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Dual Token Blockchains †

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Abstract: It is standard for blockchain platforms to issue native tokens, currencies, that users must own to operate within the platform. Some blockchains however decided to issue two tokens, a dual system, one typically for governance and the other for implementing functions on the blockchain, such as executing transactions or smart contracts. Therefore, the two tokens are used for different activities. Typically, owning the governance tokens gives right to receive the other token for free, as a reward for participating to the blockchain decision-making process. However, both tokens can also be traded on some exchange nodes, which means that platform functions could be implemented even without owning governance tokens. In the paper we discuss some economic fundamentals of dual-token blockchain platforms. In particular, we investigate the market *relative attractiveness* of the two tokens by introducing some simple numerical indicators, based on prices, traded and circulating monetary quantities. Such indicators, which are meant to reflect the platform view on the tokens' market desirability, could be computed in real time and used to support the platform's policy making.

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Introduction

In recent years there has been a remarkable growth in the number of blockchain platforms providing a variety of services. Typically, platforms, are endowed with a native currency, token, which is used to perform a variety of functions: implement transactions and smart contracts, obtain voting rights for governance decisions and others. In such blockchains, market demand for the unique token can be considered as an indication of the desirability of the platform. However, with a unique token the market cannot distinguish, for example, between a request for implementing transactions from demand for voting rights.

Some platforms, such as VeChain (2019) and NEO (2023), opted instead for a dual token economy, that is for introducing two different tokens performing different functions. One of the reasons for doing so has been to isolate transaction fees from market oscillations separating the main native token, needed for governance participation, from the one used to pay for operating on the platform (Takemiya, 2019; Hardin-Kotz, 2021; Manolache et al., 2022; Mayer, 2022). However, it is necessary to observe, that if both tokens are traded on the market stabilization of transaction fees may be compromised, since both prices may oscillate. In any case, unlike the one-token blockchains, if traded on the market the dual tokens model can provide more detailed information on the attractiveness and desirability of the different activities, related to the platform. Indeed, some users could be more interested in governance while others only in implementing services on the blockchain.

In this paper, we investigate some economic fundamentals of dual tokens blockchains.

In particular, we shall discuss how to define economic indicators to quantify the relative attractiveness, desirability, of the two tokens. These, we believe, may provide useful numerical representations on the degree of economic success of such tokens and, more in general, of the entire platform. The challenge is to find indicators that are both sufficiently simple while, at the same time, effective in expressing the platform view on how to evaluate the tokens desirability. Indeed, if so, they could be easy to compute as well as useful for supporting the platform policy making.

Some *natural* variables to consider, for constructing such indicators, are the market prices and the exchanged quantities on the market. Yet, several other variables could also be informative and of interest such as the block size, the average transaction size, transaction fees, the average time that tokens are held in the users' wallets and others.

In the work we shall focus on market prices together with circulating and traded quantities, as privileged variables to build indicators representing the platform view on the attractiveness of the two tokens. As a matter of fact, such indicators should reflect what the blockchain thinks is the current degree of desirability of the tokens, so that they could possibly act consequently. The paper is structured as follows. In Section 2 we discuss the *economic attractiveness* of the two tokens, and introduce some economic indicators, based on price and quantities taken separately. In Section 3 we present some indicators where price and quantities are combined. Section 4 concludes the paper.

1. Two-Token Economies

Two-token economies (TTE), as NEO, exhibit some resemblances with standard economies, and with other blockchain platforms, but also differences. We begin the paper discussing in this section some of the main economic features of TTE. With reference to NEO, we shall indicate the two tokens respectively with N(eo) and $G(as)$, where N is the governance token and G the token needed to operate on the platform.

1.1. The economic meaning of N and G

The standard economic interpretation, evaluation, of N and G hinges on their market price, where the price is typically computed in terms of fiat currencies, or of the main cryptocurrencies. Indeed, their price is supposed to embody the degree of *absolute desirability* of the two tokens by the market, that is desirability expressed in terms of a currency *external* to, outside, the platform.

Therefore it seems natural to think that appropriate combinations of the two prices, such as their ratio, can be interpreted to be as an indication of *relative desirability* of the two tokens, that is how much the market is valuing one token as compared to the other.

However, price combinations neither contain explicit information on the exchanged volumes of tokens that induced that price, that is quantities, nor on the number of circulating quantities which may also inform on the tokens' attractiveness. In what follows we introduce the above indicators and discuss their meaning.

