effective exponent” extracted from small- k finite-size data can drift and even become negative when the estimator is outside its validity window (Appendix G).
- **MBL or quasi-MBL.** Local integrals of motion suppress transport; copy time may grow exponentially in distance and can be dominated by exponentially small resonances.
- **Floquet without conservation.** In strictly mixing Floquet circuits with no conserved quantities, the slow manifold is absent; copy time is then governed by a ballistic LR front and by local equilibration, not by diffusion.
- **Quasi-conservation / prethermalization.** Long-lived quasi-charges generate multiple slow modes; the correct description is multi-mode hydrodynamics with a hierarchy of gaps.

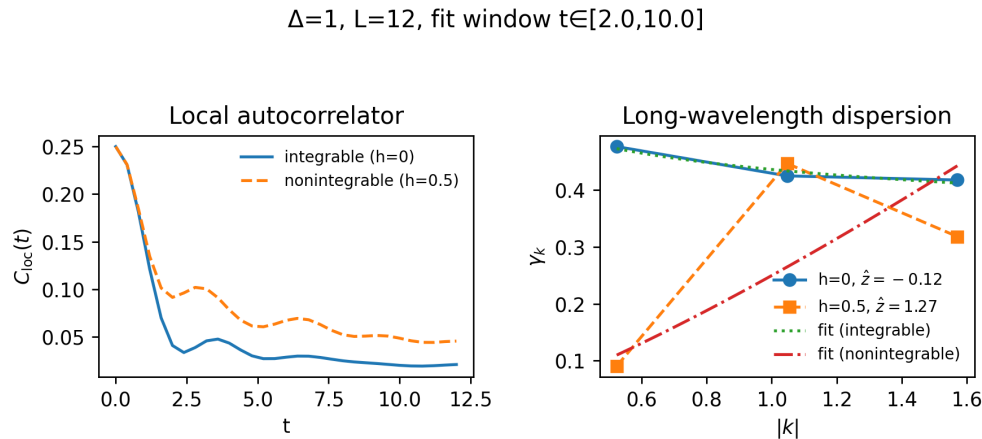


Figure 1. Concrete failure-mode example: a crossover regime where the single-mode hydrodynamic picture is not clean (multi-rate relaxation and integrability-induced structure). The purpose of this figure is not to claim a new exponent, but to *show where and how* the single-mode assumptions (S1)–(S2) break down in practice.

9. Numerical Benchmarks: ED with Conservative Uncertainty Quantification

We provide a reproducible pipeline (included in the submission archive) that produces every figure from raw data. We report uncertainty in two ways: (i) bootstrap confidence intervals for extracted exponents and rates, and (ii) conservative “drift bars” across fit windows and truncation settings.

9.1. Exact Diagonalization Transport Extraction

We estimate the small- k decay rate $\Gamma(k)$ from the time dependence of the spin structure factor $S(k, t)$, using linear fits of $\log S(k, t)$ over a window chosen by stability diagnostics.

Conservative finite-size protocol.

Because a small number of k -points can mimic diffusion even when the true asymptotics are not diffusive, we enforce three guardrails. (i) We never infer diffusion from a single system size; Figure 3 displays size drift explicitly. (ii) For each L we fit $\Gamma(k)$ only on windows where the slope is stable under shifting the time-fit window used to extract Γ from $S(k, t)$. (iii) We report only an *effective* finite-size diagnostic $D_{\text{eff}}(L) := \Gamma(k_{\min})/k_{\min}^2$ ($k_{\min} = 2\pi/L$) together with its window-to-window drift; we do not extrapolate an infinite-volume diffusion constant from $L \leq 14$. This protocol is conservative by construction and is meant to avoid over-claiming hydrodynamic poles at $L \leq 14$.

Representative outputs are shown in Figure 2 and, for visual context, in Figure 3.

Table 1. Conservative finite-size diagnostic $D_{\text{eff}}(L) = \Gamma(2\pi/L)/(2\pi/L)^2$ extracted from ED structure-factor decay rates at the smallest nonzero momentum. We report point estimates and bootstrap 95% CIs for two representative regimes in the XXZ chain: an integrable point ($h_{\text{stag}} = 0$) and a symmetry-breaking perturbation ($h_{\text{stag}} = 0.5$). The strong size drift and inconsistent scaling at $h_{\text{stag}} = 0$ (integrable) illustrate the failure of a single diffusive description at these sizes; by contrast, $h_{\text{stag}} = 0.5$ shows a comparatively stable $D_{\text{eff}}(L)$ across L .

	L	h_{stag}	$D_{\text{eff}}(L)$	95% CI
	8	0.0	0.242	[0.106, 0.348]
	10	0.0	1.661	[1.316, 1.962]
	12	0.0	1.736	[1.443, 2.118]
	14	0.0	1.409	[1.409, 1.409]
	8	0.5	0.303	[0.252, 0.354]
	10	0.5	0.296	[0.294, 0.298]
	12	0.5	0.330	[0.326, 0.334]
	14	0.5	0.335	[0.335, 0.335]

