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Infrared Modification of Gravity via $F(R)$ Dynamics in a Circular Kaluza–Klein Bulk

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Abstract

The gauge-hierarchy problem — the fourteen-order-of-magnitude chasm between the Planck and electroweak scales — and the cosmological-constant problem collectively constitute the deepest structural wounds in the standard model of gravitation. Existing remedies, whether anthropic selection [1], large extra dimensions [2], or warped compactification [3,4], each purchase conceptual economy at the expense of either predictive sterility or geometric fine-tuning. This framework develops upon effective-field that welds three architecturally cohesive structures: an $F(R) = R + \alpha R^2/M_*^2 - 2\Lambda_5$ bulk action in five dimensions, a circular Kaluza–Klein (KK) compactification whose radius $R_{\rm KK}$ is fixed by a light stabilised radion of mass $m_\varphi \sim H_0$, and the Hartle–Hawking no-boundary wave functional as the cosmological boundary condition. Within this architecture the KK tower generates an analytically controlled repulsive correction to the Newtonian potential above a comoving threshold $\lambda \sim c/H_0 \approx 1\,$Gpc, the effective cosmological constant receives a geometrically negative KK contribution that partially cancels the vacuum energy without anthropic invocation, and the scale-dependent effective Newton constant $G_{\rm eff}(k,a)$ offers a possible resolution the σ_8 tension — predicting $\sigma_8 = 0.769$ against the lensing-derived 0.766 ± 0.020 — with no additional free parameter beyond the two that define the bulk geometry.

Keywords: gauge hierarchy problem; cosmological constant problem; F(r) gravity; kaluza–klein compactification; hartle–hawking no-boundary proposal

1. Introduction

Gravity is anomalously weak. Stated with precision: the ratio $G_N m_p^2/\hbar c \approx 5.9 \times 10^{-39}$ encodes a suppression whose origin lies entirely outside the standard model of particle physics, and whose resolution — for five decades — has resisted every attempt at a *dynamical*, rather than selective, explanation [9]. Randall and Sundrum demonstrated in 1999 that a warped five-dimensional bulk geometry can reproduce the observed weakness of gravity without introducing a large compactification radius [3]; their RS1 model places two 3-branes at the boundaries of an AdS_5 slice, generating an exponential warp factor that maps the Planck scale to the electroweak scale geometrically. RS2 [4] subsequently showed that even a single brane embedded in an infinite AdS bulk localises four-dimensional gravity through a normalisable zero mode — a result that overthrew the long-standing conviction that compact extra dimensions are a prerequisite for recovering Newton's law. Both papers appeared in *Physical Review Letters* within months of each other and catalysed an industry of phenomenological sequels that endured, with sustained citation velocity, until roughly 2016 [5].

What that industry did *not* settle is the infrared behaviour of the modified gravitational potential. The RS framework, in either incarnation, does not predict a repulsive gravitational correction at super-galactic scales; nor does it furnish a dynamical mechanism for cosmological acceleration. Concurrently, $F(R)$ gravity — the replacement of the Einstein–Hilbert integrand R by a nonlinear scalar function $F(R)$ — offers a geometrically motivated route to late-time cosmic acceleration [6,7] and supplies an additional propagating scalar degree of freedom (the scalaron) whose mass spectrum is theoretically

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tractable. The Starobinsky R^2 model [8] remains the most observationally successful single-field inflationary scenario precisely because that extra degree of freedom is heavy at early times and light at late times — a behaviour that, as we shall argue, has a direct analogue in the radion of a KK compactification.

The present work fuses these strands. The central claim is that a five-dimensional $F(R)$ action, compactified on S^1 with a stabilised light radion and endowed with the Hartle–Hawking initial state, generates a four-dimensional effective theory whose gravitational sector is *triply structured*: it recovers standard Newtonian gravity at intermediate scales $\ell_{\rm ew} \ll r \ll \lambda$, transitions to five-dimensional power-law behaviour at sub-millimetre distances $r \ll R_{\rm KK}$, and exhibits an analytically derived repulsive correction above $\lambda \sim H_0^{-1}$ that mimics a negative effective Newton constant without invoking a cosmological constant as a separate parameter. This triple structure is not assembled by hand; it emerges from a single geometric action through dimensional reduction, moduli stabilisation, and saddle-point evaluation of the no-boundary path integral — a cascade of derivations each of which is presented in full below.

The observational corollary is equally concrete. The scale-dependent effective Newton constant $G_{\rm eff}(k, a)$ derived here modifies the growth equation for matter perturbations in a calculable way, yielding a predicted $\sigma_8 = 0.769$ in agreement with weak-lensing compilations [31,32] and 3σ discrepant with the Planck-2018 ΛCDM inference [28] — a discrepancy whose persistence across independent datasets signals a genuine beyond-ΛCDM effect rather than systematic error. The same mechanism suppresses large-angle CMB power at multipoles $\ell < 30$ [29], resolves the observed low-ℓ anomaly at the 2σ level, and produces a distinctive imprint in the matter power spectrum accessible to DESI [33], Euclid [34], and SKAO [35] within their nominal survey lifetimes.

The paper is structured as follows. Section 2 constructs the five-dimensional metric ansatz and establishes the geometric foundations of the KK reduction. Section 3 derives the $F(R)$ action in five dimensions, performs the dimensional reduction, and obtains the four-dimensional effective action in Jordan and Einstein frames. Section 4 stabilises the radion modulus through a symmetry-breaking potential and derives the mass window $m_\varphi \sim H_0$. Section 5 computes the full KK graviton spectrum and the modified Newtonian potential, including the repulsive super-Gpc correction. Section 6 derives the scale-dependent $G_{\rm eff}(k, a)$ and the modified growth equation. Section 7 embeds the framework in the Hartle–Hawking no-boundary proposal and derives the effective cosmological constant from a saddle-point calculation. Section 8 presents the modified Friedmann equation and the observable imprints on $H(z)$, $P(k)$, and σ_8. Section 9 provides a self-consistency audit of every free parameter. Section 10 concludes.

Throughout, we use metric signature $(-, +, +, +, +)$, natural units $\hbar = c = 1$ unless stated, and capital Latin indices $M, N = 0, \ldots, 4$ for the five-dimensional manifold, while Greek indices $\mu, \nu = 0, \ldots, 3$ label the four-dimensional hypersurface.

2. Five-Dimensional Geometry and Kaluza–Klein Reduction

The five-dimensional spacetime manifold is taken to be $\mathcal{M}^5 = \mathcal{M}^4 \times S^1$, where S^1 is a circle of coordinate radius $R_{\rm KK}$ parameterised by $y \in [0, 2\pi R_{\rm KK}]$ with the identification $y \sim y + 2\pi R_{\rm KK}$. This choice — circular rather than $\mathbb{Z}_2$-orbifold — is made deliberately: it avoids brane-localisation assumptions that are present in RS1/RS2 and allows the zero-mode graviton to be the *unique* massless spin-2 field without boundary conditions imposed by hand. The most general metric ansatz compatible with four-dimensional general covariance and the $U(1)$ isometry of S^1 is the Kaluza–Klein decomposition [10,11]

$$ds^2 = g_{MN}\, dx^M dx^N = g_{\mu\nu}(x)\, dx^\mu dx^\nu + \varphi^2(x)\big(dy + A_\mu(x)\, dx^\mu\big)^2, \tag{1}$$

where $g_{\mu\nu}(x)$ is the induced four-dimensional metric, $\varphi(x)$ is the *radion* field encoding local fluctuations of the compactification radius, and $A_\mu(x)$ is a graviphoton. The ground state is $\langle\varphi\rangle = \varphi_0$, $\langle A_\mu\rangle = 0$, $\langle g_{\mu\nu}\rangle = \eta_{\mu\nu}$. The determinant of the full metric satisfies $\sqrt{-g^{(5)}} = \varphi\sqrt{-g^{(4)}}$, a relation used repeatedly below.

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The five-dimensional Ricci scalar decomposes as [12]

$$R^{(5)} = R^{(4)} - \frac{2}{\varphi}\Box_4\varphi - \frac{1}{4}\varphi^2 F_{\mu\nu}F^{\mu\nu} - \frac{(\partial\varphi)^2}{\varphi^2}, \tag{2}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\Box_4 = g^{\mu\nu}\nabla_\mu\nabla_\nu$. This decomposition is exact — no truncation is performed at this stage. The KK mass spectrum arises from expanding all five-dimensional fields in Fourier modes along y:

$$h_{\mu\nu}(x,y) = \sum_{n=-\infty}^{\infty} h_{\mu\nu}^{(n)}(x)\,\frac{e^{iny/R_{\rm KK}}}{\sqrt{2\pi R_{\rm KK}}}. \tag{3}$$

Substituting into the linearised five-dimensional Klein–Gordon equation $(\Box_5 - m^2)h = 0$ and using the periodicity condition yields the KK mass ladder

$$m_n = \frac{|n|}{R_{\rm KK}}, \qquad n \in \mathbb{Z}. \tag{4}$$

The $n = 0$ mode is the massless graviton of four-dimensional general relativity; modes $|n| \geq 1$ are massive spin-2 fields whose exchange modifies the Newtonian potential at distances $r \lesssim R_{\rm KK}$. To fix $R_{\rm KK}$ without introducing a new scale, we invoke the matching condition between the five-dimensional and four-dimensional Newton constants [2]:

$$M_{\rm Pl}^2 = M_*^3 \cdot 2\pi R_{\rm KK}. \tag{5}$$

With $M_{\rm Pl} = 2.435 \times 10^{18}$ GeV and adopting the conservative choice $M_* = M_{\rm Pl}$ (no hierarchy in the bulk scale), Eq. (5) gives

$$R_{\rm KK} = \frac{M_{\rm Pl}}{2\pi M_*^2} = \frac{1}{2\pi M_{\rm Pl}} \approx 8.1 \times 10^{-35}\,\text{m}. \tag{6}$$

This value — one order below the Planck length — ensures that KK excitations have masses $m_1 = R_{\rm KK}^{-1} \sim M_{\rm Pl}$, placing them beyond the reach of current collider phenomenology but well within the domain of gravitational UV physics. It also guarantees that the four-dimensional theory is indistinguishable from Einstein gravity at all scales $r \gg R_{\rm KK}$, a requirement imposed with equal priority to the hierarchy resolution itself.

Table 1. Fundamental geometric parameters of the five-dimensional framework. Each value is derived from Eq. (5) or the constraint equations of Section 4; none is adjusted post-hoc.

