

Article

Not peer-reviewed version

Spontaneous U(1) Symmetry Breaking and Phase Transitions in Rotating Interacting Bose Gases

[Wenming Sun](#) *

Posted Date: 25 September 2025

doi: 10.20944/preprints202509.2134.v1

Keywords: U(1) symmetry breaking; rotating Bose gas; phase transition



Preprints.org is a free multidisciplinary platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This open access article is published under a Creative Commons CC BY 4.0 license, which permit the free download, distribution, and reuse, provided that the author and preprint are cited in any reuse.

Disclaimer/Publisher's Note: The statements, opinions, and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.

Article

Spontaneous U(1) Symmetry Breaking and Phase Transitions in Rotating Interacting Bose Gases

Sun Wenming

Graduate School of Science, University of Tokyo, Japan 113-8654; ywtbsykg@pgu.edu.pl

Abstract

We investigate spontaneous U(1) symmetry breaking and the associated phase transitions in rotating interacting Bose gases. Using a theoretical framework that combines mean-field analysis with rotational dynamics, we analyze how rigid rotation modifies the condensate structure and critical behavior. The study identifies the emergence of Goldstone modes and clarifies their role in the low-energy excitation spectrum. The results provide insight into the interplay between symmetry, rotation, and many-body interactions, contributing to a deeper theoretical understanding of phase structures in Bose systems.

Keywords: U(1) symmetry breaking; rotating Bose gas; phase transition

PACS: 05.30.Jp;11.30.Qc;67.85.-d;64.60.-i

I. Introduction

One of the primary goals of modern Heavy Ion Collision (HIC) experiments is to study matter under extreme conditions and its transitions through various phases. In Quantum Chromodynamics (QCD), these phases range from the deconfined quark-gluon plasma to the confined hadron phase, which consists of mesons and baryons. Mesons, as composite particles made up of a quark and an antiquark, are often regarded as (pseudo-)Goldstone bosons arising from the spontaneous breaking of chiral symmetry. Key questions related to the phase transition of matter created in HIC experiments focus in particular on the order of the phase transition and the location of the critical endpoint [1–5]. Answers to these questions provide valuable insights into astrophysical and cosmological models of the early universe [6,7]. Both of these properties are affected by external conditions, such as external electromagnetic fields and rotation. Intense magnetic fields are believed to be generated in the early stages of noncentral HICs. Depending on the initial conditions, the strength of the magnetic fields is estimated to be approximately $B \sim 10^{18} - 10^{20}$ Gauß in the early stages after these collisions [8,9]. In recent years, several studies have explored the QCD phase diagram in the presence of magnetic fields. Novel effects, such as magnetic and inverse magnetic catalysis are associated with the effect of constant background magnetic fields on the nature of the chiral phase transition and the location of the critical point [10–12]. Recently, several studies have investigated the effect of rotation on quark matter created in HIC experiments. This matter is believed to experience extremely high vorticity, with an angular velocity reaching up to 10^{22} Hz [13,14]. Extensive research has focused on how rotation influences the thermodynamic properties of relativistic fermionic systems [15–20]. One notable example is the chiral vortical effect, which is related to the transport properties of the quark matter produced after HICs and provides insights into the topological aspects of QCD [21]. When examining the thermodynamic properties of rotating Fermi gases using field theoretical methods, it is advantageous to assume rigid rotation with a constant angular velocity [22,23]. The impact of rigid rotation on QCD phase transitions, including chiral and

confinement/deconfinement, has been studied with and without boundary conditions, e.g., in [15,24]. In [24], it is shown that at finite temperature the phase diagram of a uniformly rotating system exhibits, in addition to a confining and a deconfining phase at low and high temperatures, a mixed inhomogeneous phase at intermediate temperatures.

Several studies have also explored both relativistic bosons [25–35] and the linear sigma model with quarks [36–39] under rigid rotation. In [26], a spin-one gluon gas under rigid rotation is analyzed, revealing that at temperatures below a certain supervortical temperature, the moment of inertia of a rotating spin-one gluon plasma becomes negative. This phenomenon indicates a thermodynamic instability and is associated with the negative Barnett effect, where the total angular momentum of the system opposes the direction of its angular velocity. For spin-zero bosons in the presence of imaginary rotation, nonionic statistics arise, modifying the standard Bose-Einstein distribution with a statistical angle. Under specific conditions, these bosons exhibit fermionic-like behavior and display fractal thermodynamics that depend on the angle of imaginary rotation [27]. A separate study in [28] investigated the thermodynamics of spin-zero complex scalar fields under rigid rotation, revealing that thermodynamic instabilities emerge at high temperatures and large coupling constants. These instabilities include negative moment of inertia and heat capacity. Finally, in [30], the Bose-Einstein (BE) condensation of a free Bose gas subjected to rigid rotation is investigated in both relativistic and nonrelativistic limits. It is demonstrated that rotation not only modifies the equation of state of the system but also impacts the transition temperature for BEC and the fraction of condensates. Specifically, it is shown that the critical temperature of a rotating Bose gas is lower than that of a nonrotating gas; however, as the angular velocity increases, the critical temperature of the rotating gas also rises. Additionally, an analysis of the heat capacity of a nonrelativistic rotating free Bose gas indicates that rotation alters the nature of the BEC phase transition from continuous to discontinuous. The present paper aims to extend these findings to an interacting Bose gas under rigid rotation.

We begin with the Lagrangian density of a complex Klein-Gordon field φ that includes a self-interaction term $\lambda(\varphi^*\varphi)^2$ with a coupling constant λ . To introduce rigid rotation we use a metric including the angular velocity Ω . In the first part of this paper, we introduce a chemical potential μ corresponding to the global U(1) symmetry of the Lagrangian. For later analysis, we expand the Lagrangian density around a classical configuration $|\langle\varphi\rangle| = v$. Following standard methods [40,41] and utilizing an appropriate Bessel-Fourier transformation [29,30], we derive the free propagator of this model. This propagator is subsequently employed to compute the thermodynamic potential as a function of μ, Ω , and the energy dispersion relation ϵ_k . As it turns out, the spontaneous breaking of U(1) symmetry occurs for $m < \mu$. In this regime, we find two distinct energy branches; one corresponding to a massive phonon and the other to a massless roton. It is noteworthy that the rotation does not alter ϵ_k at low momentum, and the results are similar to the nonrotating case [42].

In the second part of this paper, we explore the impact of rotation on the spontaneous breaking of U(1) symmetry, focusing specifically on the case of zero chemical potential. Our primary emphasis is on the T and Ω dependence of the critical temperature of the corresponding phase transition, as well as two masses m_1 and m_2 , which are identified with the masses of the σ and π mesons, respectively. We begin by considering the thermodynamic potential discussed in the first part of this paper. Apart from a classical part, it consists of a thermal and a vacuum contributions. By employing a novel method for summing over the quantum number ℓ related to rotation, we perform a high-temperature expansion. Combining the classical and the thermal parts, we derive an analytical expression for the critical temperature of U(1) phase transition T_c , which is found to be proportional to $\Omega^{1/3}$. Furthermore, we show that the minima of this potential are proportional to $(1 - t^3)$, where $t \equiv T/T_c$ is the reduced temperature. This contrasts with the behavior observed in a nonrotating Bose gas, where the minima are described by the factor $(1 - t_0)$ with $t_0 \equiv T/T_c^{(0)}$. (Here, sub- and superscripts zero correspond

to nonrotating Bose gas.) We also demonstrate that when substituting these minima into m_1 and m_2 , they become imaginary in the symmetry-restored phase, analogous to the behavior in a nonrotating Bose gas. This issue is addressed by adding the thermal masses that arise from one-loop perturbative contributions to m_1 and m_2 . By following this method, we confirm that the Goldstone theorem is satisfied in the symmetry-restored phase.

We then compute the vacuum part of the potential by adding the appropriate counterterms and performing dimensional regularization. Our findings extend the results from [43], where the vacuum contribution to the effective action for a $\lambda\phi^4$ theory was computed. We add this potential to the classical and thermal parts of the potential, minimize the resulting expression, and examine how the minima depend on temperature T for fixed angular velocity Ω . We show that, similar to the behavior observed in a noninteracting Bose gas [30], rotation reduces the critical temperature of the phase transition, which then increases as Ω rises. Additionally, by plugging these minima into the corresponding expressions to m_1 and m_2 (or equivalently m_σ and m_π) we investigate the T dependence of σ and π meson masses for fixed Ω . As expected, in the symmetry-restored phase, we find $m_\sigma = m_\pi$. This equality indicates that at T_c the minima of the corresponding potential vanish, suggesting a second-order phase transition, even in the presence of rigid rotation.

Finally, we focus on the nonperturbative ring contribution to the potential described above. We present a full derivation of the ring potential in the presence of rotation. Based on the findings in [43], we expect that the addition of the ring potential will alter the order of the phase transition. Our results indicate that when rotation is absent ($\Omega = 0$), a discontinuous phase transition occurs at a specific temperature. In contrast, when rotation is present ($\Omega \neq 0$), the phase transition remains continuous. Furthermore, we define a σ dissociation temperature, denoted by T_{diss} , which is characterized by $m_\sigma(T_{\text{diss}}) = 2m_\pi(T_{\text{diss}})$ and show that T_{diss} is less than the critical temperature.

The organization of this paper is as follows: In Sec. II, we introduce the rigid rotation in the Lagrangian density of a complex scalar field in the presence of a finite chemical potential. We derive the corresponding free propagator, determine the full thermodynamic potential of this model, and explore how rotation affects the spontaneous breaking of global $U(1)$ symmetry. In Sec. III, we focus on the special case of $\mu = 0$ and systematically determine the full thermodynamic potential, which consists, apart from the classical part, of a thermal and a vacuum contribution. After examining the effect of rotation on the Goldstone theorem, we add the nonperturbative ring contribution to this potential, which is explicitly derived for the case of a rotating complex scalar field. In Sec. IV, we numerically solve the corresponding gap equation for the full potential with and without the ring potential. We investigate the T dependence of the corresponding minima for fixed Ω . Additionally, we determine the T and Ω dependence of m_σ and m_π , along with the σ dissociation temperatures. Section V concludes the paper with a compact summary of our findings. In Appendix A, we present the high-temperature expansion in the presence of a rigid rotation. Notably, we apply a method introduced in [30] to sum over ℓ . Appendices B and C contain derivations of formulas (III.27) and (III.34), while the derivation of (III.44) is detailed in Appendix D.

II. Interacting Charged Scalars Under Rigid Rotation

A. The Free Propagator

We start with the Lagrangian density of a charged scalar field ϕ

$$\mathcal{L} = g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2, \quad (\text{II.1})$$

with the metric

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (\text{II.2})$$

describing a rigid rotation. Here, m is the rest mass and $0 < \lambda < 1$ is the coupling constant, describing the strength of the interaction. The spacetime coordinate is described by $x^\mu = (t, x, y, z)$ and $r^2 \equiv x^2 + y^2$. Moreover, Ω is the constant angular velocity of a rigid rotation around the z -axis. The above Lagrangian is invariant under global $U(1)$ transformation

$$\varphi(x) \rightarrow e^{-i\alpha}\varphi(x), \quad \varphi^*(x) \rightarrow e^{+i\alpha}\varphi^*(x), \quad (\text{II.3})$$

with α a real constant phase. Plugging the metric into (II.1), we obtain

$$L = |(\partial_0 - i\mu - i\Omega L_Z)\varphi|^2 - |\nabla\varphi|^2 - m^2|\varphi|^2 - \lambda|\varphi|^4, \quad (\text{II.4})$$

where the chemical potential μ corresponding to the global $U(1)$ symmetry (II.3) is introduced. The z -component of the angular momentum, L_Z , is defined by $L_Z = i(y\partial_x - x\partial_y)$. To investigate the spontaneous breaking of $U(1)$ symmetry, we rewrite L in terms of real fields φ_1 and φ_2 appearing in $\varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$ and perform the shift $\varphi_i \rightarrow \Phi_i + \varphi_i$ with

$$\Phi = \begin{pmatrix} v \\ 0 \end{pmatrix} \quad \text{and} \quad v = \text{const.} \quad \text{We arrive at}$$

$$\mathcal{L} = \sum_{i=0}^4 \mathcal{L}_i, \quad (\text{II.5}) \quad \text{with}$$

$$\mathcal{L}_0 = \frac{1}{2}(\mu^2 - m^2)v^2 - \frac{\lambda}{4}v^4,$$

$$L_1 = (\mu^2 - m^2)v\varphi_1 - \mu v\partial_0\varphi_2 - \lambda v^3\varphi_1 + i\mu\Omega v L_Z\varphi_2,$$

$$\mathcal{L}_2 = \frac{1}{2}\left\{(\partial_0\varphi_1)^2 + (\partial_0\varphi_2)^2 - (\nabla\varphi_1)^2 - (\nabla\varphi_2)^2 + (\mu^2 - m^2)(\varphi_1^2 + \varphi_2^2) + 2\mu(\varphi_2\partial_0\varphi_1 - \varphi_1\partial_0\varphi_2) - \lambda(3v^2\varphi_1^2 + v^2\varphi_2^2) - \Omega^2\right.$$

$$\left. [(L_Z\varphi_1)^2 + (L_Z\varphi_2)^2] - 2i\Omega[(\partial_0\varphi_1 + \mu\varphi_2)L_Z\varphi_1] - 2i\Omega[(\partial_0\varphi_2 - \mu\varphi_1)L_Z\varphi_2] \right\},$$

$$L_3 = -\lambda v\varphi_1(\varphi_1^2 + \varphi_2^2),$$

$$\mathcal{L}_4 = -\frac{\lambda}{4}(\varphi_1^2 + \varphi_2^2)^2. \quad (\text{II.6})$$

The classical part of the Lagrangian, L_0 , defines the classical (zero mode) potential

$$\mathcal{V}_{cl}(v) \equiv -\mathcal{L}_0 = \frac{1}{2}(m^2 - \mu^2)v^2 + \frac{\lambda}{4}v^4. \quad (\text{II.7})$$

The free propagator arises from the quadratic term L_2 in the fluctuating fields φ_1 and φ_2 . To derive the free propagator in the momentum space, we use the Fourier-Bessel transformation

$$\varphi_i(x) = \sqrt{\frac{\beta}{V}} \sum_{n,\ell,k} e^{i(\omega_n\tau + \ell\phi + k_z z)} J_\ell(k_\perp r) \tilde{\varphi}_i(k),$$

$$(\text{II.8})$$

with $i = 1, 2$. The cylindrical symmetry is implemented by introducing the cylinder coordinate system described by $x^\mu = (t, x, y, z) = (t, r \cos\phi, r \sin\phi, z)$, with r the radial coordinate, ϕ the azimuthal angle, and z the height of the cylinder. The conjugate momenta, corresponding to these coordinates at finite temperature T , are given by the bosonic Matsubara frequency $\omega_n = 2\pi nT$, discrete quantum number ℓ , which is the eigenvalue of L_Z , continuous momentum k_z , and $k_\perp \equiv |\mathbf{k}_\perp| \equiv (k_x^2 + k_y^2)^{1/2}$ in cylindrical coordinates. The Bessel function $J_\ell(k_\perp r)$ captures the radial dependence in this transformation and $\tau \equiv it$. Plugging (II.8) into L_2 and performing an integration over cylindrical coordinates, according to

$$\int_X \equiv \int_0^\beta d\tau \int_0^\infty r dr \int_0^{2\pi} d\phi \int_{-\infty}^\infty dz, \quad (\text{II.9})$$

we arrive after some manipulations at

$$\int_X \mathcal{L}_2 = -\frac{V}{2} \sum_{n,\ell,k} (\tilde{\varphi}_1(-k) \tilde{\varphi}_2(-k)) (\beta^2 \mathcal{D}_\ell^{-1}(k)) \begin{pmatrix} \tilde{\varphi}_1(k) \\ \tilde{\varphi}_2(k) \end{pmatrix}, \quad (\text{II.10})$$

with the free propagator

$$\beta^2 \mathcal{D}_\ell^{-1}(k) = \begin{pmatrix} (\omega_n + i\ell\Omega)^2 + \omega_1^2 - \mu^2 & -2\mu(\omega_n + i\ell\Omega) \\ 2\mu(\omega_n + i\ell\Omega) & (\omega_n + i\ell\Omega)^2 + \omega_2^2 - \mu^2 \end{pmatrix}. \quad (\text{II.11})$$

Here, $\omega_i^2 \equiv \mathbf{k}^2 + m_i^2$, $i = 1, 2$, with $m_1^2(v) \equiv 3\lambda v^2 + m^2$ and $m_2^2(v) \equiv \lambda v^2 + m^2$, the corresponding masses to two fields ϕ_1 and ϕ_2 . In cylinder coordinate system, we have $\mathbf{k}^2 \equiv \mathbf{k}_\perp^2 + k_z^2$. In Sec. III, we break the global U(1) symmetry by choosing $m^2 = -c^2$ with $c^2 > 0$ and show that after considering the quantum corrections, ϕ_2 become a massless Goldstone mode.

A comparison with similar results for a nonrotating charged Bose gas at T and μ shows that while $\ell\Omega$ is said to play a role analogous to that of the chemical potential μ [23], the manner in which it is incorporated into the free propagator and the thermodynamic potential differs significantly (as discussed below).