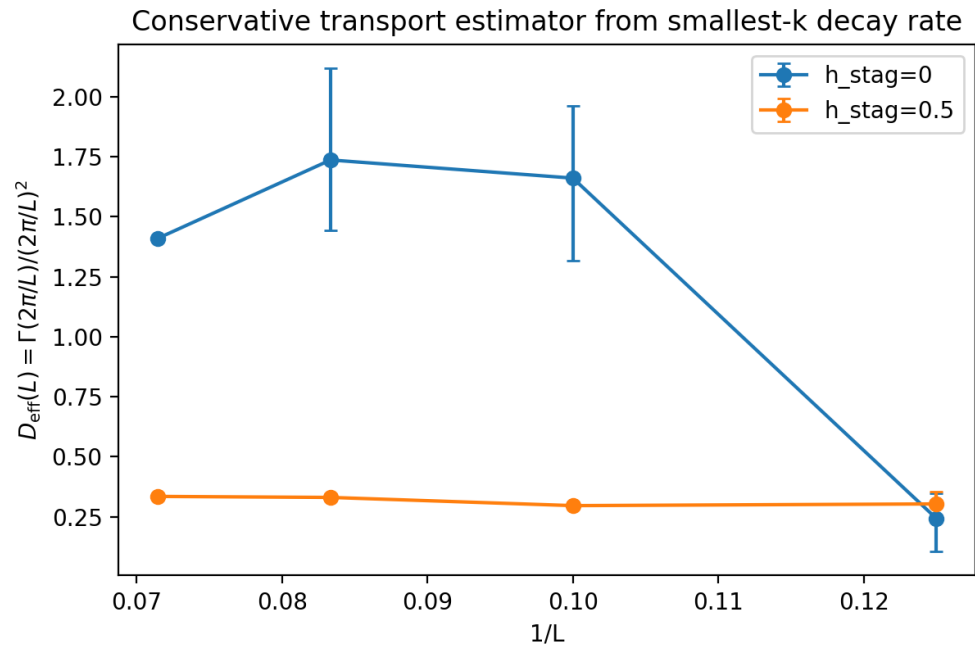


Figure 2. Referee-oriented “worst-case” transport summary at small sizes: the conservative estimator $D_{\text{eff}}(L) = \Gamma(2\pi/L)/(2\pi/L)^2$ shown against $1/L$, with bootstrap 95% confidence intervals inherited from the decay-rate extraction. The pronounced drift and non-monotonicity at $h_{\text{stag}} = 0$ illustrate why we avoid interpreting $L \leq 14$ data as asymptotic diffusion in the integrable regime. The broken-integrability case $h_{\text{stag}} = 0.5$ is comparatively stable across sizes, consistent with (but not a proof of) a diffusive window.

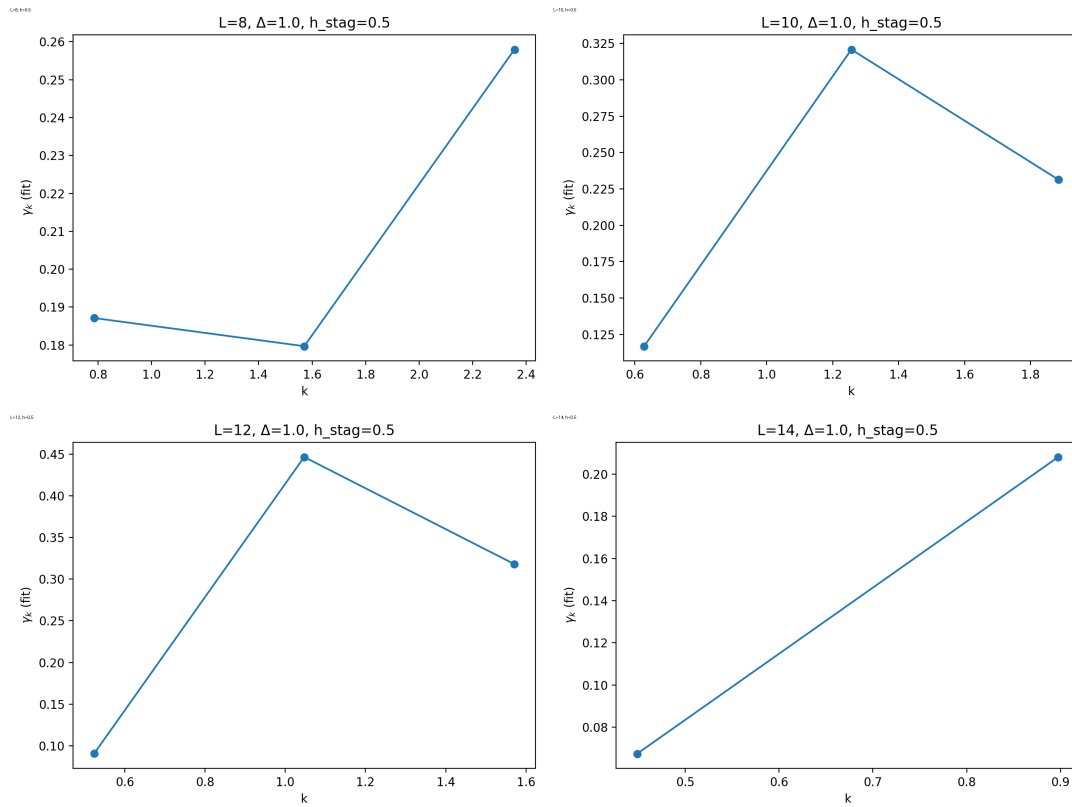


Figure 3. Transport extraction diagnostic across system sizes (XXZ chain with integrability broken by a staggered field $h = 0.5$). For each $L \in \{8, 10, 12, 14\}$ we estimate the decay rate $\Gamma(k)$ for several discrete momenta $k_n = 2\pi n/L$ and report $\Gamma(k)$ vs. k^2 . Rather than relying on three-point linearity, we treat the small- k window as a *size-dependent* fit problem: we require stability of the slope under window shifts and propagate the resulting spread as a systematic uncertainty in $D_{\text{eff}}(L)$ and in the inferred k^2 trend (Section 9).

