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On Covariance for Entire-Function Deformations of Relativistic Field Theories

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Abstract

We prove a what we call a general covariance theorem for entire-function deformations of relativistic field theories, the result is that if an undeformed theory is covariant or invariant under some symmetry group, and the deformation is defined by an entire function of a differential operator that intertwines the symmetry group-action, then the deformed theory remains covariant or invariant with respect to that same symmetry group. We will establish this result in both abstract and geometric forms and derive corollaries for Poincaré symmetry, gauge covariance, and continuum diffeomorphism covariance. In the gauge-fixed sector we as well prove a finite-dimensional BRST theorem with Ward and Slavnov–Taylor identities for admissible truncations in the theory.

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1. Introductory Remarks

We know that in local relativistic quantum field theory and general relativity we organize the theory by symmetry principles since symmetries act as fundamental, structural constraints that detrime the mathematical form of physical laws, enforce conservation laws, and generate forces, for example in flat spacetime the relevant kinematic symmetry is the Poincaré group, consisting of translations and Lorentz transformations [1–10]. In Yang–Mills theory we will further impose gauge covariance or gauge invariance to ensure theoretical consistency, renormalizability, and physical validity of non-Abelian interactions, and in gravity we impose covariance under spacetime diffeomorphisms [3,11–13,13–21]. In the usual local framework, these symmetries coexist with strict point-supported locality and, at the operator level, with exact microcausality and the familiar subtleties surrounding relativistic localization [1,2,22]. So the purpose of this paper is to prove the symmetry statement that underlies nonlocal quantum field theory like those used in [23–34]. In ultraviolet-complete nonlocal deformations we replace a local field or local kinetic operator by an expression involving an entire function of a covariant second-order differential operator, $\Phi^{(F)}(x)$ where $F : \mathbb{C} \to \mathbb{C}$ is entire, $\Lambda_* > 0$ is the nonlocality scale, and $\Box$ is the relevant covariant d'Alembertian or Laplace–Beltrami operator, because F is entire the deformation contains infinitely many derivatives and is nonlocal in position space [23–28]. The natural question then is whether this nonlocality breaks the fundamental symmetries of the undeformed theory, we come to the conclusion and prove that the fundamental symmetries are uneffected.

The symmetries of the undeformed theory is preserved provided that the operator used to define the nonlocal deformation is as well covariant with respect to the underlying symmetry. The general statement is givin in terms of a representation-theoretic description, and it is that if the undeformed theory carries an action of a group G and a differential operator D used in the deformation intertwines that action, then the entire functional calculus $F(D)$ will also intertwine with the action. So this means that any field equation, algebra of observables, or action functional that is deformed by $F(D)$ will remain G-covariant or G-invariant.

Throughout this paper we work in natural units $\hbar = c = 1$. Greek indices μ, ν, ρ, σ that run over spacetime coordinates $0, 1, 2, 3$ in Lorentzian signature and over $1, \ldots, d$ in Euclidean lattice formulas, where the intended the meaning will always be made clear from the context.

2. Preliminary Framework

We will first review the undeformed symmetry framework and notation as to not cause any confusion later in this paper. So, for spacetime, fields, and group actions we will take M be a smooth oriented spacetime manifold, so in the flat case we take M to be a $3+1$ dimensional manifold defined as:

$$M = \mathbb{R}^{1,3}, \tag{1}$$

this manifold is equipped with the standard Minkowski metric:

$$\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1). \tag{2}$$

We should note that if $x = (x^0, x^1, x^2, x^3) \in \mathbb{R}^{1,3}$ and $y = (y^0, y^1, y^2, y^3) \in \mathbb{R}^{1,3}$, then we see the squared spacetime interval between two events x and y in special relativity as defined by:

$$(x - y)^2 := \eta_{\mu\nu}(x - y)^\mu (x - y)^\nu, \tag{3}$$

we call separation is spacelike when $(x - y)^2 < 0$. We let $E \to M$ where E is the total space and M is the base space, and this is a complex vector bundle of finite rank. A classical field is a section, this means that the classical field Φ is a section of the vector bundle and we write this as $\Phi \in \Gamma(E)$, so Φ will pick one vector in the fiber above each point x and this is the geometric meaning of a classical field. When $M = \mathbb{R}^{1,3}$ and the field transforms in a finite-dimensional representation S of the Lorentz group, we write the field as $\Phi_A(x)$, where the A index will label components in the representation space. We defined G to be a symmetry group acting on the field space, we know the action is described by a representation U of the symmetry group where:

$$U : G \to \mathrm{Aut}(\Gamma(E)), \tag{4}$$

in this equation $\mathrm{Aut}(\Gamma(E))$ is the invertible linear map on $\Gamma(E)$, so we will then write the transformed field as:

$$(U_g \Phi)(x), \qquad g \in G, \tag{5}$$

and this describes the operation where we first apply the symmetry transformation U_g to the field Φ and then we get a new field $U_g \Phi \in \Gamma(E)$, and then we evaluate that transformed field at that spacetime point where $x \in M$.

Definition 1 (Covariance of a field equation). *We let $\mathscr{E}[\Phi] = 0$ be a field equation where $\mathscr{E}$ maps $\Gamma(E)$ to sections of another bundle $E' \to M$. So $\mathscr{E}$ is the operator that defines the equation of motion and $\mathscr{E}[\Phi] = 0$ just means that the field Φ is a solution of our system. Now we call an equation G-covariant if for every $g \in G$,*

$$\mathscr{E}[\Phi] = 0 \quad \Longrightarrow \quad \mathscr{E}[U_g \Phi] = 0. \tag{6}$$

If $E = E'$ and $\mathscr{E}$ intertwines the G-action, this condition is equivalent to:

$$\mathscr{E} \circ U_g = U_g \circ \mathscr{E}. \tag{7}$$

Definition 2 (Invariance of an action functional). *We will take $S[\Phi]$ to be an action functional on the field space. It is called G-invariant if*

$$S[U_g \Phi] = S[\Phi] \tag{8}$$

for all $g \in G$ and all fields Φ in the domain of S.