B. The Thermodynamic Potential

To derive the thermodynamic potential V, corresponding to this model, we follow the standard procedure and define this potential by

$$\mathcal{V} = -\frac{T}{V} \ln \mathcal{Z}, \quad (\text{II.12})$$

with

$$\ln \mathcal{Z} = -\frac{1}{2} \ln \det (\beta^2 \mathcal{D}_\ell^{-1}(k)). \quad (\text{II.13})$$

Let us first focus on $\ln \mathcal{Z}$ with $\mathcal{Z}$ the partition function of this model. Plugging $\mathcal{D}_\ell^{-1}$ from (II.11) into (II.13), we arrive first at

$$\ln \mathcal{Z} = -\frac{1}{2} \sum_{e=\pm} \sum_{n,\ell,\mathbf{k}} \ln |\beta^2 [(\epsilon_k^e)^2 + (\omega_n + i\ell\Omega)^2]|, \quad (\text{II.14})$$

with $\epsilon_k^\pm$ given by

$$\epsilon_k^\pm \equiv \left(E_k^2 + \mu^2 \mp \sqrt{4\mu^2 E_k^2 + \delta M^4} \right)^{1/2}, \quad (\text{II.15})$$

$E_k^2 = \mathbf{k}^2 + M^2$, and

$$M^2 \equiv \frac{1}{2} (m_1^2 + m_2^2), \quad \delta M^2 \equiv \frac{1}{2} (m_1^2 - m_2^2). \quad (\text{II.16})$$

Following standard steps, it is possible to show that

$$\ln \mathcal{Z} = -\frac{1}{4} \sum_{e=\pm} \sum_{n,\ell,\mathbf{k}} \left\{ \ln \left(\beta^2 [\omega_n^2 + (\epsilon_k^e + \ell\Omega)^2] \right) + \ln \left(\beta^2 [\omega_n^2 + (\epsilon_k^e - \ell\Omega)^2] \right) \right\}. \quad (\text{II.17})$$

Performing the Matsubara sum with

$$\sum_{n=-\infty}^{+\infty} \ln ((2\pi n)^2 + \eta^2) = \eta + 2 \ln (1 - e^{-\eta}), \quad (\text{II.18})$$

we arrive at

$$\ln \mathcal{Z} = -\frac{V}{2} \sum_{e=\pm} \sum_{\ell} \int d\tilde{\mathbf{k}} \left\{ \beta \epsilon_k^e + \ln \left(1 - e^{-\beta(\epsilon_k^e + \ell\Omega)} \right) + \ln \left(1 - e^{-\beta(\epsilon_k^e - \ell\Omega)} \right) \right\}, \quad (\text{II.19})$$

where the summation over $\mathbf{k}$ is replaced with the integration over $\mathbf{k}$ in the cylinder coordinate system,

$$\sum_{\mathbf{k}} \rightarrow V \sum_{n,\ell} \int d\tilde{\mathbf{k}}, \quad \text{with} \quad \int d\tilde{\mathbf{k}} \equiv \int \frac{k_\perp dk_\perp dk_z}{(2\pi)^3}. \quad (\text{II.20})$$

Here, $k_\perp \equiv |\mathbf{k}_\perp|$. Using (II.12), the thermodynamic potential V is given by

$$V = V_{\text{vac}} + VT, \quad (\text{II.21})$$

with the vacuum part

$$V_{\text{vac}} \equiv \frac{1}{2} \sum_{\ell} \int d\tilde{k} (\epsilon_k^+ + \epsilon_k^-), \quad (\text{II.22})$$

and the matter (thermal) part

$$\mathcal{V}_T = \frac{T}{2} \sum_{e=\pm} \sum_{\ell} \int d\tilde{k} \left\{ \ln \left(1 - e^{-\beta(\epsilon_k^e + \ell\Omega)} \right) + \ln \left(1 - e^{-\beta(\epsilon_k^e - \ell\Omega)} \right) \right\}. \quad (\text{II.23})$$

Adding V with $V_{\text{cl}}(v)$ from (II.7), to include the zero mode

contribution, we obtain the full thermodynamic potential V_{tot} ,

$$\begin{aligned} \mathcal{V}_{\text{tot}} = & \frac{1}{2}(m^2 - \mu^2)v^2 + \frac{\lambda}{4}v^4 + \frac{1}{2} \sum_{\ell} \int d\tilde{k} (\epsilon_k^+ + \epsilon_k^-) \\ & + \frac{T}{2} \sum_{e=\pm} \sum_{\ell \neq 0} \int d\tilde{k} \left\{ \ln \left(1 - e^{-\beta(\epsilon_k^e + \ell\Omega)} \right) + \ln \left(1 - e^{-\beta(\epsilon_k^e - \ell\Omega)} \right) \right\}. \end{aligned} \quad (\text{II.24})$$

C. Spontaneous Breaking of Global $U(1)$ Symmetry

Let us consider the classical potential (II.7). Assuming $m^2 > \mu^2$, the coefficient of v^2 in this expression is positive and, as it turns out, V_{cl} possesses one single minimum at $\bar{v}_0 = 0$ and the system is in its symmetric phase.

In this case, $m_1^2(v=0) = m_2^2(v=0) = m^2$, $\delta M^2 = 0$ and $\epsilon_k^\pm$ is

$$\epsilon_k^\pm = \sqrt{k^2 + m^2} \mp \mu. \quad (\text{II.25})$$

Here, m is a mass gap and $\Delta\epsilon_k \equiv \epsilon_k^- - \epsilon_k^+ = 2\mu$. In Figure 1(a), $\epsilon_k^\pm$ is plotted for generic mass $m = 1$ MeV and chemical potential $\mu = 0.6$ MeV ($\mu < m$).

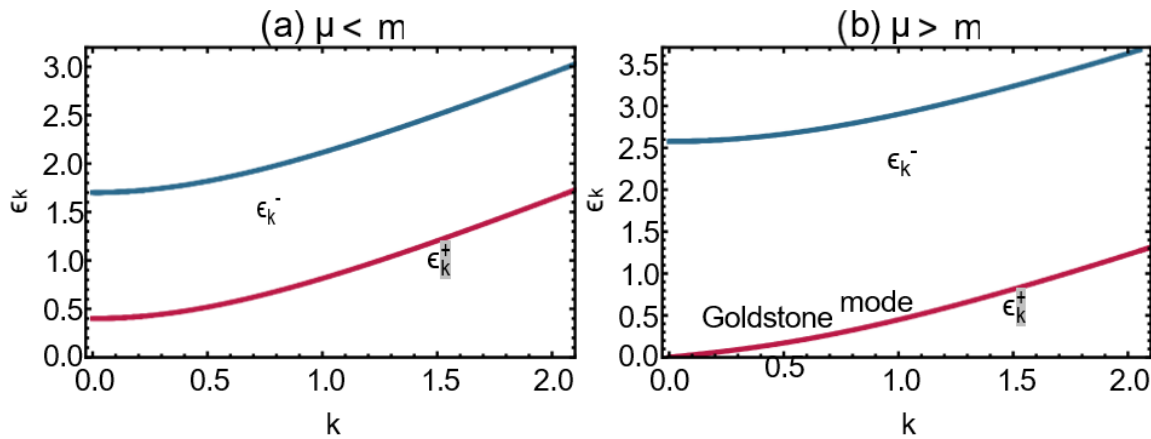


Figure 1. (color online). The k dependence of the energy dispersion $\epsilon_k^\pm$ from (II.25) and (II.26) in the $U(1)$ symmetric phase (panel a) and the symmetry-broken phase (panel b), characterized by $\mu < m$ and $\mu > m$, respectively. As demonstrated, in the symmetry-broken phase, there is a massless Goldstone mode. These findings remain unchanged regardless of any rotation.

In the symmetry-broken phase characterized by $m^2 < \mu^2$, however, extremizing V_{cl} yields a maximum at $v_a = 0$ and two minima at

$$\bar{v}_b = \pm \sqrt{\frac{\mu^2 - m^2}{\lambda}}.$$

The masses $m_1^2(v=b) = 3\mu^2 - 2m^2$ and $m_2^2(v=b) = \mu^2$. We thus have $M^2 = 2\mu^2 - m^2$ and $\delta M^2 = \mu^2 - m^2$ leading to

$$\epsilon_k^\pm = \sqrt{\mathbf{k}^2 + (3\mu^2 - m^2)} \mp \sqrt{4\mu^2 \mathbf{k}^2 + (3\mu^2 - m^2)}. \quad (\text{II.26})$$

In Figure 1(b), $\epsilon_k^\pm$ is plotted for generic $\mu = 1.1$ MeV and $m = 1$ MeV ($\mu > m$). As it is shown, whereas ϵ_k^- is quadratic in $k \equiv |\mathbf{k}|$, $\epsilon_k^\pm \sim 0$ for $k \sim 0$. This behavior indicates the presence of a massless Goldstone mode. By expanding $\epsilon_k^\pm$ in the orders of $k \sim 0$, we obtain

$$\begin{aligned} \epsilon_k^- &\simeq \sqrt{2(3\mu^2 - m^2)} + \frac{5\mu^2 - m^2}{2\sqrt{2(3\mu^2 - m^2)^3}} \mathbf{k}^2, \\ \epsilon_k^+ &\simeq \sqrt{\frac{\mu^2 - m^2}{3\mu^2 - m^2}} |\mathbf{k}|. \end{aligned} \quad (\text{II.27})$$

According to these results, $\epsilon_k^\pm$ and ϵ_k^- correspond to phonon and roton modes in the symmetry-broken phase $m < \mu$, respectively.

As it is shown in this section, $\ell\Omega$ appears in the thermal part of the effective potential V_T from (II.23) and does not modify neither $m_1^2(v)$ nor the energy dispersion $\epsilon_k^\pm$. Hence, a comparison with analogous results for nonrotating bosons [42] shows that rigid rotation has no effect on the behavior of $\epsilon_k^\pm$ at $k \sim 0$.

D. Two Special Cases

In what follows, we consider two special cases $\lambda = 0, \mu \neq 0$ and $\lambda \neq 0, \mu = 0$:

Case 1: For the special case of noninteracting rotating Bose gas with $\lambda = 0$ and $\mu \neq 0$, we have $m_1 = m_2 = m$,

$$\begin{aligned} E_k^\pm &= \mathbf{k}^2 + m^2, \text{ and } \delta M = 0. \text{ We thus have} \\ \epsilon_k^\pm|_{\lambda=0, \mu \neq 0} &= \sqrt{\mathbf{k}^2 + m^2} \mp \mu, \quad (\text{II.28}) \text{ and therefore} \\ \mathcal{V}_{\text{tot}}|_{\lambda=0, \mu \neq 0} &= \frac{1}{2}(m^2 - \mu^2)v^2 + \sum_\ell \int d\tilde{k} \left\{ E_k \right. \\ &\quad \left. + T \left[\ln(1 - e^{-\beta(E_k - \mu_{\text{eff}})}) + \ln(1 - e^{-\beta(E_k + \mu_{\text{eff}})}) \right] \right\}, \end{aligned} \quad (\text{II.29})$$

with $\mu_{\text{eff}} \equiv \mu + \ell\Omega$. This potential is exactly the same potential arising in [30]. Using this potential, the effect of rotation on the BE condensation of a relativistic free Bose gas is studied.

Case 2: Another important case is characterized by

$\lambda \neq 0$ and $\mu = 0$. In this case, $\epsilon_k^\pm$ are given by

$$\begin{aligned} \epsilon_k^+ &= \sqrt{\mathbf{k}^2 + m_2^2} = \omega_2, \\ \epsilon_k^- &= \sqrt{\mathbf{k}^2 + m_1^2} = \omega_1. \end{aligned} \quad (\text{II.30})$$

Plugging (II.30) into (II.24) and choosing $\mu = 0$ and $m^2 = -c^2$ with $c^2 > 0$, the total thermodynamic potential is given by

$$\mathcal{V}_{\text{tot}}|_{\lambda \neq 0, \mu=0} = \mathcal{V}_{\text{cl}} + \mathcal{V}_{\text{vac}} + \mathcal{V}_T, \quad (\text{II.31}) \text{ with the classical part}$$

$$\mathcal{V}_{\text{cl}} = -\frac{c^2 v^2}{2} + \frac{\lambda v^4}{4}, \quad (\text{II.32}) \text{ the vacuum part}$$

$$\mathcal{V}_{\text{vac}} = \frac{1}{2} \sum_\ell \int d\tilde{k} (\omega_1 + \omega_2), \quad (\text{II.33}) \text{ and the thermal part}$$

$$\mathcal{V}_T = \frac{1}{2} \sum_{i=1,2} (\mathcal{V}_i^+ + \mathcal{V}_i^-), \quad (\text{II.34}) \text{ where}$$

$$\mathcal{V}_i^\pm \equiv T \sum_{\ell \neq 0} \int d\tilde{k} \ln(1 - e^{-\beta(\omega_i \mp \ell\Omega)}). \quad (\text{II.35})$$

Here, $\omega_i, i = 1, 2$ are given in (II.30). Let us notice that in (II.35), the $\ell = 0$ contribution is excluded, because the zero mode contribution is already captured by $\mathcal{V}_{\text{cl}}$ from (II.32). It is possible to limit the integration over ℓ

in $V_T^\pm$ from (II.35). Having in mind that the arguments of $\ln(1 - e^{-\beta(\omega_i \mp \ell\Omega)})$ are to be positive, the summation over ℓ in $\ln(1 - e^{-\beta(\omega_i - \ell\Omega)})$ is over $\ell \in (-\infty, -1]$ and in $\ln(1 - e^{-\beta(\omega_i + \ell\Omega)})$ is over $\ell \in [1, \infty)$ [30]. Performing a change $\ell \rightarrow -\ell$, we thus have

$$\mathcal{V}_i^+ = T \sum_{\ell=1}^{\infty} \int d\tilde{k} \ln(1 - e^{-\beta(\omega_i + \ell\Omega)}) = \mathcal{V}_i^- \quad (\text{II.36})$$

reads

$$\mathcal{V}_T = T \sum_{i=1,2} \sum_{\ell=1}^{\infty} \int d\tilde{k} \ln(1 - e^{-\beta(\omega_i + \ell\Omega)}) \quad (\text{II.37})$$

III. Spontaneous Breaking of Global U(1) Symmetry in a Rigidly Rotating Bose Gas

A. The Critical Temperature of U(1) Phase Transition; Analytical Result

In this section, we study the effect of rigid rotation on the spontaneous breaking of global U(1) symmetry in an interacting charged Bose gas. Before starting, we add a new term

$$\tilde{\mathcal{L}}_0 = \frac{1}{2} m_0^2 (\varphi_1 + v) v, \quad (\text{III.1})$$

to L from (II.5). This leads to an additional mass term in the classical potential V_{cl} . We define a new mass $a^2 \equiv c^2 + m_0^2$, which replaces c^2 in (II.32). Minimizing the resulting expression, the (classical) minimum of V_{cl} is thus given by

$$v_0^2 \equiv \frac{a^2}{\lambda}. \quad (\text{III.2})$$

At this minimum, the masses of $m_1^2(v) = 3\lambda v^2 - c^2$ and $m_2^2(v) = \lambda v^2 - c^2$ are given by

$$m_1^2(v_0) = 3a^2 - c^2, \quad m_2^2(v_0) = a^2 - c^2. \quad (\text{III.3})$$

For $m_0 = 0$, we have $m_2 = 0$ and φ_2 becomes a massless Goldstone mode. The position of this (classical) minimum changes, once the contribution of the thermal part of the thermodynamic potential, $\mathcal{V}_T$, is considered. To show this, we first define $V_a \equiv V_{cl} + \mathcal{V}_T$ and use the high-temperature expansion of $\mathcal{V}_T$ by making use of the results presented in Appendix A. Considering only the first two terms of (A.13) and plugging the definitions of $m_1^2(v)$ and $m_2^2(v)$ into it, the high-temperature expansion of V_a reads

$$\begin{aligned} \mathcal{V}_a(v, T, \Omega) = & -\frac{a^2 v^2}{2} \left(1 - \frac{2\lambda T^3 \zeta(3)}{a^2 \pi^2 \Omega} \right) + \frac{\lambda v^4}{4} \\ & - \frac{2T^5 \zeta(5)}{\pi^2 \Omega} - \frac{c^2 T^3 \zeta(3)}{2\pi^2 \Omega} + \dots \quad (\text{III.4}) \end{aligned}$$

Setting the coefficient of v^2 equal to zero, the critical temperature of global U(1) phase transition is determined,

$$T_c = \left(\frac{a^2 \pi^2 \Omega}{2\lambda \zeta(3)} \right)^{1/3}. \quad (\text{III.5})$$

In [30], the BE transition in a noninteracting Bose gas under rigid rotation is studied. It is shown that in nonrelativistic regime $T_c \propto \Omega^{2/5}$ and in ultrarelativistic regime $T_c \propto \Omega^{1/4}$. In the present case of interacting Bose gas, similar to that noninteracting cases, the critical temperature increases with increasing Ω .