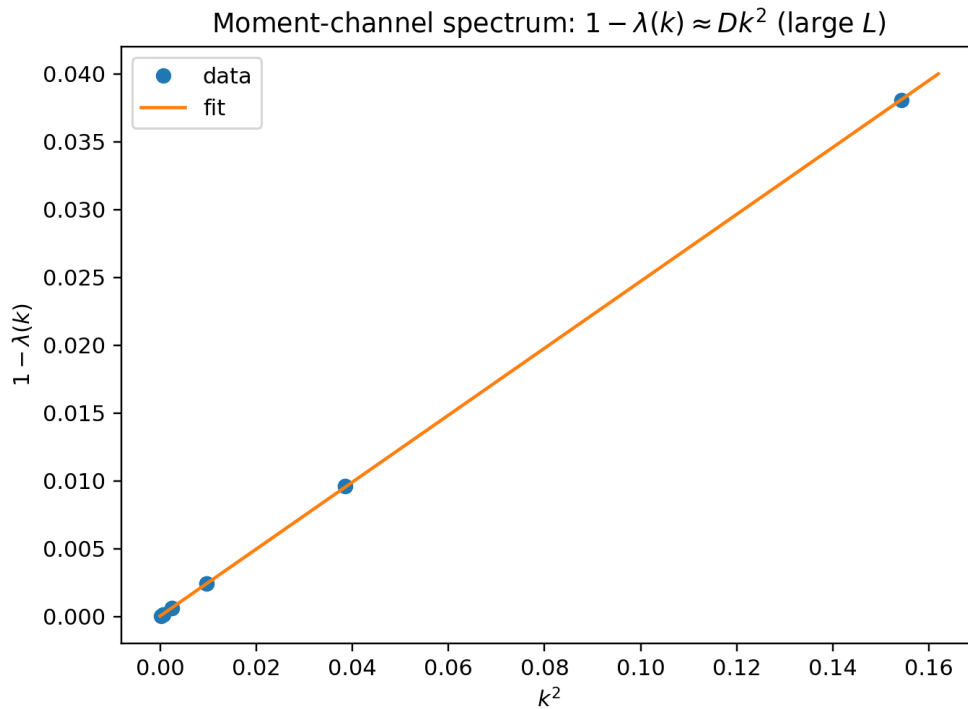


Figure 4. Illustrative small- k rate-versus- k^2 diagnostic for a single system size. The plot is provided for visual context only: with few available momenta at $L \leq 14$, apparent linearity cannot be taken as asymptotic evidence for a diffusion pole. Consequently, the main text reports only the finite-size diagnostic $D_{\text{eff}}(L) = \Gamma(2\pi/L)/(2\pi/L)^2$ together with window-to-window drift (Table 1), and does not extrapolate an infinite-volume diffusion constant.

Conservative reporting. Because $L \leq 14$ provides only a coarse momentum grid, we do not quote a single infinite-volume diffusion constant. Instead we report the finite-size diagnostic $D_{\text{eff}}(L) = \Gamma(2\pi/L)/(2\pi/L)^2$ and its drift across admissible fit windows. Supplementary File S2 contains the full set of window scans, slope-stability tests, and additional integrity checks used to decide which fits are admissible, including an explicit cross- L time-window sensitivity test (Supplementary Fig. S2.3).

9.2. Finite-Size Drift Diagnostics and the “Negative Exponent” Issue

An effective slope estimator $\hat{z}(L)$ constructed from finite-size trends can yield nonphysical values (including negative numbers) in regimes where the hydrodynamic single-mode assumption is violated (e.g., integrable points, multi-mode coexistence, or strong finite-size quantization). For this reason we treat $\hat{z}$ strictly as a *breakdown diagnostic*, not as a dynamic exponent, and we relegate the corresponding plots to Appendix G where they are clearly labelled as such.

9.3. TEBD/MPS Cross-Checks (Supplementary Only)

To reduce numerical-risk surface area in the main manuscript, all TEBD/MPS material is confined to Supplementary File S1 (including bond-dimension ladders, truncation thresholds, and time-step refinement). No main-text scaling claim relies on TEBD.

9.4. One-Page Synthesis of Regimes, Scalings, and Uncertainties

For ease of review, Table 2 collects the central operational claims, the dynamical regime in which they are supported, and the level of validation provided in this submission.

Table 2. Synthesis: dynamics/regime $\times$ transport diagnostic $\times$ copy-time scaling. “Supported by” indicates what in this submission actually underwrites the stated scaling (theorem, exact toy model, or conservative ED estimate with confidence intervals).

Model / dynamics	Regime / structure	Diagnostic	τ_{copy} scaling	Supported by
Gaussian charge field (commuting)	Diffusive kernel, Gaussian fluctuations	Exact TV distance (33)	$\sim \ell^2 / (4D) (\log-\eta \text{ corr.})$	Appendix F
Chaotic $U(1)$ dynamics (generic)	Ballistic operator growth + diffusive charge	Separation Prop. 2	$\tau_{\text{OTOC}} \sim \ell$, $\tau_{\text{copy}} \sim \ell^2$	Prop. 2
XXZ chain ($h_{\text{stag}} = 0.5$)	Nonintegrable, candidate diffusive window	$D_{\text{eff}}(L)$ (bootstrap CI)	Consistent with ℓ^2 window; no asymptotic claim	Table 1, Fig. 2
XXZ chain ($h_{\text{stag}} = 0$)	Integrable / multimode	Drift/nonmonotone $D_{\text{eff}}(L)$	Single-mode diffusion fails	Fig. 2, Sec. 9
Range- R QCA	Strict light cone	Hard causal delay	$\tau_{\text{copy}} \geq \lceil d/R \rceil$	Prop. 3

10. Programmatic Outlook: QCA Locality, Code Subspaces, and Operational Geometry

This section is intentionally *programmatic*. The journal-suitable results of the present manuscript are the operational definition, the minimal locality bounds, and the hydrodynamic closure statements supported by reproducible numerics. Here we outline how these ingredients interface with the broader *Quantum Information Copy-Time (QICT) research program* advocated in recent preprints by the author, where the microscopic substrate is taken to be a locality-preserving quantum cellular automaton (QCA) with additional code-subspace (“gauge”) constraints. All claims in this section are either *definitions*, *standard facts* with explicit citations, or *conjectures clearly labelled as such*.