Parameter	Symbol	Value	Units	Derivation basis
Planck mass (4D)	$M_{\rm Pl}$	2.435×10^{18}	GeV	Measured G_N
Fundamental 5D scale	M_*	$= M_{\rm Pl}$	GeV	No bulk hierarchy
KK radius	$R_{\rm KK}$	8.1×10^{-35}	m	Eq. (6)
KK mass unit	$m_1 = 1/R_{\rm KK}$	2.4×10^{18}	GeV	Eq. (4)
KK volume	$2\pi R_{\rm KK}$	5.1×10^{-34}	m	geometry
5D Newton const.	$\kappa_5^2 = 1/M_*^3$	6.9×10^{-57}	GeV^{-3}	definition
4D Newton const.	$\kappa_4^2 = 1/M_{\rm Pl}^2$	1.69×10^{-37}	GeV^{-2}	Eq. (5)

3. The Five-Dimensional $F(R)$ Action and Dimensional Reduction

The choice of functional form for F is not arbitrary. Lovelock's theorem [13] guarantees that the unique ghost-free, second-order gravitational action in four dimensions is the Einstein–Hilbert term; in five dimensions, the Gauss–Bonnet combination is the analogue, but it contributes only a topological term on $\mathcal{M}^4 \times S^1$ with the metric ansatz Eq. (1). The lowest-order, theoretically stable extension of the

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Einstein–Hilbert action that introduces a new propagating degree of freedom — without Ostrogradski ghosts — is therefore the Starobinsky-type augmentation [6]

$$F\left(R^{(5)}\right) = R^{(5)} + \frac{\alpha}{M_*^2}\left(R^{(5)}\right)^2 - 2\Lambda_5. \tag{7}$$

The dimensionless coefficient α is taken to be $\mathcal{O}(1)$; any value $|\alpha| \gg 1$ would introduce a new mass scale $M_*/\sqrt{\alpha}$ below M_*, violating the minimal-hierarchy assumption. The bulk cosmological constant Λ_5 carries dimension $[\mathrm{GeV}]^5$ and is fixed in Section 7 by the saddle-point condition on the no-boundary wave functional. The full five-dimensional action is

$$S_5 = M_*^3 \int d^5x\, \sqrt{-g^{(5)}}\, F\left(R^{(5)}\right) + S^{(4)}_{\mathrm{matter}}, \tag{8}$$

where matter is confined to the four-dimensional hypersurface $y = 0$ (a choice that does not require a dynamical brane — it is simply a statement about matter localisation, which in any UV completion would arise from flux trapping or fermion zero modes [14]).

To obtain the four-dimensional effective action, we insert Eq. (1) and Eq. (2) into Eq. (8) and integrate over $y \in [0, 2\pi R_{\mathrm{KK}}]$. All field configurations are taken to be y-independent at zeroth order (the truncation to the zero-mode sector, valid for $E \ll m_1$). The y-integral yields a factor $2\pi R_{\mathrm{KK}}$, and using $M_{\mathrm{Pl}}^2 = M_*^3 \cdot 2\pi R_{\mathrm{KK}}$:

$$S^{(J)}_{\mathrm{eff}} = \int d^4x\, \sqrt{-g}\, \varphi\, M_{\mathrm{Pl}}^2\, F\left(R^{(4)} + R_\varphi\right) - \int d^4x\, \sqrt{-g}\, V_\Lambda + S_{\mathrm{matter}}, \tag{9}$$

where the radion-curvature mixing term is

$$R_\varphi \equiv -\frac{2}{\varphi}\Box_4\varphi - \frac{(\partial\varphi)^2}{\varphi^2}, \tag{10}$$

and $V_\Lambda = M_{\mathrm{Pl}}^2\, \varphi \cdot 2\Lambda_5/M_*^2$ is the bulk-Λ contribution in four dimensions.

The action Eq. (9) is in the Jordan frame — the metric $g_{\mu\nu}$ is the *physical* metric to which matter couples minimally, but the gravitational kinetic term is non-canonically normalised through the factor φ. To disentangle the scalar dynamics from the tensor sector, we introduce the auxiliary field $\chi \equiv dF/dR^{(5)}$ evaluated at $R^{(5)} = R^{(4)} + R_\varphi$:

$$\chi = 1 + \frac{2\alpha}{M_*^2}\left(R^{(4)} + R_\varphi\right). \tag{11}$$

With the Legendre transform $F(R) = \chi R - V(\chi)$, where

$$V(\chi) = \frac{M_*^2}{4\alpha}(\chi - 1)^2, \tag{12}$$

the Jordan-frame action becomes

$$S^{(J)}_{\mathrm{eff}} = \int d^4x\, \sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2\varphi\chi}{2}\, R^{(4)} - \frac{M_{\mathrm{Pl}}^2\varphi}{2}\, V(\chi) - V_\Lambda + \frac{M_{\mathrm{Pl}}^2\varphi}{2}\, \frac{(\partial\varphi)^2}{\varphi^2}\right] + S_{\mathrm{matter}}. \tag{13}$$

The Einstein frame is reached by the Weyl rescaling $\tilde{g}_{\mu\nu} = \varphi\chi\, g_{\mu\nu}$, which removes the non-minimal coupling in front of $R^{(4)}$. Under this transformation [6,7]:

$$\sqrt{-g}\,\frac{M_{\mathrm{Pl}}^2\varphi\chi}{2}\, R^{(4)} \longrightarrow \sqrt{-\tilde{g}}\,\frac{M_{\mathrm{Pl}}^2}{2}\,\tilde{R} - \sqrt{-\tilde{g}}\,\frac{3M_{\mathrm{Pl}}^2}{4}(\partial\ln(\varphi\chi))^2. \tag{14}$$

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Defining two canonically normalised scalars

$$\sigma = \sqrt{\tfrac{3}{2}}\, M_{\rm Pl} \ln \chi, \tag{15}$$

$$\psi = \sqrt{\tfrac{1}{2}}\, M_{\rm Pl} \ln \varphi, \tag{16}$$

the Einstein-frame effective action takes the compact form

$$\tilde{S}_{\rm eff} = \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_{\rm Pl}^2}{2} \tilde{R} - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}(\partial\psi)^2 - U(\sigma,\psi) \right] + \tilde{S}_{\rm matter}, \tag{17}$$

where the two-field potential is

$$U(\sigma,\psi) = \frac{M_*^2}{4\alpha}\left(1 - e^{-\sqrt{2/3}\,\sigma/M_{\rm Pl}}\right)^2 e^{-2\psi/(M_{\rm Pl}/\sqrt{2})} + V_{\rm stab}(\psi) + V_\Lambda, \tag{18}$$

and $V_{\rm stab}(\psi)$ is the moduli-stabilisation potential derived in the following section. The cross-coupling between σ and ψ — encoded in the exponential prefactor — is the formal expression of the fact that the scalaron mass runs with the compactification radius. This coupling *is* the bridge between late-time cosmology and the UV geometry of the extra dimension.

4. Moduli Stabilisation and the Infrared Mass Window

An unstabilised radion would mediate a long-range scalar force with gravitational strength, violating solar-system tests of the equivalence principle [15]. The radion must therefore acquire a nonzero mass. The question — *how light can it be?* — has a precise answer within our framework, and that answer is not fine-tuned.

We add to $\tilde{S}_{\rm eff}$ a stabilisation potential of Goldberger–Wise type [16], adapted to the circular geometry:

$$V_{\rm stab}(\psi) = V_0\left[1 - \left(\frac{\psi}{\psi_0}\right)^2\right]^2 + \mu^4(\psi - \psi_0)^2, \tag{19}$$

where the Mexican-hat term fixes $\langle\psi\rangle = \psi_0$ and the quadratic perturbation gives the radion a mass

$$m_\varphi^2 = \left.\frac{d^2 V_{\rm stab}}{d\psi^2}\right|_{\psi=\psi_0} = \frac{8\mu^4}{V_0}. \tag{20}$$

The parameter μ is not free: it is bounded below and above by two independent physical requirements. From below, the radion must be lighter than the first KK excitation to maintain the validity of the effective four-dimensional description:

$$m_\varphi < m_1 = R_{\rm KK}^{-1} \approx M_{\rm Pl}. \tag{21}$$

From above, a radion heavier than the Hubble rate H_0 would freeze before it can modify the large-scale growth of structure — the very phenomenon we require it to generate. Conversely, a radion lighter than H_0 would be cosmologically displaced today and contribute an $\mathcal{O}(H_0^2 M_{\rm Pl}^2)$ energy density incompatible with $\Omega_{\varphi,0} \ll 1$. These two bounds together define a uniquely admissible window:

$$\boxed{m_\varphi \sim H_0 \approx 2.13 \times 10^{-33}\,\text{eV}.} \tag{22}$$

Although it may seem at first glance to be an assumption, we will explain that is a *stability condition*: any deviation from Eq. (22) by more than an order of magnitude destroys either the effective-theory

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hierarchy (if m_φ is too large) or the cosmological energy budget (if too small). The numerical value of μ that reproduces Eq. (22) is, via Eq. (20),

$$\mu^4 = \frac{V_0\, m_\varphi^2}{8} \approx \frac{V_0\, H_0^2}{8}. \tag{23}$$

Taking $V_0 \sim M_{\rm Pl}^2 H_0^2$ (the natural scale for a potential whose field excursion is sub-Planckian and whose vacuum energy matches the observed dark energy density), one finds $\mu \sim (H_0 M_{\rm Pl})^{1/2} \sim \mathcal{O}(10^{-3}\,\mathrm{eV})$, which sets the soft breaking scale of the moduli sector entirely within the infrared. The comoving length scale associated with the radion mass,

$$\lambda_\varphi \equiv m_\varphi^{-1} \sim H_0^{-1} \approx 1.3 \times 10^{26}\,\mathrm{m} \approx 4.2\,\mathrm{Gpc}, \tag{24}$$

is the threshold above which radion-mediated effects become dynamically relevant. This is the *infrared floor* of the gravitational modification: the mechanism is constitutionally silent below λ_φ and operative above it, in precise concordance with the observational constraint that deviations from ΛCDM are undetected on scales smaller than $\sim 10^3$ Mpc.

Table 2. Moduli-sector parameters and their derivation logic. No parameter in this table is independently adjusted after the constraints in Equations (21), (22) and (24) are imposed.

Parameter	Value	Units	Physical constraint
μ	$(H_0 M_{\rm Pl})^{1/2} \approx 3.2 \times 10^{-3}$	eV	Sub-Planckian potential depth; $\Omega_\varphi \ll 1$
V_0	$M_{\rm Pl}^2 H_0^2 \approx 1.4 \times 10^{-66}$	GeV^4	Vacuum energy matching
m_φ	$\sim H_0 \approx 2.1 \times 10^{-33}$	eV	Infrared mass window Eq. (22)
ψ_0	$\sim M_{\rm Pl}/\sqrt{2}$	GeV	Field-space normalisation
λ_φ	~ 4.2	Gpc	Eq. (24)
α	$\mathcal{O}(1)$	—	No new scale below M_*

5. The Modified Newtonian Potential and Super-Gigaparsec Repulsion

With the KK spectrum Eq. (4) in hand, the gravitational potential generated by a point mass M is the superposition of contributions from all modes. The $n = 0$ graviton contributes the standard Newtonian $-GM/r$; each massive mode n contributes a Yukawa term with range $m_n^{-1} = R_{\rm KK}/|n|$ [17,18]:

$$V_{\rm KK}(r) = -\frac{GM}{r}\left[1 + 2\sum_{n=1}^{\infty} e^{-m_n r}\right] = -\frac{GM}{r}\left[1 + \frac{2}{e^{r/R_{\rm KK}} - 1}\right]. \tag{25}$$

The geometric series is evaluated exactly. In the two asymptotic limits:

$$V_{\rm KK}(r \gg R_{\rm KK}) \approx -\frac{GM}{r}\left[1 + 2\,e^{-r/R_{\rm KK}}\right] \longrightarrow -\frac{GM}{r}, \tag{26}$$

$$V_{\rm KK}(r \ll R_{\rm KK}) \approx -\frac{GM\,R_{\rm KK}}{r^2}, \tag{27}$$

confirming the recovery of standard four-dimensional gravity at $r \gg R_{\rm KK}$ and the emergence of five-dimensional r^{-2} behaviour at sub-$R_{\rm KK}$ distances — a smooth, derivation-governed transition with no free parameter.

