Article

Not peer-reviewed version

About Stability of SAIRP Epidemic Model Under Stochastic Perturbations of the Type of Poisson's Jumps

Leonid Shaikhet *

Posted Date: 27 April 2025

doi: 10.20944/preprints202504.2254.v1

Keywords: equilibria; stability in probability; asymptotic mean square stability; Lyapunov function; Poisson's jumps; Linear Matrix Inequality (LMI); numerical simulation; MATLAB

Article

About Stability of SAIRP Epidemic Model Under Stochastic Perturbations of the Type of Poisson's Jumps

Leonid Shaikhet

Department of Mathematics, Ariel University, Ariel 40700, Israel; leonid.shaikhet@usa.net

Abstract: Asymptotic properties of the known SAIRP epidemic model are studied under stochastic perturbations. It is assumed that the stochastic perturbations, given by a combination of the white noise and Poisson's jumps, are proportional to the deviation of a current state of the system under consideration from one of the system equilibria. Sufficient conditions of stability in probability for two different equilibria of the considered system are formulated via a simple linear matrix inequality (LMI), that can be easily studied via MATLAB. Two demonstrative examples illustrate the obtained results via numerical simulation of solutions of the considered system of five nonlinear stochastic differential equations. The research method used here can be applied to a lot of other more complicated models in various applications.

Keywords: equilibria; stability in probability; asymptotic mean square stability; Lyapunov function; Poisson's jumps; linear matrix inequality (LMI); numerical simulation; MATLAB

---

1. Introduction

The so-called SAIRP epidemic model is very popular in research (see, for instance, [1–4] and the references therein). This epidemic model is described by the following system of five ordinary differential equations:

$$\begin{aligned}
\dot{S}(t) &= \Lambda - \left[\beta(1-p(1-u))\frac{\theta A(t)+I(t)}{N(t)} + \psi p(1-u) + \mu\right]S(t) + \omega P(t),\\
\dot{A}(t) &= \beta(1-p(1-u))\frac{\theta A(t)+I(t)}{N(t)}S(t) - (\nu+\mu)A(t),\\
\dot{I}(t) &= \nu A(t) - (\delta+\mu)I(t),\\
\dot{R}(t) &= \delta I(t) - \mu R(t),\\
\dot{P}(t) &= \psi p(1-u)S(t) - (\omega+\mu)P(t).
\end{aligned} \tag{1}$$

Here it is supposed that the total population

$$N(t) = S(t) + A(t) + I(t) + R(t) + P(t),$$

is subdivided into five distinct classes:

- susceptible individuals ($S(t)$);
- asymptomatic infected individuals ($A(t)$);
- active infected individuals ($I(t)$);
- removed (including recovered and deceased) individuals ($R(t)$);
- protected individuals ($P(t)$).

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 27 April 2025 doi:10.20944/preprints202504.2254.v1

The total population $N(t)$ has a variable size, the recruitment rate Λ and the natural death rate μ in (1) are assumed to be constant. The susceptible individuals $S(t)$ become infected by contact with active infected $I(t)$ and asymptomatic infected individuals $A(t)$, at a rate of infection

$$\beta\frac{\theta A(t)+I(t)}{N(t)},$$

where θ represents a modification parameter for the infectiousness of the asymptomatic infected individuals $A(t)$. It is supposed also that all parameters of the system (1) are positive and, besides, $p<1, u<1$.

In [1–3] some properties of the system (1) are studied in the deterministic case. In [4] stability in probability of two equilibria of the system (1) is investigated by the assumption that the considered system is exposed to stochastic perturbations of the white noise type [5,6].

In particular, in [4] it is shown that the equilibria of the system (1) are defined by the system of five algebraic equations

$$\begin{aligned}\Lambda-\left[\beta(1-p(1-u))\frac{\theta A+I}{N}+\psi p(1-u)+\mu\right]S+\omega P=0,\\ \beta(1-p(1-u))\frac{\theta A+I}{N}S-(\nu+\mu)A=0,\\ \nu A-(\delta+\mu)I=0,\\ \delta I-\mu R=0,\\ \psi p(1-u)S-(\omega+\mu)P=0,\end{aligned}\tag{2}$$

with the two solutions:

1) disease-free equilibrium

$$E_0^*=(S_0^*,A_0^*,I_0^*,R_0^*,P_0^*)$$

with

$$S_0^*=\frac{(\omega+\mu)\Lambda}{\mu[\omega+\mu+\psi p(1-u)]},$$

$$A_0^*=I_0^*=R_0^*=0,\tag{3}$$

$$P_0^*=\frac{\psi p(1-u)\Lambda}{\mu[\omega+\mu+\psi p(1-u)]},$$

and

2) endemic equilibrium

$$E_+^*=(S_+^*,A_+^*,I_+^*,R_+^*,P_+^*)$$

with

$$\begin{aligned}S_+^*&=\frac{(\omega+\mu)\Lambda}{\mu[\omega+\mu+\psi p(1-u)]}R_0^{-1},\\ A_+^*&=\frac{\Lambda}{\nu+\mu}(1-R_0^{-1}),\\ I_+&=\frac{\nu\Lambda}{(\nu+\mu)(\delta+\mu)}(1-R_0^{-1}),\\ R_+^*&=\frac{\delta\nu\Lambda}{\mu(\nu+\mu)(\delta+\mu)}(1-R_0^{-1}),\\ P_+^*&=\frac{\psi p(1-u)\Lambda}{\mu[\omega+\mu+\psi p(1-u)]}R_0^{-1},\end{aligned}\tag{4}$$

 doi:10.20944/preprints202504.2254.v1

where the basic reproduction number

$$R_0 = \frac{\beta(1-p(1-u))(\theta(\delta+\mu)+\nu)(\omega+\mu)}{(\nu+\mu)(\delta+\mu)(\omega+\mu+\psi p(1-u))} > 1. \tag{5}$$

Note also that, summing all equations of the system (2), we obtain $N^* = \dfrac{\Lambda}{\mu}$ for the both equilibria (3) and (4).

Below, stability of the both equilibria is studied by the assumption that the system (1) is exposed to stochastic perturbations, given by a combination of the white noise and Poisson's jumps, which are directly proportional to the deviation of the state of the system (1) from one of the equilibrium.

2. Stochastic Perturbations

Let $\{\Omega, \mathfrak{F}, \mathbf{P}\}$ be a complete probability space, $\{\mathfrak{F}_t, t \geq 0\}$ be a nondecreasing family of sub-σ-algebras of $\mathfrak{F}$, i.e., $\mathfrak{F}_{t_1} \subset \mathfrak{F}_{t_2} \subset \mathfrak{F}$ for $t_1 < t_2$, $\mathbf{E}$ be the mathematical expectation with respect to the measure $\mathbf{P}$.

Let us suppose that the system (1) is exposed to stochastic perturbations of the type of

$$\xi_i(t) = \sigma_i w_i(t) + \gamma_i \tilde{\nu}_i(t), \quad i = 1, ..., 5, \tag{6}$$

where σ_i and γ_i are arbitrary constants,

$$\tilde{\nu}_i(t) = \nu_i(t) - \lambda_i t, \quad i = 1, ..., 5,$$

$w_i(t)$ and $\nu_i(t)$ are respectively $\mathfrak{F}_t$-measurable and mutually independent the Wiener and the Poisson processes, $\mathbf{E}\nu_i(t) = \lambda_i t$, $\lambda_i > 0$ [5–8].

Remark 1. *Note that the Wiener processes describe continuous stochastic perturbations of the Brownian motion type, while the Poisson processes describe stochastic perturbations of the jumps type. In [4] in the similar problem stochastic perturbations are considered in the form (6) with $\gamma_i = 0$, i.e., without the Poisson jumps.*

Let us suppose also that the stochastic perturbations (6) are directly proportional to the deviation of the system state $(S(t), A(t), I(t), R(t), P(t))$ from one of the equilibria $(S^*, A^*, I^*, R^*, P^*)$. As a result we obtain the system of stochastic differential equations [5,6]

$$\begin{aligned}
dS(t) &= \left[\Lambda - \left(\beta(1-p(1-u))\frac{\theta A(t) + I(t)}{N(t)}\psi p(1-u) + \mu\right)S(t) + \omega P(t)\right]dt \\
&\quad + (S(t) - S^*)d\xi_1(t), \\
dA(t) &= \left[\beta(1-p(1-u))\frac{\theta A(t) + I(t)}{N(t)}S(t) - (\nu+\mu)A(t)\right]dt \\
&\quad + (A(t) - A^*)d\xi_2(t), \\
dI(t) &= [\nu A(t) - (\delta+\mu)I(t)]dt + (I(t) - I^*)d\xi_3(t), \\
dR(t) &= [\delta I(t) - \mu R(t)]dt + (R(t) - R^*)d\xi_4(t), \\
dP(t) &= [\psi p(1-u)S(t) - (\omega+\mu)P(t)]dt + (P(t) - P^*)d\xi_5(t).
\end{aligned} \tag{7}$$

Note that the equilibrium $(S^*, A^*, I^*, R^*, P^*)$ of the deterministic system (1) is also the solution of the system of stochastic differential equations (7). Stochastic perturbations of this type were first proposed in [9] for SIR epidemic model and later also for some other mathematical models in various applications (see, for instance, [10–14] and the references therein).

