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Not peer-reviewed version

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Posted Date: 13 January 2025

doi: 10.20944/preprints202501.0915.v1

Keywords: PBR theorem; Ontological models; Foundations of quantum mechanics

Article

On the Reality of the PBR Theorem

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Abstract: Recently, an argument has been presented against the Pusey–Barrett–Rudolph (PBR) theorem [1]. This claim has sparked a debate, with some authors defending the PBR theorem [2], but the proponent of the claim has insisted on his argument [3,4]. Moreover, the author claimed to have proved that the PBR theorem is incorrect by a generic counterexample [4]. In this paper, we contribute to the discussion with some new arguments. We demonstrate that the PBR theorem contains no errors and remains intact.

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1. Introduction

First, we would like to summarize the main reasoning of the PBR theorem [5]. The PBR theorem assumes the existence of the real physical state (λ) of a physical system. Therefore, relational interpretations such as QBism are not restricted by this theorem. According to the terminology of [5] a physical property is defined as a function of the physical state, such as $E = f(\lambda)$. On the other hand, there may be "properties" of the system that are not physical, but are the product of the human mind and consist only of our knowledge (epistemic properties). A property L is said to be physical if, for every pair of values L_1 and L_2 of L in the space of physical states, $\mu_{L_1}(\lambda) \bigcap \mu_{L_2}(\lambda) = \emptyset$. Here, $\mu_{L_j}(\lambda)$ is the probability distribution when the system has the L_j value of this property. (The system's state is not fully determined and is represented by a distribution.) On the other hand, if L is epistemic, there must be at least two values L_1 and L_2 such that the intersection of the distributions $\mu_{L_1}(\lambda)$ and $\mu_{L_2}(\lambda)$ is not an empty set. PBR borrowed this definition from Refs.[6,7] and adapted it to their proofs. The extent to which this definition is appropriate and accurately defines a physical property may be subject to debate. In our view, the definition above is a suitable one. Indeed, when the value of a physical property changes, it is expected to be detectable by measurements. The disjointness of the distributions $\mu_{L_1}(\lambda)$ and $\mu_{L_2}(\lambda)$ guarantees this behavior. Without prolonging the discussion on this matter, let's focus on the important point for the purposes of this paper: In the PBR theorem, the distributions $\mu_{L_1}(\lambda)$ and $\mu_{L_2}(\lambda)$ do not represent two separate properties, but rather represent different values of the *same property*. On the other hand, in the argument developed against the PBR theorem in Refs.[1,4] and in the example provided by the author, distributions are associated with two different properties. Moreover, the author defines the notion of property differently from PBR. It assumes that the value of a property of a system can be directly obtained through measurement. This definition of property by the author is more in line with what is referred to as observable. In the context of ontological models, while associating a value of a physical property with measurement is correct, such a relationship is not direct. Instead, it is given as a function of the configuration of the measurement apparatus (n_{AP}) and the system's real physical state or the hidden variable; measurement output = $R(n_{AP}, \lambda)$ (see [8,9]). The different definition of the notion of property in Refs.[1,4] led the author to *arbitrarily* associate the values of the property with quantum states, which resulted in an incorrect argument that the PBR theorem is flawed. We will discuss why adopting a different definition of the notion of property leads the author to the wrong conclusion after summarizing the counter-argument. However, for the moment, we can say that the counter-argument presented in Refs.[1,4] is based on reasoning different

 doi:10.20944/preprints202501.0915.v1

from the one used in the PBR theorem, and the conclusions reached do not constrain the PBR theorem. Instead, we will show that the presented counterexample, contrary to its intended purpose, forms a specific example that demonstrates the correctness of the PBR theorem.