In flat spacetime the Poincaré group is the semidirect product:

$$\mathcal{P} = \mathbb{R}^{1,3} \rtimes O(1,3), \tag{9}$$

where $O(1,3)$ is taken to be the Lorentz group and $\mathbb{R}^{1,3}$ acts by translations. For a field transforming in a finite-dimensional Lorentz representation S, the standard action is [1,35]:

$$(U(a,\Lambda)\Phi)_A(x) = S(\Lambda)_A{}^B\,\Phi_B(\Lambda^{-1}(x-a)) \tag{10}$$

in this $a \in \mathbb{R}^{1,3}$ is the translation parameter, $\Lambda \in O(1,3)$ is the Lorentz matrix, and $S(\Lambda)$ is the matrix of the representation acting on the internal tensor or spinor indices of the corresponding field. If Φ is a scalar field then $S(\Lambda) = 1$ and:

$$(U(a,\Lambda)\Phi)(x) = \Phi(\Lambda^{-1}(x-a)). \tag{11}$$

We let $P \to M$ be a principal bundle with structure group we will call K, and we let $E = P \times_\rho V$ be an associated vector bundle for a finite-dimensional representation $\rho : K \to \mathrm{GL}(V)$, a gauge transformation is a bundle automorphism covering the identity on M [11,16,17]. In a local trivialization it acts by a map by:

$$u : M \to K, \tag{12}$$

on a field $\Phi \in \Gamma(E)$ the gauge action is:

$$\Phi(x) \mapsto \rho(u(x))\,\Phi(x). \tag{13}$$

A gauge connection A_μ defines a gauge-covariant derivative:

$$D_\mu = \nabla_\mu^{\mathrm{LC}} + A_\mu, \tag{14}$$

when acting on tensor-spinor fields that also carry gauge indices. Here ∇_μ^{LC} is the Levi–Civita covariant derivative associated with our spacetime metric, while A_μ acts in the gauge representation. If M is a smooth manifold with metric g, then a diffeomorphism $\varphi \in \mathrm{Diff}(M)$ acts on tensor fields by pullback. If T is a tensor field, its pullback can be written as:

$$\varphi^* T. \tag{15}$$

A geometric equation such as:

$$\mathscr{E}[g,\Phi] = 0 \tag{16}$$

is diffeomorphism covariant if [3,36]:

$$\mathscr{E}[g,\Phi] = 0 \quad \Longrightarrow \quad \mathscr{E}[\varphi^* g, \varphi^*\Phi] = 0 \tag{17}$$

for all $\varphi \in \mathrm{Diff}(M)$.

3. Entire-Function Deformations

Now will define the minimal nonlocal deformation precisely before we go over the general covariance theorem, first we let D be a linear differential operator acting on sections of $E \to M$, this we write as:

$$D : \Gamma(E) \to \Gamma(E). \tag{18}$$

The basic example we can use is the covariant Laplace–Beltrami or d'Alembertian operator built from the gauge- and diffeomorphism-covariant derivative given by:

$$\Box_g := -g^{\mu\nu} D_\mu D_\nu. \tag{19}$$

Here $g^{\mu\nu}$ is the inverse metric tensor, and D_μ is the total covariant derivative appropriate to the bundle on which the field lives. In flat spacetime with vanishing background connection of $g_{\mu\nu} = \eta_{\mu\nu}$ and $D_\mu = \partial_\mu$ we recover the Minkowski d'Alembertian:

$$\Box := \eta^{\mu\nu}\partial_\mu\partial_\nu. \tag{20}$$

We assume $F : \mathbb{C} \to \mathbb{C}$ is entire then F has a globally convergent power series of:

$$F(z) = \sum_{n=0}^{\infty} a_n z^n, \qquad a_n = \frac{F^{(n)}(0)}{n!}. \tag{21}$$

Whenever the operator D is such that the series is well-defined on the chosen invariant domain of $\Gamma_0(E) \subseteq \Gamma(E)$, we have [37,38]:

$$F(D) := \sum_{n=0}^{\infty} a_n D^n. \tag{22}$$

In the minimal nonlocal construction the deformation is given by:

$$F(D/\Lambda_*^2), \tag{23}$$

where $\Lambda_* > 0$ will have the dimension of mass and then Λ_*^{-1} will have the dimension of length, in this theory the quantity Λ_*^{-1} is the characteristic nonlocality length of our deformation.

Definition 3 (Admissible regulator)**.** *We call an entire function $F : \mathbb{C} \to \mathbb{C}$ admissible if it satisfies these conditions:*

$$F(0) = 1, \tag{24}$$
$$F(\bar{z}) = \overline{F(z)} \quad \forall z \in \mathbb{C}, \tag{25}$$
$$F(z) \neq 0 \quad \forall \text{ finite } z \in \mathbb{C}, \tag{26}$$

and if F decays sufficiently strongly along the negative real axis to render Euclidean loop integrals finite. These conditions state that in the low energy limit the regulator will produce the local theory, that the regulator will keep the deformation physically real on the real spectrum, and do not add new finite-plane singularities or extra propagating states. A simple example for our regulator is an exponential:

$$F(z) = e^z \tag{27}$$

when argument is Euclidean $z = -p_E^2/\Lambda_*^2 \leq 0$, we can write this more generally as:

$$F(z) = e^{z^N}, \qquad N \in \mathbb{N}, \tag{28}$$

with the branch chosen so that $F(-p_E^2/\Lambda_*^2)$ decays on the Euclidean axis.

To state it more formally (24) ensures that the soft or infrared sector is unchanged at zero momentum. (25) will ensure that the reality of the deformation on real sections when the operator has real spectrum on the relevant axis. And condition (26) ensures for us that no new finite-plane poles are

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introduced by multiplying propagators by F or by its inverse [23–28]. Now given a field Φ we define its minimally deformed version as:

$$\Phi^{(F)} := F(D/\Lambda_*^2)\Phi, \tag{29}$$

now given some local observable density $\mathcal{O}(x)$, we then can write its deformed counterpart is:

$$\mathcal{O}^{(F)}(x) := F(D/\Lambda_*^2)\mathcal{O}(x). \tag{30}$$

By replacing every occurrence of the relevant local building block by its deformed counterpart we relize a nonlocal QFT. This will hold for action functionals where every occurence of Φ is multipled by $F(D/\Lambda_*^2)$.

4. The General Covariance Theorem

We now will explain and prove the that an entire-function nonlocal deformation does not destroy symmetry of the system, so long as the operator used in said deformation already respects that underlying symmetry. The proof of abstract commutation is trivial but it gives us the exact conditions under which nonlocal deformation preserves the symmetry architecture of the original theory, and because it sharply identifies locality, not covariance as the sector that is deformed. To start we let V be a complex vector space and G be a group, and have that:

$$U : G \to \mathrm{GL}(V) \tag{31}$$

is a representation of G on V, and let $D : V \to V$ be a linear operator. Before we state the general theorem lets review the definition of the intertwining operator.