Introducing the reduced temperature $t = T/T_c$, with $T_c = T_c(\Omega)$ from (III.5), and minimizing V_a from (III.4) with respect to v , the new nontrivial minimum is given by

$$\begin{aligned} v_{\min}^2(T, \Omega) = & \begin{cases} \frac{a^2}{\lambda} (1 - t^3) & t < 1, \\ 0 & t \geq 1. \end{cases} \quad (\text{III.6}) \end{aligned}$$

When comparing with a similar result for a nonrotating charged Bose gas [40], it turns out that the power of t in (III.6) changes once the gas is subjected to small rotation. In Sec. IV, we

numerically study the effect of rotation on the spontaneous breaking of global U(1) symmetry. For this purpose, we employ a phenomenological model that includes σ and π mesons, replacing φ_1 and φ_2 fields in the above computation. We set $m_1^2(v_0) = 3\lambda v_0^2 - c^2 =$

m_σ^2 and $m_2^2(v_0) = \lambda v_0^2 - c^2 = m_\pi^2$ with v_0 the classical minimum from (III.2). Moreover, we choose m_0 in (III.1) equal to m_π . For $m_\sigma = 400$ MeV, and $m_\pi = 140$ MeV, we obtain

$$c = \left(\frac{m_\sigma^2 - 3m_\pi^2}{2} \right)^{1/2} \simeq 225 \text{ MeV. (III.7)}$$

Moreover, $a = (c^2 + m_\pi^2)^{1/2} \simeq 265$ MeV. We also choose $\lambda = 0.5$. Using these quantities the function

$$\Delta V_a \equiv V_a(v, T, \Omega) - V_a(0, T, \Omega) = -\frac{a^2 v^2}{2} (1 - t^3) + \frac{\lambda v^4}{4}, \text{ (III.8)}$$

is plotted in Figure 2 at $t = 0.6, 0.8$ in the symmetry-broken phase and $t = 1.2$ in the symmetry-restored phase. At $t = 1$ a phase transition from the symmetry-broken phase to a symmetry-restored phase occurs. Let us notice, that the effect of rotation consists of changing the power of t in (III.6) and (III.8) from t^2 to t^3 . This is apart from the Ω dependence of the critical temperature T_c from (III.5) (see Figure 7).

The result indicates a continuous phase transition from a symmetry-broken phase at $t < 1$ to a symmetry-restored phase at $t \geq 1$. To scrutinize this conclusion, let us consider the pressure P arising from V_a from (III.4).

It is given by $P = -V_a$. Denoting the pressures below and above T_c with $P_<(v, T, \Omega)$ and $P_>(v, T, \Omega)$, we have $P_<(v_{\min}, T, \Omega) = -\frac{a^4}{2\lambda}t^3 + \frac{c^2 T^3 \zeta(3)}{2\pi^2 \Omega} + \frac{2T^5 \zeta(5)}{\pi^2 \Omega} + \frac{a^4}{4\lambda}t^6$, $P_>(0, T, \Omega) = \frac{c^2 T^3 \zeta(3)}{2\pi^2 \Omega} + \frac{2T^5 \zeta(5)}{\pi^2 \Omega} - \frac{a^4}{4\lambda}$. (III.9)

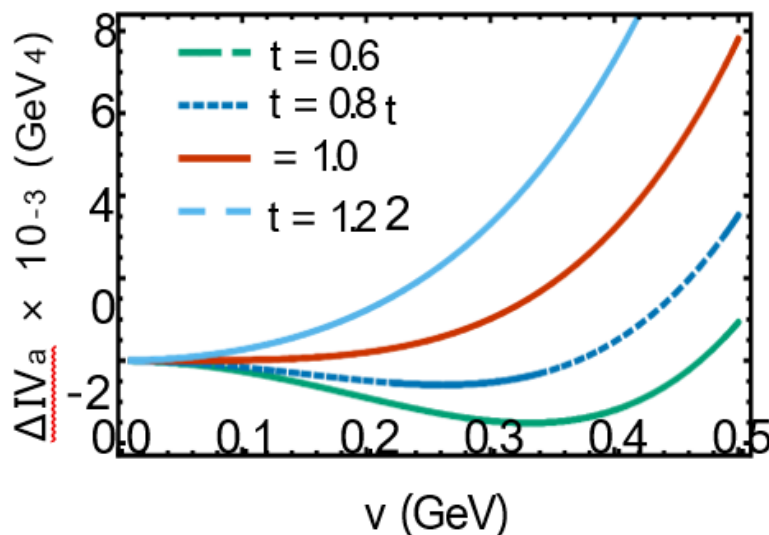


Figure 2. (color online). The v dependence of ΔV_a from (III.8) is plotted at $t = 0.6, 0.8, 1, 1.2$. At $t < 1$ the global U(1) symmetry is broken and ΔV_a possesses nontrivial minima at $v_{\min}^2 = a^2 (1 - t^3)/\lambda$. At $t = 1$ the symmetry is restored and at $t \geq 1$ a single minimum at $v_{\min} = 0$ appears (see (III.6)).

Here, we have added a term $-a^4/4\lambda$ to $P_<$ and $P_>$ in order to guarantee $P_<(v_{\min}^2, 0, \Omega) = 0$ and $P_< = P_>$ at the transition temperature T_c . At $T = T_c$, the pressure is given by

$$P_{<}(v_{\min}^2, T_c, \Omega) = P_{>}(0, T_c, \Omega) \\ = -\frac{a^4}{4\lambda} + \frac{a^2 c^2}{4\lambda} + \frac{a^{10/3} \pi^{4/3} \Omega^{2/3} \zeta(5)}{2^{2/3} \lambda^{5/3} [\zeta(3)]^{5/3}} \quad (\text{III.10})$$

For $m_0 = 0$ (or $a = c$), the first two terms cancel, resulting in an increase in pressure as Ω increases. Moreover, whereas the entropy (dP/dT) is continuous at $T = T_c$, $\left. \frac{dP_{<}}{dT} \right|_{T_c} = \left. \frac{dP_{>}}{dT} \right|_{T_c}$,
(III.11)

the heat capacity ($d^2 P/dT^2$) is discontinuous

$$\left. \frac{d^2 P_{<}}{dT^2} \right|_{T_c} - \left. \frac{d^2 P_{>}}{dT^2} \right|_{T_c} = \frac{9c^{8/3} [\zeta(3)]^{2/3}}{2^{1/3} \pi^{4/3} \lambda^{1/3} \Omega^{2/3}}. \quad (\text{III.12})$$

Hence, according to Ehrenfest classification, this is a second order phase transition. In comparison to the non-rotating case [40], although rotation alters the critical temperature, the order of the phase transition remains unchanged. It is noteworthy that the discontinuity in the heat capacity decreases with increasing Ω .

Plugging at this stage, $v_{\min}^2$ from (III.6) into $m_1^2(v) = 3\lambda v^2 - c^2$ and $m_2^2(v) = \lambda v^2 - c^2$, we arrive at

$$m_1^2(v_{\min}) = \begin{cases} 3a^2(1-t^3) - c^2, & t < 1, \\ c^2, & t \geq 1, \end{cases} \\ m_2^2(v_{\min}) = \begin{cases} a^2(1-t^3) - c^2, & t < 1, \\ -c^2, & t \geq 1. \end{cases} \quad (\text{III.13})$$

Hence, as it turns out, at $t \geq 1$, after the symmetry is restored, m_1^2 and m_2^2 become negative. Contrary to our expectation, for $a = c$, i.e., in the chiral limit $m_0 = 0$, the Goldstone boson '2' acquires a negative mass $-c^2 t^3$ in the symmetry-broken phase at $t < 1$. In what follows, we compute the one-loop tadpole diagram contributions to masses m_1 and m_2 . We show, in particular, that by considering the thermal mass, the one-loop corrected mass of the Goldstone mode '2' vanishes in chiral limit $m_0 = 0$.

B. One-Loop Corrections to $m_1(v)$ and $m_2(v)$

To calculate the one-loop corrections to m_1 and m_2 , let us consider $\mathcal{L}_4$ from (II.4). Three vertices, corresponding to three terms in $\mathcal{L}_4 = -\frac{\lambda}{4}(\varphi_1^2 + \varphi_2^2)^2$, are to be considered in this computation (see Figure 3),

$$-\frac{\lambda}{4} \varphi_1^4 \rightarrow -\frac{\lambda}{4}; \\ -\frac{\lambda}{4} \varphi_2^4 \rightarrow -\frac{\lambda}{4}; \\ -\frac{\lambda}{2} \varphi_1^2 \varphi_2^2 \rightarrow -\frac{\lambda}{2}. \quad (\text{III.14})$$

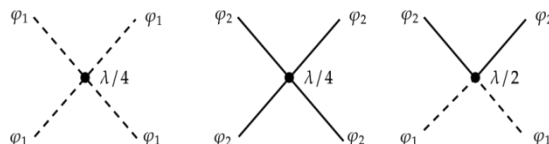


Figure 3. Three vertices arising from L4 from (II.4). Dashed and solid lines correspond to φ_1 and φ_2 fields, respectively.

They lead to two different tadpole contributions to $\langle \Omega | T (\varphi_1(x) \varphi_1(y)) | \Omega \rangle$ and $\langle \Omega | T (\varphi_2(x) \varphi_2(y)) | \Omega \rangle$ that correct m_1 and m_2 perturbatively. They are denoted by Π_{ij} with the first index, $i = 1; 2$, corresponds to whether φ_1 or φ_2 are in the external legs, and the second index $j = 1; 2$ to whether the internal loop is built from φ_1 or φ_2 (see Figure 4, where Π_{ij} are plotted). Hence, according to this notation, the one-loop perturbative corrections to m_1^2 and m_2^2 arise from

$$\begin{aligned} m_1^2(v) &\rightarrow m_1^2(v) + \Pi_{11} + \Pi_{12}; \\ m_2^2(v) &\rightarrow m_2^2(v) + \Pi_{21} + \Pi_{22}. \end{aligned} \quad (\text{III.15})$$

At this stage, we introduce

$$\Pi_i(T, \Omega, m_i) \equiv T \sum_{n=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} \int d\tilde{k} D_{\ell}(\omega_n, \omega_i), \quad (\text{III.16})$$

with free boson propagator

$$D_{\ell}(\omega_n, \omega_i) \equiv \frac{1}{(\omega_n - i\ell\Omega)^2 + \omega_i^2}, \quad (\text{III.17})$$

arising from (II.11) with $\omega = 0$. Here, $\omega_i^2 = \mathbf{k}_{\perp}^2 + k_z^2 + m_i^2$ and $i = 1; 2$. Using this notation, it turns out that

$$\begin{aligned} \Pi_{11} &= 3\Pi_1; \Pi_{12} = \Pi_2; \\ \Pi_{22} &= 3\Pi_2; \Pi_{21} = \Pi_1. \end{aligned} \quad (\text{III.18})$$

Hence, the perturbative corrections of masses are given by

$$\begin{aligned} m_1^2(v) &\rightarrow m_1^2(v) + 3\Pi_1 + \Pi_2; \\ m_2^2(v) &\rightarrow m_2^2(v) + 3\Pi_2 + \Pi_1. \end{aligned} \quad (\text{III.19})$$

To evaluate Π_i from (III.16), we follow the same steps as presented in [30]. The Matsubara summation is evaluated with

$$\sum_n D_{\ell}(\omega_n, \omega_i) = \frac{1}{2T\omega_i} [n_b(\omega_i + \ell\Omega) + n_b(\omega_i - \ell\Omega) + 1], \quad (\text{III.20})$$

where $n_b(\omega) \equiv 1/(e^{\beta\omega} - 1)$ is the BE distribution function. In what follows, we insert (III.20) into (III.16) and focus only on the matter (T and Ω dependent) part of Π_i ,

$$\Pi_i^{\text{mat}} = \frac{1}{2} \sum_{\ell=\pm} \sum_{\omega_i \neq 0} \int d\tilde{k} \frac{n_b(\omega_i + \ell\Omega)}{\omega_i}. \quad (\text{III.21})$$

Having in mind that in $n_b(\omega_i \pm \ell\Omega)$, we must have $e^{\beta(\omega_i \pm \ell\Omega)} - 1 > 0$, it is possible to limit the summation over ℓ . We thus obtain

$$\Pi_i^{\text{mat}} = \sum_{\ell=1}^{\infty} \int d\tilde{k} \frac{n_b(\omega_i + \ell\Omega)}{\omega_i}. \quad (\text{III.22})$$

Let us notice that in the term including $n_b(\omega_i - \ell\Omega)$ an additional shift $\ell \rightarrow -\ell$ is performed. To carry out the summation over ℓ and eventually the integration over $k_{\perp}$ and k_z , we use

$$n_b(\omega_i + \ell\Omega) = T \frac{d}{d\omega_i} \ln(1 - e^{-\beta(\omega_i + \ell\Omega)}), \quad (\text{III.23})$$

and arrive first at

$$\Pi_i^{\text{mat}} = T \sum_{\ell=1}^{\infty} \int d\tilde{k} \frac{1}{\omega_i} \frac{d}{d\omega_i} \ln(1 - e^{-\beta(\omega_i + \ell\Omega)}). \quad (\text{III.24})$$

Using, at this stage, (A.2), we then obtain

$$\begin{aligned} \Pi_i^{\text{mat}} &= -T \sum_{\ell=1}^{\infty} \sum_{j=1}^{\infty} \int d\tilde{k} \frac{1}{\omega_i} \frac{d}{d\omega_i} \frac{e^{-\beta\omega_i j} e^{-\beta\ell\Omega j}}{j} \\ &= \sum_{\ell=1}^{\infty} \sum_{j=1}^{\infty} \int d\tilde{k} \frac{e^{-\beta\omega_i j} e^{-\beta\ell\Omega j}}{\omega_i}. \end{aligned} \quad (\text{III.25})$$

The summation over ℓ can be performed by making use of (A.4). Assuming $\Omega < 1$ and using (A.5), Π_1^{mat} reads

$$\Pi_i^{\text{mat}} = -\frac{1}{\Omega} \sum_{j=1}^{\infty} \frac{1}{j} \int d\tilde{k} \frac{e^{-\beta\omega_i j}}{\omega_i}. \quad (\text{III.26})$$

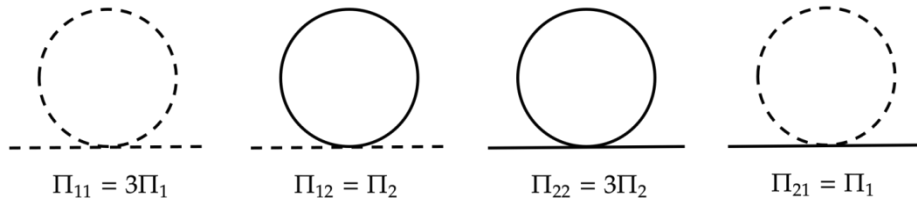


Figure 4. The tadpole diagrams contributing to the one-loop corrections of m_1 and m_2 . Dashed and solid lines correspond to φ_1 and φ_2 fields, respectively.

Following the method presented in Appendix B, we finally arrive at

$$\Pi_i^{\text{mat}} = \frac{\lambda T^3 \zeta(3)}{2\pi^2 \Omega} + \dots \quad (\text{III.27})$$

The first term in (III.27) is analogous to the thermal mass $\lambda T^2/3$ in a nonrotating interacting Bose gas [40] and the ellipsis includes higher order corrections of Π_i^{mat} in βm_i .

At high temperature, it is enough to consider only the first term in (III.27), which is independent of m_i . We thus have

$$\Pi_1^{\text{mat}} = \Pi_2^{\text{mat}} = \frac{\lambda T^3 \zeta(3)}{2\pi^2 \Omega}, \quad (\text{III.28}) \text{ and therefore}$$

$$m_1^2(v) \rightarrow m_1^2(v) + 4\Pi_1^{\text{mat}} = m_1^2(v) + a^2 t^3,$$

$$m_2^2(v) \rightarrow m_2^2(v) + 4\Pi_2^{\text{mat}} = m_2^2(v) + a^2 t^3,$$

$$(\text{III.29})$$

with $t = T/T_c$ and T_c from (III.5).

C. Goldstone Theorem

Let us consider again the result presented in (III.13). Adding the contribution of thermal mass (III.28) to

$m_1^2(v_{\min})$ and $m_2^2(v_{\min})$ according to (III.29), we obtain

$$m_1^2(v_{\min}) = \begin{cases} 2c^2(1-t^3) + 3m_0^2 \left(1 - \frac{2t^3}{3}\right), & t < 1, \\ c^2(t^3 - 1) + m_0^2 t^3, & t \geq 1, \end{cases}$$

$$m_2^2(v_{\min}) = \begin{cases} m_0^2, & t < 1, \\ c^2(t^3 - 1) + m_0^2 t^3, & t \geq 1, \end{cases} \quad (\text{III.30})$$

where $a^2 = c^2 + m_0^2$ is used. Assuming $m_0 = 0$, m_2 vanishes at $t < 1$. This indicates that the Goldstone theorem is valid when the thermal mass corrections to m_1 and m_2 are taken into account. Moreover, we observe that $m_1^2(v_{\min}) = m_2^2(v_{\min})$ in the symmetry-restored phase at $t \geq 1$. In Figure 5, the t dependence of $m_1^2(v_{\min})$ and $m_2^2(v_{\min})$ from (III.30) is plotted. These masses are identified with m_σ^2 and m_π^2 , respectively. We use $c \simeq 0.225$ GeV from (III.7) and $m_0 = 0.140$ GeV, as described in Sec. III B and observe that in the symmetry-broken phase, at $t < 1$, m_σ decreases with increasing temperature, while m_π remains constant. As expected, at symmetry-restored phase at $t \geq 1$, m_σ and m_π are equal and increase with increasing temperature. It is noteworthy that the effect of rotation, apart from affecting the value of the critical temperature T_c from (III.5), consists of changing the power of t in (III.30) from t^2 to t^3 (see [40]).