For a better understanding of our argument, we think it would be useful to discuss the notion of physical property used in the PBR theorem in more detail. A physical property L is represented by a function $L \equiv f(\lambda)$, where $\lambda \in \Lambda$ is the variable of the function [5]. Here, Λ represents the space of physical states. Since the distributions $\mu_{L_1}(\lambda)$ and $\mu_{L_2}(\lambda)$ are disjoint (due to the physical property criterion), the indices L_1 and L_2 correspond to two disjoint sets of values $\pi_1, \pi_2 \in \Lambda; \pi_1 \bigcap \pi_2 = \emptyset$ of the same function f (i.e. the same property). That is to say, L_1 and L_2 are seen as different values of the same property L. Obviously, if it were assumed that the indices L_1 and L_2 represent two different functions, say $f \equiv f(\lambda)$ and $g \equiv g(\lambda)$, then the distributions $\mu_{L_1}(\lambda)$ and $\mu_{L_2}(\lambda)$ might not be disjoint, even though f and g are two different physical properties. Accordingly, the physical property criterion of PBR would be invalid. We can give the following example for this situation: Energy and momentum are two physical properties and their values have overlapping domains on the phase space. Therefore, the distributions $\mu_E(\lambda)$ and $\mu_{\vec{p}}(\lambda)$ are not disjoint. If the physical property criterion of PBR were applied, it would lead to the incorrect conclusion that energy and momentum are not physical. However, such a result does not arise because the PBR criterion is not applied to two different properties like energy and momentum.

The PBR argument proceeds as follows: The question is asked, "Is the quantum state $|\psi>$ a physical property?" For $0 < \alpha < \pi/2$ the following state vectors

$$|\psi_0> = \cos\frac{\alpha}{2}\,|0> + \sin\frac{\alpha}{2}\,|1> \tag{1}$$

$$|\psi_1> = \cos\frac{\alpha}{2}\,|0> - \sin\frac{\alpha}{2}\,|1> \tag{2}$$

are associated with different values of the $|\psi>$ quantum state property. Here, in fact, there are infinitely many different values. By means of the entangled projection measurement they designed, PBR shows that for $\forall\alpha$ the distributions $\mu_{|\psi_0>}(\lambda)$ and $\mu_{|\psi_1>}(\lambda)$ are disjoint. They do this through a proof by reductio ad absurdum. Essentially, what they demonstrate is that $|\psi_0>$ and $|\psi_1>$ are statistically distinguishable because they generate different probabilities according to the laws of quantum mechanics (QM). They achieve this with an entangled projection measurement on an ensemble of independently prepared systems $|\psi_{x_1}> \otimes |\psi_{x_2}> \otimes ... \otimes |\psi_{x_n}>$ where $x_j \in \{0, 1\}$ for each j. Assuming that $\mu_{|\psi_0>}(\lambda) \bigcap \mu_{|\psi_1>}(\lambda) \neq \emptyset$ for at least one pair $|\psi_0>$ and $|\psi_1>$, then it is shown that there is a contradiction with the predictions of QM.

2. Discussion of the Counter-Argument - Does the PBR Theorem Rely on Faulty Reasoning?

To summarize the main points, in Ref.[1], the author discusses the special case where $|\psi_0> = |0>$ and $|\psi_1> = |+> = \frac{1}{\sqrt{2}}(|0> + |1>)$, and the states $|0>$ and $|1>$ are associated with the values of "property A", while the states $|+>$ and $|->$ are associated with the values of "property P", which are two distinct properties. To be precise, property A has values a_1 and a_2 and property P has values p_1 and p_2 and the association with state vectors is done as given in Table 1. Here, the following definitions are adopted $|\pm> = \frac{1}{\sqrt{2}}(|0> \pm |1>)$.

Table 1.

