

Article

SLIDING MODE CONTROL AND GEOMETRIZATION CONJECTURE IN SEISMOLOGY

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Abstract: The purpose of this paper is to study the sliding mode control as a Ricci flow process in the context of a three-story building structure subjected to seismic waves. The stability conditions result from two Lyapunov- functions, the first associated with slipping in a finite period of time, and the second with convergence of trajectories to the desired state. Simulation results show that the Ricci flow control leads to the minimization of the displacements of the floors. 3D Ricci solitons projection via a semi-conformal mapping to a surface is also studied.

Keywords: Ricci flow, Ricci solitons, geodesics, seismic waves

1. Introduction

In recent years, the algorithms applied to control the building systems subjected to seismic loads have been studied extensively [1-3]. The sliding mode control arises as a variable control which constrains the structure to lie within a neighborhood of the switching function [4]. The advantage of the control is to tailor the structure behavior with respect to a choice of the switching function in terms of insensitive to any uncertainties [5]. Let us introduce the vector of displacement of the i -th story relative to ground $x(t)=[x_1, x_2, \dots, x_n]^T$ and the state vector of the structure $z(t)=[x(t), \dot{x}(t)]^T$. The sliding mode controller designed for a dynamical system is given by

$$\dot{z}(t) = A[z(t) - z_d] + Bu(t) + f(z, u, t) \quad (1)$$

where z_d is the reference desired trajectory and $u(t)$ is the input to the system

$$u = -k \operatorname{sgn}(s) + u_{eq} \quad (2)$$

with u_{eq} the equivalent control used when the system is in the sliding mode [6, 7], k is a constant representing the maximum output of the controller and s is the switching function given by

$$s = \dot{e} + pe, \quad e = z - z_d \quad (3)$$

where p is a constant. The sliding mode control is schematically shown in Figure 1.

The control function $f(z, u, t)$ depends on z , time t and the m control forces $u(t)=[u_1, u_2, \dots, u_m]^T$. We introduce a Riemannian 2-manifold (M^2, g) in order to define the sign on the two sides of this manifold. The condition $s = 0$ becomes

$$\dot{z} + pz - C = 0, \quad (4)$$

where $C = \dot{z}_d + pz_d$. The control uses the sign on the two sides of (M^2, g) .

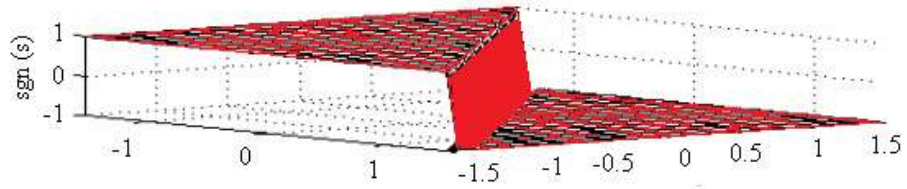


Figure 1. The sliding mode control.

The matrices A and B from (1) are given by

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}B_1 \end{bmatrix}, \quad (5)$$

with M, C and K the mass, the damping and the stiffness matrices, respectively, and B_1 the location matrix of control forces. The aim of the control is to oblige the system trajectories to stay and move on the Riemannian 2-manifold (M^2, g) for which (4) is verified for any initial condition $z(0) = z_0$.

An advantage of sliding-mode control consists in its stability property for dynamic systems subjected to large loads. On the negative side, the sliding-mode control use large and rapidly switching control signals which consumes a lot of energy and damages the control actuators [8].

To overcome this disadvantage, we use the the Ricci flow and the trajectories moving on a manifold to model the sliding mode control and also the structure response to large loadings. Many results for Ricci flow are related to the mean curvature flow of hypersurfaces. The condition of the system trajectories to stay and move on the Riemannian 2-manifold (M^2, g) for which $s = 0$, can be modeled as the Ricci flow defined on a manifold (\bar{M}, g) as

$$\frac{\partial g}{\partial t} = -2\text{Ricci}(g). \quad (6)$$

The Ricci soliton was introduced by Hamilton in order to proof the 3D sphere theorem [9]. In 2002 and 2003, Perelman stated a new version of the Hamilton's method [10, 11]. He was awarded a Fields medal in 2006 for his contributions but he declined to accept it. The Ricci flow is often thought as a tensor written in local coordinates by simple formulae involving the first and second derivatives of the metric tensor [12-17].