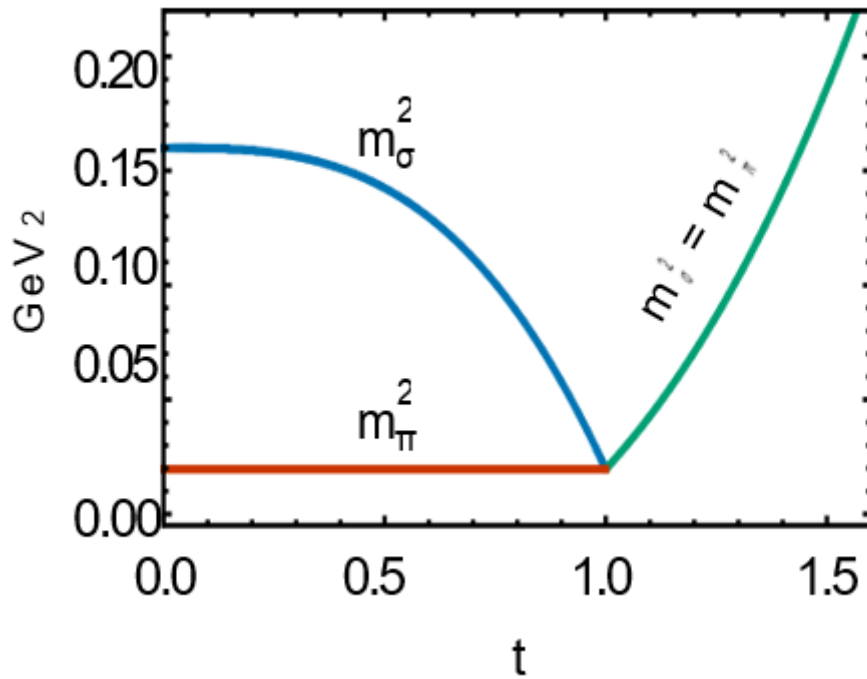


Figure 5. (color online). The t dependence of m_1^2 and m_2^2 from (III.30) at $v_{\min}^2$ from (III.6) is plotted. These masses are identified with σ and π meson masses. In the symmetry-broken phase, at $t < 1$, m_σ decreases with increasing temperature, while m_π remains constant. At symmetry-restored phase at $t \geq 1$, m_σ and m_π are equal and increase with increasing t .

D. Vacuum Potential

In what follows, we compute the contribution of the vacuum part of the thermodynamic potential, V_{vac} from (II.33) to V_{tot} . Let us first consider the summation over $\ell \in (-\infty, +\infty)$ in this expression. This sum is divergent and need an appropriate regularization. To perform the summation over ℓ , we use

$$\begin{aligned} \sum_{\ell=-\infty}^{\infty} 1 &= \lim_{x \rightarrow 0} \sum_{\ell=-\infty}^{\infty} e^{-\ell^2 x} \\ &= \lim_{x \rightarrow 0} \left(1 + 2 \sum_{\ell=1}^{\infty} e^{-\ell^2 x} \right) = 1 + \lim_{x \rightarrow 0} \frac{1}{1 - e^{-x}} \\ &= 1 + \text{divergent term. (III.31)} \end{aligned}$$

Neglecting the divergent term, we obtain

$$\mathcal{V}_{\text{vac}} = \frac{1}{2} \int d\tilde{k} (\omega_1 + \omega_2). \quad (\text{III.32})$$

The above regularization guarantees that rotation does not alter V_{vac} . To perform the integration over $k_\perp$ and k_z , let us consider the integral

$$I(m) \equiv \frac{\bar{\mu}^\epsilon}{2} \int d\tilde{k} (\mathbf{k}^2 + m^2)^{1/2}, \quad (\text{III.33}) \quad \text{with } \epsilon = 3-d. \quad \text{Here, } d \text{ is the dimension of spacetime}$$

and $\bar{\mu}$ denotes an appropriate energy scale. Later, we show that $\bar{\mu}$ can be eliminated from the computation. Utilizing

$$\begin{aligned} \Phi(m, d, n) &= \int \frac{d^d k}{(2\pi)^d} \frac{1}{(\mathbf{k}^2 + m^2)^n} \\ &= \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(n - d/2)}{\Gamma(n)} \frac{1}{(m^2)^{n-d/2}}, \quad (\text{III.34}) \end{aligned}$$

to perform a d dimensional regularization, we obtain for $\Phi(m, 3 - \epsilon, -1/2)$, (In Appendix C, we derive (III.24) in cylinder coordinate system).

$$I(m) = -\frac{m^4}{64\pi^2} \left(\frac{2}{\epsilon} + \frac{3}{2} - \gamma_E - \ln \frac{m^2}{4\pi\bar{\mu}^2} \right). \quad (\text{III.35})$$

The vacuum part of the thermodynamic potential (III.32) is thus given by

$$\begin{aligned} V_{\text{vac}} &= I(m_1) + I(m_2) \\ &= -\frac{(m_1^4 + m_2^4)}{64\pi^2} \left(\frac{2}{\epsilon} + \frac{3}{2} - \gamma_E \right) + \frac{m_1^4}{64\pi^2} \ln \frac{m_1^2}{4\pi\bar{\mu}^2} \\ &\quad + \frac{m_2^4}{64\pi^2} \ln \frac{m_2^2}{4\pi\bar{\mu}^2}. \end{aligned} \quad (\text{III.36})$$

In what follows, we regularize this potential by following

the method presented in [43]. To do this, we first define $V_b \equiv V_{\text{cl}} + V_{\text{vac}} + V_{\text{CT}}$, (III.37)

with V_{cl} from (II.32) with c^2 replaced with $a^2 = c^2 + m_0^2$ and V_{vac} from (III.36). The counterterm potential is given by

$$V_{\text{CT}} = \frac{Av^2}{2} + \frac{Bv^4}{4} + C. \quad (\text{III.38})$$

The coefficients A and B are determined by utilizing two prescriptions

$$\left. \frac{\partial V_b}{\partial v} \right|_{v_0^2} = 0, \quad \left. \frac{\partial^2 V_b}{\partial v^2} \right|_{v_0^2} = m_1^2(v_0). \quad (\text{III.39})$$

Here, v_0 from (III.2) is the classical minimum and $m_1^2(v_0)$ from (III.3). Let us note that the first prescription guarantees that the position of the classical minimum does not change by considering the vacuum part of the potential. The term C in (III.38) includes all terms which are independent of v. Using (III.39), we arrive at

$$\begin{aligned} A &= -\frac{m_0^2}{2} + \frac{3c^2\lambda}{8\pi^2} + \frac{c^2\lambda\gamma_E}{4\pi^2} + \frac{5m_0^2\lambda}{8\pi^2} - \frac{c^2\lambda}{2\pi^2\epsilon} \\ &\quad + \frac{c^2\lambda}{16\pi^2} \ln \left(\frac{m_0^2}{4\pi\bar{\mu}^2} \right) + \frac{3c^2\lambda}{16\pi^2} \ln \left(\frac{2c^2 + 3m_0^2}{4\pi\bar{\mu}^2} \right), \\ B &= \frac{m_0^2\lambda}{2a^2} - \frac{5\lambda^2\gamma_E}{8\pi^2} + \frac{5\lambda^2}{4\pi^2\epsilon} - \frac{\lambda^2}{16\pi^2} \ln \left(\frac{m_0^2}{4\pi\bar{\mu}^2} \right) \\ &\quad - \frac{9\lambda^2}{16\pi^2} \ln \left(\frac{2c^2 + 3m_0^2}{4\pi\bar{\mu}^2} \right). \end{aligned} \quad (\text{III.40})$$

Plugging A and B from (III.40) into V_{CT} from (III.38) and choosing

$$\begin{aligned} C &= \frac{c^4}{16\pi^2\epsilon} - \frac{c^4}{64\pi^2} \ln \left(\frac{m_0^2}{4\pi\bar{\mu}^2} \right) - \frac{c^4}{64\pi^2} \ln \left(\frac{2c^2 + 3m_0^2}{4\pi\bar{\mu}^2} \right) \\ &\quad + \frac{3c^4}{64\pi^2} - \frac{c^4\gamma_E}{32\pi^2}, \end{aligned} \quad (\text{III.41})$$

the counterterm potential from (III.38) is determined. These counterterms eliminate the divergent terms in the vacuum potential, as expected. The total potential V_b from (III.37) is thus given by

$$\begin{aligned} V_b &= -\frac{a^2v^2}{2} + \frac{\lambda v^4}{4} - \frac{m_0^2v^2}{4} + \frac{3c^2\lambda v^2}{8\pi^2} + \frac{5m_0^2\lambda v^2}{16\pi^2} \\ &\quad - \frac{15\lambda^2v^4}{64\pi^2} + \frac{m_0^2\lambda v^4}{8a^2} \\ &\quad + \frac{m_1^4}{64\pi^2} \ln \left(\frac{m_1^2}{2c^2 + 3m_0^2} \right) + \frac{m_2^4}{64\pi^2} \ln \left(\frac{m_2^2}{m_0^2} \right). \end{aligned} \quad (\text{III.42})$$

As mentioned earlier, the energy scale $\bar{\mu}$ does not appear in the final expression of V_b . Additionally, a nonzero m_0 is necessary to specifically regularize the last term in V_b from (III.42).

E. Ring Potential

We finally consider the nonperturbative ring potential V_{ring} . As mentioned in the previous paragraphs, the Lagrangian is written in terms of ϕ_1 and ϕ_2 , three type of vertices appear in the $\lambda(\phi^\dagger\phi)$ model (see Figure 3). We thus have four different types of ring diagrams:

- Type A: A ring with N insertions of Π_2 and N propagators $D_\ell(\omega_n, \omega_1)$ propagators, V_{ring}^A ,
 - Type B: A ring with N insertions of Π_1 and N propagators $D_\ell(\omega_n, \omega_2)$ propagators, V_{ring}^B ,
 - Type C: A ring with r insertions of Π_2 and s insertions of Π_1 with N propagators $D_\ell(\omega_n, \omega_2)$, V_{ring}^C .
- Here, $r \geq 1$ and $r + s = N$.
- Type D: A ring with r insertions of Π_1 and s insertions of Π_2 with N propagators $D_\ell(\omega_n, \omega_1)$, V_{ring}^D .

Similar to the previous case, $r \geq 1$ and $r + s = N$.

Here, $\Pi_i(T, \Omega, m_i)$ and $D_\ell(\omega_n, \omega_i)$, $i = 1, 2$ are defined in (III.16) and (III.17), respectively. In Figure 6, these different types of ring potentials are demonstrated. The full contribution of the ring potential is given by

$$V_{\text{ring}} = \sum_{I = \{A, \dots, D\}} V_{\text{ring}}^I. \quad (\text{III.43})$$

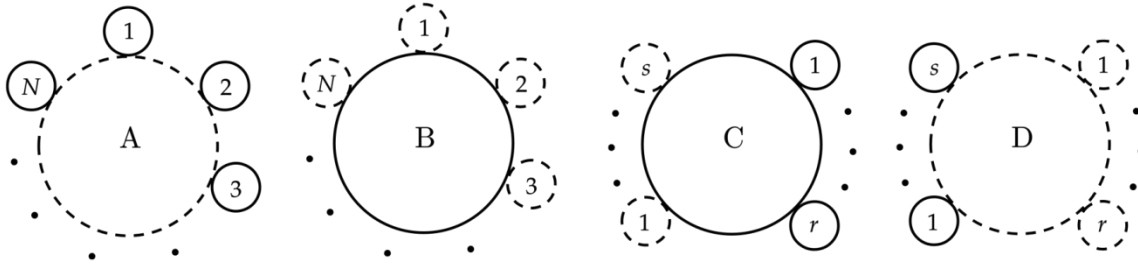


Figure 6. Ring diagrams of Type A, B, C, and D contributing to the nonperturbative ring potential V_{ring} . Dashed and solid lines correspond to φ_1 and φ_2 , respectively. They are given by the expressions from (III.44).

Following standard field theoretical method, it is possible to determine the combinatorial factors leading to the standard form of the ring potential [40]. In Appendix D, we outline the derivation of V_{ring}^I , $I = A, \dots, D$. They

are given by

$$\begin{aligned} V_{\text{ring}}^A &= -\frac{T}{2} \sum_{n, \ell} \int d\tilde{k} \sum_{N=2}^{\infty} \frac{1}{N} (-\Pi_2 D_1)^N, \\ V_{\text{ring}}^B &= -\frac{T}{2} \sum_{n, \ell} \int d\tilde{k} \sum_{N=2}^{\infty} \frac{1}{N} (-\Pi_1 D_2)^N, \\ V_{\text{ring}}^C &= -\frac{T}{2} \sum_{n, \ell} \int d\tilde{k} \sum_{N=2}^{\infty} \sum_{r=1}^N \frac{(N-r)!(r-1)!}{N!} \\ &\times [(-\Pi_2)^r (-\Pi_1)^{N-r} D_2^N], \\ V_{\text{ring}}^D &= -\frac{T}{2} \sum_{n, \ell} \int d\tilde{k} \sum_{N=2}^{\infty} \sum_{r=1}^N \frac{(N-r)!(r-1)!}{N!} \\ &\times [(-\Pi_1)^r (-\Pi_2)^{N-r} D_1^N]. \end{aligned} \quad (\text{III.44})$$

Here, the notation $D_i \equiv D_\ell(\omega_n, \omega_i)$ is used. To evaluate

V_{ring}^A and V_{ring}^B , we introduce a simplifying notation

$$V_{\text{ring}}^{(i,j)} = -\frac{T}{2} \sum_{n, \ell} \int d\tilde{k} \sum_{N=2}^{\infty} \frac{1}{N} (-\Pi_i D_j)^N. \quad (\text{III.45})$$

Here, $(i, j) = (2, 1)$ and $(i, j) = (1, 2)$ correspond to V_{ring}^A and V_{ring}^B , respectively. Plugging D_j from (III.17) into (III.45) and focusing on $n = 0$ as well as $\ell \neq 0$ contributions in the summation over n and ℓ , we arrive first at

$$\mathcal{V}_{\text{ring}}^{(i,j)} = T \sum_{\ell=1}^{\infty} \int d\tilde{k} \sum_{N=2}^{\infty} \frac{(-1)^{N+1}}{N} (u_j^2)^{-N} (\Pi_i)^N, \quad (\text{III.46})$$

with $u_j^2 \equiv \mathbf{k}_{\perp}^2 + k_z^2 + m_j^2 - \ell^2 \Omega^2$. Plugging then

$$(u_j^2)^{-N} = \frac{1}{\Gamma(N)} \int_0^{\infty} dt t^{N-1} e^{-m_j^2 t} e^{-(\mathbf{k}_{\perp}^2 + k_z^2)t} e^{\ell^2 \Omega^2 t}, \quad (\text{III.47})$$

into (III.46), the integration over $\mathbf{k}_{\perp}$ and k_z can be carried out by making use of (A.9). To limit the summation over ℓ from below, we use the fact that the summand is even in ℓ . To perform the integration over $\mathbf{k}_{\perp}$ and k_z , we use the Mellin transformation of $(u_j^2)^{-N}$,

$$(u_j^2)^{-N} = \frac{1}{\Gamma(N)} \int_0^{\infty} dt t^{N-1} e^{-m_j^2 t} e^{-(\mathbf{k}_{\perp}^2 + k_z^2)t} e^{\ell^2 \Omega^2 t}, \quad (\text{III.48})$$

and (A.9) to arrive first at

$$\mathcal{V}_{\text{ring}}^{(i,j)} = \frac{T}{8\pi^{3/2}} \sum_{N=2}^{\infty} \frac{(-1)^{N+1} \Pi_i^N}{N \Gamma(N)} \int_0^{\infty} dt t^{N-5/2} e^{-m_j^2 t} I(\Omega), \quad (\text{III.49})$$

where

$$I(\Omega) \equiv \sum_{\ell=1}^{\infty} e^{\ell^2 \Omega^2 t}. \quad (\text{III.50})$$

To evaluate the summation over ℓ , we expand $e^{\ell^2 \Omega^2 t}$ in a Taylor

expansion and obtain

$$I(\Omega) = \sum_{r=0}^{\infty} \frac{(\Omega^2 t)^r}{r!} \zeta(-2r), \quad (\text{III.51}) \quad \text{with} \quad \sum_{\ell=1}^{\infty} \ell^{2r} = \zeta(-2r) \quad \text{and} \quad \zeta(z) \quad \text{the Riemann } \zeta\text{-}$$

function. Since for $r \in \mathbb{N}$, we have $\zeta(-2r) = 0$, the only nonvanishing contribution to the summation over r arises from $r = 0$. We thus use $\zeta(0) = -\frac{1}{2}$ to arrive at

$$I(\Omega) = -\frac{1}{2}. \quad (\text{III.52})$$

Plugging this result into (III.49), using

$$\int_0^{\infty} dt t^{N-5/2} e^{-m_j^2 t} = (m_j^2)^{-j+3/2} \Gamma(j-3/2), \quad (\text{III.53})$$

and performing the summation over N , we arrive at

$$\mathcal{V}_{\text{ring}}^{(i,j)} = \frac{T}{24\pi} \left(2 (m_j^2 + \Pi_i)^{3/2} - 2m_j^3 - 3m_j \Pi_i \right). \quad (\text{III.54})$$