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 27 April 2025 doi:10.20944/preprints202504.2254.v1

3. Linear Approximation

Consider the nonlinear differential equation

$$\dot{x}(t) = F(x(t)), \tag{8}$$

where $x(t) \in \mathbf{R}^n$ and the equation $F(x) = 0$ has a solution x^* that is an equilibrium of the differential Equation (8). Using the new variable $y(t) = x(t) - x^*$, represent the Equation (8) in the form

$$\dot{y}(t) = F(x^* + y(t)). \tag{9}$$

It is clear that stability of the zero solution of the Equation (9) is equivalent to stability of the equilibrium x^* of the Equation (8).

Let $J_F = \left\| \frac{\partial F_i}{\partial x_j} \right\|$, $i, j = 1, ..., n$, be the Jacobian matrix of the function $F = \{F_1, ..., F_n\}$ and $\lim_{|y|\to 0} \frac{|o(y)|}{|y|} = 0$, where $|y|$ is the Euclidean norm in $\mathbf{R}^n$. Using Taylor's expansion in the form

$$F(x^* + y) = F(x^*) + J_F(x^*)y + o(y)$$

and the equality $F(x^*) = 0$, we obtain the linear approximation

$$\dot{z}(t) = J_F(x^*)z(t) \tag{10}$$

of the nonlinear differential Equation (9). So, a condition for the asymptotic stability of the zero solution of the linear Equation (10) is also a condition for the local stability of the equilibrium x^* of the initial nonlinear Equation (8).

To construct the linear approximation of the system (7) let us put

$$\begin{aligned} x(t) &= (S(t), A(t), I(t), R(t), P(t))', \\ x^* &= (S^*, A^*, I^*, R^*, P^*)', \\ y(t) &= x(t) - x^*, \\ N^* &= S^* + A^* + I^* + R^* + P^*. \end{aligned} \tag{11}$$

Here and everywhere below $'$ is the sign of transpose.

Representing the system (1) in the form (8) and calculating the Jacobian matrix, we obtain the linear part of the system (7) in the form

$$dz(t) = Az(t)dt + \sum_{i=1}^{5} B_i z(t) dw_i(t) + \sum_{i=1}^{5} C_i z(t) d\tilde{v}_i(t), \tag{12}$$

where $z(t) \in \mathbf{R}^5$, B_i and C_i are the 5×5-matrices with all zero elements besides of respectively $b_{ii} = \sigma_i$ and $c_{ii} = \gamma_i$, $i = 1, ..., 5$.

Note that for $C_i = 0$, $i = 1, ..., 5$, the linear Equation (12) was obtained in [4] with

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ 0 & \nu & -(\delta + \mu) & 0 & 0 \\ 0 & 0 & \delta & -\mu & 0 \\ \psi p(1-u) & 0 & 0 & 0 & -(\omega + \mu) \end{bmatrix}, \tag{13}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 27 April 2025 doi:10.20944/preprints202504.2254.v1

where

$$\begin{aligned}
a_{11} &= -\left[\beta(1-p(1-u))\frac{\theta A^* + I^*}{N^*}\left(1-\frac{S^*}{N^*}\right) + \psi p(1-u) + \mu\right],\\
a_{12} &= -\beta(1-p(1-u))\frac{S^*}{N^*}\left(\theta - \frac{\theta A^* + I^*}{N^*}\right),\\
a_{13} &= -\beta(1-p(1-u))\frac{S^*}{N^*}\left(1 - \frac{\theta A^* + I^*}{N^*}\right),\\
a_{14} &= \beta(1-p(1-u))\frac{S^*(\theta A^* + I^*)}{(N^*)^2},\\
a_{15} &= a_{14} + \omega,
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
a_{21} &= \beta(1-p(1-u))\frac{\theta A^* + I^*}{N^*}\left(1-\frac{S^*}{N^*}\right),\\
a_{22} &= \beta(1-p(1-u))\frac{S^*}{N^*}\left(\theta - \frac{\theta A^* + I^*}{N^*}\right) - (\nu + \mu),\\
a_{23} &= \beta(1-p(1-u))\frac{S^*}{N^*}\left(1 - \frac{\theta A^* + I^*}{N^*}\right),\\
a_{24} &= a_{25} = -\beta(1-p(1-u))\frac{S^*(\theta A^* + I^*)}{(N^*)^2}.
\end{aligned} \tag{15}$$