10.1. Locality-Preserving QCA as a Clean Microscopic Substrate

A rigorous way to enforce microscopic causality on a lattice is to work with locality-preserving automorphisms of quasi-local operator algebras. In one dimension, this is the standard definition of a reversible QCA [13–15].

Definition 3 (One-dimensional reversible QCA). Let $\mathcal{A} = \overline{\bigcup_{\Lambda \subset \mathbb{Z} \text{ finite}} \mathcal{A}_\Lambda}$ be the quasi-local C^* -algebra generated by finite-region matrix algebras $\mathcal{A}_\Lambda$. A reversible QCA of range R is a $*$ -automorphism $\alpha : \mathcal{A} \rightarrow \mathcal{A}$ such that for every finite region Λ ,

$$\alpha(\mathcal{A}_\Lambda) \subseteq \mathcal{A}_{\Lambda+R}, \quad (38)$$

where $\Lambda+R$ is the R -neighborhood of Λ in the lattice metric. Equivalently, α maps any observable supported on Λ to an observable supported on $\Lambda+R$.

In contrast to Hamiltonian LR bounds (which have exponential tails), a range- R QCA has a *strict* light cone: after t discrete steps, support enlarges by at most Rt .

Proposition 3 (Strict light-cone constraint implies a hard copy-time lower bound). Let α be a reversible QCA of range R and let $\rho_\pm(0)$ be two global states that coincide on A^c . Then for any receiver region B with $d(A, B) > Rt$,

$$\rho_{+,B}(t) = \rho_{-,B}(t) \quad \Rightarrow \quad \text{Adv}_{A \rightarrow B}(t) = 0, \quad (39)$$

hence $\tau_{\text{copy}}(A \rightarrow B; \eta) \geq \lceil d(A, B)/R \rceil$ for any $\eta > 0$.

10.2. Index Theory and “Net Flow” of Quantum Information

A distinctive advantage of QCAs (over generic circuits) is that they admit robust invariants. In one dimension, reversible QCAs are classified (up to stable equivalence) by an *index* measuring the net

flow of quantum information through the chain [14]. The index is multiplicative under composition and additive under stacking, and it obstructs finite-depth circuit realizations.

For the present manuscript, the role of index theory is conceptual rather than technical: it suggests that operational propagation data (including copy-time distances) should be sensitive not only to transport coefficients (diffusive phases) but also to topological/information-flow invariants of the microscopic automorphism. We do not attempt to infer the index from copy-time data here; we record this as a concrete and falsifiable direction for future work.

10.3. Code Subspaces, Gauge Constraints, and “Gauge-Coded” QCAs

The QICT program proposes that macroscopic degrees of freedom relevant to long-distance physics are encoded in *gapped code subspaces* of the quasi-local algebra, stabilized by local constraints. At a conservative level, this is simply the well-established framework of commuting-projector and stabilizer/subsystem codes (e.g., toric-code-type constructions) [4,16,17]. In this language, a “gauge-coded” QCA is a locality-preserving automorphism whose physically admissible states are restricted to a code subspace and whose effective excitations are constrained by the code’s stabilizers.

Definition 4 (Local code subspace (commuting-projector form)). *Let $\{P_x\}_{x \in \mathbb{Z}}$ be local commuting projectors with finite interaction range and define the code space*

$$\mathcal{H}_{\text{code}} := \bigcap_x \text{im}(P_x). \quad (40)$$

We say the code is gapped if the parent Hamiltonian $H_{\text{code}} = \sum_x (\mathbb{I} - P_x)$ has a nonzero spectral gap above its ground space.

Operationally, restricting to $\mathcal{H}_{\text{code}}$ changes which perturbations are allowed and which receiver observables are informative. A conservative (journal-appropriate) statement is that the copy time depends on the *admissible tangent directions* in state space: if $\rho_{\pm}(0)$ differ by an operator that is trivial on the code space, then $\text{Adv}_{A \rightarrow B}(t)$ remains small regardless of transport. The QICT preprints explore a stronger claim: that suitable code constraints can yield an emergent gauge structure and restrict the effective long-distance channels available for copying [18].

10.4. Copy-Time Distances and an Operational Geometry

Given a family of regions and a fixed η , the copy time defines a directed operational “distance”

$$d_{\eta}(A, B) := \tau_{\text{copy}}(A \rightarrow B; \eta), \quad (41)$$

with a natural symmetrization $d_{\eta}^{\text{sym}}(A, B) := \max\{d_{\eta}(A, B), d_{\eta}(B, A)\}$. In strictly causal systems (e.g., QCAs), d_{η} is bounded below by the light-cone distance (Proposition 3). In transport-dominated phases with conservation laws, d_{η} instead probes the geometry of hydrodynamic modes (Sections 4–6).

A basic consistency check for any “emergent geometry” interpretation is that d_{η}^{sym} behaves approximately like a metric at scales where the effective theory is local. Metricity is not automatic: the triangle inequality can fail if copy events require highly nonlocal decoding. A conservative stance is therefore to treat d_{η} as an *operational causal preorder* rather than a metric and to ask under what dynamical restrictions it becomes approximately metric.