The fixed points of (6) are called Ricci solitons. In order to find the fixed points of (5) on the diffeomorphism of \bar{M} and scaling of g , (5) is rewritten as

$$-2\text{Ricci}(g) = L_E g + 2\lambda g, \quad (7)$$

where $L_E g$ is Lie derivation of g with respect to E and $\lambda > 0$ is a constant.

On the Riemannian manifold (\bar{M}, g) we have for any vector X the condition

$$(L_X g)_{ij} = \nabla_i X_j + \nabla_j X_i \leq -\eta |L_X g|, \quad (8)$$

which ensures that the vector belongs to \bar{M} , where ∇ is the Levi-Civita connection of the metric g .

Our approach is to reduce the Ricci soliton equation (6) to the state equation of a n -story building subjected to seismic loads. This does not imply that we no longer apply the modal analysis but the fact that the sliding control for vibration control is performed simply and efficiently.

2. Sliding mode control

A solution of (6) is given by

$$g(t) = c(t)\psi_t^*g \quad (9)$$

where $c(t)$ is a scalar function of t and ψ_t is a family of diffeomorphisms with $c(0) = 1$, $\psi(0) = id_0$ with $d_0 = \min d(r), \forall P \subset M^2$ and $\lambda = \frac{c'(0)}{2}$.

For $\lambda < 0$, $\lambda = 0$ and $\lambda > 0$ the soliton is called shrinking, stationary or expanding, respectively. If $E = \text{grad}F$ for an arbitrary function F we have

$$L_E g = 2\nabla dF. \quad (10)$$

The system trajectories of the structure stay and move on the Riemannian 2-manifold (M^2, g) for any point $P \in M^2$ if and only if the condition (8) is verified. This control law can be written as

$$s\dot{s} \leq -\eta|s|, \quad (11)$$

with $\eta > 0$ a constant.

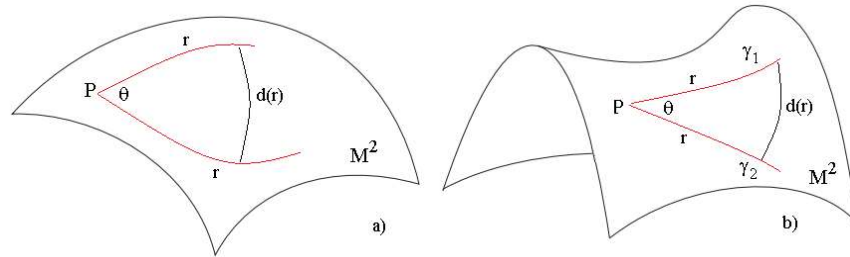


Figure 2. Geodesics in a 2-manifold of positive curvature (a) and negative curvature (b).

The sliding mode control of the structures verifies (11) which represents the necessary and sufficient condition for the system trajectories to stay and move on the Riemannian 2-manifold (M^2, g) for which (4) is verified for any initial condition $z(0) = z_0$. We note that each point P means a set of initial conditions.

The condition (11) guarantees the control by switching the sign on the two sides of the switching surface $s = 0$. The Ricci flow process ensures and guarantees the control law (11) such that the Riemannian 2-manifold (M^2, g) exists and is reachable along the system trajectories. The law (11) forces the system to stay and move on (M^2, g) which give to the system desirable features.