In particular, for the equilibrium E_0^* the elements (14) and (15) of the matrix (13) are respectively

$$\begin{aligned}
a_{11} &= -(\psi p(1-u) + \mu),\\
a_{12} &= -\theta\beta(1-p(1-u))\frac{S^*}{N^*},\\
a_{13} &= -\beta(1-p(1-u))\frac{S^*}{N^*},\\
a_{14} &= 0, \qquad a_{15} = \omega,
\end{aligned} \tag{16}$$

and

$$\begin{aligned}
a_{21} &= a_{24} = a_{25} = 0,\\
a_{22} &= \theta\beta(1-p(1-u))\frac{S^*}{N^*} - (\nu + \mu),\\
a_{23} &= \beta(1-p(1-u))\frac{S^*}{N^*}.
\end{aligned} \tag{17}$$

Remark 2. *Let the function $V(z)$, $z \in \mathbf{R}^5$, has two derivatives $\nabla V(z)$ and $\nabla^2 V(z)$. The generator L of the Equation (12) has the form [5,6,10]*

$$\begin{aligned}
LV(z) =& (\nabla V(z))' Az + \frac{1}{2}\sum_{i=1}^{5} z' B_i \nabla^2 V(z) B_i z\\
&+ \sum_{i=1}^{5} \lambda_i [V(z + C_i z) - V(z) - (\nabla V(z))' C_i z].
\end{aligned} \tag{18}$$

4. Stability

Definition 1. *Put*

$$\begin{aligned}
y(t) &= (S(t), A(t), I(t), R(t), P(t)) - (S^*, A^*, I^*, R^*, P^*)\\
&= (S(t) - S^*, A(t) - A^*, I(t) - I^*, R(t) - R^*, P(t) - P^*).
\end{aligned}$$

The solution $(S^*, A^*, I^*, R^*, P^*)$ of the system (7) is called stable in probability if for any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ there exists $\delta > 0$ such that $y(t)$ satisfies the condition $\mathbf{P}\{\sup_{t\geq 0} |y(t)| > \varepsilon_1\} < \varepsilon_2$ for any $y(0)$, such that $\mathbf{P}\{|y(0)| < \delta\} = 1$.

Definition 2. *The zero solution of the Equation (12) is called:*

- *mean square stable if for each $\varepsilon > 0$ there exists a $\delta > 0$ such that $\mathbf{E}|z(t)|^2 < \varepsilon$, $t \geq 0$, provided that $\mathbf{E}|z(0)|^2 < \delta$;*
- *asymptotically mean square stable if it is mean square stable and for each initial value $z(0)$, such that $\mathbf{E}|z(0)|^2 < \infty$, the solution $z(t)$ of the Equation (12) satisfies the condition $\lim_{t\to\infty} \mathbf{E}|z(t)|^2 = 0$.*

Remark 3. *It is known [10] that sufficient conditions for asymptotic mean square stability of the zero solution of the linear part of a stochastic nonlinear system with the order of nonlinearity higher than one at the same time are sufficient conditions for stability in probability of the solution of the initial nonlinear system. So, for investigation of stability in probability of the equilibrium $(S^*, A^*, I^*, R^*, P^*)$ of the nonlinear system (7) it is enough to get conditions for asymptotic mean square stability of the zero solution of the linear Equation (12).*

Theorem 1 ([10])**.** *Let there exist a function $V(z)$ and positive constants c_1, c_2, c_3, such that the following conditions hold:*

$$\mathbf{E}V(z(t)) \geq c_1\mathbf{E}|z(t)|^2, \quad \mathbf{E}V(z(0)) \leq c_2|z(0)|^2,$$
$$\mathbf{E}LV(z(t)) \leq -c_3\mathbf{E}|z(t)|^2.$$

Then the zero solution of the Equation (12) is asymptotically mean square stable.