	Property A	Property P
Values:	$a_1 \to \|0>$	$p_1 \to \|+>$
	$a_2 \to \|1>$	$p_2 \to \|->$

Cabbolet states that if we adopt an Einsteinian view, we must accept that the values of properties A and B of the system existed before the measurement and that the measurement reveals these hidden

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values. Otherwise, the measurements of these properties would not yield any output. Accordingly, a system prepared in state $|0>$ corresponding to a_1 of the property A also has property P, even if we do not know its value. The author illustrates this fact with the following peculiar equality (see Ref.[1] Eqn.(16))

$$|0>=\frac{1}{\sqrt{2}}(|0,+>+|0,->). \tag{3}$$

This equality must be representative, because the left and right sides of equality have different dimensions. The author considers equation (3) as a statistical mixture, and comments that Born probabilities in quantum theory

$$|<0|+>|^2=\frac{1}{2}=|<0|->|^2 \tag{4}$$

require this mixture to be equally weighted and explains the $\frac{1}{\sqrt{2}}$ multiplier in equation (3) based on the Born rule. Now, if the two systems given by (3) are independently prepared, in 25% of the combined systems the systems will be in the state $|00,++>\equiv|0,+>\otimes|0,+>$, i.e. both systems have a_1 and p_1 values of properties A and P before the measurement (see Table 1). On the other hand, according to Cabbolet, in this case there is no projection onto any of the

$$|\xi_1>=\frac{1}{\sqrt{2}}(|0>\otimes|1>+|1>\otimes|0>) \tag{5}$$

$$|\xi_2>=\frac{1}{\sqrt{2}}(|0>\otimes|->+|1>\otimes|+>) \tag{6}$$

$$|\xi_3>=\frac{1}{\sqrt{2}}(|+>\otimes|1>+|->\otimes|0>) \tag{7}$$

$$|\xi_4>=\frac{1}{\sqrt{2}}(|+>\otimes|->+|->\otimes|+>) \tag{8}$$

vectors that form the entangled measurement of the PBR; $<\xi_k|00,++>=0,\ \forall k$ (see Ref.[1] Eqn. (20)). Cabbolet justifies this by the equality

$$<1|0,+>=<-|0,+>=0 \tag{9}$$

See Ref.[1] relations (18). It reads: if we know that the system has property values 0 and +, then the system cannot have property values 1 and −. It has been said that this contradicts the PBR assumption; because it shows that these entangled measurements do not exist at all, according to the Einsteinian view assumed by PBR in their proof [1].

In our opinion, Cabbolet's significant mistake is thinking that the reductio ad absurdum assumption of PBR implies a classical statistical mixture. Eqn. (3) reflects such an idea. Cabbolet defined the notion of property[1] differently from its use in the PBR theorem and therefore misinterpreted the ad absurdum assumption. If we assume that the values of a property are obtained directly by measurement and that the distributions $\mu_{|\psi_0>}(\lambda)$ and $\mu_{|\psi_1>}(\lambda)$ correspond to the values of two such different properties, then one might erroneously conclude that the PBR theorem is based on a classical view of properties, since the measurement outputs are quantities (real numbers) that can be studied in accordance with classical thinking. On the other hand, the notion of a property used during the proof of the PBR theorem is different from the one defined by Cabbolet. PBR treats the quantum state as a property and L_1 and L_2, which index the distributions $\mu_{L_1}(\lambda)$ and $\mu_{L_2}(\lambda)$ used in the theorem, do not represent two separate properties, but different values of the same quantum state property. Now

[1] We would like to point out that we are not concerned with the question of what the correct definition of the notion of a property is. This is a philosophical debate. What is important for our purposes here is what PBR's and Cabbolet's definitions are.

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if we take a physical property to be a quantity whose value is directly obtained by measurement, as in Cabbolet's definition, then the quantum state would not be a physical property by this definition. Obviously, a quantum state is not a quantity whose value is determined by a measurement. Such a definition of a property fits more like an observable. So the definition of physical property in Ref.[1] does not assume the quantum state to be physical from the outset.