Consider the kinetic energy

$$T(x, y) = \frac{1}{2} \|y\|^2 = \frac{1}{2} g_{ij}(x) y^i y^j, \quad (12)$$

and the smooth functions $f_1, f_2 : [0, \infty) \rightarrow R$ such that $f_1(t) > 0$ and $f_1(t) + 2f_2(t) > 0$, $\forall t$. In this case the symmetric tensor

$$G_{ij} = f_1(t)g_{ij} + f_2(t)g_{0i}g_{0j}, \quad (13)$$

is positive definite, and the pseudo-Riemannian metric can be written as

$$\begin{aligned} G\left(\frac{\delta}{\delta x^i}, \frac{\delta}{\delta x^j}\right) &= G\left(\frac{\partial}{\partial y^i}, \frac{\partial}{\partial y^j}\right) = 0, \\ G\left(\frac{\delta}{\delta x^i}, \frac{\partial}{\delta y^j}\right) &= G\left(\frac{\partial}{\partial y^j}, \frac{\delta}{\delta x^i}\right) = 0. \end{aligned} \quad (14)$$

We note that any vector field is independent of any Riemannian metric on the base manifold

$$\Gamma = y^i \frac{\partial}{\partial y^i}. \quad (15)$$

The Riemannian metric g is related to the geodesic spray by

$$\Gamma_g = y^a \frac{\delta}{\delta x^a}. \quad (16)$$

The stability of the sliding-mode control results from the Lyapunov stability analysis [17]. Two Lyapunov functions are introduced as two continuous nonnegative functions: $V_1: R^n \rightarrow [0, \infty)$ and $V_2: R^n \rightarrow [0, \infty)$. We add a continuous nonnegative function $W: R^n \rightarrow [0, \infty)$ and a manifold (M^2, g) of the state space containing the origin which is the desired equilibrium.

These functions satisfy the conditions:

1. $V_1(P) = 0$, $V_1(0) = V_2(0) = W(0) = 0$ if and only if the point $P \in M^2$,
2. The restriction of V_1 to $R^n \setminus M^2$ and the restriction of V_2 to $R^n \setminus \{0\}$ are continuously differentiable, but the function V_1 is not differentiable at points in M^2 .

The role of V_1 is to drive the state arbitrarily close to M^2 in finite time and the function V_2 models the motion of the state to a neighbourhood of the origin.

The conditions 1 and 2 are related to the curvature at a point $P \in M^2$ of the Riemannian 2-manifold (M^2, g) . Two tangent vectors v_1, v_2 , $|v_1| = |v_2| = 1$ with respect to g are introduced.

3. Results

Let $\gamma_i: [0, 1] \rightarrow M^2$ $i=1, 2$, be two geodesics in M^2 that start at P , with $\dot{\gamma}_i(0) = v_i$. If $d(r): R^+ \rightarrow R^+$ is the distance from $\gamma_1(r)$ to $\gamma_2(r)$ along a circle with centre P and radius r . The radius is measured with respect to the metric d on M^2 induced by the Riemannian metric g . Let θ be the angle between v_1 and v_2 . Then, we have $d(r) \approx \theta r$, $d'(0) = \theta$.

If we take into account (12-16), the conditions 1 and 2 are verified by the geodesics in a 2-manifold M^2 of positive and negative curvature.

These geodesics are displayed in Figure 2. The movements of the structure during deformation are made along these geodesics and then, the minimization of displacements and the deformation of the structure are automatically fulfilled.

We say that the condition (11) guarantees the control by switching the sign on the two sides of the switching surface $s = 0$. Also, the Ricci flow process guarantees (11) such that the Riemannian 2-manifold (M^2, g) is reachable along the system trajectories. The law (11) forces the system to stay and move on (M^2, g) which give to the system the benefit of a minimal deformation and movements.

To show this, we compute the displacements achieved by each floor with and without the sliding control and represent these results in Figure 3. We see that the level of displacements is smaller for the sliding control compared to the case without control.