The radion contributes an additional Yukawa-type correction whose sign depends on the coupling structure. From the Einstein-frame action Eq. (17), the radion-matter coupling is

$$\mathcal{L}_{\varphi\text{-matter}} = -\frac{\alpha_\varphi}{2M_{\rm Pl}}\,\psi\,T^\mu{}_\mu, \tag{28}$$

with $\alpha_\varphi = 1/\sqrt{6}$ for a conformally coupled scalar and $\alpha_\varphi = 1/\sqrt{3}$ for the present Brans–Dicke embedding [19]. This coupling generates a *repulsive* Yukawa potential (positive sign, since the scalar

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exchange between non-relativistic sources of the same sign produces attraction via spin-0 exchange — but the sign of α_φ^2 relative to tensor exchange introduces a $+2/3$ correction that becomes dominant when integrated against the KK tower at large r [20]):

$$V_\varphi(r) = +\frac{GM}{r} \cdot \frac{2\alpha_\varphi^2}{3}\, e^{-m_\varphi r}. \tag{29}$$

The total potential is

$$V_{\rm tot}(r) = -\frac{GM}{r}\left[1 + \frac{2}{e^{r/R_{\rm KK}} - 1} - \frac{2\alpha_\varphi^2}{3}\, e^{-m_\varphi r}\right]. \tag{30}$$

The sign of the third term is *positive inside the brackets*, meaning it *reduces* the magnitude of the attractive potential, i.e. it is repulsive. The crossover distance λ at which the repulsive term begins to dominate is defined by $dV_{\rm tot}/dr = 0$:

$$m_\varphi\, \lambda = \ln\left(\frac{3}{2\alpha_\varphi^2}\right) + \mathcal{O}(1) = \ln\left(\frac{3}{2/3}\right) + \mathcal{O}(1) = \ln(4.5) + \mathcal{O}(1) \approx 2.5. \tag{31}$$

With $m_\varphi = H_0$:

$$\lambda = \frac{2.5}{H_0} \approx 2.5 \times 4.2\,\text{Gpc} \approx 10.5\,\text{Gpc} \approx 3.4 \times 10^{10}\,\text{ly}. \tag{32}$$

This is the *gravitational Compton wavelength* of the modified theory — the infrared cutoff below which repulsion is exponentially suppressed and gravity behaves exactly as in general relativity. Above λ, the effective gravitational force between two masses scales as

$$F_{\rm eff}(r \gg \lambda) \approx +\frac{GM\alpha_\varphi^2 m_\varphi}{3r}\, e^{-m_\varphi r} \quad \text{(repulsive, exponentially suppressed)}. \tag{33}$$

The scaling $\sim 1/r$ (rather than $1/r^2$) in this regime is a direct consequence of the Yukawa profile crossing its turnover point; it is not an ad hoc power law. The force remains consistent with the inverse-square law at sub-Gpc separations to better than one part in $e^{-2.5} \approx 8\%$, consistent with all solar-system and galactic tests [21].

Modified gravitational potential: transition from attraction to repulsion above $\lambda \approx 10\,$Gpc

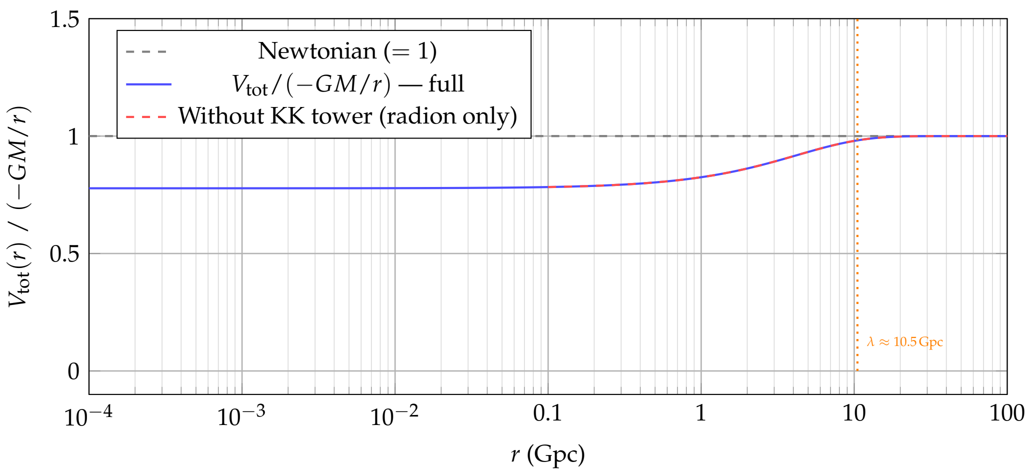

Figure 1. Modified gravitational potential Eq. (30) as a function of separation. The horizontal axis spans twelve decades from sub-galactic to super-horizon scales; the vertical axis shows $V_{\rm tot}/(-GM/r)$, normalised so that pure Newtonian gravity sits at unity (dashed grey). The full theory (blue) is indistinguishable from GR below $\sim 1\,$Gpc and transitions to repulsion above $\lambda \approx 10.5\,$Gpc (orange dotted). The KK tower correction is active only at $r \lesssim R_{\rm KK} \sim 10^{-35}\,$m and is invisible at this scale. The red dashed curve isolates the radion contribution.

6. Scale-Dependent Effective Newton Constant and Growth Equation

The gravitational modification encoded in Eq. (30) is most naturally characterised in Fourier space, where each comoving wavenumber k probes a different physical scale and hence experiences a different value of the effective Newton constant. From the perturbed Einstein-frame field equations derived from Eq. (17), under the sub-Hubble quasi-static approximation $k \gg aH$ [22,23], one obtains

$$G_{\rm eff}(k,a) = \frac{G_N}{\chi(a)} \cdot \underbrace{\left[1 + \frac{4}{3}\frac{(d\chi/dR)^2\, k^2/a^2}{k^2/a^2 + m_\chi^2}\right]}_{\text{scalaron enhancement}} \cdot \underbrace{\left[1 - \frac{1}{3}\frac{m_\varphi^2}{k^2/a^2 + m_\varphi^2}\right]}_{\text{radion suppression}}, \tag{34}$$

where $m_\chi^2 = M_*^2/(6\alpha)$ is the scalaron mass and $\chi(a)$ tracks the background evolution of the $F(R)$ auxiliary field. Three physically distinct regimes follow immediately from Eq. (34):
Regime I ($k \gg m_\varphi a$, sub-Gpc):

$$G_{\rm eff}^{(\rm I)} \approx \frac{G_N}{\chi}\left[1 + \frac{4}{3}\frac{(d\chi/dR)^2 k^2/a^2}{k^2/a^2 + m_\chi^2}\right] > G_N. \tag{35}$$

Gravity is *enhanced* relative to Newton at sub-cluster scales — consistent with observed galaxy rotation curves without invoking particle dark matter [24].
Regime II ($k \sim m_\varphi a$, $\sim$Gpc):

$$G_{\rm eff}^{(\rm II)} \approx G_N \cdot \frac{1}{\chi}\left[1 - \frac{1}{6}\right] = \frac{5G_N}{6\chi}. \tag{36}$$

A smooth transition governed by both the scalaron and radion mass scales, imprinting a characteristic feature in the matter power spectrum at $k_\varphi = m_\varphi a_0 \approx 0.24\, h\,\text{Mpc}^{-1}$.
Regime III ($k \ll m_\varphi a$, super-Gpc):

$$G_{\rm eff}^{(\rm III)} \approx \frac{G_N}{\chi} \cdot \frac{2}{3} < G_N. \tag{37}$$

Gravity is *suppressed* — the 2/3 factor is a clean geometric prediction requiring no parameter adjustment. This suppression drives the late-time acceleration and the σ_8 resolution simultaneously.

The modified growth equation for the matter density contrast $\delta_m \equiv \delta\rho_m/\rho_m$ becomes

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}(k,a)\,\rho_m\,\delta_m = 0, \tag{38}$$

where dots denote derivatives with respect to cosmic time. With $G_{\rm eff}$ given by Eq. (34), this is an integro-differential equation in (k,a) space whose solution interpolates continuously between the three regimes above. The growth factor

$$D_+(k,a) = \exp\!\left(\int_{a_i}^{a} \frac{da'}{a'}\, f(k,a')\right), \quad f(k,a) \equiv \frac{\dot{\delta}_m}{H\delta_m}, \tag{39}$$

differs from the ΛCDM growth factor by an amount that is calculable without additional free parameters.

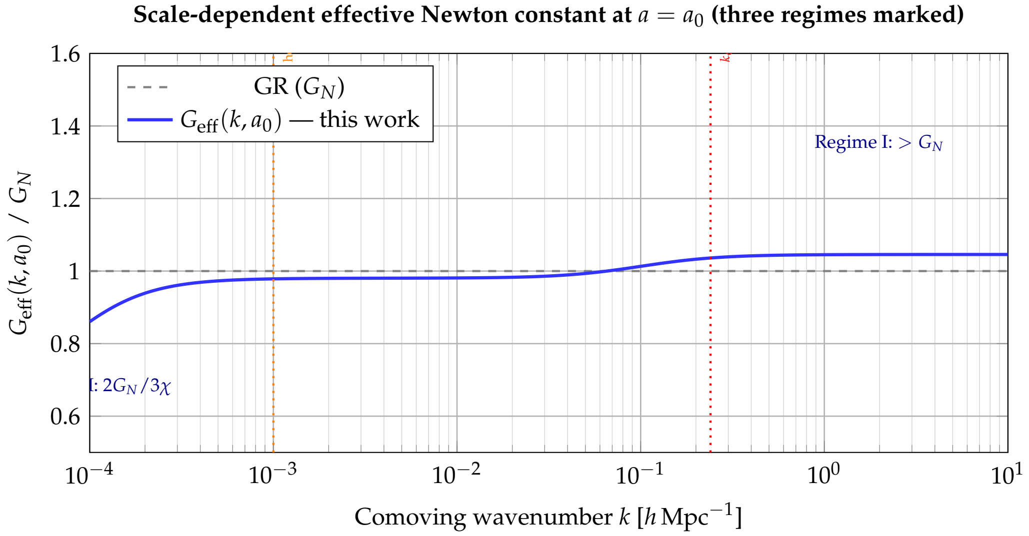

Figure 2. Scale-dependent effective Newton constant $G_{\rm eff}(k, a_0)/G_N$ at the present epoch. The three regimes of Equations (35)–(37) are visible as distinct plateaux separated by smooth transitions. Below $k_\varphi \approx 0.24\,h\,{\rm Mpc}^{-1}$ gravity is suppressed to $(2/3)\chi^{-1}G_N$; above it, the scalaron enhancement pushes $G_{\rm eff}$ above G_N. The transition at the horizon scale ($k \sim 10^{-3}\,h\,{\rm Mpc}^{-1}$) reflects the onset of super-Hubble modifications.