Theorem 2. *Let for the matrices A, B_i and C_i, $i = 1, ..., 5$, of the Equation (12) there exists a positive definite matrix Q, such that the following LMI*

$$QA + A'Q + \sum_{i=1}^{5}(B_i'QB_i + \lambda_i C_i'QC_i) < 0 \tag{19}$$

holds. Then the equilibrium $(S^*, A^*, I^*, R^*, P^*)$ of the system (7) is stable in probability.

Proof. Using the generator (18) for the Lyapunov function $V(z) = z'Qz$, $Q > 0$, we have

$$\begin{aligned} LV(z) =& 2z'QAz + \sum_{i=1}^{5} z'B_i'QB_iz + \sum_{i=1}^{5} \lambda_i z'C_i'QC_iz \\ =& z'\left[QA + A'Q + \sum_{i=1}^{5}\left(B_i'QB_i + \lambda_i C_i'QC_i\right)\right]z. \end{aligned} \tag{20}$$

So, if the LMI (19) holds then via (20)

$$LV(z) \leq -c|z|^2$$

for some $c > 0$ and, therefore, via Theorem 1 the zero solution of the linear stochastic differential Equation (12) is asymptotically mean square stable.

Via Remark 3 it means that the appropriate equilibrium $(S^*, A^*, I^*, R^*, P^*)$ of the nonlinear system (7) is stable in probability. The proof is completed. □

5. Numerical Simulations

5.1. Difference Analogue

For numerical simulation of solutions of the system (7) let us construct the difference analogue of this system. Put

$$\begin{gathered} t_j = \Delta j, \quad \Delta > 0, \\ S_j = S(t_j), \quad A_j = A(t_j), \quad I_j = I(t_j), \quad R_j = R(t_j), \quad P_j = P(t_j), \\ w_{i,j} = w_i(t_j), \quad \nu_{i,j} = \nu_i(t_j), \quad i = 1, ..., 5, \quad j = 0, 1, 2, ... \end{gathered} \tag{21}$$

 doi:10.20944/preprints202504.2254.v1

Via (21) the difference analogue of the system (7) takes the form

$$
\begin{aligned}
S_{j+1} &= S_j + \left[\Lambda - \left(\beta(1-p(1-u))\frac{\theta A_j + I_j}{N_j} + \psi p(1-u) + \mu\right)S_j + \omega P_j\right]\Delta \\
&\quad + (S_j - S^*)[\sigma_1(w_{1,j+1} - w_{1,j}) + \gamma_1(\nu_{1,j+1} - \nu_{1,j} - \lambda_1\Delta)], \\
A_{j+1} &= A_j + \left[\beta(1-p(1-u))\frac{\theta A_j + I_j}{N_j}S_j - (\nu+\mu)A_j\right]\Delta \\
&\quad + (A_j - A^*)[\sigma_2(w_{2,j+1} - w_{2,j}) + \gamma_2(\nu_{2,j+1} - \nu_{2,j} - \lambda_2\Delta)], \\
I_{j+1} &= I_j + [\nu A_j - (\delta+\mu)I_j]\Delta \\
&\quad + (I_j - I^*)[\sigma_3(w_{3,j+1} - w_{3,j}) + \gamma_3(\nu_{3,j+1} - \nu_{3,j} - \lambda_3\Delta)], \\
R_{j+1} &= R_j + [\delta I_j - \mu R_j]\Delta \\
&\quad + (R_j - R^*)[\sigma_4(w_{4,j+1} - w_{4,j}) + \gamma_4(\nu_{4,j+1} - \nu_{3,j} - \lambda_4\Delta)], \\
P_{j+1} &= P_j + [\psi p(1-u)S_j - (\omega+\mu)P_j]\Delta \\
&\quad + (P_j - P^*)[\sigma_5(w_{5,j+1} - w_{5,j}) + \gamma_5(\nu_{5,j+1} - \nu_{5,j} - \lambda_5\Delta)], \\
& j = 0, 1, 2, \ldots \ .
\end{aligned}
\tag{22}
$$

5.2. Examples

Here two demonstrative numerical examples are considered.

