Brief Report

Empirical Mass Formula for Charged Leptons and Cosmology

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Article

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Abstract

An explicit mass formula for charged leptons is proposed which employs the $(2\ell + 1)$ degeneracy of the $SU(2)$ Laplacian of quantum mechanics. Possible cosmological implications of such "Fermion vortices" are briefly discussed.

Keywords: particle physics; leptons; generations

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1. Introduction

The muon was already 1936 detected in cosmic rays, whereas the tau was found in 1975. The immense difference between the individual masses of charged leptons and the occurrence of generations in particle physics still appears to be a mystery.

This short paper intents to shed some light on the appearance of generations in particle physics. A mass formula for charged leptons is proposed which interpolates between De Broglie–Vigier ideas on "hidden" internal degrees of freedom and (gauge) group concepts, cf. the preprint doi: 10.20944/preprints202408.1309.v1.

2. Product Formula

Here we propose the *product* formula

$$m_N = \Pi_{\ell=0}^{N-1} 2^{(2\ell-1)}(2\ell+1)^2 \, \text{MeV} \tag{1}$$

for the charged leptons, where ℓ is an angular momentum eigenvalue and $N \geq 3$ the number of generations. This is only aproximate (in units of $c = 1$) and in the need of "radiative corrections".

In a nonlinear spinor equation, such a formula may arise from the nonlinear self-interaction $(\overline{\psi}\psi)^2$. Beginning with the muon, the angular degeneracy $2^\ell(2\ell+1)$ of some sort of spin-weighted spherical harmonics [9], p. 246, could come into the play. For the tau and higher charged leptons, an "entanglement" of nonlinear vortices or angular momentum kinks may arise, in a multiplicative manner, in spinor solitons. The instability [13] of such "Fermion vortices" would increase via the Heisenberg uncertainty relation $\Delta\tau \simeq \hbar/(m_N - m_e)^2$ for the muon or higher generation leptons.

The Yukawa type coupling $\lambda(\overline{\psi}\Phi\psi)$ to the Higgs field Φ would likewise provide mass to the fermions, involving a free parameter λ, though. After symmetry breaking, the vacuum expectation v of the Higgs would penetrate the whole cosmos, resembling a Lorentz invariant "aether", as envisioned already 1963 by de Broglie [3].

On the other hand, hypothetical *topological excitations* of the electron would also lead to another product formula

$$\tilde{m}_N = 4\pi(16\pi/3)^{N-1} m_e \tag{2}$$

for $N \geq 2$ in terms of the volumes of apropiate fibre bundles [5].

3. Next gEneration Lepton Masses?

In view of (1), the electron starts with $m_e = 0.5$ MeV, about the experimentally established value. Accordingly, the tau lepton is $2 \times 9 = 18$ times heavier than the muon which itselve is $8 \times 25 = 200$ more massive than the electron, cf. Ne'eman et al. [11], p. 247.

Thus we find the masses $0.5, 100, 1800$ MeV which are rather close to the experimental values of the known charged leptons:

Table 1. Charged Lepton Masses (MeV/c^2).

Electron	Muon	Tau
e	μ	τ
0.5	100	1800
0.511	105.66	1776.9

If a next generation lepton $L^{\pm}$ would exist, according to our formula (1), it would be $32 \times 49 = 1568$ times heavier than the tau, i.e., $m_L = 5.6448$ TeV. This is well above the lower bound of 100.8 GeV from current searches in particle accelerators, cf. the Particle Data Group [10].

A hypothetical 5th generation lepton would aquire an "astronomically" large mass of $128 \times 81 = 10,368$ times the predicted value of the $L^{\pm}$ as it has been dubbed here. Even in the decay of cosmic rays, this PeV range would remain rather difficult to detect even in the future.

Let us compare this with the rather precise lepton mass formula of Barut

$$\tilde{m}_\ell / m_e = 1 + \frac{1}{20\alpha}(2\ell+1)\ell(\ell+1)[3\ell(\ell+1)-1] \tag{3}$$

in the equivalent representation [8] for $\ell = N - 1$, where α is Sommerfeld's fine structure constant. However, it would predict a rather low mass of about $m_L \simeq 10$ GeV for next generation charged leptons and should have already been seen in current searches.

4. New Cosmos?

Our rather exotic formula (1) would indicate an internal "hidden" sub-structure of leptons, as considered earlier by Vigier et al. [1]. As in the angular momentum operator of $SU(2) \approx SO(3)$, the eigenfunctions of the electron excitations would be $2\ell + 1$ degenerate.

On the other hand, if nature restricts herself to *precisely* three generations, the known leptons could be accomodated in irreducible spinor representations of a $SO(10)$ unification of gauge groups. This may also indicate a lower dimensional topology of the early Universe and a Chern-Simons like gravitational term [2], cf. also the Mielke–Baekler model [9]. This might, as well, facilitate the conformal representation of the initial condition ("Big Bang") in Penrose's model [12] of Conformal Cyclic Cosmology. It is not clear if conformal transformations a la Weyl could resolve discrepancies [4] in the cosmological parameters like the Hubble constant H_0 or the recent observations of massive galaxies at high redshift via the James Webb Space Telescope (JWST).

The scale-invariant normalized sum and product of known charged lepton masses can be related to vacuum expectations values $<>$ of a hypothetical $U(3)$ nonet of scalar fields. Then the empirical mass spectrum can be understood as originating from a specific choice of scalar potentials [7]. In this scheme, possible higher generations are so far lacking, however.

A beyond third generation model would require also heavy quarks and cannot co-exist with the relatively light Higgs particle of 125 GeV in the standard model, cf. Holdom [6].

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