Definition 4 (Intertwining operator)**.** *We say that D intertwines the G-action if*

$$U_g D = D U_g \qquad \forall\ g \in G. \tag{32}$$

If U_g is invertible, this is

$$U_g D U_g^{-1} = D. \tag{33}$$

Theorem 1 (General covariance theorem, abstract form)**.** *We let V be a complex vector space carrying a representation U of a group G, $D : V \to V$ is a linear operator satisfying the intertwining relation (32). Recall that $F : \mathbb{C} \to \mathbb{C}$ is taken to be entire and suppose that the power-series definition*

$$F(D) = \sum_{n=0}^{\infty} a_n D^n$$

is well-defined on a G-invariant domain $V_0 \subseteq V$. Then:

1. *$F(D)$ also intertwines the G-action:*

$$U_g F(D) = F(D) U_g \qquad \forall\ g \in G. \tag{34}$$

2. *If $\Phi^{(F)} := F(D)\Phi$, then $\Phi^{(F)}$ transforms covariantly in the same representation:*

$$U_g \Phi^{(F)} = F(D)\, U_g \Phi. \tag{35}$$

3. *If $S[\Phi]$ is a G-invariant functional on V_0, then the deformed functional*

$$S_F[\Phi] := S[F(D)\Phi] \tag{36}$$

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is also G-invariant:

$$S_F[U_g\Phi] = S_F[\Phi]. \tag{37}$$

4. *If $\mathscr{E} : V_0 \to V_0$ is a G-covariant field equation and the deformed equation is defined by*

$$\mathscr{E}_F[\Phi] := \mathscr{E}[F(D)\Phi], \tag{38}$$

then $\mathscr{E}_F$ is G-covariant.

The proof is simple, since D intertwines the G-action, repeated application gives is the relationship:

$$U_g D^n = D^n U_g \qquad \forall \; n \in \mathbb{N}_0 \tag{39}$$

by induction on n. The case $n = 0$ is trivial because $D^0 = \mathrm{id}$, but if $U_g D^n = D^n U_g$, then:

$$U_g D^{n+1} = U_g D^n D = D^n U_g D = D^n D U_g = D^{n+1} U_g. \tag{40}$$

Now we use the power series $F(z) = \sum_{n=0}^{\infty} a_n z^n$, on the domain V_0:

$$\begin{aligned}
U_g F(D)\Phi &= U_g \left(\sum_{n=0}^{\infty} a_n D^n \Phi \right) && (41)\\
&= \sum_{n=0}^{\infty} a_n U_g D^n \Phi && (42)\\
&= \sum_{n=0}^{\infty} a_n D^n U_g \Phi && (43)\\
&= F(D) U_g \Phi. && (44)
\end{aligned}$$

This proves (34). The second statement follows immediately:

$$U_g \Phi^{(F)} = U_g F(D)\Phi = F(D) U_g \Phi. \tag{45}$$

For the action functional we know S is G-invariant so:

$$\begin{aligned}
S_F[U_g\Phi] &= S[F(D)U_g\Phi] && (46)\\
&= S[U_g F(D)\Phi] && (47)\\
&= S[F(D)\Phi] && (48)\\
&= S_F[\Phi], && (49)
\end{aligned}$$

this proves (37). If $\mathscr{E}$ is G-covariant, then:

$$\mathscr{E}[U_g\Psi] = U_g \mathscr{E}[\Psi] \tag{50}$$

for $\Psi \in V_0$, where Ψ is a generic element of the domain of the full vector space on which the group acts V_0. Setting $\Psi = F(D)\Phi$ will give us:

$$\begin{aligned}
\mathscr{E}_F[U_g\Phi] &= \mathscr{E}[F(D)U_g\Phi] && (51)\\
&= \mathscr{E}[U_g F(D)\Phi] && (52)\\
&= U_g \mathscr{E}[F(D)\Phi] && (53)\\
&= U_g \mathscr{E}_F[\Phi], && (54)
\end{aligned}$$

this shows that $\mathscr{E}_F$ is G-covariant. This theorem is completely general as it does not require that D be second order, nor that G be finite-dimensional, snd not that V arise from a field theory. The only structural input is that the deformation is defined by some entire functional calculus and that the operator in that calculus intertwines the group action. The abstract theorem is sufficient for Poincaré symmetry and internal symmetries on a fixed background, but for diffeomorphism covariance the operator itself depends on geometric data such as the metric and the connection. To show this theormem for diffeomorphism covariance we start by letting $E_g \to M$ denote a natural bundle built from the metric g, and let:

$$D_g : \Gamma(E_g) \to \Gamma(E_g) \tag{55}$$

be a differential operator depending on g. For a diffeomorphism $\varphi \in \mathrm{Diff}(M)$, the pullback acts as:

$$\varphi^* : \Gamma(E_g) \to \Gamma(E_{\varphi^* g}). \tag{56}$$

Definition 5 (Natural operator)**.** *The family $\{D_g\}_g$ is called natural if [36,39]*

$$\varphi^* \circ D_g = D_{\varphi^* g} \circ \varphi^* \qquad \forall\ \varphi \in \mathrm{Diff}(M). \tag{57}$$

Theorem 2 (General covariance theorem in a geometric form)**.** *We let D_g be a natural family of differential operators such as (57). F be entire and assume that $F(D_g)$ is defined by the convergent power series on a pullback-invariant domain, then*

$$\varphi^* \circ F(D_g) = F(D_{\varphi^* g}) \circ \varphi^* \qquad \forall\ \ \varphi \in \mathrm{Diff}(M). \tag{58}$$

If the undeformed field equation or action is diffeomorphism covariant or invariant, then the corresponding entire-function-deformed equation or action is also diffeomorphism covariant or invariant.

From (57) we obtain by induction:

$$\varphi^* \circ D_g^n = D_{\varphi^* g}^n \circ \varphi^* \qquad \forall\ \ n \in \mathbb{N}_0. \tag{59}$$

It then will follow that:

$$\begin{aligned} \varphi^* F(D_g) &= \varphi^* \left(\sum_{n=0}^{\infty} a_n D_g^n \right) && (60)\\ &= \sum_{n=0}^{\infty} a_n \varphi^* D_g^n && (61)\\ &= \sum_{n=0}^{\infty} a_n D_{\varphi^* g}^n \varphi^* && (62)\\ &= F(D_{\varphi^* g}) \varphi^*. && (63) \end{aligned}$$

The covariance or invariance of the deformed equation or action then follows exactly as in Theorem 1.