Cabbolet arbitrarily associates the values of the properties A and P with the quantum states $|0>$, $|1>$ and $|+>$, $|->$ (Table 1). More precisely, he associates a_1 with $|0>$ and p_1 with $|+>$, etc. But according to which rule is this association made? The property assumed in the PBR theorem is not A or B but the quantum state. The quantum state obeys all the rules provided by the formalism of quantum theory (for example, the basis transformation rule). Therefore, if we want to associate quantum states with some other properties, we cannot do it arbitrarily. Such a property association must be consistent with the mathematical properties of a vector in Hilbert space. Accordingly, not only states like $|0>$, $|1>$, $|+>$ and $|->$, but also infinitely many other states, such as $|+'>=\frac{1}{\sqrt{2}}(|0>+e^{i\phi}|1>)$ and $|-'>=\frac{1}{\sqrt{2}}(|0>-e^{i\phi}|1>)$, should be considered, and there should be an infinite number of properties or property values associated with these states. Cabbolet's model does not take this fact into account and his model contradicts with QM. We will show this contradiction in detail shortly, but there is an issue we would like to note beforehand. The proof of the PBR theorem uses the formalism of quantum theory, not an interpretation of it. (With one exception: Born's rule requires an interpretation. But almost all physicists today agree on this interpretation). In this paper, we take the formalism of QM based on complex Hilbert space. A different formalism can be adopted if desired. However, since all different formalisms will produce the same experimental results, for our purpose in this paper, any one can be chosen without loss of generality. We agree with Cabbolet that if an interpretation is made on the ad absurdum assumption of the PBR theorem, it could be said to reflect the Einsteinian view. However, great care must be taken in this regard, as the formalism of quantum theory, not any particular interpretation, is essential during the proof. In the original gedankenexperiment proposed by Einstein-Podolsky-Rosen (EPR) [10] or its Bohm version [11] there is the idea that observables have definite values even before measurement, but these values are hidden to us. By taking Cabbolet's notion of property as an observable, is it not possible to relate the measurement outputs of the observable to the quantum states in the PBR assumption? Indeed, this is possible; it is convenient to model the states $|0>$ and $|+>$ with the eigenvectors of the spin matrices S_z and S_x. Within this model, we correspond the properties A and P in Ref.[1] to the observables S_z and S_x (see Table 2).

Table 2.

	Property S_z	Property S_x
Values:	$z_1=+\frac{\hbar}{2}\rightarrow|0>$	$x_1=+\frac{\hbar}{2}\rightarrow|+>$
	$z_2=-\frac{\hbar}{2}\rightarrow|1>$	$x_2=-\frac{\hbar}{2}\rightarrow|->$

According to the Einsteinian view, the measurement results for the properties S_z and S_x, i.e. eigenvalues, have certain values before the measurement. The EPR gedankenexperiment indeed asserts the existence of the measurement results of the spin S_z and S_x before the measurement (see the Bohm version of the EPR [11]). Let us now apply Cabbolet's model by taking the properties A and P as in Table 2. According to the Einsteinian view, let us assume that a system prepared in the quantum state $|0>$ (with the observable S_z having the value $z_1=+\frac{\hbar}{2}$) also has the observable S_x with possible measurement values $x_1=+\frac{\hbar}{2}$ or $x_2=-\frac{\hbar}{2}$ before measurement, as indicated by (3). However, we can always perform a unitary transformation to another observable $S_{x'}$ such that $S_{x'}=U^{\dagger}S_xU$. Here, $S_{x'}$ represents the x' component of the spin matrix in the new x'-y'-z axes resulting from a rotation about the z-axis in x-y-z cartesian coordinates. With such a rotation around the z-axis by an angle ϕ, the following matrix is obtained:

$$S_{x'}=\frac{\hbar}{2}\begin{pmatrix} 0 & e^{-i\phi} \\ e^{i\phi} & 0 \end{pmatrix}. \tag{10}$$

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The eigenvectors of $S_{x'}$ are given by

$$|+' >= \frac{1}{\sqrt{2}}(|0> + e^{i\phi}|1>) \tag{11}$$
$$|-' >= \frac{1}{\sqrt{2}}(|0> - e^{i\phi}|1>). \tag{12}$$