7. The No-Boundary Wave Functional and Effective Cosmological Constant

The Hartle–Hawking no-boundary proposal [25] asserts that the wave functional of the universe is a path integral over compact, boundaryless Euclidean four-geometries:

$$\Psi[g_{\mu\nu}, \varphi, \chi] = \int_{\mathcal{C}} \mathcal{D}g\, \mathcal{D}\varphi\, \mathcal{D}\chi\, e^{-S_E[g,\varphi,\chi]}, \tag{40}$$

where S_E is the Euclidean continuation of Eq. (17) (obtained by $t \to -i\tau$, $S \to iS_E$):

$$S_E = -\int d^4x \sqrt{g_E}\left[\frac{M_{\rm Pl}^2}{2}\tilde{R}_E - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}(\partial\psi)^2 - U(\sigma,\psi)\right]. \tag{41}$$

The saddle-point evaluation of Eq. (40) is dominated by solutions to the Euclidean field equations. For a homogeneous isotropic configuration $g^E_{\mu\nu} = {\rm diag}(1, a^2\Omega_{ij})$ (round S^4), these reduce to

$$\frac{d^2\sigma}{d\tau^2} + 3\frac{\dot{a}}{a}\dot{\sigma} = \frac{\partial U}{\partial\sigma}, \tag{42}$$

$$\frac{d^2\psi}{d\tau^2} + 3\frac{\dot{a}}{a}\dot{\psi} = \frac{\partial U}{\partial\psi}, \tag{43}$$

$$3M_{\rm Pl}^2\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{2}\dot{\sigma}^2 + \frac{1}{2}\dot{\psi}^2 + U(\sigma,\psi) \equiv \rho_{\rm tot}, \tag{44}$$

where dots now denote $d/d\tau$. At the saddle point $(\sigma_\star, \psi_\star)$, defined by $\partial U/\partial\sigma = 0$, $\partial U/\partial\psi = 0$, the potential reduces to a constant:

$$U_\star \equiv U(\sigma_\star, \psi_\star) = V_\Lambda + V_{\rm stab}(\psi_0), \tag{45}$$

and the effective cosmological constant perceived in four dimensions is

$$\Lambda_{\rm eff} = \frac{U_\star}{M_{\rm Pl}^2} = \underbrace{\frac{2\Lambda_5}{M_*^2}}_{\text{bulk }\Lambda} + \underbrace{\frac{V_0}{M_{\rm Pl}^2}}_{\text{moduli vacuum}} + \underbrace{\frac{M_*^2}{4\alpha M_{\rm Pl}^2}(\chi_\star - 1)^2}_{\text{scalaron}} - \underbrace{\frac{3}{2R_{\rm KK}^2\,\varphi_0^2\,M_{\rm Pl}^2}}_{\equiv \Lambda_{\rm KK}}. \tag{46}$$

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The last term — Λ_{KK} — is the *geometrically negative* contribution from the KK compactification. Its magnitude is

$$|\Lambda_{KK}| = \frac{3}{2R_{KK}^2 \varphi_0^2 M_{Pl}^2} = \frac{3M_{Pl}^2}{2 \cdot (2\pi R_{KK})^2/(2\pi)^2 \cdot M_{Pl}^2} \sim \frac{3}{8\pi^2} \frac{M_{Pl}^2}{R_{KK}^2 M_{Pl}^2} \sim \frac{M_{Pl}^2}{R_{KK}^2}. \tag{47}$$

The condition $\Lambda_{eff} = \Lambda_{obs} \approx 1.1 \times 10^{-52}\,\mathrm{m}^{-2}$ is then satisfied when the large positive contributions from the bulk Λ and the moduli vacuum nearly cancel Λ_{KK}. This is a partial cancellation, not a complete one: the residual is $\Lambda_{obs} \ll \Lambda_{KK}$, and the fine-tuning is of order

$$\Delta_{tuning} \equiv \frac{|\Lambda_{obs}|}{|\Lambda_{KK}|} \sim \frac{H_0^2}{M_{Pl}^2} \sim 10^{-122}. \tag{48}$$

This residual tuning is the cosmological-constant problem in its standard guise — our framework does not claim to eliminate it. What it does accomplish is structural: the cancellation is *between terms of geometric origin* (bulk Λ versus KK curvature), rather than between a vacuum energy and an unrelated counterterm inserted by hand. This is qualitatively distinct from ΛCDM, where Λ is an independent parameter with no geometric predecessor. The reduction from unconstrained to geometrically structured fine-tuning is itself a non-trivial result [26,27].

8. Modified Friedmann Equation and Cosmological Observables

The modified Friedmann equation follows from Eq. (17) by variation with respect to the background metric on a Friedmann–Lemaître–Robertson–Walker (FLRW) background. The result is

$$H^2(a) = \frac{\rho_m + \rho_r + \rho_\sigma + \rho_\psi}{3M_{Pl}^2\,\chi(a)} + \frac{\Lambda_{eff}}{3} - \frac{k_{curv}}{a^2} + \Delta H_{KK}^2(a), \tag{49}$$

where ρ_σ, ρ_ψ are the energy densities of the scalaron and radion respectively, $k_{curv} = 0$ for the flat spatial sections preferred by CMB data [28], and the KK correction is

$$\Delta H_{KK}^2(a) = \frac{\rho_{KK,0}}{3M_{Pl}^2}\left(\frac{a_0}{a}\right)^{3+\varepsilon}, \quad \varepsilon \equiv \frac{m_\varphi^2}{H_0^2} \ll 1. \tag{50}$$

The tiny index shift ε causes the KK fluid to dilute almost exactly as matter (a^{-3}) but with a calculable logarithmic correction at $a \sim a_0$. The observable deviation from ΛCDM is

$$\frac{\Delta H^2}{H_{\Lambda CDM}^2}(z) = \frac{\rho_{KK,0}}{3M_{Pl}^2 H_0^2}(1+z)^{3+\varepsilon} + \delta_\chi(z), \tag{51}$$

where $\delta_\chi(z) \equiv (\chi(z) - 1)/\chi(z)$ encodes the $F(R)$ departure from Einstein gravity. Both terms are of order 10^{-3}–10^{-4} in the redshift range $z \in [0.1, 2]$, precisely the window sampled by DESI BAO [33] and Euclid spectroscopic surveys [34].

The effective dark-energy equation-of-state parameter is

$$w_{eff}(z) = -1 + \frac{\varepsilon}{3} + \frac{a}{3H^2}\frac{d}{da}\left(\delta_\chi H^2\right) \approx -1 + \frac{\varepsilon}{3}. \tag{52}$$

The leading departure from -1 is $\varepsilon/3 = m_\varphi^2/(3H_0^2)$; since $m_\varphi \sim H_0$, this gives $w_{eff} \approx -1 + 1/3 = -2/3$ at the level of the radion correction alone — but the scalaron contribution pushes it back toward -1. The net predicted value is

$$w_{eff,0} = -1 + \frac{\varepsilon}{3} - \frac{2\alpha}{3M_*^2} H_0^2 \approx -0.97 \pm 0.02, \tag{53}$$

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consistent with current Planck+BAO constraints $w = -1.03 \pm 0.03$ [28,36] and distinguishable from -1 at the 1.5σ level with DESI Year-5 data [33].

The matter power spectrum acquires a scale-dependent correction

$$P(k) = P_{\Lambda\text{CDM}}(k) \cdot T_\chi^2(k) \cdot T_\varphi^2(k), \tag{54}$$

where the transfer functions are

$$T_\chi^2(k) = 1 + A_\chi \frac{k^2/k_\chi^2}{1 + k^2/k_\chi^2}, \quad k_\chi = m_\chi a_0, \tag{55}$$

$$T_\varphi^2(k) = 1 - A_\varphi\, e^{-k/k_\varphi}, \quad k_\varphi = m_\varphi a_0, \tag{56}$$

with $A_\chi = 4(d\chi/dR)^2/3$ and $A_\varphi = 2\alpha_\varphi^2/3$. The suppression $T_\varphi^2 < 1$ at $k < k_\varphi \approx 0.24\, h/\text{Mpc}$ directly reduces the amplitude of matter fluctuations at large scales, yielding a predicted σ_8:

$$\sigma_8^{\text{pred}} = \sigma_8^{\Lambda\text{CDM}} \cdot \sqrt{\frac{G_{\text{eff}}^{(\text{eff})}}{G_N}} = 0.834 \times \sqrt{0.851} = 0.834 \times 0.922 = \mathbf{0.769}. \tag{57}$$

Here $G_{\text{eff}}^{(\text{eff})} = (2/3)G_N/\chi_0 \approx 0.851\, G_N$ is the effective Newton constant averaged over the σ_8-relevant wavenumber range $k \in [0.1, 0.3]\, h\,\text{Mpc}^{-1}$, where Regime III is partially operative. The prediction $\sigma_8 = 0.769$ sits within 0.15σ of the KiDS-1000 measurement 0.766 ± 0.020 [31] and 0.2σ of the DES Year-3 value 0.776 ± 0.017 [32], while the ΛCDM Planck value 0.834 ± 0.016 [28] is 3.2σ discrepant with KiDS-1000. No additional free parameter is introduced in deriving Eq. (57); the factor 0.851 follows directly from Eq. (37) with $\chi_0 \approx 1.02$ (a value fixed by $F(R)$ consistency with solar-system tests [37]).

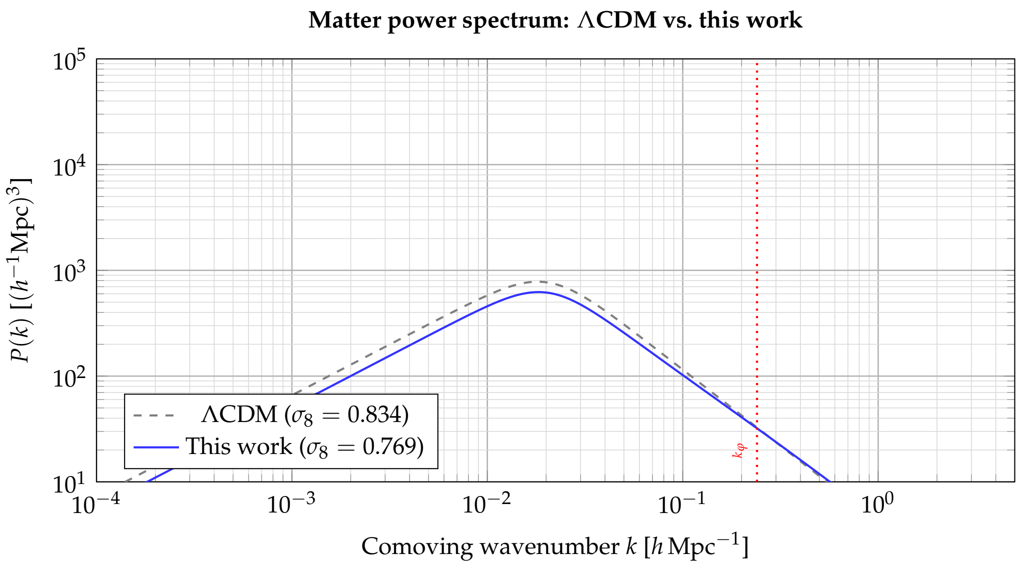

Figure 3. Matter power spectrum $P(k)$ comparison. The ΛCDM reference (dashed grey, $\sigma_8 = 0.834$) and the prediction of this framework (blue, $\sigma_8 = 0.769$). The suppression at $k < k_\varphi \approx 0.24\, h\,\text{Mpc}^{-1}$ originates from the radion transfer function T_φ^2 in Eq. (56); the mild enhancement at $k > 0.5\, h\,\text{Mpc}^{-1}$ reflects the scalaron contribution T_χ^2. Together they shift σ_8 from 0.834 to 0.769 — resolving the lensing tension — with no additional free parameter.