5. Consequences for Continuum Field Theory

We will now apply the general theorem to the cases relevant for the minimal nonlocal construction to show how our new theoroms can prove symmetries remain unchanged for the deformations we want to apply. We start with Lorentz covariance and Lorentz invariance as these are important to maintain to show our theory is consistent with the general theory of relativity and special theory of relativity [1,35].

Proposition 1 (Lorentz covariance of a deformed field)**.** *We take our manifold to be $M = \mathbb{R}^{1,3}$ with the Minkowski metric η, and*

$$\Box = \eta^{\mu\nu} \partial_\mu \partial_\nu, \tag{64}$$

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and let $U(\Lambda)$ be the Lorentz action

$$(U(\Lambda)\Phi)_A(x) = S(\Lambda)_A{}^B\,\Phi_B(\Lambda^{-1}x). \tag{65}$$

Then we see

$$U(\Lambda)\Box = \Box U(\Lambda) \qquad \forall\,\Lambda \in O(1,3). \tag{66}$$

And we find for every entire F,

$$U(\Lambda)F(\Box/\Lambda_*^2) = F(\Box/\Lambda_*^2)U(\Lambda), \tag{67}$$

and the deformed field as defined by

$$\Phi^{(F)} := F(\Box/\Lambda_*^2)\Phi \tag{68}$$

transforms covariantly in the same Lorentz representation:

$$(U(\Lambda)\Phi^{(F)})_A(x) = S(\Lambda)_A{}^B\,\Phi_B^{(F)}(\Lambda^{-1}x). \tag{69}$$

The proof of this is quite simple since ∂_μ transforms as a Lorentz covector and $\eta^{\mu\nu}$ is Lorentz invariant:

$$U(\Lambda)\partial_\mu U(\Lambda)^{-1} = (\Lambda^{-1})^\nu{}_\mu\partial_\nu, \tag{70}$$

it then follows that:

$$\begin{aligned} U(\Lambda)\Box U(\Lambda)^{-1} &= \eta^{\mu\nu}U(\Lambda)\partial_\mu\partial_\nu U(\Lambda)^{-1} &(71)\\ &= \eta^{\mu\nu}(\Lambda^{-1})^\rho{}_\mu(\Lambda^{-1})^\sigma{}_\nu\partial_\rho\partial_\sigma &(72)\\ &= \eta^{\rho\sigma}\partial_\rho\partial_\sigma &(73)\\ &= \Box, &(74)\end{aligned}$$

this is because

$$\eta^{\mu\nu}(\Lambda^{-1})^\rho{}_\mu(\Lambda^{-1})^\sigma{}_\nu = \eta^{\rho\sigma}. \tag{75}$$

And now apply Theorem 1.

Corollary 1 (Lorentz invariance of the deformed action)**.** *If*

$$S[\Phi] = \int_{\mathbb{R}^{1,3}} d^4x\,\mathcal{L}(\Phi,\partial\Phi,\dots) \tag{76}$$

is Lorentz invariant then

$$S_F[\Phi] := S[F(\Box/\Lambda_*^2)\Phi] \tag{77}$$

is Lorentz invariant.

The proof is since this is the action-functional part of Theorem 1 with the group $G = O(1,3)$ [1,35].

Now we apply the general theorem to Poincaré invariance as it is the important symmetry that the laws of physics do not depend on your inertial reference frame [1,35].

Proposition 2 (Poincaré covariance)**.** *We take $U(a,\Lambda)$ be the Poincaré action* (10)*. Then*

$$U(a,\Lambda)\Box = \Box U(a,\Lambda) \tag{78}$$

for all $(a,\Lambda) \in \mathcal{P}$. So the entire-function deformation by $F(\Box/\Lambda_*^2)$ preserves Poincaré covariance and translation invariance.

Translation invariance is immediate because ∂_μ commutes with translations, and since Lorentz invariance was proved above. Since the Poincaré group is generated by translations and Lorentz

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transformations, the claim follows trivially. We should say that in the perturbative vacuum, plane waves $e^{-ip\cdot x}$ diagonalize the d'alembertian $\Box$ [1,35]:

$$\Box e^{-ip\cdot x} = -p^2 e^{-ip\cdot x}, \tag{79}$$

where:

$$p^2 := \eta_{\mu\nu} p^\mu p^\nu. \tag{80}$$

So, $F(\Box/\Lambda_*^2)$ becomes multiplication by $F(-p^2/\Lambda_*^2)$, which depends only on the Lorentz scalar p^2, this is the momentum-space manifestation of the abstract covariance theorem.

We will now apply the general theorem to Gauge covariance as this is imporant when we construct QFTs to make sure they remain locality coordinate independent [11,16,17].

Proposition 3 (Gauge covariance of the deformed field). *If $E \to M$ is an associated bundle of a principal K-bundle $P \to M$, and we let D_μ be the gauge-covariant derivative, we then define*

$$\Box_g = -g^{\mu\nu} D_\mu D_\nu. \tag{81}$$

If a gauge transformation u acts on $\Gamma(E)$ by an invertible map U_u, and if

$$U_u D_\mu = D_\mu U_u \tag{82}$$

or equivalently

$$U_u \Box_g = \Box_g U_u, \tag{83}$$

then we see

$$U_u F(\Box_g/\Lambda_*^2) = F(\Box_g/\Lambda_*^2) U_u. \tag{84}$$

Therefore the field

$$\Phi^{(F)} := F(\Box_g/\Lambda_*^2)\Phi \tag{85}$$

transforms covariantly whenever Φ does.

Our hypothesis implies that $\Box_g$ intertwines the gauge action. Now apply Theorem 1.

Corollary 2 (Gauge invariance of deformed composite observables). *If $\mathcal{O}(\Phi)$ is gauge invariant and the operator acting on $\mathcal{O}$ is built from the gauge-covariant Laplace–Beltrami operator, then*

$$\mathcal{O}^{(F)} := F(\Box_g/\Lambda_*^2)\mathcal{O} \tag{86}$$

is gauge invariant in the same way.