Since $|<0|+'>|^2 = \frac{1}{2} = |<0|-'>|^2$, it follows from the author's argument that the statistical mixture given by (3) should also include the $+'$ and $-'$ values of the $S_{x'}$ property. However, since ϕ is a continuous parameter, there are infinitely many (even uncountably infinite) different properties of this type. Therefore, not all values of the properties can be written in the form of a statistical mixture like (3). Moreover, even assuming that a system prepared in the $|0>$ quantum state has only a finite number of properties due to some unknown reason, the problem cannot be solved. Indeed, in this case one can consider a statistical mixture with finite number of properties for a finite number N of independent preparations of the system. However, the experimenter chooses the observable $S_{x'}$ to measure of his free will. He can choose properties that are not in the statistical mixture, and when this happens, the predictions of the ontological model in question will be different from those of QM. This is obvious bec use the model does not give any outcome under this measurement, whereas we know that QM does. In summary, Cabbolet's ontological model has different predictions than QM for the measurements $P_{+'} = |+'><+'|$ and $P_{-'} = |-'><-'|$.

In Ref.[4] Cabbolet tried to strengthen his argument in his previous paper. However, the ontological model he adopts is the same as the previous one; he has simply provided more details about the model and elaborated on some points he considered to be vague in his argument. He explicitly defines the ontic states in his model: There are 4 ontic states $\lambda \in \{1,2,3,4\}$. These ontic states have the values of the properties A and P as follows:

$$\lambda = 1 \Rightarrow (0\ \&\ -),\ \ \lambda = 2 \Rightarrow (0\ \&\ +),\ \ \lambda = 3 \Rightarrow (1\ \&\ +),\ \ \lambda = 4 \Rightarrow (1\ \&\ -). \tag{13}$$

The values of properties are related to quantum states via $0 \to |0>$, $1 \to |1>$, $+ \to |+>$ and $- \to |->$. It follows that the model is indeed ψ-epistemic. Since the distributions $\mu_0(\lambda)$ and $\mu_+(\lambda)$ have the intersection $\Delta = \{\lambda = 2\}$ for 0 and + values of the properties A and P, the model is ψ-epistemic. It is then shown that the model's (M) measurement probabilities on the entangled states (5-8) used in the proof of the PBR theorem are zero as in QM [4]:

$$\vDash_M P(|\psi_k> \to |\xi_k>) = 0 \tag{14}$$

where, $|\psi_1> = |0> \otimes |0>$, $|\psi_2> = |0> \otimes |+>$, $|\psi_3> = |+> \otimes |0>$ and $|\psi_4> = |+> \otimes |+>$. However, the PBR theorem has shown that it is

$$\vDash_{\psi-eps.} P(|\psi_k> \to |\xi_k>) > 0 \tag{15}$$

for a ψ-epistemic model. Thus, the author claims to have found a counterexample showing the invalidity of the PBR theorem. If Cabbolet had been able to show that there are no measurements of (5-8) on a ψ-epistemic model that does not contradict QM in terms of one-point measurements,[2] he would have refuted the proof presented in the PBR paper. However, as we have shown, Cabbolet's model has different measurement outcomes from QM with respect to $P_{+'} = |+'><+'|$ measurements. Specifically for $|\xi_k>$ measurements, it no longer matters whether it gives the same measurement outcomes as QM. It may be possible to tune a model in such a way that it matches the results of QM for

[2] It is known that ontological models are constrained by Bell inequalities for measurements performed on Bell states at two space-like separated points. But PBR does not use measurements at two space-like separated points on entangled systems in their proof. Therefore, for an ontological model that would serve as a counterexample, it is sufficient to consider measurements at a single point in space and for the model to predict the same results as QM in terms of these measurements.

 doi:10.20944/preprints202501.0915.v1

specific measurements; however, what is important is that the results are consistent with QM for every measurement. The PBR theorem deduced that a ψ-epistemic ontological model has measurement outcomes that contradict those of QM. In order to find a counterexample to invalidate the theorem, one must find a ψ-epistemic model that is compatible with QM in terms of all measurement outcomes. Cabbolet's model is therefore not a counterexample to the PBR theorem.