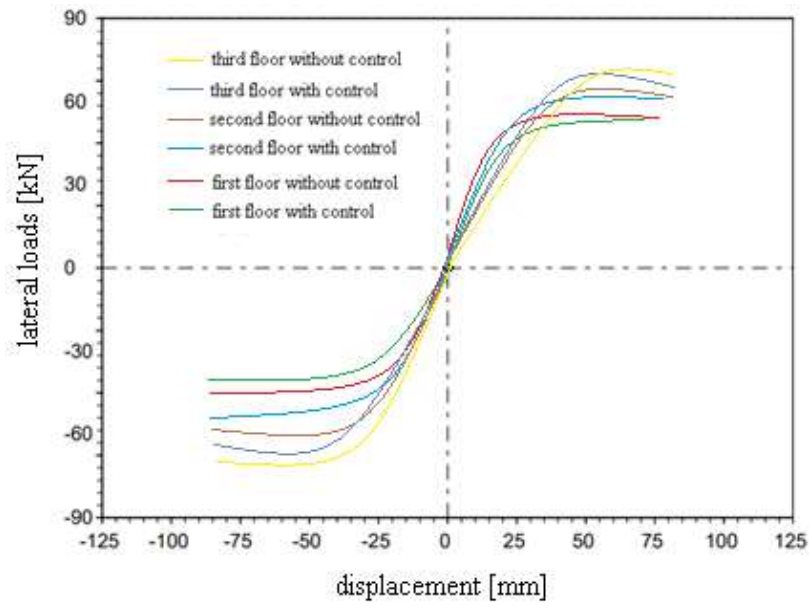


Figure 3. Constitutive curves for the structure.

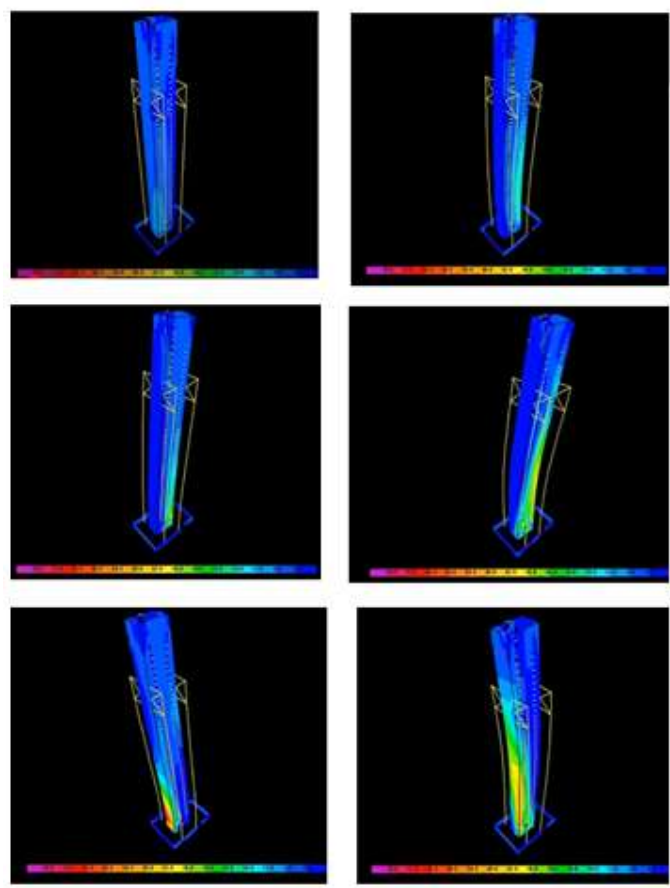


Figure 4. The structure in the first mode of vibration – longitudinal (2.08Hz) and transversal (1.56Hz) directions, with damping -longitudinal 2.5% and transversal 1.3%.

Let us to compute the vibration modes of the structure. The dynamics of the structure in the first mode of vibration is shown in Figure 4, for the longitudinal direction (2.08Hz) and the transversal direction (1.56Hz) with damping 2.5% (longitudinal) and 1.3% (transversal). The second mode of vibration appears at 7Hz for longitudinal direction with damping 2.5% and 6.6 Hz for transversal direction with 4% the damping. The third mode of vibration appears at 12.8Hz for longitudinal direction with damping 12% and 13.5 Hz for transversal direction with 5% the damping.

4. Conclusions

An disadvantage of the sliding-mode control is that it usually involves the use of large and rapidly switching control signals, sometimes leading to high energy consumption or short lifetime of the control actuators. To overcome this disadvantage, the Ricci flow and the trajectories moving on a manifold are introduced. In this way, we clarify the important ideas behind the sliding mode motion control and the structure response. The sliding-mode control involves selection of a sliding manifold such that the system trajectory intersects, move and stays on this manifold. The sliding mode control law has the ability to drive trajectories to the sliding motions in a finite period of time. That means, the stability of the sliding manifold is better than the asymptotic stability.

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