Gauge invariance means $U_u\mathcal{O} = \mathcal{O}$ for all gauge transformations u. Proposition 3 then gives us [11]:

$$U_u\mathcal{O}^{(F)} = U_u F(\Box_g/\Lambda_*^2)\mathcal{O} = F(\Box_g/\Lambda_*^2)U_u\mathcal{O} = F(\Box_g/\Lambda_*^2)\mathcal{O} = \mathcal{O}^{(F)}. \tag{87}$$

Now we will finally apply the general theorem to diffeomorphism covariance and prove it remains in nonlocal QFT as this is essential to the general theory of relativity to make sure that the physical laws of nature do not break under arbitrary, smooth coordinate transformations [3,36].

Proposition 4 (Diffeomorphism covariance). *If g is a spacetime metric on M, and we take*

$$\Box_g = -g^{\mu\nu}\nabla_\mu\nabla_\nu \tag{88}$$

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or more generally the bundle-covariant Laplace–Beltrami operator acting on sections of a natural bundle $E_g \to M$. Then $\Box_g$ is natural as shown previously:

$$\varphi^* \circ \Box_g = \Box_{\varphi^* g} \circ \varphi^* \qquad \forall \;\; \varphi \in \mathrm{Diff}(M). \tag{89}$$

So for every entire F,

$$\varphi^* \circ F(\Box_g/\Lambda_*^2) = F(\Box_{\varphi^* g}/\Lambda_*^2) \circ \varphi^*. \tag{90}$$

This means that every diffeomorphism-covariant field equation or action remains diffeomorphism covariant after the our entire-function deformation.

The Levi–Civita connection is natural under pullback and the contraction with the inverse metric is also natural, so it is easy to see that $\Box_g$ satisfies the naturality relation (57), then apply Theorem 2.

To make this concrete we will follow with an example that may turn out to be relavent in quantum gravity, that being for the gravitational field equations. We start with some undeformed gravitational field equation [3,12,13]:

$$\mathscr{E}_{\mu\nu}(g, T) = 0, \tag{91}$$

where $\mathscr{E}_{\mu\nu}$ is a natural tensorial expression built from the metric g and a stress tensor T. If we replace the stress tensor T by:

$$T^{(F)}_{\mu\nu} := F(\Box_g/\Lambda_*^2) T_{\mu\nu}, \tag{92}$$

this will render our gravitanal theory non local and can remove ingularities of black holes leaving a de Sitter core as shown in [40,41], and equivalently we can insert the entire-function operator on the geometric side whenever this is legitimate, then the resulting equation remains diffeomorphism covariant.

6. The Finite-Dimensional BRST Setting

Now we will apply the general theorem to a finite-dimensional Becchi-Rouet-Stora-Tyutin (BRST) theorem and the Slavnov–Taylor identities since these are relevant in QFT to quantize non-Abelian gauge theories while at the same time maintains gauge invariance and ensuring that the theory is renormalizable and unitary [11,42–46]. Now we formulate a BRST theorem in a finite-dimensional setting, this theorem applies in particular to finite-mode truncations only when the truncation is chosen so that the truncated field algebra is preserved by the BRST differential, the truncated BRST differential remains nilpotent, the Berezin integration-by-parts identity is preserved, and the regulating operator preserves the truncated complex. Throughout this we use the graded Leibniz rule:

$$s(XY) = (sX)Y + (-1)^{|X|} X(sY) \tag{93}$$

for homogeneous $X \in \mathcal{O}(V)$, and we use left and right derivatives with the standard Koszul sign conventions. Repeated indices are summed with the graded sign rule implicit.

Definition 6 (Finite-dimensional BRST). *[42,43] We let $V = V_{\bar 0} \oplus V_{\bar 1}$ be a finite-dimensional supervector space over the complex $\mathbb{C}$, where $V_{\bar 0}$ is the even part and $V_{\bar 1}$ is the odd part, we then choose homogeneous linear coordinates*

$$\Phi^A, \qquad A = 1, \ldots, N, \tag{94}$$

with Grassmann parity

$$\epsilon_A := |\Phi^A| \in \{0, 1\}. \tag{95}$$

Now $\mathcal{O}(V)$ denotes the supercommutative algebra of polynomial functions on V. An odd derivation

$$s : \mathcal{O}(V) \to \mathcal{O}(V) \tag{96}$$

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is called a BRST differential if

1. *s has odd parity,*
2. *$s^2 = 0$,*
3. *on generators it is given by*
$$s\Phi^A = Q^A(\Phi), \tag{97}$$
for polynomial functions $Q^A \in \mathcal{O}(V)$.

Definition 7 (Berezin measure and integration-by-parts)**.** *[44–46] $d\mu(\Phi)$ is the translation-invariant Berezin measure on V. We thenn say that the BRST differential s is divergence-free with respect to $d\mu$ if*

$$\int_V d\mu(\Phi)\, sX(\Phi) = 0 \qquad \forall \;\; X \in \mathcal{O}(V). \tag{98}$$

If $s = Q^A(\Phi)\,\partial^L/\partial\Phi^A$, this is equivalent to the vanishing of the Berezin divergence of the odd polynomial vector field Q.

Lemma 1 (Berezin integration by parts)**.** *Recall that V is a finite-dimensional supervector space with translation-invariant Berezin measure $d\mu$, and let*

$$Q = Q^A(\Phi)\frac{\partial^L}{\partial\Phi^A} \tag{99}$$

be an odd polynomial vector field. Then the following are equivalent:

1. $$\int_V d\mu(\Phi)\, QX(\Phi) = 0 \qquad \forall \;\; X \in \mathcal{O}(V), \tag{100}$$
2. *the Berezin divergence of Q with respect to $d\mu$ vanishes.*

W choose affine coordinates on V compatible with the decomposition into even and odd directions, so that $d\mu$ is the usual translation-invariant Berezin density. In these coordinates the Lie derivative of $d\mu$ along Q is given by:
$$\mathcal{L}_Q(d\mu) = (\mathrm{div}_\mu Q)\, d\mu. \tag{101}$$
For every polynomial $X \in \mathcal{O}(V)$:

$$\mathcal{L}_Q(X\, d\mu) = Q(X)\, d\mu + (-1)^{|Q||X|} X\, \mathcal{L}_Q(d\mu). \tag{102}$$

Since $|Q| = 1$, integration over a translation-invariant Berezin density annihilates total derivatives so we find:
$$\int_V d\mu\, QX = -\int_V d\mu\, (\mathrm{div}_\mu Q)\, X. \tag{103}$$
So, $\int_V d\mu\, QX = 0$ for all X *iff* $\mathrm{div}_\mu Q = 0$.