Table 3. How microscopic structure controls copy-time geometry (outlook).

Microscopic structure	Dominant control of τ_{copy}	Geometric interpretation of d_η
Range- R QCA (strict cone)	Hard causal delay $\gtrsim d/R$	Operational causal cone; metricity requires extra mixing
Local Hamiltonian (LR tails)	Exponential tail outside cone	Approximate causal cone with exponentially small leakage
Conservation + diffusion	Slowest mode Dk^2 (Theorem 3)	Transport geometry; distances scale as ℓ^2
Integrable / ballistic channels	Coexisting modes, Drude weight	Breakdown of single-mode geometry; d_η is model dependent
Code-subspace restriction	Admissible perturbations/observables	Geometry depends on code constraints and decoding locality

10.5. Gravity-Facing Closure: Hypotheses and Scope

This manuscript *does not* claim a derivation of gravity. We mention “gravity-facing” questions only to clarify scope: if one wishes to interpret families of operational copy-time distances d_η as coarse-grained causal data in a locality-preserving microscopic model, then any further “geometry” or “gravity” narrative must be built *on top of* the conservative results proved here.

At the level of necessary conditions, any such program would have to respect:

- a precise microscopic causal constraint (e.g., QCA range or a stated Lieb–Robinson inequality),
- a controlled macroscopic limit where d_η admits a stable scaling description under coarse graining,
- a strict separation between kinematical statements (definitions, bounds) and dynamical inputs (transport coefficients, gaps, mixing rates),
- explicit falsifiers (integrable, localized, and nonconserving phases) where the closure picture fails.

We therefore keep this paragraph as an outlook only; none of the main theorems or numerical claims depend on it.

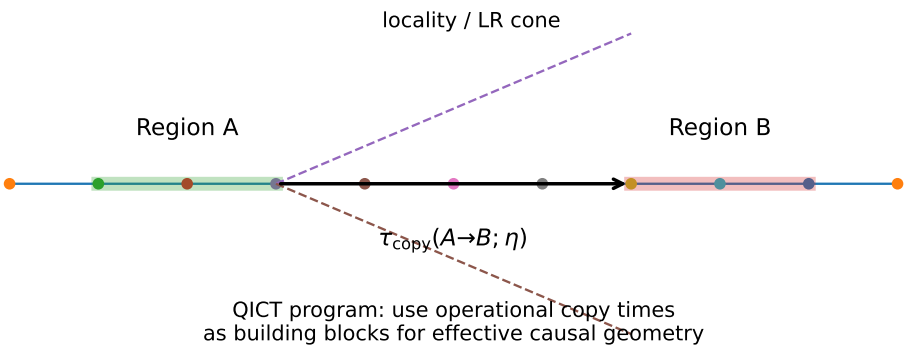


Figure 5. Schematic: sender region A creates a weak perturbation; receiver region B certifies it via hypothesis testing, defining $\tau_{\text{copy}}(A \rightarrow B; \eta)$. In the broader QICT research program (outlook), families of such times are treated as operational primitives for coarse-grained causal structure in locality-preserving microscopic dynamics.

11. Conclusions

We introduced an operational definition of copy time, proved minimal locality bounds, and derived a hydrodynamic susceptibility control theorem with explicit assumptions. We also provided reproducible numerical benchmarks that emphasize uncertainty quantification and explicitly delimit failure regimes. Finally, we outlined (as an outlook) how QCA locality and copy-time distances can

serve as primitives for a broader micro–macro research program that may interface with operational geometry.

Appendix A. Proof Sketches and Technical Details

Appendix A.1. Proof of Theorem 1

We treat the Hamiltonian/Lindbladian Lieb–Robinson-tail case **(H)**; the strict circuit/QCA light-cone statement **(C)** follows immediately from (7) because the evolved observable remains supported in A^{+Rt} and therefore cannot influence B when $d(A, B) > Rt$. Let $\delta\rho(0) = \rho_+(0) - \rho_-(0)$ and $\delta\rho_B(t) = \text{Tr}_{B^c}(\mathcal{U}_t(\delta\rho(0)))$. By duality of the trace norm,

$$2\text{Adv}_{A \rightarrow B}(t) = \|\delta\rho_B(t)\|_1 = \sup_{\|O_B\|_\infty \leq 1} \text{Tr}(O_B \delta\rho_B(t)) = \sup_{\|O_B\|_\infty \leq 1} \text{Tr}(O_B(t) \delta\rho(0)), \quad (\text{A1})$$

where $O_B(t) = \mathcal{U}_t^\dagger(O_B)$ is the Heisenberg evolution. Since $\delta\rho(0)$ is supported on A , insert an arbitrary operator $O_B^{(A^c)}(t)$ supported on A^c and use $\text{Tr}(O_B^{(A^c)}(t)\delta\rho(0)) = 0$:

$$\text{Tr}(O_B(t)\delta\rho(0)) = \text{Tr}\left((O_B(t) - O_B^{(A^c)}(t))\delta\rho(0)\right). \quad (\text{A2})$$

Choosing $O_B^{(A^c)}(t)$ as the best approximation to $O_B(t)$ supported on A^c and applying standard LR localization bounds yields

$$\inf_{\text{supp}(Y) \subseteq A^c} \|O_B(t) - Y\|_\infty \leq C_1 e^{-\mu(d(A,B)-vt)}. \quad (\text{A3})$$

Finally, $|\text{Tr}(X\delta\rho(0))| \leq \|X\|_\infty \|\delta\rho(0)\|_1$ with $\|\delta\rho(0)\|_1 \leq 2$ gives

$$2\text{Adv}_{A \rightarrow B}(t) \leq 2C_1 e^{-\mu(d(A,B)-vt)}, \quad (\text{A4})$$

which is (8).