Example 1. *Putting*

$$
\begin{gathered}
\Lambda = 15, \quad \mu = 1, \quad \theta = 1, \quad \psi = 0.4, \quad \nu = 0.15, \\
\delta = 0.033, \quad \omega = 0.0013, \quad p = 0.7, \quad u = 0.3, \quad \beta = 1.5,
\end{gathered}
\tag{23}
$$

from (3) *we obtain* $N_0^* = \dfrac{\Lambda}{\mu} = 15$ *and*

$$
(S_0^*, A_0^*, I_0^*, R_0^*, P_0^*) = (12.5445, 0, 0, 0, 2.4555).
\tag{24}
$$

Via MATLAB it was shown that for the values of the parameters

$$
\begin{gathered}
\sigma_1 = 1.4, \quad \sigma_2 = 0.93, \quad \sigma_3 = 1.2, \quad \sigma_4 = 1.4, \quad \sigma_5 = 1.4, \\
\gamma_i = \lambda_i = 1, \quad i = 1, ..., 5,
\end{gathered}
\tag{25}
$$

the LMI (19) *holds and, therefore, the equilibrium* (24) *is stable in probability.*

In Figure 1 *50 trajectories of the solution of the system* (7)*, obtained via the difference analogue* (22) *with the parameters* (23), (25) *and* $\Delta = 0.06$*, are shown with the initial values*

$$
S(0) = 22, \quad A(0) = 11, \quad I(0) = 4, \quad R(0) = 8, \quad P(0) = 17.
\tag{26}
$$

All trajectories ($S(t)$*-brown,* $A(t)$*-violet,* $I(t)$*-blue,* $R(t)$*-red,* $P(t)$*-green) converge to the stable in probability equilibrium* (24)*.*

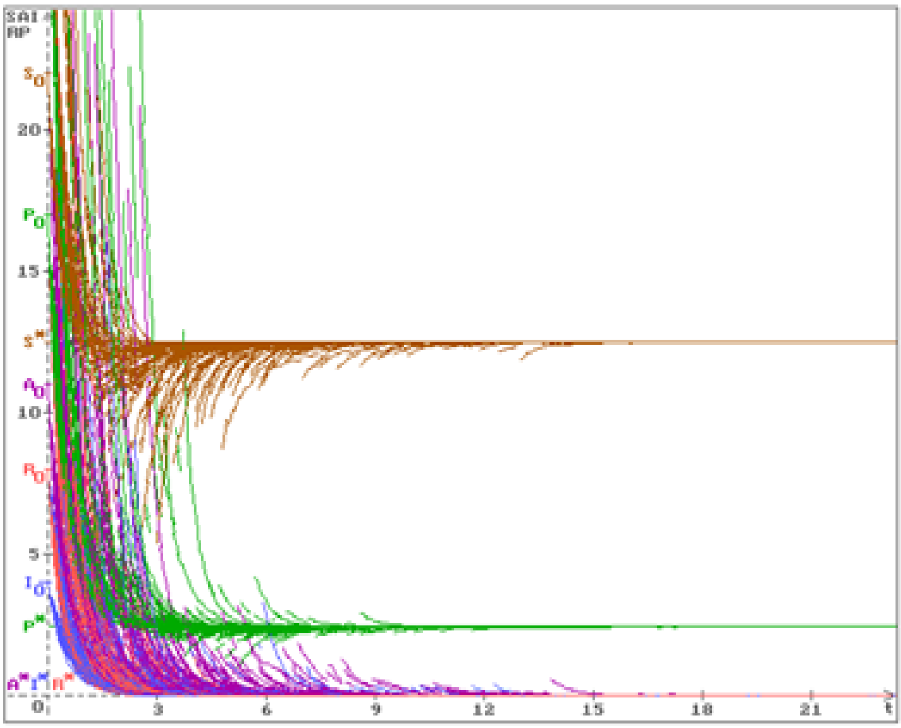

Figure 1. 50 trajectories of the solution of the system (7) with the parameters (23), (25), (26) converge to the stable equilibrium (24).

Example 2. *Putting*

$$\begin{aligned} \Lambda = \ 15, \quad \mu = 1, \quad \theta = 1, \quad \psi = 0.08, \quad \nu = 0.18, \\ \delta = 0.33, \quad \omega = 0.0013, \quad p = 0.4, \quad u = 0.3, \quad \beta = 2, \end{aligned} \tag{27}$$

from (5) *and* (4) *we obtain* $R_0 = 1.3541 > 1$, $N_0^* = \dfrac{\Lambda}{\mu} = 15$ *and*

$$(S_+^*, A_+^*, I_+^*, R_+^*, P_+^*) = (10.8264, 3.3317, 0.4509, 0.1488, 0.2422). \tag{28}$$