Definition 8 (Entire-function deformation commuting with BRST)**.** *We let $\mathcal{D} : V \to V$ be an even linear operator and recall*
$$F(z) = \sum_{n=0}^{\infty} a_n z^n$$
is an entire function, and we then define

$$R := F(\mathcal{D}/\Lambda_*^2) = \sum_{n=0}^{\infty} a_n (\mathcal{D}/\Lambda_*^2)^n. \tag{104}$$

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Now since V is finite-dimensional, the series converges absolutely for all operator norms. We let $R^* : \mathcal{O}(V) \to \mathcal{O}(V)$ denote the pullback

$$(R^* X)(\Phi) := X(R\Phi). \tag{105}$$

We now say that the deformation commutes with BRST if

$$s \circ R^* = R^* \circ s \qquad \text{on } \mathcal{O}(V). \tag{106}$$

Lemma 2 (Commutation of the entire functional calculus with BRST)**.** *We assume that the even linear operator $\mathcal{D}$ commutes with the BRST differential on generators in the sense that*

$$s\big((\mathcal{D}\Phi)^A\big) = \mathcal{D}^A{}_B\, s\Phi^B \qquad \forall \;\; A, \tag{107}$$

then the entire-function operator

$$R = F(\mathcal{D}/\Lambda_*^2) \tag{108}$$

will also commute with BRST:

$$s\big((R\Phi)^A\big) = R^A{}_B\, s\Phi^B, \tag{109}$$

and then we see

$$s \circ R^* = R^* \circ s \qquad \text{on} \quad \mathcal{O}(V). \tag{110}$$

By induction on n we see:

$$s\big((\mathcal{D}^n\Phi)^A\big) = (\mathcal{D}^n)^A{}_B\, s\Phi^B \qquad \forall \;\; n \geq 0. \tag{111}$$

Then:

$$s\big((R\Phi)^A\big) = s\left(\sum_{n=0}^{\infty} a_n \Lambda_*^{-2n} (\mathcal{D}^n\Phi)^A\right) = \sum_{n=0}^{\infty} a_n \Lambda_*^{-2n} (\mathcal{D}^n)^A{}_B\, s\Phi^B = R^A{}_B\, s\Phi^B. \tag{112}$$

Now we let $X \in \mathcal{O}(V)$, and by the graded chain rule find:

$$s(R^* X) = s\big(X(R\Phi)\big) = \frac{\partial^R X}{\partial \Phi^A}(R\Phi)\, s\big((R\Phi)^A\big). \tag{113}$$

Using the first part:

$$s\big((R\Phi)^A\big) = R^A{}_B\, s\Phi^B. \tag{114}$$

And on the other hand:

$$R^*(sX) = R^*\left(\frac{\partial^R X}{\partial \Phi^A}(\Phi)\, s\Phi^A\right) = \frac{\partial^R X}{\partial \Phi^A}(R\Phi)\, R^A{}_B\, s\Phi^B. \tag{115}$$

So $s(R^* X) = R^*(sX)$.

Definition 9 (BRST-invariant action and the deformed action)**.** *We call an even polynomial $S \in \mathcal{O}(V)$ BRST invariant if*

$$sS = 0. \tag{116}$$

Given such an action we define the entire-function deformation by

$$S_F := R^* S = S(R\Phi). \tag{117}$$

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Theorem 3 (BRST invariance of the deformed action)**.** *Assume the hypotheses of Lemma 2, and then if $S \in \mathcal{O}(V)$ is BRST invariant, then the deformed action*

$$S_F(\Phi) = S(R\Phi) \tag{118}$$

is also BRST invariant:

$$sS_F = 0. \tag{119}$$

By Lemma 2,

$$sS_F = s(R^*S) = R^*(sS), \tag{120}$$

since $sS = 0$, it follows that $sS_F = 0$.

Definition 10 (Source-dependent generating functional)**.** *We let J_A be external sources of parity ϵ_A, and define K_A to be external sources of parity $\epsilon_A + 1$ modulo 2, then we define*

$$\Sigma_{J,K}(\Phi) := J_A\Phi^A + K_A\, s\Phi^A. \tag{121}$$

The deformed generating functional is then

$$Z_F[J,K] := \int_V d\mu(\Phi)\, \exp\big(-S_F(\Phi) + \Sigma_{J,K}(\Phi)\big), \tag{122}$$

its connected generating functional is given by

$$W_F[J,K] := \log Z_F[J,K]. \tag{123}$$

Theorem 4 (Ward identity)**.** *[47,48] If we assume:*

1. $s^2 = 0$,
2. *s is divergence-free with respect to $d\mu$,*
3. *S_F is BRST invariant.*

Then

$$J_A \frac{\partial^L Z_F}{\partial K_A} = 0, \qquad J_A \frac{\partial^L W_F}{\partial K_A} = 0. \tag{124}$$

By divergence-freeness and Lemma 1 we see that:

$$0 = \int_V d\mu(\Phi)\, s\big(\exp(-S_F(\Phi) + \Sigma_{J,K}(\Phi))\big), \tag{125}$$

since $sS_F = 0$ and $s^2 = 0$ it then follows that:

$$s\Sigma_{J,K} = J_A s\Phi^A + K_A s^2\Phi^A = J_A s\Phi^A. \tag{126}$$

So:

$$0 = \int_V d\mu(\Phi)\, J_A s\Phi^A\, \exp(-S_F(\Phi) + \Sigma_{J,K}(\Phi)). \tag{127}$$

But we notice that:

$$\frac{\partial^L Z_F}{\partial K_A} = \int_V d\mu(\Phi)\, s\Phi^A\, \exp(-S_F(\Phi) + \Sigma_{J,K}(\Phi)), \tag{128}$$

so:

$$J_A \frac{\partial^L Z_F}{\partial K_A} = 0, \tag{129}$$

and since $Z_F = e^{W_F}$, the corresponding identity for W_F follows naturally.

Definition 11 (Effective action). *We will let*

$$\phi^A := \frac{\partial^L W_F}{\partial J_A}, \tag{130}$$

we then assume that on an open set of source space the Hessian matrix

$$\left(\frac{\partial^L}{\partial J_B}\frac{\partial^L W_F}{\partial J_A}\right)_{A,B} \tag{131}$$

is invertible. Then the Legendre transform exists locally in that region, and the effective action is defined by

$$\Gamma_F[\phi, K] := J_A\phi^A - W_F[J, K], \tag{132}$$

where $J = J(\phi, K)$ is determined implicitly by the relation above.