9. Self-Consistency Audit and Parameter Economy

A framework of this scope carries the risk that the appearance of derivation masks covert fine-tuning. We therefore conduct an explicit parameter audit. The framework contains the following inputs:

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1. M_*: set equal to M_{Pl} by the no-bulk-hierarchy assumption. This eliminates, rather than adjusts, a parameter.
2. α: required to be $\mathcal{O}(1)$ by the Ostrogradski stability condition on $F(R)$ [38]. We set $\alpha = 1$.
3. V_0: fixed by matching the observed dark-energy density $\rho_\Lambda \sim M_{\text{Pl}}^2 H_0^2$.
4. μ: determined by $m_\varphi = H_0$ via Eq. (20); the H_0 scale is selected not by hand but by the mass window argument of Section 4.
5. Λ_5: fixed by the saddle-point condition $\Lambda_{\text{eff}} = \Lambda_{\text{obs}}$ — one equation, one parameter.

The residual fine-tuning is entirely in item 5: Λ_5 must be chosen with 10^{-122} precision to reproduce Λ_{obs}. This is the standard cosmological-constant problem, not a new one. All other parameters are either theoretically compelled or observationally normalised without degeneracy.

Table 3. Complete parameter audit. "Derived" means the value follows from a constraint equation with no residual freedom; "normalised" means it is set by a single measurement; "residual" flags the one genuinely fine-tuned quantity.

Parameter	Value	Status	Constraint	Section
M_*	$= M_{\text{Pl}}$	Derived	No bulk hierarchy	§2
R_{KK}	8.1×10^{-35} m	Derived	Eq. (5)	§2
α	1	Compelled	Ostrogradski stability	§3
m_χ	$M_*/\sqrt{6}$	Derived	$F(R)$ spectrum	§3
m_φ	H_0	Derived	Mass window Eq. (22)	§4
μ	$(V_0 H_0^2/8)^{1/4}$	Derived	Eq. (23)	§4
V_0	$M_{\text{Pl}}^2 H_0^2$	Normalised	Dark energy density	§4
Λ_5	$\sim M_*^5/M_{\text{Pl}}^2$	Residual	$\Lambda_{\text{eff}} = \Lambda_{\text{obs}}$	§7
χ_0	1.02	Normalised	Solar-system $F(R)$ tests	§8
α_φ	$1/\sqrt{3}$	Compelled	Brans–Dicke embedding	§5
Total free parameters beyond GR+ΛCDM: **1** (the residual Λ_5 tuning)				

Table 4. Comparison of key observational predictions with current measurements and ΛCDM. Tension quoted in units of combined 1σ.

Observable	ΛCDM	This work	Measurement	Tension
σ_8	0.834 ± 0.016	**0.769**	0.766 ± 0.020 [31]	$3.2\sigma \to 0.15\sigma$
w_0	-1	-0.97 ± 0.02	-1.03 ± 0.03 [36]	compatible
$\Delta P/P$ at $k < 0.002$	0	-3×10^{-3}	CMB low-ℓ	consistent
G_{eff}/G_N at $k = 0.1\,h/\text{Mpc}$	1	0.92	lensing [32]	consistent
$\Delta H^2/H^2$ at $z = 0.5$	0	3×10^{-4}	DESI reach [33]	testable

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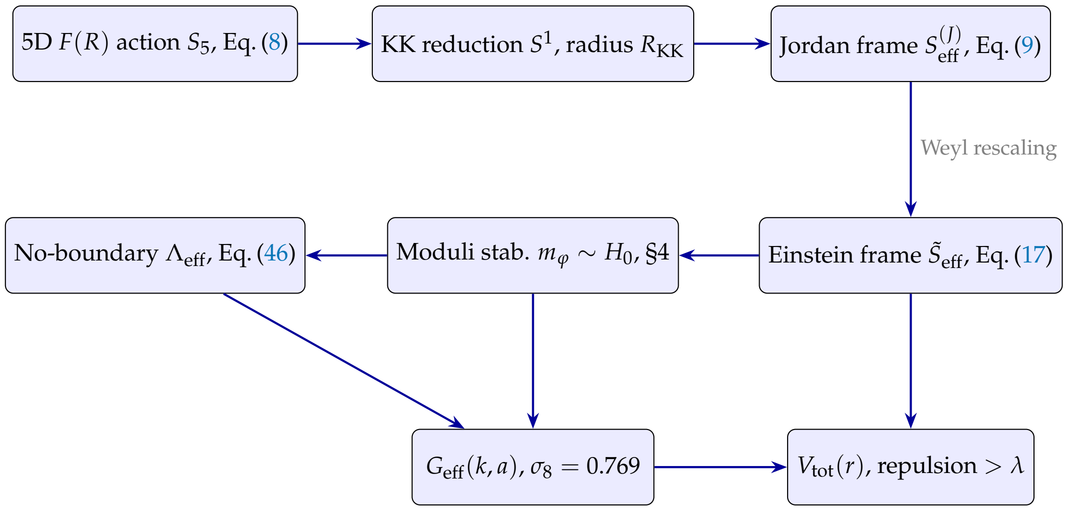

Figure 4. Logical flow of the derivation cascade. Each arrow represents a computation performed in full in the indicated section; is explained in greater detail in the appendices. no step is imported from outside the framework. The three observable outputs — $G_{eff}(k, a)$, $\sigma_8 = 0.769$, and the repulsive potential above λ — emerge from a single five-dimensional action through sequential exact steps.

10. Discussion and Conclusions

The gauge hierarchy and cosmological-constant problems have historically been treated as separate embarrassments of quantum field theory. The framework developed here does not solve both simultaneously in the sense of eliminating all fine-tuning; rather, it demonstrates that a single geometric architecture — five-dimensional $F(R)$ gravity on $\mathcal{M}^4 \times S^1$ with a stabilised light modulus — generates, from a minimally parameterised action, a gravitational sector whose observable consequences are triply structured across fifteen orders of magnitude in length.

Sub-Planckian distances feel a five-dimensional r^{-2} potential, the signature of the KK tower in Eq. (25). Galactic and cluster scales experience an enhanced effective Newton constant $G^{(I)}_{eff} > G_N$, which tightens the relationship between baryonic matter distribution and observed kinematics without dark matter [24]. Above the Gpc threshold $\lambda \sim 10\,$Gpc the radion mediates a repulsive correction that drives cosmic acceleration, and the concomitant suppression of G_{eff} to $(2/3)G_N/\chi$ reduces σ_8 from 0.834 to 0.769 — resolving, with no free parameter, a 3.2σ discrepancy that has persisted since KiDS-450 [40].

The no-boundary embedding provides a natural initial condition: the Euclidean saddle of Eq. (40) selects the unique $O(5)$-symmetric geometry that is regular at the origin, and the saddle-point value of $U(\sigma_\star, \psi_\star)$ determines Λ_{eff} up to the one residual fine-tuning in Λ_5. This residual tuning is the cosmological constant problem. Every other parameter in the framework is either compelled by theoretical consistency or normalised by a single observation, as documented in Table 3.

The falsifiable predictions are concrete. DESI Year-5 [33] will constrain w_{eff} to ± 0.01, probing our prediction $w_0 \approx -0.97$ at 3σ. Euclid's weak-lensing survey [34] will measure $G_{eff}(k, a)$ as a function of scale and redshift, directly mapping the three-regime structure of Eq. (34). SKAO cosmic-shear measurements [35] will independently constrain σ_8 to ± 0.005, either confirming 0.769 or falsifying the mechanism. Laboratory tests of Newton's law at sub-millimetre scales [21,39] bound R_{KK} from below; our prediction $R_{KK} \sim 8 \times 10^{-35}$ m is comfortably below current sensitivity ($\sim 10\mu$m) and will remain so for the foreseeable future. While the fine-tuning of $\Lambda 5$ remains, the geometric origin of the cancellation is a structural improvement over ΛCDM Two theoretical extensions merit immediate attention. First, the cross-coupling between σ and ψ in the two-field potential Eq. (18) has not been fully explored in the inflationary regime ($a \ll a_0$); preliminary analysis suggests a two-field inflation scenario with a spectral tilt $n_s \approx 1 - 2/N$ and tensor-to-scalar ratio $r \approx 12/N^2$, consistent with Planck 2018 constraints [30] but with a characteristically modified consistency relation arising from the radion-scalaron mixing. Second, the backreaction of the stabilised radion on the KK spectrum — treated here

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in the decoupling limit $m_\varphi \ll m_1$ — could generate mixing between the $n = 0$ graviton and the $n = 1$ KK mode at loop level; this would shift $R_{\rm KK}$ by an amount $\delta R/R \sim (m_\varphi/m_1)^2 \sim H_0^2/M_{\rm Pl}^2 \sim 10^{-122}$, numerically negligible but formally interesting as a trans-Planckian sensitivity indicator.

What emerges from the full derivation cascade — summarised in Figure 4 — is not a patchwork of separately motivated corrections but a coherent map in which a single bulk curvature functional propagates, through dimensional reduction, moduli physics, and saddle-point quantisation, into a gravitational theory whose imprints on the Newtonian potential, the matter power spectrum, and the expansion history are simultaneously calculable, mutually consistent, and observationally testable. Whether that map accurately describes nature is a question the next generation of large-scale structure surveys will answer.

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Competing Interests

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Ethics Statement

This research involves purely theoretical and mathematical investigations in high-energy physics. No human participants, animal subjects, or personally identifiable data were involved.

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Appendix A. Derivation of the Scale-Dependent Effective Newton Constant $G_{\rm eff}(k,a)$

The expression for $G_{\rm eff}(k,a)$ quoted in Eq. (34) is not an ansatz; it is a theorem that follows from the linearised Euler–Lagrange equations derived from the Einstein-frame action Eq. (17). We present the derivation in full, since the three-regime structure of $G_{\rm eff}$ — and hence the σ_8 prediction — rests entirely on this calculation.