Via MATLAB it was shown that for the values of the parameters

$$\begin{aligned} \sigma_1 = 1.5, \quad \sigma_2 = 0.79, \quad \sigma_3 = 1.2, \quad \sigma_4 = 1.3, \quad \sigma_5 = 1.3, \\ \gamma_1 = 1.1, \quad \gamma_2 = 0.95, \quad \gamma_3 = 1.1, \quad \gamma_4 = 0.5, \quad \gamma_5 = 1, \\ \lambda_1 = 1.1, \quad \lambda_2 = 1.59, \quad \lambda_3 = 1.1, \quad \lambda_4 = 1, \quad \lambda_5 = 1.1, \end{aligned} \tag{29}$$

the LMI (19) *holds and, therefore, the equilibrium* (28) *is stable in probability.*

In Figure 2 *50 trajectories of the solution of the system* (7), *obtained via the difference analogue* (22) *with the parameters* (27), (29) *and* $\Delta = 0.06$, *are shown with the initial values are shown with the initial values*

$$S(0) = 7, \quad A(0) = 4.5, \quad I(0) = 9, \quad R(0) = 5.5, \quad P(0) = 2.7. \tag{30}$$

All trajectories ($S(t)$*-brown,* $A(t)$*-violet,* $I(t)$*-blue,* $R(t)$*-red,* $P(t)$*-green) converge to the stable in probability equilibrium* (28).

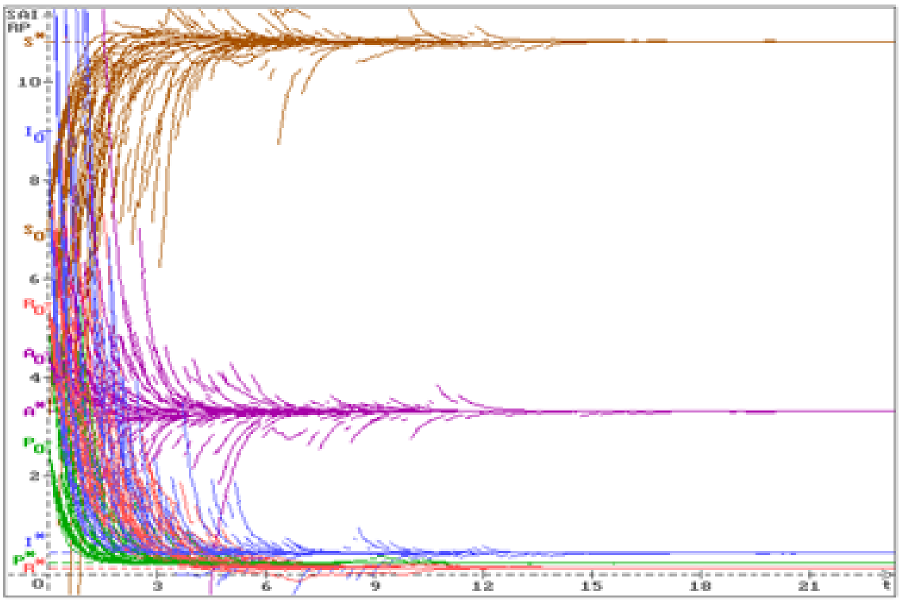

Figure 2. 50 trajectories of the solution of the system (7) with the parameters (27), (29), (30) converge to the stable equilibrium (28).

Remark 4. *Note that for the numerical simulation of trajectories of the Wiener processes* $w_i(t)$*,* $i = 1, ..., 5$*, in Examples 1 and 2 the special algorithm has been used, described in detail in [10] (p.29-31).*

Remark 5. *For the numerical simulation of the Poisson processes* $\nu_i(t)$*,* $i = 1, ..., 5$*, similarly to [7,8] the continuous random variable* ζ_i *is used, uniformly distributed on the interval* $(0, 1)$*:* $\nu_{i,j+1} - \nu_{i,j} = 1$ *if* $\zeta_i < \lambda_i \Delta$ *and* $\nu_{i,j+1} - \nu_{i,j} = 0$ *in the contrary case.*

One can see that in difference from the similar pictures in [4], where only stochastic perturbations of the white noise type are considered, here in Figures 1 and 2 the trajectories of all processes have discontinuities, that is a consequence of jumps in Poisson's processes.