We arrive eventually at

$$V_{\text{ring}}^{\text{A}} = V_{\text{ring}}^{(2,1)}, \quad V_{\text{ring}}^{\text{B}} = V_{\text{ring}}^{(1,2)}. \quad (\text{III.55})$$

To evaluate $V_{\text{ring}}^{\text{C}}$ and $V_{\text{ring}}^{\text{D}}$, we introduce

$$V_{\text{ring}}^{(i,j)} = -\frac{T}{2} \sum_{n,\ell} \int d\tilde{k} \sum_{N=2}^{\infty} \sum_{r=1}^N \frac{(-1)^N (N-r)!(r-1)!}{N!} \\ \times \Pi_i^r \Pi_j^{N-r} D_i^N. \quad (\text{III.56})$$

Here, $(i,j) = (2,1)$ corresponds to $V_{\text{ring}}^{\text{C}}$ and $(i,j) = (1,2)$ to $V_{\text{ring}}^{\text{D}}$. Plugging D_i from (III.17) into (III.56) and focusing on $n = 0$ and $\ell \neq 0$ contributions in the summation over n and ℓ , we obtain

$$V_{\text{ring}}^{(i,j)} = T \sum_{\ell=1}^{\infty} \int d\tilde{k} \sum_{N=2}^{\infty} \sum_{r=1}^N \frac{(-1)^N (N-r)!(r-1)!}{N!} \\ \times \Pi_i^r \Pi_j^{N-r} (u_i^2)^{-N}, \quad (\text{III.57})$$

where u_j^2 is defined below (III.46). Following, at this stage, the same steps as described in previous paragraph, we arrive first at

$$V_{\text{ring}}^{(i,j)} = \frac{m_i^3 T}{16\pi^{3/2}} \sum_{N=2}^{\infty} \sum_{r=1}^N \frac{(-1)^N}{N!} \frac{(N-r)!(r-1)!}{\Gamma(N)} \times \Gamma(N-3/2) \Pi_i \Pi_j^{N-r} (m_i^2)^{-N}. \quad (\text{III.58})$$

To perform the summation over N and r , we use the relation

$$\sum_{N=2}^{\infty} \sum_{r=1}^N f(N, r) = \sum_{N=2}^{\infty} f(N, N) + \sum_{r=1}^{\infty} \sum_{N=r+1}^{\infty} f(N, r). \quad (\text{III.59})$$

We thus obtain

$$V_{\text{ring}}^{(i,j)} = V^{(i)} + V^{(i,j)}, \quad (\text{III.60})$$

with

$$V^{(i)} \equiv \frac{m_i^3 T}{16\pi^{3/2}} \sum_{N=2}^{\infty} \frac{(-1)^N}{N} \frac{\Gamma(N-3/2) \Pi_i^N (m_i^2)^{-N}}{\Gamma(N)}$$

$$V^{(i,j)} \equiv \frac{m_i^3 T}{16\pi^{3/2}} \sum_{r=1}^{\infty} \sum_{N=r+1}^{\infty} \frac{(-1)^N}{N!} \frac{(N-r)!(r-1)!}{\Gamma(N)} \times \Gamma(N-3/2) \Pi_i \Pi_j^{N-r} (m_i^2)^{-N}. \quad (\text{III.61})$$

For $V^{(i)}$, the summation over N can be carried out and yields

$$V^{(i)} = \frac{T}{24} \left(2(m_i^2 + \Pi_i)^{3/2} - 2m_i^3 - 3m_i \Pi_i \right). \quad (\text{III.62})$$

As concerns $V^{(i,j)}$, we perform the summation over N and arrive at

$$V^{(i,j)} = \sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{r} \frac{\Gamma(r-1/2)}{\Gamma(r+2)} \Pi_i^r \Pi_j (m_i^2)^{-r-1} \times {}_3F_2 \left((1, 2, r-1/2); (r+1, r+2); -\frac{\Pi_j}{m_i^2} \right), \quad (\text{III.63})$$

where ${}_pF_q(\mathbf{a}; \mathbf{b}; z)$ is the generalized hypergeometric function having the following series expansion

$${}_pF_q(\mathbf{a}; \mathbf{b}; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k \cdots (a_p)_k}{(b_1)_k \cdots (b_q)_k} \frac{z^k}{k!}. \quad (\text{III.64})$$

Here, $\mathbf{a} = (a_1, \dots, a_p)$, $\mathbf{b} = (b_1, \dots, b_q)$ are vectors with p and q components. Moreover, $(a)_k \equiv \Gamma(a+k)/\Gamma(a)$ is the Pochhammer symbol. For our purposes, it is sufficient to focus on the contribution at $r=1$ in (III.63).

$$V^{(i,j)}|_{r=1} = \frac{T}{24\pi} \left(2(m_i^2 + \Pi_j)^{3/2} - 2m_i^3 - 3m_i \Pi_j \right) \frac{\Pi_i}{\Pi_j}. \quad (\text{III.65})$$

Having in mind that the one-loop contribution to the self-energy Π_i , which is determined in Sec. III B is of order $O(\lambda)$, the contributions corresponding to $r \geq 2$ are of order $O(\lambda^2)$ and can be neglected at this stage. We thus have

$$V_{\text{ring}}^C = V_{\text{ring}}^{(2,1)} = V^{(2)} + V^{(2,1)}|_{r=1} + O(\lambda^2), \quad V_{\text{ring}}^D = V_{\text{ring}}^{(1,2)} = V^{(1)} + V^{(1,2)}|_{r=1} + O(\lambda^2). \quad (\text{III.66})$$

The final result for V_{ring} is given by plugging $V_{\text{ring}, I}^C, I = A, \dots, D$ from (III.55) and (III.65) into (III.43),

$$V_{\text{ring}} = \frac{T}{24\pi} \left\{ \left(2(m_1^2 + \Pi_2)^{3/2} - 2m_1^3 - 3m_1 \Pi_2 \right) + \left(2(m_2^2 + \Pi_1)^{3/2} - 2m_2^3 - 3m_2 \Pi_1 \right) + \left(2(m_2^2 + \Pi_2)^{3/2} - 2m_2^3 - 3m_2 \Pi_2 \right) + \left(2(m_1^2 + \Pi_1)^{3/2} - 2m_1^3 - 3m_1 \Pi_1 \right) + \left(2(m_2^2 + \Pi_1)^{3/2} - 2m_2^3 - 3m_2 \Pi_1 \right) \frac{\Pi_2}{\Pi_1} \right\}$$

$$+ \left(2(m_1^2 + \Pi_2)^{3/2} - 2m_1^3 - 3m_1\Pi_2 \right) \frac{\Pi_1}{\Pi_2} \Big\} \\ + \mathcal{O}(\lambda^2). \quad (\text{III.67})$$

Focusing only on the first perturbative correction to Π_i and using $\Pi_i^{\text{mat}}, i = 1, 2$ from (III.28), the above results is simplified as

$$\mathcal{V}_{\text{ring}} \approx \frac{T}{8\pi} \sum_{i=1}^2 \left(2(m_i^2 + \Pi^{\text{mat}})^{3/2} - 2m_i^3 - 3m_i\Pi^{\text{mat}} \right), \quad (\text{III.68})$$

where $\Pi^{\text{mat}} \equiv \Pi_1^{\text{mat}} = \Pi_2^{\text{mat}} = \frac{\lambda T^3 \zeta(3)}{2\pi^2 \Omega}$.

F. Summary of Analytical Results in Sec. III

In this section, we summarize the main findings. According to these results, the total thermodynamic potential of a rigidly rotating Bose gas, V_{tot} , including the classical potential V_{cl} from (II.32) with c^2 replaced with a^2 , the vacuum potential (II.33), the thermal part (II.34), and the ring potential (III.43) is given by

$V_{\text{tot}} = V_{\text{cl}} + V_{\text{vac}} + V_T + V_{\text{ring}}$, (III.69) with

$$\begin{aligned} V_{\text{cl}} &= -\frac{a^2 v^2}{2} + \frac{\lambda v^4}{4}, \\ V_{\text{vac}} &\approx -\frac{m_0^2 v^2}{4} + \frac{3c^2 \lambda v^2}{8\pi^2} + \frac{5m_0^2 \lambda v^2}{16\pi^2} - \frac{15\lambda^2 v^4}{64\pi^2} + \frac{m_0^2 \lambda v^4}{8a^2}, \\ V_T &\approx -\frac{2T^5 \zeta(5)}{\pi^2 \Omega} + \frac{\lambda T^3 v^2 \zeta(3)}{\pi^2 \Omega} - \frac{c^2 T^3 \zeta(3)}{2\pi^2 \Omega}, \\ V_{\text{ring}} &\approx +\frac{T}{8\pi} \sum_{i=1}^2 \left(2(m_i^2 + \Pi^{\text{mat}})^{3/2} - 2m_i^3 - 3m_i \Pi^{\text{mat}} \right). \end{aligned} \quad (\text{III.70})$$

Here, $a^2 = c^2 + m_0^2$, $m_1^2(v) = 3\lambda v^2 - c^2$ and $m_2^2(v) = \lambda v^2 - c^2$, and $\Pi^{\text{mat}} = \lambda T^3 \zeta(3)/2\pi^2 \Omega$. We notice that the logarithmic terms appearing in V_{vac} from (III.42) are skipped in (III.70).

In the next section, we study the effect of rotation on the formation of condensate and the critical temperature of the global U(1) phase transition. To this purpose, we compare our results with the results arising from the full thermodynamic potential of a nonrotating Bose gas. According to [40], it is given by (Subscripts (0) correspond to $\Omega = 0$.)

$V_{\text{tot}}^{(0)} = V_{\text{cl}} + V_{\text{vac}} + V_T^{(0)} + V_{\text{ring}}^{(0)}$, (III.71) where V_{cl} and V_{vac} are given in (III.70), while $V_T^{(0)}$ and $V_{\text{ring}}^{(0)}$ read

$$V_T^{(0)} \approx -\frac{\pi^2 T^4}{45} + \frac{\lambda T^2 v^2}{6} - \frac{c^2 T^2}{12}, \quad (\text{III.72})$$

and

$$V_{\text{ring}}^{(0)} \approx -\frac{T}{4\pi} \sum_{i=1}^2 \left(2(m_i^2 + \Pi_0)^{3/2} - 2m_i^3 - 3m_i \Pi_0 \right), \quad (\text{III.73})$$

with the one-loop self-energy correction $\Pi_0^{\text{mat}} = \lambda T^2/3$ [40] and $m_i^2, i = 1, 2$ given as above.

IV. Numerical Results

In this section, we explore the effect of rotation on different quantities related to the spontaneous breaking

of global U(1) symmetry. To this purpose, we consider different parts of V_{tot} from (III.69).

In Sec. III A, we derived the minimum of the potential

V_a including V_{cl} and V_T . We arrived at $v_{\text{min}}^2(T, \Omega)$ from

(III.6). Replacing V_T with $V_T^{(0)}$ from (III.72) for a nonrotating Bose gas and following the same steps leading from (III.4) to (III.6), we arrive at the critical temperature

$$T_c^{(0)} = \left(\frac{3a^2}{\lambda} \right)^{1/2}, \quad (\text{IV.1})$$

and the T dependent minima

$$v_{\text{min}}^2(T) = \begin{cases} \frac{a^2}{\lambda} (1 - t^2) & t < 1 \\ 0, & t \geq 1, \end{cases} \quad (\text{IV.2})$$

with the reduced temperature $t_0 = T/T_c^{(0)}$ and $T_c^{(0)}$ from (IV.1). In Figure 7, v_{min}^2 is plotted for $\Omega = 0$ [see (IV.2)] and $\Omega \neq 0$ [see (III.6)] as function of the corresponding reduced temperature t_0 and t . The difference between these two plots arises mainly from different exponents of the corresponding reduced temperatures t_0 and t in (IV.2) and (III.6). The reason of this difference lies in different results for the high-temperature expansion of $V_T^{(0)}$ for $\Omega = 0$ [see (III.72)] and V_T for $\Omega \neq 0$ [see (III.70)].

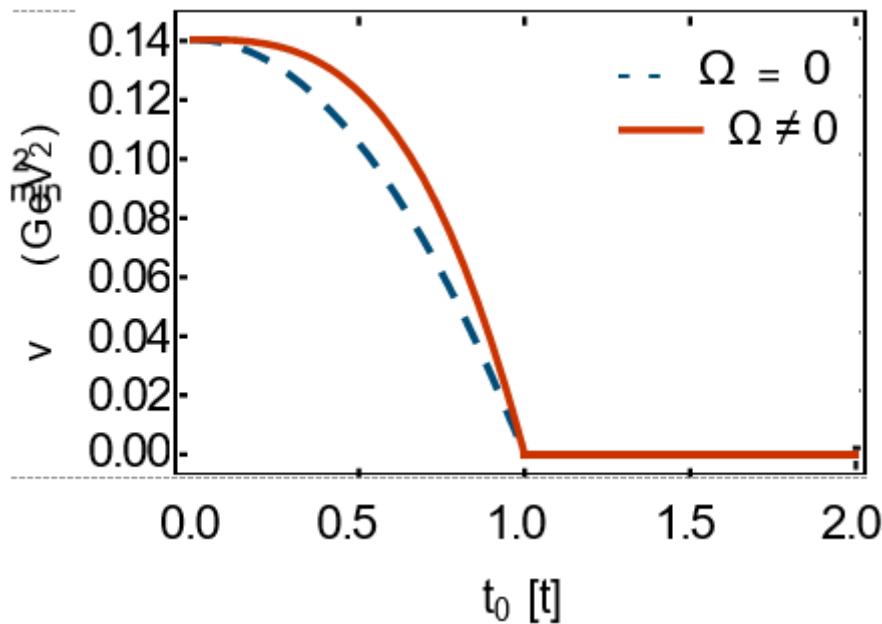


Figure 7. (color online). The t_0 [t] dependence of $v_{\text{min}}^2(T)$ and $v_{\text{min}}^2(T, \Omega)$ for nonrotating ($\Omega = 0$) and rotating ($\Omega \neq 0$) Bose gas [see (III.6) and (IV.2)]. For $\Omega = 0$ and $\Omega \neq 0$, the reduced temperature t_0 or t is defined by $t_0 = T/T_c^{(0)}$ and $t = T/T_c$, respectively.

Let us consider $V_{\text{tot}} - V_{\text{ring}} = V_{\text{cl}} + V_{\text{vac}} + V_T$ from (III.69). By minimizing this potential with respect to v , and solving the resulting gap equation,

$$\frac{d}{dv} (V_{\text{tot}} - V_{\text{ring}}) \Big|_{\bar{v}_{\min}} = 0, \quad (\text{IV.3})$$

it is possible to determine numerically the T dependence the minima, denoted by $v_{\min}(T, \Omega)$, for fixed Ω . To this purpose, we use the quantities $a \simeq 0.265$ GeV, $c \simeq 0.225$ GeV, and $\lambda = 0.5$ given in (III.7). In Figure 8, the $T/T_c^{(0)}$ dependence of $\bar{v}_{\min}$ is demonstrated for $\beta\Omega = 0.1, 0.2, 0.3$

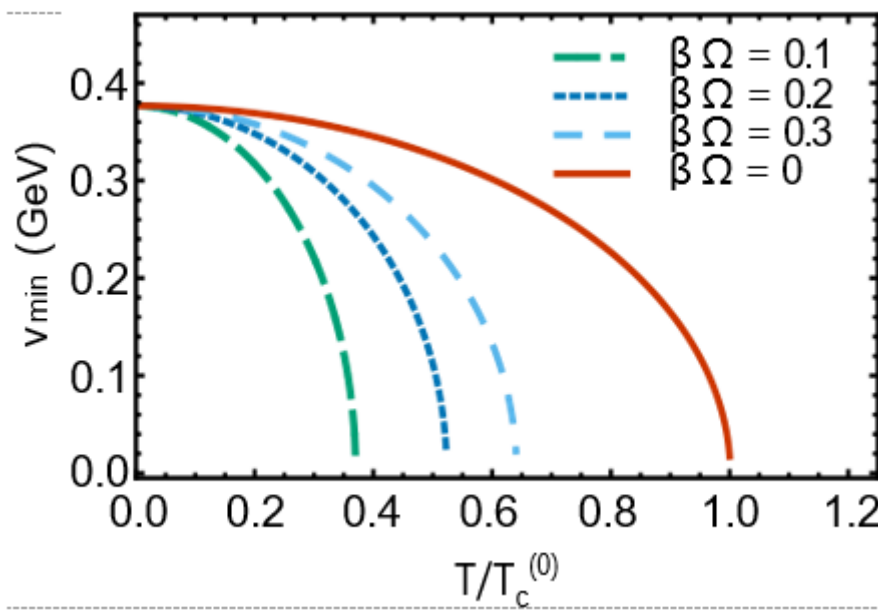


Figure 8. (color online). The $T/T_c^{(0)}$ dependence of $\bar{v}_{\min}$ is plotted for $\beta\Omega = 0, 0.1, 0.2, 0.3$. For $\Omega \neq 0$ and $\Omega = 0$, $v_{\min}(T)$ arises by solving the gap equation (IV.3) and (IV.4), respectively. The temperature T is rescaled with $T_c^{(0)} = 0.681$ GeV, the Ω independent critical temperature of a nonrotating Bose gas. It turns out that $T_c < T_c^{(0)}$ and for $\beta\Omega \neq 0$, T_c increases by increasing $\beta\Omega$.