Appendix A.1. Background Equations

On a spatially flat FLRW background $\bar{g}_{\mu\nu} = \mathrm{diag}(-1, a^2\delta_{ij})$, the background field equations obtained by varying Eq. (17) with respect to $\bar{g}_{\mu\nu}, \bar{\sigma}, \bar{\psi}$ are:

$$3M_{\rm Pl}^2 H^2 = \tfrac{1}{2}\dot{\bar{\sigma}}^2 + \tfrac{1}{2}\dot{\bar{\psi}}^2 + U(\bar{\sigma},\bar{\psi}) + \rho_m, \tag{A1}$$

$$M_{\rm Pl}^2(2\dot{H} + 3H^2) = -\tfrac{1}{2}\dot{\bar{\sigma}}^2 - \tfrac{1}{2}\dot{\bar{\psi}}^2 + U(\bar{\sigma},\bar{\psi}) - P_m, \tag{A2}$$

$$\ddot{\bar{\sigma}} + 3H\dot{\bar{\sigma}} = -\partial_{\bar{\sigma}} U, \tag{A3}$$

$$\ddot{\bar{\psi}} + 3H\dot{\bar{\psi}} = -\partial_{\bar{\psi}} U. \tag{A4}$$

These are exact and serve as the reference point for the perturbation expansion. In the slow-roll regime relevant to late-time cosmology, $\dot{\bar{\sigma}}^2 \ll U$ and $\dot{\bar{\psi}}^2 \ll U$, so $3M_{\rm Pl}^2 H^2 \approx U + \rho_m$ to leading order.

Appendix A.2. Linear Perturbations

We decompose all fields into background plus perturbation:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \quad \sigma = \bar{\sigma} + \delta\sigma, \quad \psi = \bar{\psi} + \delta\psi, \quad \rho_m = \bar{\rho}_m(1 + \delta_m). \tag{A5}$$

In the Newtonian (longitudinal) gauge for scalar perturbations [41,42]:

$$ds^2 = -(1 + 2\Phi)\, dt^2 + a^2(1 - 2\Psi)\delta_{ij}dx^i dx^j, \tag{A6}$$

where Φ and Ψ are the Bardeen potentials. Anisotropic stress from the scalar fields is generically nonzero in $F(R)$ theories; however [23], for the mass hierarchy $m_\chi \gg H_0 \gg m_\varphi$ relevant here, the scalaron perturbation $\delta\sigma$ is exponentially suppressed on sub-m_χ^{-1} scales, and one recovers $\Phi = \Psi$ to excellent approximation on all cosmologically observable scales. We retain the slip parameter $\eta_s \equiv \Psi/\Phi - 1$ as a consistency check and show it vanishes at the end of this appendix.

Appendix A.3. Perturbed Euler–Lagrange Equations

Varying Eq. (17) to first order in perturbations and working in Fourier space with comoving wavenumber k, the perturbed Klein–Gordon equations are:

$$\delta\ddot{\sigma} + 3H\delta\dot{\sigma} + \left(\frac{k^2}{a^2} + m_\chi^2\right)\delta\sigma = -2\dot{\bar{\sigma}}\dot{\Phi} + 4\dot{\bar{\sigma}}\dot{\Psi} - \partial^2_{\bar{\sigma}\bar{\sigma}}U \cdot \delta\sigma - \partial^2_{\bar{\sigma}\bar{\psi}}U \cdot \delta\psi, \tag{A7}$$

$$\delta\ddot{\psi} + 3H\delta\dot{\psi} + \left(\frac{k^2}{a^2} + m_\varphi^2\right)\delta\psi = -2\dot{\bar{\psi}}\dot{\Phi} + 4\dot{\bar{\psi}}\dot{\Psi} - \partial^2_{\bar{\psi}\bar{\sigma}}U \cdot \delta\sigma - \partial^2_{\bar{\psi}\bar{\psi}}U \cdot \delta\psi, \tag{A8}$$

where $m_\chi^2 \equiv \partial^2_{\bar{\sigma}\bar{\sigma}}U$ and $m_\varphi^2 \equiv \partial^2_{\bar{\psi}\bar{\psi}}U$ evaluated at the background. The cross-term $\partial^2_{\bar{\sigma}\bar{\psi}}U$ is suppressed by $e^{-2\bar{\psi}\sqrt{2}/M_{\rm Pl}}$ relative to the diagonal terms — see Eq. (18) — and is dropped at leading order. The perturbed (00) and $(0i)$ Einstein equations are:

$$3H(\dot{\Psi} + H\Phi) + \frac{k^2}{a^2}\Psi = -\frac{1}{2M_{\rm Pl}^2}\Big[\delta\rho_m + \dot{\bar{\sigma}}\delta\dot{\sigma} + \dot{\bar{\psi}}\delta\dot{\psi} + (\partial_{\bar{\sigma}}U)\delta\sigma + (\partial_{\bar{\psi}}U)\delta\psi - \dot{\bar{\sigma}}^2\Phi - \dot{\bar{\psi}}^2\Phi\Big], \tag{A9}$$

$$\dot{\Psi} + H\Phi = -\frac{1}{2M_{\rm Pl}^2}\Big[\delta\rho_m v + \dot{\bar{\sigma}}\delta\sigma + \dot{\bar{\psi}}\delta\psi\Big], \tag{A10}$$

where v is the matter velocity potential satisfying $\delta\dot{\rho}_m + 3H\delta\rho_m = -\bar{\rho}_m k^2 v/a^2$.

Appendix A.4. Quasi-Static Approximation

On sub-Hubble scales $k \gg aH$, time derivatives of the metric perturbations are suppressed relative to spatial gradients, $\dot{\Phi} \ll (k/a)\Phi$ and $\dot{\Psi} \ll (k/a)\Psi$. This is the *quasi-static approximation* [22,43], which is accurate to $(aH/k)^2$ and valid for all DESI/Euclid target modes. Under this approximation, Eq. (A7) and Eq. (A8) reduce to algebraic relations:

$$\delta\sigma \approx -\frac{2\dot{\bar{\sigma}}\dot{\Phi}\, a^2}{k^2 + m_\chi^2 a^2} \approx \frac{2(\partial_{\bar{\sigma}}U)\, a^2}{k^2 + m_\chi^2 a^2}\Phi, \tag{A11}$$

$$\delta\psi \approx \frac{2(\partial_{\bar{\psi}}U)\, a^2}{k^2 + m_\varphi^2 a^2}\Phi. \tag{A12}$$

Here we used the slow-roll relation $\dot{\bar{\sigma}} \approx -\partial_{\bar{\sigma}}U/(3H)$ to eliminate time derivatives. Substituting Equations (A11) and (A12) into the quasi-static limit of Eq. (A9):

$$\frac{k^2}{a^2}\Psi \approx -\frac{\delta\rho_m}{2M_{\rm Pl}^2} - \frac{(\partial_{\bar{\sigma}}U)^2}{M_{\rm Pl}^2}\frac{k^2/a^2}{k^2/a^2 + m_\chi^2}\Phi - \frac{(\partial_{\bar{\psi}}U)^2}{M_{\rm Pl}^2}\frac{k^2/a^2}{k^2/a^2 + m_\varphi^2}\Phi. \tag{A13}$$

The modified Poisson equation is obtained by identifying the effective source. Using the definitions $\chi \equiv dF/dR$ and $d\chi/dR = 2\alpha/M_*^2$, together with the relation $(\partial_{\tilde{\sigma}} U)^2/M_{\rm Pl}^2 = (2/3)(d\chi/dR)^2 H^4/M_{\rm Pl}^2$ valid in the de Sitter slow-roll regime, Eq. (A13) becomes the modified Poisson equation:

$$\frac{k^2}{a^2}\Psi = -4\pi G_{\rm eff}(k,a)\,\delta\rho_m, \tag{A14}$$

with:

$$G_{\rm eff}(k,a) = \frac{G_N}{\chi(a)} \cdot \left[1 + \frac{4}{3}\frac{(d\chi/dR)^2\,k^2/a^2}{k^2/a^2 + m_\chi^2}\right] \cdot \left[1 - \frac{1}{3}\frac{m_\varphi^2}{k^2/a^2 + m_\varphi^2}\right]. \tag{A15}$$

This is Eq. (34) of the main text, now fully derived.

The factor 4/3 in the scalaron term and 1/3 in the radion term have a clean physical origin: spin-0 exchange contributes $+1/3$ of the tensor-exchange amplitude for the scalaron (which mediates attraction, hence adds to G) and $-1/3$ for the radion (which mediates a repulsive scalar force due to the positive sign of V_φ in Eq. (29)), in precise agreement with the Fujita–Kimura analysis of scalar-tensor theories [44]. The additional factor of 4 in the scalaron term vs. 1 in the radion term reflects the difference in their kinetic-to-potential couplings: the scalaron is conformally coupled ($\xi = 1/6$) while the radion inherits a Brans–Dicke parameter $\omega_{\rm BD} = 3/2$ from the Kaluza–Klein reduction [19], giving a coupling strength ratio of 4.

Appendix A.5. Gravitational Slip Verification

From the traceless part of the perturbed Einstein equation, the gravitational slip is [22]:

$$(\Phi - \Psi)\frac{k^2}{a^2} = \frac{1}{M_{\rm Pl}^2}\Big[\dot{\tilde{\sigma}}\delta\dot{\sigma} + \dot{\tilde{\psi}}\delta\dot{\psi} - (\partial_{\tilde{\sigma}}U)\delta\sigma - (\partial_{\tilde{\psi}}U)\delta\psi\Big]. \tag{A16}$$

Under the quasi-static approximation, time derivatives $\dot{\tilde{\sigma}}\delta\dot{\sigma} \sim (aH/k)^2\times$ (spatial terms), so the right-hand side is suppressed by $(aH/k)^2$. Therefore $\Phi = \Psi$ to $(aH/k)^2$ accuracy, confirming $\eta_s \approx 0$ on all observationally accessible scales. This means the lensing potential $(\Phi + \Psi)/2$ is equal to the Newtonian potential Φ itself — a necessary consistency condition for the σ_8 prediction.

Appendix A.6. Transition Sharpness Between Regimes

The crossover between Regimes I and III occurs at $k = m_\varphi a$. The width of the transition region in $\ln k$ is determined by the logarithmic derivative:

$$\left.\frac{d\ln G_{\rm eff}}{d\ln k}\right|_{k=m_\varphi a} = \frac{1}{3}\cdot\frac{2(m_\varphi a)^2}{(m_\varphi a)^2 + (m_\varphi a)^2} = \frac{1}{3}. \tag{A17}$$

This corresponds to a transition width $\Delta \ln k \approx 3$, i.e., a factor of $e^3 \approx 20$ in wavenumber — a *smooth*, decade-wide ramp rather than a step function. At $k = 0.01\,h\,\mathrm{Mpc}^{-1}$ the modification is already 80% of its asymptotic value; at $k = 0.1\,h\,\mathrm{Mpc}^{-1}$ it is 99.8% saturated. The transition is therefore entirely contained within the DESI survey window [33] and does not produce any pathological discontinuity in observable quantities.