6. Conclusions

Asymptotic properties of the known SAIRP epidemic model, described by a system of five nonlinear differential equations, are studied under stochastic perturbations, given by a combination of the white noise and Poisson's jumps. It is shown that a sufficient condition of stability in probability for two equilibria of the considered system is formulated in the form of a simple linear matrix inequality (LMI) that can be easily studied via MATLAB. Two demonstrative examples illustrate the obtained results via numerical simulation of solutions of the considered system. The research method used here can be applied to a lot of other more complicated models in different applications.

Funding: This research received no external funding.

Data Availability Statement: No new data were created or analyzed in this study. Data sharing is not applicable to this article.

Conflicts of Interest: The author declares no conficts of interest.

References

1. Silva, C.J.; Cantin, G.; Cruz, C.; Fonseca-Pinto, R.; Fonseca, R.P.; Santos, E.S.; Torres, D.F.M. Complex network model for COVID-19: human behavior, pseudo-periodic solutions and multiple epidemic waves, Journal of Mathematical Analysis and Applications, **2021**, Art. 125171, 23 p. Available online: doi:10.1016/j.jmaa.2021.125171

 doi:10.20944/preprints202504.2254.v1

2. Silva, C.J.; Cruz, C.; Torresetal, D.F.M. Optimal control of the COVID-19 pandemic: controlled sanitary deconfinement in Portugal, Scientic Reports, **2021**, 11(1), Art. 3451, 15 p. Available online: doi:10.1038/s41598-021-83075-6.
3. Cantin, G.; Silva, C.J.; Banos, A. Mathematical analysis of a hybrid model: impacts of individual behaviors on the spreading of an epidemic, Networks & Heterogeneous Media, **2022**, 17(3), 333-357. Available online: doi:10.3934/nhm.2022010
4. Shaikhet, L. On stability of one mathematical model of the epidemic spread under stochastic perturbations. Biomedical Journal of Scientific & Technical Research, **2023**, 53(3), 44630-44635. Available online: https://biomedres.us/pdfs/BJSTR.MS.ID.008392.pdf
5. Gikhman, I.I.; Skorokhod, A.V. Stochastic Differential Equations; Springer: Berlin/Heidelberg, Germany, 1972.
6. Gikhman, I.I.; Skorokhod, A.V. The Theory of Stochastic Processes, v.III; Springer: Berlin/Heidelberg, Germany, 1979.
7. Shaikhet, L. Stability of the neoclassical growth model under perturbations of the type of Poisson's jumps: Analytical and numerical analysis. Communications in Nonlinear Science and Numerical Simulation, **2019**, 72, 78-87.
   Available online: https://doi.org/10.1016/j.cnsns.2018.12.008
8. Shaikhet, L. About stabilization by Poisson's jumps for stochastic differential equations. Applied Mathematics Letters, **2024**, 153 Art. 109068, 6 p. Available online: https://authors.elsevier.com/sd/article/S0893-9659(24)00088-0 (accessed on 28 December 2024).
9. Beretta, E.; Kolmanovskii, V.; Shaikhet, L. Stability of epidemic model with time delays influenced by stochastic perturbations, Mathematics and Computers in Simulation, Special Issue "Delay Systems", **1998**, 45(3-4), 269-277.
   Available online: doi:10.1016/S0378-4754(97)00106-7
10. Shaikhet, L. Lyapunov Functionals and Stability of Stochastic Functional Differential Equations; Springer Science & Business Media: Berlin/Heidelberg, Germany, 2013. Available online: https://link.springer.com/book/10.1007/978-3-319-00101-2 (accessed on 28 December 2024)
11. Santonja, F-J., Shaikhet, L. Probabilistic stability analysis of social obesity epidemic by a delayed stochastic model. Nonlinear Analysis: Real World Applications, **2014**, 17, 114-125. Available online: http://dx.doi.org/10.1016/j.nonrwa.2013.10.010
12. Shaikhet, L. Improving stability conditions for equilibria of SIR epidemic model with delay under stochastic perturbations, MDPI, Mathematics, **2020**, 8(8), Art. 1302, 13 p. Available online: https://www.mdpi.com/2227-7390/8/8/1302
13. Shaikhet, L. Some generalization of the method of stability investigation for nonlinear stochastic delay differential equations. MDPI, Symmetry, **2022**, 14(8), Art.1734, 13p. Available online: https://www.mdpi.com/2073-8994/14/8/1734
14. Shaikhet, L. Stability analysis of Cryptosporidiosis disease SIR model under stochastic perturbations. Communications in Mathematical Biology and Neuroscience, **2025**, 2025:7, 15 p. Available online: https://scik.org/index.php/cmbn/article/view/8983