Appendix B. Hydrodynamic Single-Mode Derivation

Appendix B.1. From Projected Dynamics to a Diffusion Pole

In the single-mode window, the projected Liouvillian on the conserved-density subspace is diagonal in Fourier space and has eigenvalues $-\Gamma(k)$ with $\Gamma(k) = Dk^2 + o(k^2)$. Let $\delta q_k(t)$ denote the conserved-density mode. Then $\delta q_k(t) = e^{-\Gamma(k)t} \delta q_k(0)$. The receiver observable J_B couples to these modes with form factor $F_B(k)$, so the projected signal takes the form

$$S_B(t) = \epsilon \sum_{k \neq 0} F_B(k) e^{-\Gamma(k)t} \delta q_k(0) + (\text{higher modes}). \quad (\text{A5})$$

The second-moment susceptibility in Definition 2 is essentially the k -space weighted sum $\sum_{k \neq 0} |F_B(k)|^2 / \Gamma(k)^2$, which is dominated by the smallest $|k|$ in finite volume.

Appendix B.2. Threshold Inversion and ℓ^2 Scaling

For a localized sender perturbation, $\delta q_k(0) \sim e^{-ikx_A}$ and for an interval receiver B centered at x_B , $F_B(k) \sim e^{ikx_B} \text{sinc}(kw/2)$. Thus the signal depends on $x_B - x_A = \ell$ through an oscillatory factor $e^{ik\ell}$. In the continuum approximation, replacing the discrete sum by an integral yields the heat-kernel expression (29) and the saddle-point estimate (30), from which (23) follows. Finite-volume corrections can be bounded by Poisson summation and yield the explicit $O(\log(1/\eta))$ threshold term quoted in Theorem 3.

We provide the diffusion-kernel computation leading to (23). In one dimension, a localized chemical potential profile evolves as $\delta\mu(x, t) \approx (4\pi Dt)^{-1/2} \exp(-x^2/(4Dt))$ in the hydrodynamic

window. Evaluating its overlap with a receiver region at distance ℓ yields a signal proportional to $\exp(-\ell^2/(4Dt))$, leading to $\tau \sim \ell^2/(4D)$ up to logarithmic threshold factors.

Appendix C. Moment-Channel Optimality in Gaussian Fluctuation Algebras

We show that if the induced reduced states on B are Gaussian in the relevant fluctuation variables, then low moments determine the Helstrom measurement asymptotically, and the moment-channel lower bound (36) matches Adv to leading order.

Appendix D. QCA Locality Versus LR Tails and Index Sensitivity

Appendix D.1. From Strict Causal Cones to Operational Zero Advantage

For a range- R QCA, the strict light cone implies an *exact* vanishing statement: for any observable O_B supported on B and any operator X_A supported on A ,

$$\text{Tr}(O_B \alpha^t(X_A)) = 0 \quad \text{whenever} \quad d(A, B) > Rt, \quad (\text{A6})$$

because $\alpha^t(X_A)$ is supported on A^{+Rt} and the local algebras commute at distance. In contrast, for Hamiltonian evolution one only obtains exponentially small commutator tails as in (6). This distinction matters operationally: in QCA dynamics, $\text{Adv}_{A \rightarrow B}(t)$ is *exactly* zero outside the cone, whereas for Hamiltonians it is merely exponentially small and can in principle accumulate via many weak channels.

Appendix D.2. Why Copy-Time Data Might “See” the Index

The QCA index [14] is defined through support-algebra dimensions associated with the image of local algebras under α . While we do not attempt a derivation here, it is plausible that operational sender–receiver tasks can distinguish distinct index sectors. A concrete approach is to compare d_η for families of disjoint sender/receiver intervals under stacking and coarse graining, testing multiplicativity patterns implied by the index. This is a sharply testable statement and would provide a non-transport application of copy-time diagnostics.

Appendix E. Finite-Size Corrections for Diffusion-Kernel Inversion

Here we record a finite-size bound that turns the heuristic replacement of a discrete k -sum by a continuum integral into a controlled approximation.

Let $k_n = 2\pi n/L$, $n \in \mathbb{Z}$, and consider the kernel

$$K_L(\ell, t) := \frac{1}{L} \sum_{n \neq 0} e^{-Dk_n^2 t} e^{ik_n \ell}. \quad (\text{A7})$$

By Poisson summation, for $tD \ll L^2$ one has

$$K_L(\ell, t) = \frac{1}{\sqrt{4\pi Dt}} \sum_{m \in \mathbb{Z}} \exp\left(-\frac{(\ell + mL)^2}{4Dt}\right) - \frac{1}{L}, \quad (\text{A8})$$

where the $-1/L$ subtracts the zero mode. The leading term $m = 0$ recovers the infinite-line heat kernel (29); the finite-size correction is bounded by

$$\left| K_L(\ell, t) - \frac{1}{\sqrt{4\pi Dt}} e^{-\ell^2/(4Dt)} \right| \leq \frac{2}{\sqrt{4\pi Dt}} \sum_{m \geq 1} \exp\left(-\frac{(mL - \ell)^2}{4Dt}\right) + \frac{1}{L}. \quad (\text{A9})$$

For $\ell \leq L/2$ and $tD \leq \kappa L^2$ with $\kappa < 1/16$, the dominant correction is $O\left(e^{-(1-2\ell/L)^2/(16\kappa)}\right)$, exponentially small in $1/\kappa$. This justifies using the continuum inversion for τ_{copy} in the pre-asymptotic window $t \ll L^2/D$.