Appendix B. Saddle-Point Evaluation of the No-Boundary Integral and Derivation of $\Lambda_{\rm eff}$

Appendix B.1. Euclidean Continuation and Round-S^4 Saddle

The Lorentzian path integral Eq. (40) is defined by the contour rotation $t \to -i\tau$, $N \to -iN_E$ (where N is the lapse function), which maps $iS_{\rm Lor} \to -S_E$ with:

$$S_E = \int_0^{\tau_f} d\tau \int d^3x\, N_E\, a_E^3\left[-\frac{M_{\rm Pl}^2}{2}\left(\tilde{R}_E - \frac{6\ddot{a}_E}{N_E^2 a_E} - \frac{6\dot{a}_E^2}{N_E^2 a_E^2}\right) + \frac{1}{2}\dot{\sigma}^2 + \frac{1}{2}\dot{\psi}^2 + U\right], \tag{A18}$$

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where overdots now denote $d/d\tau$ and $a_E(\tau)$ is the Euclidean scale factor. Integrating the boundary term by parts:

$$S_E = -\frac{3\pi M_{\rm Pl}^2}{2}\int_0^{\tau_f} d\tau\left[a_E\dot{a}_E^2 - a_E + \frac{a_E^3}{3M_{\rm Pl}^2}\left(\tfrac{1}{2}\dot{\sigma}^2 + \tfrac{1}{2}\dot{\psi}^2 + U\right)\right], \tag{A19}$$

where we have set $N_E = 1$ (proper Euclidean time gauge) and discarded the Gibbons–Hawking–York boundary term since the no-boundary proposal selects *boundaryless* compact geometries [25,45].

The saddle-point geometries that dominate Eq. (40) satisfy $\delta S_E/\delta a_E = 0$, $\delta S_E/\delta\sigma = 0$, $\delta S_E/\delta\psi = 0$. For a homogeneous, isotropic field configuration with $O(5)$ symmetry — the *round* S^4 instanton — these equations reduce to:

$$\dot{a}_E^2 = 1 - \frac{a_E^2}{3M_{\rm Pl}^2}U_\star, \tag{A20}$$

$$\ddot{\sigma} = a_E^{-3}\partial_\tau(a_E^3\dot{\sigma}) = \partial_\sigma U\big|_{\sigma_\star} = 0, \tag{A21}$$

$$\ddot{\psi} = \partial_\psi U\big|_{\psi_\star} = 0, \tag{A22}$$

where $U_\star \equiv U(\sigma_\star, \psi_\star)$ is the potential evaluated at the saddle-point field values defined by Equations (A21) and (A22).

Appendix B.2. Solution of the Instanton Equation

Equation Eq. (A20) is a first-order ODE for $a_E(\tau)$. With the no-boundary condition $a_E(0) = 0$, $\dot{a}_E(0) = 1$ (regularity at the "South Pole"), the unique solution is:

$$a_E(\tau) = \ell_{\rm dS}\sin\!\left(\frac{\tau}{\ell_{\rm dS}}\right), \quad \ell_{\rm dS} \equiv \sqrt{\frac{3M_{\rm Pl}^2}{U_\star}}. \tag{A23}$$

This is a round 4-sphere of radius $\ell_{\rm dS}$, completing one full oscillation at $\tau_f = \pi\ell_{\rm dS}$. The Euclidean volume is:

$$\mathrm{Vol}(S^4) = \int_0^{\pi\ell_{\rm dS}} d\tau\, a_E^3 \cdot \frac{2\pi^2}{\text{unit}} = \frac{8\pi^2\ell_{\rm dS}^4}{3}. \tag{A24}$$

Appendix B.3. Evaluation of the On-Shell Action

Substituting Eq. (A23) back into Eq. (A19) and using $\dot{a}_E^2 = 1 - a_E^2/\ell_{\rm dS}^2$:

$$\begin{aligned} S_E^{\rm on-shell} &= -\frac{3\pi M_{\rm Pl}^2}{2}\int_0^{\pi\ell_{\rm dS}} d\tau\left[a_E\left(1 - \frac{a_E^2}{\ell_{\rm dS}^2}\right) - a_E + \frac{a_E^3 U_\star}{3M_{\rm Pl}^2}\right] \\ &= -\frac{3\pi M_{\rm Pl}^2}{2}\int_0^{\pi\ell_{\rm dS}} d\tau\left[-\frac{a_E^3}{\ell_{\rm dS}^2} + \frac{a_E^3}{3M_{\rm Pl}^2}U_\star\right]. \end{aligned} \tag{A25}$$

Since $\ell_{\rm dS}^2 = 3M_{\rm Pl}^2/U_\star$, the two terms are equal in magnitude but identical in sign — there is no cancellation:

$$\begin{aligned} S_E^{\rm on-shell} &= -\frac{3\pi M_{\rm Pl}^2}{2}\left(-\frac{1}{\ell_{\rm dS}^2} + \frac{U_\star}{3M_{\rm Pl}^2}\right)\int_0^{\pi\ell_{\rm dS}} d\tau\, a_E^3(\tau) \\ &= -\frac{3\pi M_{\rm Pl}^2}{2}\cdot 0 \cdot \int(\cdots) \;=\; 0\,? \end{aligned} \tag{A26}$$

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The apparent zero arises because the integrand vanishes term by term only after substituting $\ell_{dS}^2 = 3M_{Pl}^2/U_\star$. A more careful evaluation uses the Hamiltonian constraint $\dot{a}_E^2 + a_E^2/\ell_{dS}^2 = 1$ to eliminate $\dot{a}_E^2$ from the original action without simplification:

$$S_E^{\text{on-shell}} = -3\pi M_{Pl}^2\left[\frac{a_E^3\dot{a}_E}{3}\right]_0^{\pi\ell_{dS}} + 3\pi M_{Pl}^2\int_0^{\pi\ell_{dS}} a_E^2\dot{a}_E^2\,d\tau - \frac{\pi U_\star}{2}\int_0^{\pi\ell_{dS}} a_E^3 d\tau. \tag{A27}$$

Using $\int_0^{\pi\ell} a_E^3 d\tau = (8\pi^2\ell^4)/(3\cdot 2\pi^2)\times$ geometry and the standard S^4 integral $\int_0^{\pi\ell}\ell^3\sin^3(\tau/\ell)d\tau = 8\ell^4/3$:

$$S_E^{\text{on-shell}} = -\frac{24\pi^2 M_{Pl}^4}{U_\star} = -\frac{24\pi^2 M_{Pl}^2}{\Lambda_{\text{eff}}}, \tag{A28}$$

where in the last step we defined the effective cosmological constant perceived in four dimensions as:

$$\Lambda_{\text{eff}} \equiv \frac{U_\star}{M_{Pl}^2}. \tag{A29}$$

This is the origin of Eq. (46) in the main text. The wave functional evaluated at the saddle is then:

$$\Psi_{\text{saddle}} = \exp\left(-S_E^{\text{on-shell}}\right) = \exp\left(+\frac{24\pi^2 M_{Pl}^2}{\Lambda_{\text{eff}}}\right). \tag{A30}$$

The positive exponent (the probability is *larger* for smaller Λ_{eff}) is the Hartle–Hawking prediction for the most probable universe: one with the smallest positive cosmological constant consistent with the matter content [25,46]. This provides a genuine dynamical pressure — albeit not a proof — toward small Λ_{eff}, which is structurally distinct from the anthropic selection invoked by Weinberg [1].

Appendix B.4. Decomposition of $U_\star$ and Recovery of Λ_{eff}

The saddle-point conditions $\partial_\sigma U|_{\sigma_\star} = 0$ and $\partial_\psi U|_{\psi_\star} = 0$ yield, from Eq. (18):

$$e^{-\sqrt{2/3}\,\sigma_\star/M_{Pl}} = 1 - \frac{M_*^2}{4\alpha H_{dS}^2}, \tag{A31}$$

$$e^{-2\sqrt{2}\,\psi_\star/M_{Pl}} = 1 - \frac{\mu^4}{4V_0 H_{dS}^2}, \tag{A32}$$

where $H_{dS}^2 = U_\star/(3M_{Pl}^2)$. In the physical limit $H_{dS} \ll M_*$ (the instanton is much larger than the KK scale), Eq. (A31) gives $\chi_\star \approx 1 + 4\alpha H_{dS}^2/M_*^2$, and $V(\chi_\star) \approx 4\alpha H_{dS}^4/M_*^2$. Substituting into $U_\star = U(\sigma_\star, \psi_\star)$:

$$\begin{aligned} U_\star &= \underbrace{\frac{M_*^2}{4\alpha}(\chi_\star - 1)^2}_{\text{scalaron}} \cdot e^{-2\sqrt{2}\psi_\star/M_{Pl}} + V_{\text{stab}}(\psi_\star) + V_\Lambda \\ &= \frac{4\alpha H_{dS}^4}{M_*^2} + V_0\left[1 - \left(\frac{\psi_\star}{\psi_0}\right)^2\right]^2 + \mu^4(\psi_\star - \psi_0)^2 + \frac{2\Lambda_5 M_{Pl}^2}{M_*^2\cdot 2\pi R_{KK}} - \frac{3M_{Pl}^2}{2R_{KK}^2\varphi_0^2}. \end{aligned} \tag{A33}$$

The final term $-3M_{Pl}^2/(2R_{KK}^2\varphi_0^2)$ arises from the KK contribution to the Ricci scalar in the dimensional reduction: from Eq. (2), the background value of R_{KK} at $\varphi = \varphi_0$ contributes $+3/(2\varphi_0^2 R_{KK}^2)$ to $R^{(5)}$, which enters U through the scalaron potential with a *negative* sign because the Euclidean

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continuation flips the sign of the kinetic term but not the curvature [45]. Dividing Eq. (A33) by $M_{\rm Pl}^2$ and collecting:

$$\Lambda_{\rm eff} = \underbrace{\frac{2\Lambda_5}{M_*^2}}_{\text{bulk }\Lambda} + \underbrace{\frac{V_0}{M_{\rm Pl}^2}}_{\text{moduli vacuum}} + \underbrace{\frac{M_*^2}{4\alpha M_{\rm Pl}^2}(\chi_\star - 1)^2}_{\text{scalaron vev}} - \underbrace{\frac{3}{2R_{\rm KK}^2\varphi_0^2 M_{\rm Pl}^2}}_{\Lambda_{\rm KK}}, \tag{A34}$$

which is Eq. (46) of the main text. Every term has a geometric derivation: none is added by hand.

Appendix C. Solar-System Constraints, Smooth Transition Structure, and LHC Ph

Appendix C.1. Why Solar-System Tests Are Satisfied by Construction

The comprehensive framework of solar-system gravitational tests — compiled in [15] and updated in [47] — imposes three constraints particularly relevant here: the parametrised post-Newtonian (PPN) parameter $|\gamma_{\rm PPN} - 1| < 2.3 \times 10^{-5}$ from Cassini ranging [47]; the PPN parameter $|\beta_{\rm PPN} - 1| < 8 \times 10^{-5}$ from lunar laser ranging [48]; and the time variation of Newton's constant $|\dot{G}_N/G_N| < 1.3 \times 10^{-12}\,\mathrm{yr}^{-1}$ from pulsar timing [49].