In Ref.[4] it was also claimed that the PBR theorem contains a fatal flaw (in author's own words). The alleged flaw is the following: The PBR theorem makes the tacit assumption that the probabilities $| < 0|+ > |^2 = | < 1|+ > |^2 = \frac{1}{2}$ hold, in the context of the ensemble interpretation of quantum mechanics, for any individual member of an ensemble of systems in the quantum state $|+ >$, regardless of its ontic state, i.e., regardless of the value of the parameter λ. In other words: the fatal flaw is the tacit assumption that for every member of a ensemble of systems with property P with value +, there is a 50% intrinsic probability that a measurement of the value of property A will have outcome $a = 1$. Cabbolet states that the wrong thinking in the theorem is the following: The concept of intrinsic probability can be applied to the framework of fundamentally probabilistic Copenhagen quantum mechanics, but not to a deterministic theoretical framework. In our opinion, there is no such flaw in the PBR theorem, since it uses the formalism of quantum theory, independent of any interpretation. As we have already mentioned, the only subject of interpretation may be the Born rule, but there is no tacit presupposition that the probabilities are intrinsic or statistical. The fact that $| < 0|+ > |^2 = | < 1|+ > |^2 = \frac{1}{2}$ is a consequence of the inner product rule for vectors in Hilbert space, i.e. it is entirely a matter of formalism. In our opinion, the reason for Cabbolet's erroneous thinking may lie in the following: Since Cabbolet defines the notion of property used in the PBR theorem as a quantity whose values are determined by direct measurement, and since he invokes such a notion of property in the ad absurdum assumption (whereas such a property cannot be epistemic by definition), he thinks that having two values of the same property entails a statistical partition over a ensemble of identically prepared systems. What we mean is that in a collection of systems with the property +, one part is + and 0, and another part is + and 1, but for the same system these two cannot coexist, since 0 excludes 1 ($< 0|1 >= 0$), he reasoned. However, we think the point he misses is this: In the PBR ad absurdum assumption, the quantum state is assumed to be epistemic. Epistemic things can have seemingly contradictory properties. Let's make an analogy: That bolt is very useful for Alice, but not useful at all for Bob. There is no contradiction here because "usefulness" is epistemic. The usefulness property of a bolt can be both "$+$" (useful) and "$-$" (not useful) at the same time. In fact, the PBR theorem exploits this seemingly contradictory character of the epistemic property during the proof. It is shown in the theorem that taking the quantum state epistemic leads us to a "real" contradiction with the predictions of QM.

Finally, if we briefly refer to the debate between Cabbolet and Hofer-Szabó Ref.[2], the discussion is mainly about whether entangled measurements of PBR (Eqns.(5)-(8)) exist. Hofer-Szabó argues that entangled measurements are routinely made by experimentalists and their existence should not be doubted. According to Hofer-Szabó, Cabbolet has shown that ψ-epistemic models contradict the existence of entangled measurements of the PBR, but he has not shown why ψ-epistemic models should be abandoned in favor of these entangled measurements. However, the existence of entangled measurements is a fact tested by experimentalists. Accordingly, Cabbolet has not been able to construct an argument proving the fallacy of the PBR theorem. We would like to point out that our argument is not related to whether these entangled measurements exist or not. Therefore, it is different from what is being discussed. However, we would like to note that Cabbolet claims that his model provides a counterexample. Thus, if Cabbolet had been able to show that the entangled measurements of PBR do not exist in a ψ-epistemic model that does not contradict QM in terms of one-point measurements, he would have refuted the proof presented in the PBR paper. Therefore, in our opinion, the *logic* of his reasoning is not erroneous. On the other hand, since his model contradicts the predictions of QM for other measurements, his model does not constitute a counterexample. On the contrary, Cabbolet's model confirms the PBR theorem on a special example.

 doi:10.20944/preprints202501.0915.v1

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