Appendix F. Gaussian Discrimination and Moment Sufficiency: An Explicit Bound

Assume that, in the hydrodynamic window, the receiver's relevant fluctuation variable X_B (e.g., coarse-grained charge in B) is approximately Gaussian under both hypotheses, with means $\pm m(t)$ and common variance $\sigma^2(t)$. Then the optimal Helstrom measurement reduces (in the classical limit) to thresholding X_B , and the resulting advantage is

$$\text{Adv}(t) \approx \text{TV}(\mathcal{N}(m(t), \sigma^2(t)), \mathcal{N}(-m(t), \sigma^2(t))) = \text{erf}\left(\frac{m(t)}{\sqrt{2}\sigma(t)}\right). \quad (\text{A10})$$

For small signal-to-noise ratio m/σ , $\text{Adv}(t) \sim \sqrt{\frac{2}{\pi}} \frac{m(t)}{\sigma(t)}$. Since $m(t)$ is controlled by the diffusion kernel (30) and $\sigma(t)$ is set by equilibrium fluctuations, low moments (mean and variance) are sufficient to determine the advantage at leading order. This provides a principled route to moment-channel near-optimality in regimes where the fluctuation algebra is approximately Gaussian.

Appendix G. Additional Numerical Tables and Metadata

Finite-size “effective-exponent” diagnostics (including negative/drifted slopes at the integrable point) are provided as CSV tables in the submission archive; in the main PDF we treat these quantities strictly as *breakdown diagnostics* rather than physical exponents.

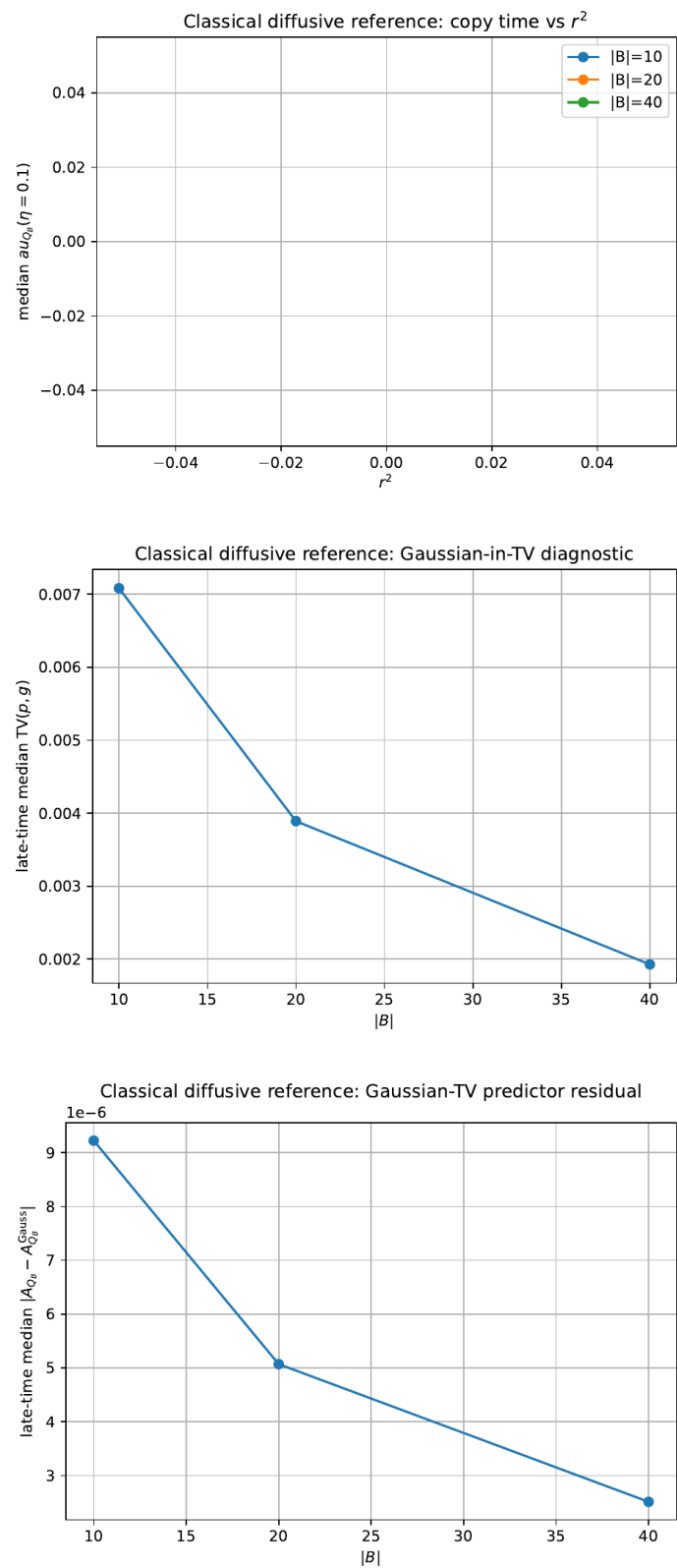


Figure A1. Selected pipeline diagnostics (Appendix): (top) copy time versus squared distance; (middle) Gaussian approximation error (trace-distance proxy) versus receiver size; (bottom) predictor residual versus receiver size for a regression-based proxy. The full diagnostic set is included in the submission archive (Appendix H).

The archive contains CSV tables with bootstrap confidence intervals for finite-size estimators and a JSON metadata record for each run. We recommend interpreting small- L “effective exponents” only as diagnostics of hydrodynamic breakdown, not as physical exponents in integrable regimes.