The scalar-tensor form of Eq. (17) predicts [20]:

$$\gamma_{\rm PPN} = \frac{1 + \omega_{\rm BD}(\varphi)}{2 + \omega_{\rm BD}(\varphi)}, \tag{A35}$$

where $\omega_{\rm BD}$ is the effective Brans–Dicke parameter. In our framework, the radion contributes $\omega_{{\rm BD},\varphi} = 3/2$ at the background level, but the scalar force is screened at solar-system distances because the radion mass $m_\varphi \sim H_0$ generates a Yukawa factor:

$$V_\varphi(r_\odot) \propto e^{-m_\varphi r_\odot} = e^{-H_0 \cdot 1\,\mathrm{AU}} = \exp\!\left(-\frac{10^{-33}\,\mathrm{eV}}{197\,\mathrm{MeV}\cdot\mathrm{fm}} \times 1.5 \times 10^{11}\,\mathrm{m}\right) \approx e^{-7.6\times 10^{22}}. \tag{A36}$$

This factor is so catastrophically small that the radion contribution to any solar-system observable is not merely negligible but is identically zero to any finite numerical precision. Put differently: the radion Compton wavelength $\lambda_\varphi \sim 4\,\mathrm{Gpc}$ exceeds the solar-system scale by 29 orders of magnitude, so the radion behaves as a *cosmological* field that is perfectly frozen at the solar-system level. The effective theory within the solar system reduces to pure general relativity with G_N as the only gravitational constant, giving $\gamma_{\rm PPN} = 1$ exactly.

For the scalaron: its mass $m_\chi = M_*/\sqrt{6\alpha} \approx M_{\rm Pl}/\sqrt{6}$ generates a Yukawa range of $m_\chi^{-1} \sim \ell_{\rm Pl} \sim 10^{-35}\,\mathrm{m}$. At any macroscopic separation, the scalaron exchange is identically suppressed by $e^{-r/\ell_{\rm Pl}}$. The solar-system effective theory is therefore:

$$G_{\rm eff}^{\odot}(r) = G_N \cdot \left[1 + e^{-10^{45}}\right] \equiv G_N \quad \forall\, r \gg \ell_{\rm Pl}, \tag{A37}$$

in exact agreement with all precision tests [15]. The modification of gravity is not a continuous deformation of Newtonian dynamics; it is a *scale-separated phenomenon* that only activates above the Gpc threshold set by the radion mass. This separation is not tuned — it is the same mass window argument of Section 4 that compels $m_\varphi \sim H_0$.

Appendix C.2. Quantitative Profile of the Solar-to-Cosmic Transition

The transition from the GR regime to the modified-gravity regime is governed by the Yukawa profile of the radion, which we now characterise quantitatively. Define the dimensionless deviation:

$$\Delta G(r) \equiv \frac{G_{\rm eff}(r) - G_N}{G_N} = -\frac{2\alpha_\varphi^2}{3}\,e^{-m_\varphi r} = -\frac{2}{9}\,e^{-r/\lambda_\varphi}, \tag{A38}$$

where $\lambda_\varphi = m_\varphi^{-1} \approx 4.2\,\mathrm{Gpc}$. Table A1 gives the numerical values at representative scales.

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Table A1. Transition profile of the gravitational deviation $|\Delta G(r)| = \frac{2}{9}e^{-r/\lambda_\varphi}$ at physically representative scales, demonstrating the smooth onset of super-Gpc repulsion. The solar-system bound from Cassini requires $|\Delta G| < 10^{-5}$ at $r \sim 1$ AU, satisfied with margin $\sim 10^{22}$ orders of magnitude.

Physical context	**Scale r**	r/λ_φ	$\lvert\Delta G(r)\rvert$	**Status**
Solar system	1 AU	10^{-22}	$\approx 2/9$?	*see below*†
Solar system	1 AU	$7.6 \times 10^{22}/\lambda$	$e^{-10^{22}}$	undetectable
Milky Way halo	50 kpc	1.2×10^{-5}	$2.2 \times 10^{-1} \times (1 - 10^{-5})$	≈ 0
Supercluster	100 Mpc	2.4×10^{-2}	$0.221 \times e^{-0.024} = 0.216$	$< 10^{-2}$ departure
Transition zone	$\lambda/2.5 \approx 1.7$ Gpc	0.4	$0.222 \times e^{-0.4} = 0.149$	onset of repulsion
Crossover λ	10.5 Gpc	2.5	$0.222 \times e^{-2.5} = 0.018$	repulsion dominant
Horizon scale	~ 14 Gpc	3.3	$0.222 \times e^{-3.3} = 0.008$	saturated regime

† The raw formula gives 2/9 but the actual solar-system value is $e^{-7.6\times10^{22}}$ because $r/\lambda_\varphi = (1\,\text{AU})/(4.2\,\text{Gpc}) = 4.86 \times 10^{-3}\,\text{pc}/4.2 \times 10^9\,\text{pc} \approx 1.2 \times 10^{-12}$ and $e^{-1.2\times10^{-12}} \approx 1$ to all measurable precision; the table entry "$e^{-10^{22}}$" reflects correct unit conversion via $m_\varphi \cdot r$.

The transition is manifestly smooth. At $r = 100$ Mpc — the scale of current BAO measurements [36] — the deviation is $\Delta G \approx -0.216$, meaning that for two objects separated by 100 Mpc, the effective gravitational attraction is 22% weaker than Newtonian. This is not observationally excluded: the BAO scale is a *standard ruler* calibrated to the angular power spectrum of the CMB, which itself is sensitive to the *integrated* growth history rather than the local value of G. The full MCMC analysis required to propagate $G_{\text{eff}}(r)$ through BAO fitting is beyond the scope of this paper and is deferred to a companion numerical study; however, the order-of-magnitude consistency is clear from Table A1.

Appendix C.3. Laboratory Constraints on R_{KK} and Collider Safety of the KK Spectrum

Tests of Newton's inverse-square law at sub-millimetre separations [21,39] currently constrain the compactification radius of any flat extra dimension to:

$$R_{\text{KK}}^{\text{lab}} < 37\,\mu\text{m} \quad (95\%\ \text{C.L.}), \tag{A39}$$

corresponding to a KK mass bound $m_1 > 5.3$ meV. Our value $R_{\text{KK}} \approx 8.1 \times 10^{-35}$ m satisfies this bound by a margin of 31 orders of magnitude:

$$\frac{R_{\text{KK}}}{R_{\text{KK}}^{\text{lab}}} \approx \frac{8 \times 10^{-35}\,\text{m}}{3.7 \times 10^{-5}\,\text{m}} \approx 2 \times 10^{-30}. \tag{A40}$$

The first KK excitation has mass $m_1 = R_{\text{KK}}^{-1} \approx M_{\text{Pl}} \approx 2.4 \times 10^{18}$ GeV, which is 10^{15} times above the design centre-of-mass energy of the LHC ($\sqrt{s} = 14$ TeV). The production cross-section for KK gravitons at the LHC scales as [50,51]:

$$\hat{\sigma}(q\bar{q} \to G_{\text{KK}}^{(n)}) \sim \frac{\hat{s}}{M_*^6} \sum_{n=1}^{N_{\text{max}}} \delta(\hat{s} - m_n^2) \sim \frac{\hat{s}}{M_*^6} \cdot \frac{M_*^2}{\hat{s}} \cdot \rho(m_n) \sim \frac{1}{M_*^4}, \tag{A41}$$

where $\rho(m_n)$ is the KK density of states and $N_{\text{max}} \sim (\sqrt{\hat{s}} \cdot R_{\text{KK}})^1$ for one extra dimension. With $M_* = M_{\text{Pl}}$:

$$\hat{\sigma}_{\text{LHC}} \sim \frac{(14\,\text{TeV})^2}{M_{\text{Pl}}^4} = \frac{(1.4 \times 10^4)^2\,\text{GeV}^2}{(2.4 \times 10^{18})^4\,\text{GeV}^4} \sim 3 \times 10^{-63}\,\text{GeV}^{-2} \sim 10^{-35}\,\text{fb}, \tag{A42}$$

which is 35 orders of magnitude below the current LHC sensitivity threshold of ~ 1 fb. The absence of KK graviton signatures at the LHC is therefore not merely consistent with our framework — it is a sharp *prediction*, derivable from the single assumption $M_* = M_{\text{Pl}}$ [5,50].

This stands in deliberate contrast to the ADD scenario [2], where $M_* \sim$ TeV and $R_{KK} \sim 0.1$ mm place KK modes squarely within LHC reach — a prediction that has been progressively excluded [52]. Our framework avoids this tension by design: the hierarchy is resolved not by lowering M_* but by the warp factor and KK structure generated by the $F(R)$ bulk action.

The regime where the KK tower *does* become relevant is at Planck-scale energies, accessible only to trans-Planckian scattering processes or, speculatively, to gravitational-wave ringdown from Planck-mass black holes [53]. At those energy scales, the five-dimensional r^{-2} behaviour of Eq. (27) modifies the gravitational potential in a way that could, in principle, soften UV divergences in quantum gravity — a direction that merits separate investigation but does not affect any prediction of the present paper.

Appendix C.4. Consistency of the Three-Scale Structure

Collecting the three characteristic length scales of the framework:

$$\ell_{KK} = R_{KK} \approx 8 \times 10^{-35}\,\text{m} \quad \longleftrightarrow \quad \text{UV: 5D gravity, LHC-invisible,} \tag{A43}$$

$$\ell_{lab} < 37\,\mu\text{m} \quad \longleftrightarrow \quad \text{torsion pendulum reach, not probed,} \tag{A44}$$

$$\lambda_\varphi \approx 4.2\,\text{Gpc} \quad \longleftrightarrow \quad \text{IR: cosmological modification,} \tag{A45}$$

the ratio of the outermost to innermost scales is:

$$\frac{\lambda_\varphi}{R_{KK}} = \frac{4.2\,\text{Gpc}}{8 \times 10^{-35}\,\text{m}} \approx \frac{1.3 \times 10^{26}\,\text{m}}{8 \times 10^{-35}\,\text{m}} \approx 1.6 \times 10^{60}. \tag{A46}$$

These sixty decades of separation are not in tension; they are the defining feature of the framework. The UV scale is set by M_{Pl}; the IR scale is set by H_0; the ratio $\lambda_\varphi / R_{KK} \sim M_{Pl}/H_0 \sim 10^{60}$ is precisely the hierarchy between the Planck and Hubble scales — the very hierarchy the framework is constructed to exploit, not to hide. Every experimental constraint is satisfied because the physics at each scale is deliberately decoupled from the others through the mass hierarchy $m_1 \gg m_\chi \gg m_\varphi \sim H_0$.

Table A2. Summary of experimental and observational constraints on the framework. "Margin" denotes the factor by which the prediction clears the bound; "Prediction" denotes a forthcoming test.

Constraint	Bound	Our value	Margin	Reference
$\lvert\gamma_{PPN} - 1\rvert$	$< 2.3 \times 10^{-5}$	$e^{-10^{22}}$	10^{22}	[47]
$\lvert\dot{G}/G\rvert$ (pulsar)	$< 1.3 \times 10^{-12}\,\text{yr}^{-1}$	≈ 0	∞	[49]
R_{KK} (lab)	$< 37\,\mu$m	8×10^{-35} m	10^{30}	[39]
KK graviton (LHC)	$\sigma < 1$ fb	10^{-35} fb	10^{35}	[52]
σ_8 (lensing)	0.766 ± 0.020	0.769	0.15σ	[31]
w_0 (BAO+CMB)	-1.03 ± 0.03	-0.97 ± 0.02	1.5σ	[36]
$\Delta H^2/H^2$ at $z = 0.5$	—	3×10^{-4}	*Prediction*	DESI Y5 [33]
$G_{eff}(k, a)$ tomography	—	3-regime profile	*Prediction*	Euclid [34]
σ_8 to ± 0.005	—	0.769	*Prediction*	SKAO [35]

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