(dashed, dotted, and dotted-dashed curves). The results are then compared with the corresponding minima for a nonrotating Bose gas (red solid curve). The latter is determined by minimizing the combination $V_{\text{tot}}^{(0)} - V_{\text{ring}}^{(0)}$, according to

$$\frac{d}{dv} (V_{\text{tot}}^{(0)} - V_{\text{ring}}^{(0)}) \Big|_{\bar{v}_{\min}} = 0, \quad (\text{IV.4})$$

with $V_{\text{tot}}^{(0)}$ from (III.71). In both cases, $T_c^{(0)} \simeq 0.681$ GeV is the critical temperature of the spontaneous $U(1)$ symmetry breaking in a nonrotating Bose gas. (The critical temperature is the temperature at which the condensate $\bar{v}_{\min}$ vanishes.)

These results indicate that rotation lowers the critical temperature of the phase transition. However, as shown in Figure 8, T_c increases with increasing Ω . It is also important to note that this same trend is observed in a noninteracting Bose gas under rigid rotation [30].

To answer the question whether the transition is continuous or discontinuous, we have to explore the shape of the potential, the value of its first and second order derivatives at temperatures below and above the critical temperature, T_c . Using the numerical values for the set of free parameters a , c , and λ as mentioned above, the transitions turns out to be continuous not only for $\Omega = 0$ but also for $\Omega \neq 0$.

To explore the effect of the ring potential on the temperature dependence of the condensate $v_{\min}$, we solved numerically the gap equation

$$\frac{dV_{\text{tot}}}{dv} \Big|_{\bar{v}_{\min}} = 0, \quad (\text{IV.5})$$

and

$$\frac{d^2V_{\text{tot}}}{dv^2} \Big|_{\bar{v}_{\min}} = 0, \quad (\text{IV.6})$$

for a rotating and a nonrotating Bose gas, respectively. The corresponding results are demonstrated in Figure 9. Because of the specific form of the ring potentials V_{ring} and $V_{\text{ring}}^{(0)}$ from (III.70) and (III.73), including in particular $(m_l^2 + \Pi^{\text{mat}})^{3/2}$, there is a certain value of v below which the potential is undefined (imaginary). Let us denote this value by $\bar{v}_*$. In both rotating and non-rotating cases $v_* \simeq 0.319$ GeV. As it is shown in Figure 9, the minima decrease with increasing temperature and converge towards $\bar{v}_*$. Let us denote the temperature at which $\bar{v}_{\min} = v_*$ with T_* for $\Omega \neq 0$ and $T_*^{(0)}$ for $\Omega = 0$. For $\Omega = 0$, $T_*^{(0)} \simeq 0.300$ GeV, and as it is shown in Figure 9, the transition to v_* is discontinuous (red circles). For $\Omega \neq 0$, however, $T_* < T_*^{(0)}$ and increases with increasing $\beta\Omega$, similar to the results presented in Figure 8. Moreover, in contrast to the case of $\Omega = 0$, the transition to $\bar{v}_*$ for all values of $\beta\Omega \neq 0$ is continuous.

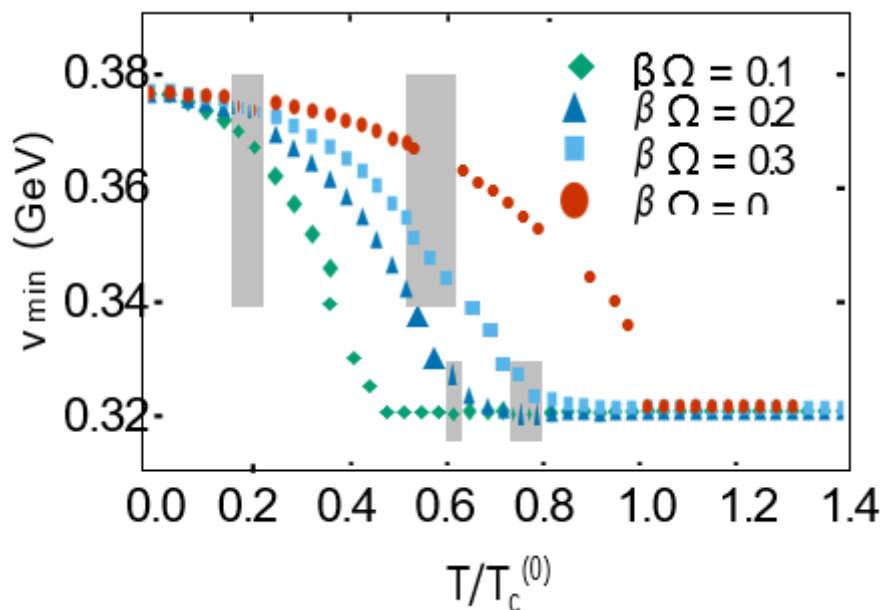


Figure 9. (color online). The $T/T_c^{(0)}$ dependence of $\bar{v}_{\min}$ is plotted for $\beta\Omega = 0, 0.1, 0.2, 0.3$. For $\Omega \neq 0$ and $\Omega = 0$, $v_{\min}(T)$ arises by solving the gap equation (IV.5) and (IV.6), respectively. Here, $\bar{v}_* = 0.319$ GeV and $T_*^{(0)} = 0.300$ GeV. It turns out that $T_* < T_*^{(0)}$ and for $\beta\Omega \neq 0$, T_* increases by increasing $\beta\Omega$.

In Figure 10, the phase diagram T_c - Ω is plotted for two cases: The blue solid curve demonstrates T_c from (III.5) arising from $V_{cl} + V_T$. Red dots denote the Ω dependence of T_c arising from the potential $V_{tot} - V_{ring}$. A comparison between these data reveals the effect of V_{vac} in increasing T_c . Apart from the Ω dependence of T_c , the Ω dependence of $T_\star$ is demonstrated in Figure 10. It arises by adding the ring contribution to $V_{cl} + V_T + V_{vac}$, as described above. According to the results demonstrated in Figure 10, considering V_{ring} decreases T_c . But, similar to T_c , $T_\star$ also increases with increasing Ω . It should be emphasized that the transition shown in Figure 8 is a crossover, since $\bar{v}_\star \neq 0$.

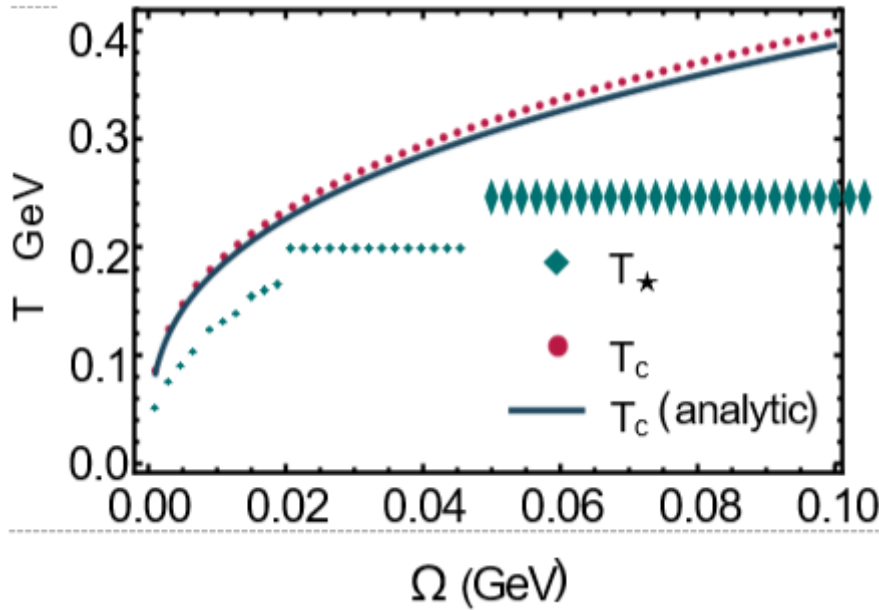


Figure 10. (color online). The Ω dependence of the transition temperatures is plotted. The blue solid line is the transition temperature $T_c \propto \Omega^{1/3}$ from (III.5). It arises from $V_{cl} + V_T$, as described in Sec. III A. Red dots correspond to the critical temperatures T_c , arising from $V_{tot} - V_{ring}$. Green diamonds denote $T_\star$, arising from V_{tot} .

In Sec. III B, the masses m_i^2 , $i = 1, 2$ including the one-loop correction are determined [see (III.29)]. Identifying m_1^2 with m_ϕ^2 and m_2^2 with m_π^2 , we arrive at

$$\begin{aligned} m_\pi^2(v) &= 3\lambda v^2 - c^2 + a^2 t^3, \\ m_\phi^2(v) &= \lambda v^2 - c^2 + a^2 t^3. \end{aligned} \quad (IV.7)$$

Using the data for $\bar{v}_{min}^2$ arising from the solution of the gap equation (IV.3) and (IV.5), and evaluating $m_\phi^2(v^2)$ and $m_\pi^2(v^2)$ from (IV.7) at $\bar{v}_{min}^2$ for a fixed $\beta\Omega$, the $t = T/T_c$ dependence of m_ϕ^2 and m_π^2 is determined. In Figure 11(a), the dependence of $m_\phi^2(\bar{v}_{min}^2)$ and $m_\pi^2(\bar{v}_{min}^2)$ with v_{min} arising from (IV.3) on the reduced temperature $t = T/T_c$ is plotted for fixed $\beta\Omega = 0.1$. Here, the contribution of the ring potential is not taken into account. Hence, a continuous phase transition occurs with the critical temperature $T_c \sim 0.399$ GeV for $\Omega = 0.1$ GeV. In contrast, in Figure 11(b), m_ϕ^2 and m_π^2 are determined by plugging the data of $\bar{v}_{min}$ arising from (IV.5), with V_{tot} including the ring potential. Hence, the difference between the plots demonstrated in Figs. 11(a) and 11(b) arises from the contribution of the nonperturbative ring potential. As we have mentioned above, when the ring potential is taken into account, the data demonstrated in Figure 9 do not describe a true transition, since $\bar{v}_\star$ is not zero. The reduced temperature in

Figure 11(b) is thus defined by $t_\star \equiv T/T_\star$, where, according to the data presented in Figure 10 $T_\star \sim 0.278$ GeV for $\Omega = 0.1$.

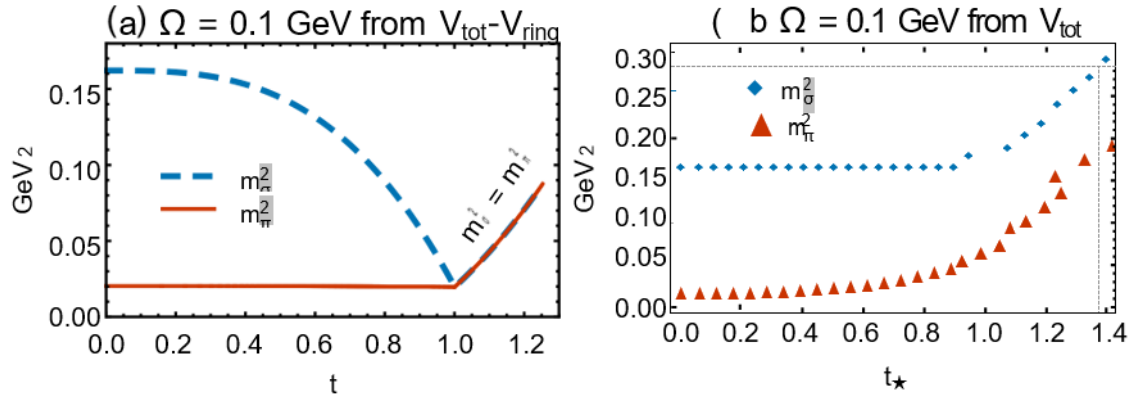


Figure 11. (color online). (Panel a) The $t = T/T_c$ dependence of $m_\sigma^2(v\text{-min})$ and $m_\pi^2(v\text{-min})$ from (IV.7) is plotted for $\Omega = 0.1$ GeV. The data of $\bar{v}_{\text{min}}$ arise by solving the gap equation (IV.3) corresponding to $V_{\text{tot}} - V_{\text{ring}}$. The critical temperature T_c for $\Omega = 0.1$ GeV is $T_c \sim 0.399$ GeV. As expected from the case of nonrotating Bose gas, in the symmetry-restored phase at $t \geq 1$, $m_\sigma^2 = m_\pi^2$. (Panel b) The $t_\star$ dependence of $m_\sigma^2(v\text{-*})$ and $m_\pi^2(v\text{-*})$ from (IV.7) is plotted for $\Omega = 0.1$ GeV. The data of $\bar{v}_{\text{min}}$ arise by solving the gap equation (IV.5), corresponding to V_{tot} which includes the nonperturbative ring potential. According to Figure 10, for $\Omega = 0.1$ GeV, we have $T_\star \sim 0.278$ GeV. At $t \geq 1$, $m_\sigma^2 - m_\pi^2 = 2\lambda\bar{v}^*$, with $\bar{v}^* \simeq 0.319$ GeV from Figure 9 and $\lambda = 0.5$.

Let us compare the results demonstrated in Figure 11(a) with that in Figure 5. In both cases, before the phase transition at $t < 1$, m_σ^2 decreases with increasing t . Moreover, whereas in Figure 5, m_π^2 remains constant, it slightly decreases once the V_{vac} contribution is taken into account.

After the transition, at $t \geq 1$, m_σ^2 becomes equal to m_π^2 and they both increase with increasing t . It is straight-forward to verify this statement using equation (IV.7). Given that, in this case, the minima of the potential at $t \geq 1$ are zero, it follows that both masses are equal, specifically $m_\sigma^2(0) = m_\pi^2(0)$, once we substitute $\bar{v}_{\text{min}} = 0$ into (IV.7).

This behavior is expected from the case of $\Omega = 0$ and in the framework of fermionic Nambu-Jona-Lasinio (NJL) model: As noted in [45], in the symmetry-broken phase, $m_\sigma^2 > m_\pi^2$. As the transition temperature is approached, m_σ^2 decreases, and at a certain dissociation temperature T_{diss} , the masses m_σ and m_π become de-generate. This temperature is characterized by $m_\sigma(T_{\text{diss}}) = 2m_\pi(T_{\text{diss}})$. (IV.8)

As it is described in [45], σ mesons dissociates into two pions because of the appearance of an s-channel pole in the scattering amplitude $\pi + \pi \rightarrow \pi + \pi$. In this process a σ meson is coupled to two pions via a quark triangle. In the symmetry-restored phase, at $t \geq 1$, m_σ becomes equal to m_π . They both increase with increasing T [45,46].

In Table 1, the σ dissociation temperatures are listed for $\Omega = 0, 0.1, 0.2, 0.3$ GeV. The data in the second (third) column correspond to T_{diss} (T_{diss}^*) for the case when $\bar{v}_{\text{min}}$ is the solution of (IV.3) [(IV.5)] for $\Omega \neq 0$ and (IV.4) [(IV.6)] for $\Omega = 0$. Comparing T_{diss} and T_{diss}^* with T_c and $T_\star$ shows that $T_{\text{diss}} < T_c$ and similarly $T_{\text{diss}}^* < T_\star$. The property $T_{\text{diss}} \neq T_c$ is because we are working with $m_\pi \neq 0$. Let us notice that, as aforementioned, the σ dissociation temperature is originally introduced in a fermionic NJL model [45]. In this model, nonvanishing m_π indicates a nonvanishing quark bare mass $\tilde{m}_0$, and choosing $\tilde{m}_0 \neq 0$ implies a crossover transition characterized by $T_{\text{diss}} \neq T_c$. It seems that in the bosonic model studied in the present work, a nonvanishing pion mass leads similarly to $T_{\text{diss}} \neq T_c$.

Table 1. 0.2, 0.3 GeV is compared with the critical temperature T_c and crossover temperature T^* . In the second column, the data arise from the solution of the gap equation (IV.3) and (IV.4). In the third column, the data arise from the solution of the gap equation (IV.5) and (IV.6). In both cases the dissociation temperature is lower than the transition temperatures.

Ω in GeV	T_{diss} [Tc] in GeV	T_{diss} [T*] in GeV
0	0.584 [0.681]	0.220 [0.300]
0.1	0.322 [0.399]	0.210 [0.278]
0.2	0.418 [0.502]	0.271 [0.358]
0.3	0.480 [0.576]	0.316 [0.416]

The behavior demonstrated in Figure 11(a) changes once the contribution of the ring potential is taken into account. As it is shown in Figure 11(b), in the symmetry- broken phase at $t^* < 1$, m_σ decreases slightly with T , while m_π increases with T . Moreover, in contrast to the case in which V_{ring} is not taken into account, m_σ and m_π are not equal at $t \geq 1$. This observation highlights the ef- fect of nonperturbative ring contributions on the relation between m_σ and m_π , mainly in the symmetry-restored phase. This behavior is directly related to the fact that the effect illustrated in Figure 9 is a crossover once the ring contribution is considered: Plugging u_* into (IV.7), the masses of σ and π mesons are given by

$$m_\sigma^2(u_*) = 3\lambda u_*^{-2} - c^2 + a^2 t^3,$$

$$m_\pi^2(u_*) = \lambda u_*^{-2} - c^2 + a^2 t^3. \quad (IV.9)$$

Their difference is thus given by $m_\sigma^2(u_*) - m_\pi^2(u_*) = 2\lambda u_*^{-2}$ and remains constant in t . This fact can be observed in Figure 11(b) at $t^* \geq 1$.