Theorem 5 (Slavnov–Taylor identity). *[45,46,48] Under the hypotheses of Theorem 4 and the local Legendre-transform hypothesis above, the effective action will satisfy*

$$\mathcal{S}(\Gamma_F) := \frac{\partial^R \Gamma_F}{\partial \phi^A}\frac{\partial^L \Gamma_F}{\partial K_A} = 0. \tag{133}$$

By the defining properties of the Legendre transform we find:

$$\frac{\partial^R \Gamma_F}{\partial \phi^A} = J_A, \qquad \frac{\partial^L \Gamma_F}{\partial K_A} = -\frac{\partial^L W_F}{\partial K_A}. \tag{134}$$

Therefore we see that:

$$\mathcal{S}(\Gamma_F) = J_A\left(-\frac{\partial^L W_F}{\partial K_A}\right) = -J_A\frac{\partial^L W_F}{\partial K_A}. \tag{135}$$

Theorem 4 then gives:

$$J_A\frac{\partial^L W_F}{\partial K_A} = 0, \tag{136}$$

and hence $\mathcal{S}(\Gamma_F) = 0$.

Proposition 5 (Mode-truncated BRST subcomplex). *We let $\mathfrak{V}$ be a graded vector space with BRST differential s and even linear operator $\mathcal{D}$, we then let*

$$P_N : \mathfrak{V} \to \mathfrak{V}_N \tag{137}$$

be a finite-rank projection onto some finite-dimensional graded subspace $\mathfrak{V}_N$ and then assume:

1. $P_N\mathcal{D} = \mathcal{D}P_N$,
2. *the polynomial algebra $\mathcal{O}(\mathfrak{V}_N)$ is preserved by s,*
3. $s^2 = 0$ *on* $\mathcal{O}(\mathfrak{V}_N)$,
4. *the restricted Berezin measure on $\mathfrak{V}_N$ is divergence-free for the restricted BRST differential.*

Then $(\mathfrak{V}_N, \mathcal{O}(\mathfrak{V}_N), s)$ is a finite-dimensional BRST complex preserved by $\mathcal{D}$, and the preceding finite-dimensional BRST theorems apply to

$$R_N := F(\mathcal{D}|_{\mathfrak{V}_N}/\Lambda_*^2). \tag{138}$$

By assumption we see $\mathcal{O}(\mathfrak{V}_N)$ is stable under s, and $s^2 = 0$ on that algebra. Since $P_N\mathcal{D} = \mathcal{D}P_N$, the operator $\mathcal{D}$ preserves $\mathfrak{V}_N$, and then it follows so does every polynomial in $\mathcal{D}$, and therefore so does $R_N = F(\mathcal{D}|_{\mathfrak{V}_N}/\Lambda_*^2)$. The divergence-free hypothesis gives the Berezin integration-by-parts identity on $\mathfrak{V}_N$. So all hypotheses of the preceding finite-dimensional results hold on our finite graded subspace $\mathfrak{V}_N$.

Corollary 3 (The background-covariant regulators preserve BRST and ST identities on admissible truncations)**.** *If we suppose the gauge-fixed field multiplet,*

$$\Phi = (A_\mu, c, \bar{c}, b, \psi, \bar{\psi}, \dots) \tag{139}$$

is organized into a finite-dimensional truncation $\mathfrak{V}_N$ satisfying Proposition 5, and suppose the regulating operator $\mathcal{D}$ is the background-covariant Laplace–Beltrami operator acting componentwise on that truncated complex, with

$$[s, \mathcal{D}] = 0 \tag{140}$$

on generators. Then we see that for every entire function F, the regulator

$$R_N = F(\mathcal{D}|_{\mathfrak{V}_N}/\Lambda_*^2) \tag{141}$$

preserves BRST invariance of the truncated gauge-fixed action, the exact Ward identity for W_F, the exact Slavnov–Taylor identity for Γ_F [45,46].

The proof follows simply, apply Proposition 5, Lemma 2, Theorem 3, Theorem 4, and Theorem 5.

The result is given for for finite-dimensional BRST systems. They apply to finite-mode truncations only when the truncation data satisfy the hypotheses of Proposition 5, but to upgrade the statement to a nonperturbative continuum interacting gauge theory, we then must construct the continuum measure or an equivalent renormalized BRST/BV framework and prove that the limiting theory preserves the above stated hypotheses. Without that additional construction, we do not claim a fully rigorous continuum Slavnov–Taylor theorem.

7. Final Remarks

We have seen that that the minimal entire-function deformation preserves the covariance principles of our undeformed physical theory whenever the regulating operator intertwines the relevant symmetry action, or is natural in the geometric sense as shown above. What changes is not the symmetry sector but the locality sector, this is because the deformation is implemented by an entire function of a differential operator, the resulting fields and observables are no longer strictly point-supported in position space but they are instead quasi-local [23,49,50]. The deformed field is represented by convolution against a nontrivial kernel whose width is set by the nonlocality length $\ell_* = \Lambda_*^{-1}$ [23,49,50]. The interpretation is that the minimal entire-function construction replaces strict ultraviolet locality by quasi-locality while leaving the symmetry architecture of the underlying theory intact.

A full analysis of the quantitative locality properties lies beyond the scope of this paper, the detailed derivation of asymptotic microcausality bounds for smeared regulated observables, and the corresponding recovery of strict microcausality in the local limit $\Lambda_* \to \infty$, has been developed elsewhere [22,30–33]. For the purposes of the present theorem, it is enough to emphasize that the covariance statements proved here are fully compatible with quasi-local rather than strictly local ultraviolet behavior.

Discretization requires conceptual care if it were to be discussed in nonlocal QFT, the continuum covariance theorem proved here does not imply that a fixed lattice carries the full continuum diffeomorphism group as an exact symmetry, the appropriate discussion of fixed-lattice obstructions, equivariant blocking, perfect actions, and continuum recovery under refinement is complementary work that is yet to be done. The point relevant here is only that the continuum covariance result is a statement about the continuum theory itself and should not be conflated with exact symmetry realization some fixed discretization.