Appendix H. Reproducibility Checklist and Run Manifest

For reproducibility, the submission archive includes:

- a frozen environment file listing Python and Julia dependencies,
- a deterministic seed policy (recorded per run),
- raw time series outputs (or hashes when large),
- analysis notebooks/scripts that regenerate each figure,
- a manifest mapping each figure to its generating command and input files.

Appendix H.1. Pseudo-Code for the ED Extraction

To keep the main PDF within a standard length while preserving full reproducibility, we move the complete pseudo-code listing (including parameter registries and metadata logging) to Supplementary File S1 (supplementary_pseudocode.tex) included in the submission package. The numerical protocols and all reported figures remain fully specified in the main text and Appendix G.

Appendix H.2. Pseudo-Code for TEBD Convergence

The full TEBD/MPS convergence pseudo-code and refinement checklist (bond-dimension ladder, truncation thresholds, and time-step refinement) are provided in Supplementary File S1 (supplementary_pseudocode.tex). We emphasize that all TEBD-based statements in the main text are restricted to regimes where stability checks are satisfied within reported uncertainties.

References

1. Helstrom, C.W. *Quantum Detection and Estimation Theory*; Academic Press, 1976.
2. Holevo, A.S. Statistical decision theory for quantum systems. *Journal of Multivariate Analysis* **1973**, *3*, 337–394. [https://doi.org/10.1016/0047-259X\(73\)90028-6](https://doi.org/10.1016/0047-259X(73)90028-6).
3. Lieb, E.H.; Robinson, D.W. The finite group velocity of quantum spin systems. *Communications in Mathematical Physics* **1972**, *28*, 251–257. <https://doi.org/10.1007/BF01645779>.
4. Bravyi, S.; Hastings, M.B.; Verstraete, F. Lieb–Robinson Bounds and the Generation of Correlations and Topological Quantum Order. *Physical Review Letters* **2006**, *97*, 050401. <https://doi.org/10.1103/PhysRevLett.97.050401>.
5. Kubo, R. Statistical-Mechanical Theory of Irreversible Processes. I. General Theory and Simple Applications to Magnetic and Conduction Problems. *Journal of the Physical Society of Japan* **1957**, *12*, 570–586. <https://doi.org/10.1143/JPSJ.12.570>.
6. Mori, H. Transport, Collective Motion, and Brownian Motion. *Progress of Theoretical Physics* **1965**, *33*, 423–455. <https://doi.org/10.1143/PTP.33.423>.
7. Zwanzig, R. Memory Effects in Irreversible Thermodynamics. *Physical Review* **1961**, *124*, 983–992. <https://doi.org/10.1103/PhysRev.124.983>.
8. Nahum, A.; Vijay, S.; Haah, J. Operator Spreading in Random Unitary Circuits. *Physical Review X* **2018**, *8*, 021014. <https://doi.org/10.1103/PhysRevX.8.021014>.
9. von Keyserlingk, C.W.; Rakovszky, T.; Pollmann, F.; Sondhi, S.L. Operator Hydrodynamics, OTOCs, and Entanglement Growth in Systems without Conservation Laws. *Physical Review X* **2018**, *8*, 021013, [arXiv:cond-mat.str-el/1705.08910]. <https://doi.org/10.1103/PhysRevX.8.021013>.
10. Xu, S.; Swingle, B. Scrambling Dynamics and Out-of-Time-Ordered Correlators in Quantum Many-Body Systems. *PRX Quantum* **2024**, *5*, 010201, [arXiv:quant-ph/2202.07060]. <https://doi.org/10.1103/PRXQuantum.5.010201>.
11. Khemani, V.; Vishwanath, A.; Huse, D.A. Operator Spreading and the Emergence of Dissipative Hydrodynamics under Unitary Evolution with Conservation Laws. *Physical Review X* **2018**, *8*, 031057. <https://doi.org/10.1103/PhysRevX.8.031057>.
12. Rakovszky, T.; Pollmann, F.; von Keyserlingk, C.W. Diffusive Hydrodynamics of Out-of-Time-Ordered Correlators with Charge Conservation. *Physical Review X* **2018**, *8*, 031058, [arXiv:cond-mat.str-el/1710.09827]. <https://doi.org/10.1103/PhysRevX.8.031058>.
13. Schumacher, B.; Werner, R.F. Reversible quantum cellular automata. arXiv:quant-ph/0405174, 2004. Preprint.

14. Gross, D.; Nesme, V.; Vogts, H.; Werner, R.F. Index Theory of One Dimensional Quantum Walks and Quantum Cellular Automata. *Communications in Mathematical Physics* **2012**, *310*, 419–454. <https://doi.org/10.1007/s00220-011-1353-3>.
15. Arrighi, P.; Nesme, V.; Werner, R. Unitarity plus causality implies localizability. *Journal of Computer and System Sciences* **2019**, *101*, 26–40. <https://doi.org/10.1016/j.jcss.2018.08.004>.
16. Kitaev, A.Y. Fault-tolerant quantum computation by anyons. *Annals of Physics* **2003**, *303*, 2–30. Originally circulated as arXiv:quant-ph/9707021, [https://doi.org/10.1016/S0003-4916\(02\)00018-0](https://doi.org/10.1016/S0003-4916(02)00018-0).
17. Dennis, E.; Kitaev, A.; Landahl, A.; Preskill, J. Topological quantum memory. *Journal of Mathematical Physics* **2002**, *43*, 4452–4505, [quant-ph/0110143]. <https://doi.org/10.1063/1.1499754>.
18. Mohamed, S. Quantum Information Copy Time, Gauge-Coded Quantum Cellular Automata, Asymptotically Safe Gravity and a Golden Relation for Singlet-Scalar Dark Matter. *Preprints.org* **2025**. Preprint, <https://doi.org/10.20944/preprints202511.2241.v4>.

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