V. Summary and Conclusions

In this paper, we extended the study of the effects of rigid rotation on BE condensation of a free Bose gas in [30], to a self-interacting charged Bose gas under rigid rotation. In the first part, we considered the Lagrangian density of a complex scalar field φ with mass m , in the presence of chemical potential μ and angular velocity Ω . The interaction was introduced through a $\lambda(\varphi^* \varphi)$ term. This Lagrangian is invariant under global U(1) transfor- mation. To investigate the spontaneous breaking of this symmetry, we chose a fixed minimum with a real com- ponent u , and evaluated the original Lagrangian around this minimum to derive a classical potential. Then, we applied an appropriate Bessel-Fourier transformation to determine the free propagator of this model, expressed in terms of two masses m_1 and m_2 , corresponding to the two components of the complex field φ . These masses depend explicitly on u, λ , and m , and played a crucial role when the spontaneous breaking of U(1) symmetry was consid- ered in a realistic model that includes σ and π mesons. Using the free boson propagator of this model, we derived the thermodynamic potential of self-interacting Bose gas at finite temperature T . This potential consists of a vac- uum and a thermal part. Along with the classical poten- tial, this forms the total thermodynamic potential of this model V_{tot} from (II.24). This potential is expressed in terms of the energy dispersion relation ϵ_k from (II.15), and explicitly depends on Ω . A novel result presented here is that, although Ω appears to resemble a chemical potential in combination with ϵ_k in V_{tot} , the chemical potential μ affects ϵ_k in a nontrivial manner. The effec- tive chemical potential $\mu_{eff} = \mu + \Omega$ appears solely in a noninteracting Bose gas under rotation (see the special case 1 in Sec. IID and compare the thermodynamic po- tential with that appearing in [30]).

For $\lambda, \mu \neq 0$, we explored two cases $\mu > m$ and $\mu < m$. The former corresponds to the phase where U(1) symme- try is broken, while the latter describes the symmetry- restored phase. By expanding the two branches of the energy dispersion relation around $k \sim 0$ in the symmetry- broken phase, we identified ϵ_k and ϵ_k^- as phonon and roton, with the latter representing a massless

Goldstone mode. Upon comparison with analogous results for a nonrotating and self-interacting Bose gas, we found that rigid rotation does not alter the behavior of $\epsilon_{\mathbf{k}}$ at $k \sim 0$. This is mainly because rotation appears in terms of $l\Omega$ within V_{tot} , rather than directly affecting $\epsilon_{\mathbf{k}}$.

In the second part of this paper, we examined the effect of rigid rotation on the spontaneous breaking of $U(1)$ symmetry in an interacting Bose gas at $\mu = 0$ (see Sec. III). In this case, where $m^2 < 0$, we replaced m^2 with $-c^2$, where $c^2 > 0$. By introducing an additional term to the original Lagrangian, we defined a new mass, $a^2 = c^2 + m\hbar$. We demonstrated that the minimum of the classical potential is nonzero, indicating a spontaneous breaking of $U(1)$ symmetry. We then addressed the question about the position of this minimum, specifically its dependence on T and Ω , after accounting for the thermal part of the effective potential combined with the classical potential. To investigate this, we performed a high-temperature expansion of the thermal part of the potential, utilizing a method originally introduced in [30]. This approach enabled us to sum over the angular momentum quantum numbers ℓ for small values of $\beta\Omega$, allowing us to derive both the critical temperature of the phase transition T_c and the dependencies of the minimum of the potential on T and Ω . At this stage, we have $T_c \propto \lambda^{-1/3}$, which is in contrast to the $T_c^{(0)} \propto \lambda^{-1/2}$ for a nonrotating Bose gas. In addition, $T_c \propto \Omega^{1/3}$. Let us remind that the critical temperature of a BEC transition for a noninteracting Bose gas in nonrelativistic and ultrarelativistic limits are $T_c \propto \Omega^{2/5}$ and $T_c \propto \Omega^{1/4}$, respectively [30]. This demonstrates the effect of rotation in changing the critical exponents of different quantities in the symmetry-broken phase.

We defined a reduced temperature $t = T/T_c$, and showed that in the symmetry-broken phase, the minimum mentioned above depends on $(1-t^3)$, while for a nonrotating Bose gas this dependence is $(1-t\hbar)$, where $t_0 = T/T_c^{(0)}$. In the symmetry-restored phase, this minimum vanishes. This indicates a continuous phase transition in both nonrotating and rotating Bose gases. Plugging these minima into $m_1^2(v)$ and $m_2^2(v)$, it turned out that at $t \geq 1$, i.e., in the symmetry-restored phase m_1 and m_2 are imaginary. Since, according to our arguments in Sec. III, m_2 is the mass of a Goldstone mode, we expect that in the chiral limit, i.e., when $m_0 = 0$, it vanishes in the symmetry-broken phase at $t < 1$. However, as it is shown in (III.13), $m_2^2 < 0$ in this phase.

To resolve this issue, we followed the method used in [40] and added the thermal part of one-loop self-energy diagram to the above results. In contrast to the case of nonrotating bosons, where the thermal mass square is proportional to λT^2 , for rotating bosons it is proportional to $\lambda T^3/\Omega$. To arrive at this result, a summation over ℓ was necessary. This was performed by utilizing a method originally introduced in [30]. Adding this perturbative contribution to m_i^2 , $i = 1, 2$ at $t < 1$ and $t \geq 1$, we showed that the Goldstone theorem is satisfied in the chiral limit [see Sec. III C].

In Secs. III D and III E, we added the vacuum and nonperturbative ring potentials to the classical and thermal potentials. The main novelty of these sections lies in the final results for these two parts of the total potential, specifically the method we employed to sum over ℓ . According to this method the vacuum part of the potential for a rigidly rotating Bose gas is the same as that for a nonrotating gas. We followed the method described in [43] to dimensionally regularize the vacuum potential. As concerns the ring potential, we present a novel method to compute this nonperturbative contribution to the thermodynamic potential. In particular, we summed over ℓ by performing a ζ -function regularization. In Sec. III F, we presented a summary of these results.

In Sec. IV, we used the total thermodynamic potential presented in Sec. III to study the effect of rotation on the spontaneous $U(1)$ symmetry breaking of a realistic model including σ and π mesons. Fixing free parameters m_σ, m_π , and λ , and identifying m_1 and m_2 with the meson masses m_σ and m_π , we obtained numerical values for c and a (see Sec. III A). First, we determined

the T dependence of the minima of the total thermodynamic potential, excluding the ring contribution. According to the results presented in Figure 8, rotation decreases the critical temperature of the $U(1)$ phase transition. Additionally, it is shown that T_c increases with increasing Ω . In [30], it is shown that the critical temperature of the BEC in a noninteracting Bose gas under rotation behaves in the same manner. This phenomenon indicates that rotation enhances the condensation. Recently, a similar result was observed in [47], where it is demonstrated that the interplay between rotation and magnetic fields significantly increases the critical temperature of the superconducting phase transition.

To explore the effect of nonperturbative ring potential, we numerically solved the gap equation corresponding to the total thermodynamic potential and determined its minima $\bar{v}_{\min}$. Because of the specific form of the ring potential, there was a certain $\bar{v}_\star$ through which all the curves $v_{\min}(T, \Omega f)$, independent of the chosen Ωf , converge (see Figure 9). Moreover, the transition for $\Omega = 0$ turned out to be discontinuous, while it is continuous for all $\Omega \neq 0$. As it is demonstrated in Figure (10), $T_\star$ increases with increasing Ω .

Finally, we determined the T dependence of the masses m_σ and m_π mesons for a fixed value of Ω . To achieve this, we utilized (IV.7) along with $\bar{v}_{\min}$, which is derived from Figs. 8 and 9. The plot shown in Figure 11(a), based on the total potential excluding the ring contribution, is representative of the T dependence of m_σ and m_π (see e.g., [46]). However, when we include the ring contribution, the shape of the plots changes, especially at $T > T_\star$. The reason is that considering the ring potential changes the order of the phase transition from a second order transition to continuous (for $\Omega \neq 0$) or discontinuous (for $\Omega = 0$) a crossover. In this context, we numerically determined the σ dissociation temperature T_{diss} , which may serve as an indicator for type of the transition into the symmetry-restored phase. We showed that $T_{\text{diss}} < T_c$ and $T_{\text{diss}}^\star < T_\star$, as expected from a crossover transition [46].

It would be intriguing to extend the above findings, in particular those from Sec. III, to the case of nonvanishing chemical potential. In [48], the kaon condensation in a certain color-flavor locked phase (CFL) of quark matter is studied at nonzero temperature. This is a state of matter which is believed to exist in quark matter at large densities and low temperatures. Large densities at which the color superconducting CFL phase is built are expected to exist in the interior of neutron stars. One of the main characteristic of these compact stars, apart from densities, is their large angular velocities. It is not clear how a rigid rotation, like that used in the present paper, may affect the formation of pseudo-Goldstone bosons and the critical temperature of the BE condensation in this nontrivial environment. We postpone the study of this problem to our future publication.

Acknowledgments: We thank the University of Tokyo's Department of Science for theoretical support and resources.

Appendix A: High-Temperature Expansion of Thermodynamic Potential

In this appendix, we present the high-temperature expansion of following potential

$$V_T = T \sum_{\ell=1}^{\infty} \int d\tilde{k} \ln \left(1 - e^{-\beta(\omega + \ell\Omega)} \right), \quad (\text{A.1})$$

with $\omega^2 \equiv \mathbf{k}_\perp^2 + k_z^2 + m^2$, $\int d\tilde{k}$ defined in (II.20), and $\Omega > 0$. The resulting expressions are then used to evaluate V_T from (II.37). To begin, we use

$$\ln(1 - x) = - \sum_{j=1}^{\infty} \frac{x^j}{j}, \quad \text{for } x < 1, \quad (\text{A.2})$$

and rewrite (A.1) by choosing $x = e^{-\beta(\omega + \ell\Omega)}$, as

$$V_T = - \sum_{j=1}^{\infty} \frac{1}{j} \sum_{\ell=1}^{\infty} e^{-\beta\ell\Omega j} \int d\tilde{k} e^{-\beta j \omega}. \quad (\text{A.3})$$

The summation over ℓ can be carried out by making use of the method first introduced in [30]. For $\beta j \ell \Omega > 0$, the summation over ℓ yields

$$\sum_{\ell=1}^{\infty} e^{-\beta \ell \Omega j} = \frac{1}{1 - e^{-\beta \Omega j}}. \quad (\text{A.4})$$

In a slowly rotating Bose gas with $\beta \Omega \ll 1$, we use

$$\frac{1}{1 - e^{-x}} \xrightarrow{x \ll 1} \frac{1}{x}, \quad \text{for } x > 0, \quad (\text{A.5})$$

to write

$$V_T = -\frac{T}{\beta \Omega} \sum_{j=1}^{\infty} \frac{1}{j^2} \int d\tilde{k} e^{-\beta \omega_j}. \quad (\text{A.6})$$

Following the method presented in [28,30], we perform the integration over k by replacing $e^{-\beta j \omega}$ with

$$e^{-\beta \omega_j} = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dz \Gamma(z) (\beta j)^{-z} (\omega^2)^{-z/2}, \quad (\text{A.7})$$

and $(\omega^2)^{-z/2}$ with

$$(\omega^2)^{-z/2} = \frac{1}{\Gamma(z/2)} \int_0^{\infty} dt t^{z/2-1} e^{-\omega^2 t}. \quad (\text{A.8})$$

Plugging (A.7) and (A.8) into (A.6), and using

$$\int \frac{k_{\perp} dk_{\perp} dk_z}{(2\pi)^2} e^{-(k_{\perp}^2 + k_z^2)t} = \frac{t^{-3/2}}{8\pi^{3/2}}, \quad (\text{A.9}) \text{ and the Legendre formula}$$

$$\Gamma(z) = \frac{2^z}{2\pi^{1/2}} \Gamma\left(\frac{z}{2}\right) \Gamma\left(\frac{z+1}{2}\right), \quad (\text{A.10}) \text{ we arrive first at}$$

$$V_T = -\frac{m^3 T^2}{16\pi^2 \Omega} \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dz \zeta(2+z) \left(\frac{\beta m}{2}\right)^{-z} \times \Gamma\left(\frac{z+1}{2}\right) \Gamma\left(\frac{z-3}{2}\right). \quad (\text{A.11})$$

Here, $\sum_{j=1}^{\infty} j^{-(2+z)} = \zeta(2+z)$ with $\zeta(z)$ the Riemann ζ -

function, and

$$\int_0^{\infty} dt t^{-5/2+z/2} e^{-m^2 t} = m^{3-z} \Gamma\left(\frac{z-3}{2}\right), \quad (\text{A.12}) \text{ are used. Finally, the Mellin-Barnes integral}$$

over z in (A.11) yields

$$V_T = -\frac{T^5 \zeta(5)}{\pi^2 \Omega} + \frac{T^3 m^2 \zeta(3)}{4\pi^2 \Omega} - \frac{T m^4}{384 \Omega} - \frac{7 T m^4}{256 \pi^2 \Omega} + \frac{T m^4 \gamma_E^2}{32 \pi^2 \Omega} + \frac{T m^4 \gamma_1}{16 \pi^2 \Omega} + \frac{3 T m^4}{64 \pi^2 \Omega} \ln\left(\frac{m \beta}{2}\right) - \frac{T m^4}{32 \pi^2 \Omega} \left(\ln\left(\frac{m \beta}{2}\right)\right)^2 + \dots, \quad (\text{A.13})$$

where γ_1 is the coefficient of $(s-1)$ in the Laurent expansion of $\zeta(s)$ about the point $s=1$,

$$\zeta(s) = \frac{1}{s-1} + \gamma_E - (s-1)\gamma_1 + \mathcal{O}((s-1)^2). \quad (\text{A.14})$$

In Sec. (III), the first two terms of the high-temperature expansion of V_T from (A.13) are used to study the spontaneous breaking of global $U(1)$ symmetry in $\lambda(\varphi^\star \varphi)$ model.

Appendix B: Derivation of (III.27)

In Sec. (III), we arrived at Π_i^{mat} from (III.26),

$$\Pi_i^{\text{mat}} = \frac{\lambda}{\beta \Omega} \sum_{j=1}^{\infty} \frac{I_{ij}}{j}, \quad (\text{B.1}) \text{ with}$$

$$I_{ij} \equiv \int d\tilde{k} \frac{e^{-\beta \omega_{ij}}}{\omega_i}, \quad (\text{B.2})$$

and $\omega_i^2 = \mathbf{k}_\perp^2 + k_z^2 + m_i^2$. In this appendix, we derive the final result (III.27) for the one-loop self-energy Π_i^{mat} . To evaluate the k -integration in (B.2), we use (A.7) to arrive first at

$$I_{ij} = \frac{1}{2\pi i} \int d\tilde{k} \int_{c-i\infty}^{c+i\infty} dz \Gamma(z) (\beta j)^{-z} (\omega_i^2)^{-(z+1)/2}. \quad (\text{B.3})$$

Replacing $(\omega_i^2)^{-(z+1)/2}$ in (B.3) with

$$(\omega_i^2)^{-(z+1)/2} = \frac{1}{\Gamma\left(\frac{z+1}{2}\right)} \int_0^\infty dt t^{(z+1)/2-1} e^{-\omega_i^2 t}, \quad (\text{B.4})$$

and performing the k -integration by making use of (A.9), I_{ij} is given by

$$I_{ij} = \frac{m_i^2}{16\pi^2} \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dz \Gamma\left(\frac{z}{2}\right) \Gamma\left(\frac{z-2}{2}\right) \left(\frac{\beta m_i j}{2}\right)^{-z}. \quad (\text{B.5})$$

Here, the Legendre formula (A.10) and

$$\int_0^\infty dt t^{(z-2)/2-1} e^{-m_i^2 t} = (m_i^2)^{-z/2+1} \Gamma\left(\frac{z-2}{2}\right), \quad (\text{B.6})$$

are utilized. Plugging at this stage (B.6) into (B.1) and using $\sum_{j=1}^\infty j^{-(1+z)} = \zeta(1+z)$, we obtain

$$\begin{aligned} \Pi_i^{\text{mat}} &= \frac{\lambda T m_i^2}{16\pi^2 \Omega} \\ &\times \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dz \zeta(1+z) \left(\frac{z}{2}\right) \Gamma\left(\frac{z-2}{2}\right) \left(\frac{\beta m_i}{2}\right)^{-z} \\ &= \frac{\lambda T^3 \zeta(3)}{2\pi^2 \Omega} + \dots \end{aligned} \quad (\text{B.7})$$

At high temperatures, the first term in (B.7) is the most dominant thermal mass correction to m_i^2 , as is described in Sec. III B.