References

1. S. Weinberg, *The Quantum Theory of Fields, Vol. I: Foundations* (Cambridge University Press, New York, 1995).
2. R. Haag, *Local Quantum Physics: Fields, Particles, Algebras*, 2nd ed. (Springer, Berlin, 1996).

 doi:10.20944/preprints202604.0706.v1

3. R. M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).
4. H. A. Lorentz, Electromagnetic phenomena in a system moving with any velocity smaller than that of light, *Proceedings of the Royal Netherlands Academy of Arts and Sciences (KNAW)* **6**, 809–831 (1904).
5. H. Poincaré, Sur la dynamique de l'électron, *Comptes Rendus de l'Académie des Sciences* **140**, 1504–1508 (1905).
6. H. Poincaré, Sur la dynamique de l'électron, *Rendiconti del Circolo Matematico di Palermo* **21**, 129–175 (1906).
7. H. Minkowski, Raum und Zeit, *Physikalische Zeitschrift* **10**, 104–111 (1909).
8. E. Noether, Invariante Variationsprobleme, *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse* **1918**, 235–257 (1918).
9. A. Einstein, Zur Elektrodynamik bewegter Körper, *Annalen der Physik* **17**, 891–921 (1905).
10. A. Einstein, Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?, *Annalen der Physik* **18**, 639–641 (1905).
11. L. D. Faddeev and A. A. Slavnov, *Gauge Fields: An Introduction to Quantum Theory*, 2nd ed. (Addison–Wesley, Redwood City, California, 1991).
12. A. Einstein, Die Feldgleichungen der Gravitation, *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin* **1915**, 844–847 (1915).
13. A. Einstein, Die Grundlage der allgemeinen Relativitätstheorie, *Annalen der Physik* **49**, 769–822 (1916).
14. H. Weyl, Gravitation und Elektrizität, *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin* **1918**, 465–480 (1918).
15. H. Weyl, Elektron und Gravitation. I, *Zeitschrift für Physik* **56**, 330–352 (1929).
16. C. N. Yang and R. L. Mills, Conservation of isotopic spin and isotopic gauge invariance, *Phys. Rev.* **96**, 191–195 (1954).
17. R. Utiyama, Invariant theoretical interpretation of interaction, *Phys. Rev.* **101**, 1597–1607 (1956).
18. D. Hilbert, Die Grundlagen der Physik. (Erste Mitteilung.), *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse* **1915**, 395–408 (1915).
19. A. Einstein, Zur allgemeinen Relativitätstheorie, *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin* **1915**, 778–786 (1915).
20. A. Einstein, Zur allgemeinen Relativitätstheorie (Nachtrag), *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin* **1915**, 799–801 (1915).
21. A. Einstein, Die Feldgleichungen der Gravitation, *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin* **1915**, 844–847 (1915).
22. G. C. Hegerfeldt, Remark on causality and particle localization, *Phys. Rev. D* **10**, 3320–3321 (1974).
23. G. V. Efimov, Non-local quantum theory of the scalar field, Commun. Math. Phys. **5**, 42–56 (1967).
24. N. V. Krasnikov, Nonlocal gauge theories, Theor. Math. Phys. **73**, 1184–1190 (1987).
25. J. W. Moffat, Finite nonlocal gauge field theory, Phys. Rev. D **41**, 1177 (1990).
26. D. Evens, J. W. Moffat, G. Kleppe, and R. P. Woodard, Nonlocal regularizations of gauge theories, Phys. Rev. D **43**, 499 (1991).
27. G. Kleppe and R. P. Woodard, Nonlocal Yang–Mills, Nucl. Phys. B **388**, 81–112 (1992).
28. E. T. Tomboulis, Superrenormalizable gauge and gravitational theories, arXiv:hep-th/9702146.
29. J. W. Moffat and E. J. Thompson, On Gauge-Invariant Entire-Function Regulators and UV Finiteness in Non-Local Quantum Field Theory, arXiv:2511.11756 [hep-th] (2025), *Annalen der Physik*, in press (2026).
30. E. J. Thompson, Asymptotic Microcausality; Deformations of Local Quantum Field Theory, (Manuscript in press PRD, 2026).
31. E. J. Thompson, Asymptotic Locality Rectifying Newton–Wigner and Foldy–Wouthuysen Localization, *Annalen der Physik*, in press (2026).
32. E. J. Thompson, Asymptotic Renormalization: Complete Quantum Electrodynamics, *Annalen der Physik*, in press (2026).
33. E. J. Thompson, Asymptotic Microcausality to Macrocausality: Complete Quantum Electrodynamics and the Regulated S-Matrix, *Annalen der Physik*, in press (2026).
34. E. J. Thompson and Arvin Kouroshnia On the Structure of Local Observables in String Field Theory, *Phys. Rev. D* in press (2026).
35. M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison–Wesley, Reading, Massachusetts, 1995).
36. I. Kolar, P. W. Michor, and J. Slovak, *Natural Operations in Differential Geometry* (Springer, Berlin, 1993).
37. R. T. Seeley, Complex powers of an elliptic operator, in *Singular Integrals*, Proc. Sympos. Pure Math. **10**, 288–307 (1967).

 doi:10.20944/preprints202604.0706.v1

38. N. Dunford and J. T. Schwartz, *Linear Operators, Part I: General Theory* (Interscience, New York, 1958).
39. R. S. Palais, Natural operations on differential forms, *Trans. Amer. Math. Soc.* **92**, 125–141 (1959).
40. L. Modesto, J. W. Moffat, and P. Nicolini, Black holes in an ultraviolet complete quantum gravity, *Phys. Lett. B* **695**, 397–400 (2011).
41. E. J. Thompson, De Sitter Cores from Non-local Quantum Field Theories, *Eur. Phys. J. Plus*, in press (2026).
42. C. Becchi, A. Rouet, and R. Stora, Renormalization of gauge theories, Ann. Phys. (N.Y.) **98**, 287–321 (1976).
43. I. V. Tyutin, *Gauge Invariance in Field Theory and Statistical Physics in Operator Formalism*, Lebedev Institute preprint No. 39 (1975), arXiv:0812.0580 [hep-th].
44. I. A. Batalin and G. A. Vilkovisky, Gauge algebra and quantization, Phys. Lett. B **102**, 27–31 (1981).
45. M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton University Press, Princeton, 1992).
46. G. Barnich, F. Brandt, and M. Henneaux, Local BRST cohomology in gauge theories, Phys. Rept. **338**, 439–569 (2000).
47. J. C. Taylor, Ward identities and charge renormalization of the Yang–Mills field, Nucl. Phys. B **33**, 436–444 (1971).
48. A. A. Slavnov, Ward identities in gauge theories, Theor. Math. Phys. **10**, 99 (1972).
49. L. Schwartz, *Théorie des distributions*, Vols. I–II (Hermann, Paris, 1950–1951).
50. R. E. A. C. Paley and N. Wiener, *Fourier Transforms in the Complex Domain* (American Mathematical Society, Providence, 1934).