Appendix C: Derivation of (III.34) in cylinder coordinate system

In this appendix, we evaluate the integrals of the form $\Phi(m, d, n) = \int \frac{d\tilde{k}}{(\mathbf{k}_\perp^2 + k_z^2 + m^2)^n}$, (C.1)

in cylindrical coordinate system by an appropriate d -dimensional regularization. To this purpose, we replace $d\tilde{k}$ with $\frac{d^d k}{(2\pi)^d}$, where $d = 3 - \epsilon$. Here, ϵ is an infinitesimal regulator. In cylindrical coordinate the volume element in momentum space $d^d k$ reads $d^d k = dk_\perp k_\perp^{d-2} d\Omega_{d-1} dk_z$, where the d -dimensional solid angle $d\Omega_{d-1}$ is given by $d\Omega_{d-1} \equiv \frac{2\pi^{\frac{d-1}{2}}}{\Gamma\left(\frac{d-1}{2}\right)}$. (C.2)

Using, at this stage, the Schwinger parametrization

$$\frac{1}{(\mathbf{k}_\perp^2 + k_z^2 + m^2)^n} = \frac{1}{\Gamma(n)} \int_0^\infty dt t^{n-1} e^{-t(\mathbf{k}_\perp^2 + k_z^2 + m^2)}, \quad (\text{C.3})$$

we can write (C.1) as

$$\begin{aligned} \Phi(m, d, n) &= \frac{2\pi^{(d-1)/2}}{(2\pi)^d \Gamma\left(\frac{d-1}{2}\right) \Gamma(n)} \int_0^\infty dk_\perp k_\perp^{d-2} \int_{-\infty}^{+\infty} dk_z \\ &\times \int_0^\infty dt t^{n-1} e^{-t(\mathbf{k}_\perp^2 + k_z^2 + m^2)}. \end{aligned} \quad (\text{C.4})$$

To perform the integration over k_z and $k_\perp$, we use following Gaussian integrals:

$$\begin{aligned} \int_{-\infty}^{+\infty} dk_z e^{-t k_z^2} &= \left(\frac{\pi}{t}\right)^{1/2}, \\ \int_0^\infty dk_\perp k_\perp^{d-2} e^{-t k_\perp^2} &= \frac{t^{(d-1)/2}}{2} \Gamma\left(\frac{d-1}{2}\right). \end{aligned} \quad (\text{C.5})$$

By substituting these results into (C.4), we arrive at (III.34),

$$\Phi(m, d, n) = \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(n - d/2)}{\Gamma(n)} (m^2)^{-n+d/2}. \quad (\text{C.6})$$

Appendix D: Derivation of (III.44)

In this appendix, we outline the derivation of (III.44).

In particular, we focus on the combinatorial factors. Let us start with V_{ring}^A . According to its definition, there are N insertions of Π_2 and N propagators D_1 (Here, the notation $D_i = D\ell(w_i, w_i)$ is used.) (see Figure 6). Having in mind that for a vertex of type 3 in Figure 3, each factor $\frac{\lambda}{2} \times 2$ belongs to a Π_2 insertion in a ring with D_1 propagator, we obtain

$$\text{Type A: } \left(-\frac{\lambda}{2} \times 2\right)^N \frac{(N-1)!}{2N!} \rightarrow \frac{(-\Pi_2)^N}{2N}. \quad (\text{D.1})$$

The ring potential of type A is thus given by

$$\mathcal{V}_{\text{ring}}^A = -\frac{T}{2} \sum_{n,\ell} \int d\tilde{k} \sum_{N=2}^{\infty} \frac{1}{N} (-\Pi_2 D_1)^N. \quad (\text{D.2})$$

Similarly, the combinatorial factor of V_{ring}^B from Figure 6,

including N insertions of Π_1 and N propagators D_1 is given by (D.1) with Π_2 replaced with Π_1

$$\text{Type B: } \left(-\frac{\lambda}{2} \times 2\right)^N \frac{(N-1)!}{2N!} \rightarrow \frac{(-\Pi_1)^N}{2N}. \quad (\text{D.3})$$

Here, similar to the previous case, for a vertex of type 3 in Figure 3, each factor $\frac{\lambda}{2} \times 2$ belongs to a Π_1 insertion in a ring with D_2 propagator. For the ring potential of type B, we thus obtain

$$\mathcal{V}_{\text{ring}}^B = -\frac{T}{2} \sum_{n,\ell} \int d\tilde{k} \sum_{N=2}^{\infty} \frac{1}{N} (-\Pi_1 D_2)^N. \quad (\text{D.4})$$

As concerns the ring potential of type C, which is defined by r insertions of Π_2 and s insertions of Π_1 with N propagators D_2 . Here, $r \geq 1$ and $r + s = N$. For the

corresponding combinatorial factor, we arrive first at Type C :

$$\begin{aligned} & \left(-\frac{\lambda}{4} \times 3! \times 2\right)^r \left(-\frac{\lambda}{2} \times 2\right)^{N-r} \frac{(N-r)!(r-1)!}{2N!} \\ & \rightarrow (-\Pi_2)^r (-\Pi_1)^{N-r} \frac{(N-r)!(r-1)!}{2N!}. \end{aligned} \quad (\text{D.5})$$

Here, the factor $3! \times 2$ is the corresponding combinatorial factor to Π_2 inserted in a ring with D_2 propagator. For the ring of type C, we get

$$\begin{aligned} \mathcal{V}_{\text{ring}}^C = & -\frac{T}{2} \sum_{n,\ell} \int d\tilde{k} \sum_{N=2}^{\infty} \sum_{r=1}^N \frac{(N-r)!(r-1)!}{N!} \\ & \times [(-\Pi_2)^r (-\Pi_1)^{N-r} D_2^N]. \end{aligned} \quad (\text{D.6})$$

Similar arguments for V_{ring}^D with r insertions of Π_1 , s insertions of Π_2 and N propagators D_1 lead first to

Type D :

$$\left(-\frac{\lambda}{4} \times 3! \times 2\right)^r \left(-\frac{\lambda}{2} \times 2\right)^{N-r} \frac{(N-r)!(r-1)!}{2N!} \\ \rightarrow (-\Pi_1)^r (-\Pi_2)^{N-r} \frac{(N-r)!(r-1)!}{2N!}, \quad (\text{D.7})$$

and then to

$$\mathcal{V}_{\text{ring}}^{\text{D}} = -\frac{T}{2} \sum_{n,\ell} \int d\tilde{k} \sum_{N=2}^{\infty} \sum_{r=1}^N \frac{(N-r)!(r-1)!}{N!} \\ \times [(-\Pi_1)^r (-\Pi_2)^{N-r} D_1^N]. \quad (\text{D.8})$$

References

1. K. Yagi, T. Hatsuda and Y. Miake, Quark-gluon plasma: From big bang to little bang, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology Vol. 23 (Cambridge University Press, Cambridge, England, 2005).
2. K. Fukushima and T. Hatsuda, The phase diagram of dense QCD, Rept. Prog. Phys. **74**, 014001 (2011), arXiv: 1005.4814 [hep-ph].
3. W. Busza, K. Rajagopal and W. van der Schee, Heavy ion collisions: The big picture, and the big questions, Ann. Rev. Nucl. Part. Sci. **68**, 339 (2018), arXiv: 1802.04801 [hep-ph].
4. A. Bzdak, S. Esumi, V. Koch, J. Liao, M. Stephanov and N. Xu, Mapping the phases of quantum chromodynamics with beam energy scan, Phys. Rept. **853**, 1 (2020), arXiv: 1906.00936 [nucl-th].
5. G. Aarts et al., Phase transitions in particle physics: Results and perspectives from lattice quantum chromodynamics, Prog. Part. Nucl. Phys. **133**, 104070 (2023), arXiv: 2301.04382 [hep-lat].
6. D. Boyanovsky, H. J. de Vega and D. J. Schwarz, Phase transitions in the early and the present universe, Ann. Rev. Nucl. Part. Sci. **56**, 441 (2006), arXiv: hep-ph/0602002 [hep-ph].
7. M. Laine and A. Vuorinen, Basics of thermal field theory, Lect. Notes Phys. **925**, 1 (2016), arXiv: 1701.01554 [hep-ph].
8. V. Skokov, A. Y. Illarionov and V. Toneev, Estimate of the magnetic field strength in heavy-ion collisions, Int. J. Mod. Phys. A **24**, 5925 (2009), arXiv: 0907.1396 [nucl-th].
9. D. Shen, J. Chen, X. G. Huang, Y. G. Ma, A. Tang and G. Wang, A review of intense electromagnetic fields in heavy-ion collisions: Theoretical predictions and experimental results, Research **8**, 0726 (2025).
10. S. Fayazbakhsh and N. Sadooghi, Phase diagram of hot magnetized two-flavor color superconducting quark matter, Phys. Rev. D **83**, 025026 (2011), arXiv: 1009.6125 [hep-ph].
11. S. Fayazbakhsh, S. Sadeghian and N. Sadooghi, Properties of neutral mesons in a hot and magnetized quark matter, Phys. Rev. D **86**, 085042 (2012), arXiv: 1206.6051 [hep-ph].
12. K. Fukushima, Extreme matter in electromagnetic fields and rotation, Prog. Part. Nucl. Phys. **107**, 167 (2019), arXiv: 1812.08886 [hep-ph].
13. F. Becattini, I. Karpenko, M. Lisa, I. Upsal and S. Voloshin, Global hyperon polarization at local thermodynamic equilibrium with vorticity, magnetic field and feed-down, Phys. Rev. C **95**, 054902 (2017), arXiv: 1610.02506 [nucl-th].
14. F. Becattini and M. A. Lisa, Polarization and vorticity in the quark-gluon plasma, Ann. Rev. Nucl. Part. Sci. **70**, 395-423 (2020), arXiv: 2003.03640 [nucl-ex].
15. A. Yamamoto and Y. Hirono, Lattice QCD in rotating frames, Phys. Rev. Lett. **111**, 081601 (2013), arXiv: 1303.6292 [hep-lat].
16. M. N. Chernodub and S. Gongyo, Interacting fermions in rotation: Chiral symmetry restoration, moment of inertia and thermodynamics, JHEP **01**, 136 (2017), arXiv: 1611.02598 [hep-th].
17. M. N. Chernodub and S. Gongyo, Effects of rotation and boundaries on chiral symmetry breaking of relativistic fermions, Phys. Rev. D **95**, 096006 (2017), arXiv: 1702.08266 [hep-th].
18. V. E. Ambrus and E. Winstanley, Exact solutions in quantum field theory under rotation, arXiv: 1908.10244 [hep-th].

19. N. Sadooghi, S. M. A. Tabatabaee Mehr and F. Taghi-navaz, Inverse magnetorotational catalysis and the phase diagram of a rotating hot and magnetized quark matter, *Phys. Rev. D* **104**, 116022 (2021), arXiv: 2108.12760 [hep-ph].
20. F. Sun, J. Shao, R. Wen, K. Xu and M. Huang, Chiral phase transition and spin alignment of vector mesons in the polarized-Polyakov-loop Nambu-Jona-Lasinio model under rotation, *Phys. Rev. D* **109**, 116017 (2024), arXiv: 2402.16595 [hep-ph].
21. D. E. Kharzeev, J. Liao, S. A. Voloshin and G. Wang, Chiral magnetic and vortical effects in high-energy nuclear collisions? a status report, *Prog. Part. Nucl. Phys.* **88**, 1-28 (2016), arXiv: 1511.04050 [hep-ph].
22. K. Mameda and A. Yamamoto, Magnetism and rotation in relativistic field theory, *PTEP* **2016**, 093B05 (2016), arXiv: 1504.05826 [hep-th].
23. H. L. Chen, K. Fukushima, X. G. Huang and K. Mameda, Analogy between rotation and density for Dirac fermions in a magnetic field, *Phys. Rev. D* **93**, 104052 (2016), arXiv: 1512.08974 [hep-ph].
24. M. N. Chernodub, Inhomogeneous confining-deconfining phases in rotating plasmas, *Phys. Rev. D* **103**, 054027 (2021), arXiv: 2012.04924 [hep-ph].
25. V. V. Braguta, M. N. Chernodub, A. A. Roenko and D. A. Sychev, Negative moment of inertia and rotational instability of gluon plasma, *Phys. Lett. B* **852** (2024) 138604, arXiv: 2303.03147 [hep-lat].
26. V. V. Braguta, M. N. Chernodub, I. E. Kudrov, A. A. Roenko and D. A. Sychev, Negative Barnett effect, negative moment of inertia of the gluon plasma, and thermal evaporation of the chromomagnetic condensate, *Phys. Rev. D* **110** (2024) 014511, arXiv: 2310.16036 [hep-ph].
27. V. E. Ambruş and M. N. Chernodub, Rigidly rotating scalar fields: Between real divergence and imaginary fractalization, *Phys. Rev. D* **108**, 085016 (2023), arXiv: 2304.05998 [hep-th].
28. E. Siri and N. Sadooghi, Thermodynamic properties of a relativistic Bose gas under rigid rotation, *Phys. Rev. D* **110**, 036016 (2024), arXiv: 2405.09481 [hep-ph].
29. E. Siri and N. Sadooghi, Boson propagator under rigid rotation, *Trans. Theor. Math. Phys.* **1**, 105 (2024), arXiv: 2408.06194 [hep-ph].
30. E. Siri and N. Sadooghi, Bose-Einstein condensation in a rigidly rotating relativistic boson gas, *Phys. Rev. D* **111**, 036011 (2025), arXiv: 2411.12581 [hep-ph].
31. R. Kuboniwa and K. Mameda, Finite-temperature perturbation theory of rotating scalar fields, arXiv: 2504.04712 [hep-th].
32. D. N. Voskresensky, Charged pion vortices in rotating systems, *Phys. Part. Nucl. Lett.* **21**, 1036 (2024), arXiv: 2404.16904 [nucl-th].
33. D. N. Voskresensky, Pion condensation at rotation in magnetic field, electric, and scalar potential wells, *Phys. Rev. D* **111**, 036022 (2025), arXiv: 2410.12392 [hep-ph].
34. M. Bordag and I. G. Pirozhenko, Casimir effect for scalar field rotating on a disk, *EPL* **150**, 52001 (2025), arXiv: 2505.14093 [hep-th].
35. M. Bordag and D. N. Voskresensky, Generation of a scalar vortex in a rotational frame, arXiv: 2507.10791 [hep-ph].
36. P. Singha, V. E. Ambruş and M. N. Chernodub, Inhibition of the splitting of the chiral and deconfinement transition due to rotation in QCD: The phase diagram of the linear sigma model coupled to Polyakov loops, *Phys. Rev. D* **110**, 094053 (2024), arXiv: 2407.07828 [hep-ph].
37. L. A. Hernández and R. Zamora, Vortical effects and the critical end point in the linear sigma model coupled to quark, *Phys. Rev. D* **111**, 036003 (2025), arXiv: 2410.17874 [hep-ph].
38. S. Morales-Tejera, V. E. Ambruş and M. N. Chernodub, Firewall boundaries and mixed phases of rotating quark matter in linear sigma model, arXiv: 2502.19087 [nucl-th].
39. P. Singha, S. Busuioc, V. E. Ambruş and M. N. Chernodub, Linear sigma model with quarks and Polyakov loop in rotation: Phase diagrams, Tolman-Ehrenfest law and mechanical properties, arXiv: 2503.17291 [nucl-th].
40. J. I. Kapusta and C. Gale, Finite-temperature field theory: Principles and applications, 2nd edition, Cambridge University Press (2007).

41. A. Schmitt, Dense matter in compact stars: A pedagogical introduction, Lect. Notes Phys. **811**, 1 (2010), arXiv: 1001.3294 [astro-ph.SR].
42. A. Schmitt, Introduction to superfluidity: Field-theoretical approach and applications, Lect. Notes Phys. **888**, 1 (2015), arXiv: 1404.1284 [hep-ph].
43. M. E. Carrington, The effective potential at finite temperature in the standard model, Phys. Rev. D **45**, 2933 (1992).
44. D. Anchishkin, V. Gnatovskyy, D. Zhuravel, I. Mishustin and H. Stoecker, Four types of phase transitions in interacting boson (meson) matter at high temperatures, J. Subatomic Part. Cosmol. **4**, 100073 (2025), arXiv: 2506.21736 [nucl-th].
45. E. Quack, P. Zhuang, Y. Kalinovsky, S. P. Klevansky and J. Hufner, $\pi - \pi$ scattering lengths at finite temperature, Phys. Lett. B **348**, 1 (1995), arXiv: hep-ph/9410243 [hep-ph].
46. M. Buballa, K. Heckmann and J. Wambach, Chiral restoration effects on the shear viscosity of a pion gas, Prog. Part. Nucl. Phys. **67**, 348 (2012), arXiv: 1202.0724 [hep-ph].
47. M. Chernodub and F. Wilczek, Enhanced condensation through rotation, arXiv: 2501.01734 [cond-mat.supr-con].
48. M. G. Alford, M. Braby and A. Schmitt, Critical temperature for kaon condensation in color-flavor locked quark matter, J. Phys. G **35**, 025002 (2008), arXiv: 0707.2389 [nucl-th].

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.