2.3. The "absolute desirability" of N and G

Let $t = 0,1,2, \ldots$ be the time index, indicating days, months etc. Furthermore, define $p_{N\$}(t)$ and $p_{G\$}(t)$ as the price of $ in terms of, respectively, N and G, with units of measurement given by, again respectively, $\frac{\$}{N}$ and $\frac{\$}{G}$. That is, how many $ are exchanged against, respectively, one unit of N and one unit of G. In general if C, with $C = 1,..,M$, is a generic fiat currency/cryptocurrency traded in the market, then $p_{NC}(t)$ and $p_{GC}(t)$ indicate the prices of the two tokens with respect to such currency. Clearly, in general, for a pair of different currencies $C' \neq C$, $p_{NC'}(t) \neq p_{NC}(t)$ and $p_{GC'}(t) \neq p_{GC}(t)$ with $p_{GC'}(t) = p_{GC}(t)p_{CC'}(t)$ and $p_{NC'}(t) = p_{NC}(t)p_{CC'}(t)$. Therefore, as indicators of absolute desirability of the tokens, prices $p_{GC}(t)$ and $p_{NC}(t)$ are *not invariant* with respect to the currency.

Thus we define the inverse prices as $p_{CN}(t) = \frac{1}{p_{NC}(t)}$ and $p_{CG}(t) = \frac{1}{p_{GC}(t)}$. If $p_{NC}(t) = 0$ then, according to the standard definition, we call N a *free good*, since N tokens can be obtained against 0 units of C. Alternatively, with any amount of C it is possible to obtain ∞ units of N. Similar considerations hold for $p_{GC}(t) = 0$.

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2.4. The "relative desirability" of N and G

A natural way to evaluate the desirability of N relative to G would be to consider the market price $p_{NG}(t)$, expressed in terms of $\frac{G}{N}$, which takes place in the market. However, exchange nodes may not have a market where N and G are exchanged *directly*, but only through a third *indirectly* market. In what follows we shall focus the discussion on indicators in absence of a *direct* market.

In this case an indicator $\varphi_{NGC}(t)$, based on prices only, can in general be defined as a function

$$\varphi_{NGC}(t) = f(p_{NC}(t), p_{GC}(t)) \quad (1)$$

where $\varphi_{NGC}(t)$ may be required to fulfil some properties. In particular, the following two are quite natural, though not always satisfied:

i) $(Equal\ Desirability\ of\ Tokens)$ $\varphi_{NGC}(t) = 1$, if $p_{NC}(t) = p_{GC}(t)$ for any C.

ii) $(Currency\ Independence\ of\ the\ Indicator)$ $\varphi_{NGC}(t) = \varphi_{NGC'}(t)$ for any $C \neq C'$

As we shall see, property (i) is inspired by the fact that when $p_{NC}(t) = p_{GC}(t)$ then, except for transaction fees, in the market one unit of N trades with one unit of G. Property (ii) means that $\varphi_{GC}(t)$ is independent of C. That is, whatever the currency of reference the index takes the same value.

Notice that (ii) could be reformulated as

(iia) $f(p_{NC}(t), p_{GC}(t)) = f(kp_{NC}(t), kp_{GC}(t))\ for\ any\ k > 0$

since, as above, changing the reference currency amounts to multiply the two prices by the same number.

Posing $k = \frac{1}{p_{GC}(t)}$ it follows that (ii)-(iia) imply

$$f(p_{NC}(t), p_{GC}(t)) = f(\frac{p_{NC}(t)}{p_{GC}(t)}, 1) \quad (2)$$

that is that $f(p_{NC}(t), p_{GC}(t))$ should depend on the prices only through their ratio rather than on their absolute values.

It is immediate to see that the number of indicators satisfying (i) and (ii) is virtually infinite. For example

$$f(p_{NC}(t), p_{GC}(t)) = (\frac{p_{NC}(t)}{p_{GC}(t)})^{(\frac{p_{NC}(t)}{p_{GC}(t)})} \text{ and } f(p_{NC}(t), p_{GC}(t)) = \left(1 + \frac{p_{NC}(t)}{p_{GC}(t)}\right)^2 \frac{1}{4\left(\frac{p_{NC}(t)}{p_{GC}(t)}\right)} \quad (3)$$

are just two examples. However, both of them have a non-obvious and, possibly, non-useful interpretation for evaluating the relative desirability of N and G, a point which we shall further develop below.

Since, according to (i)-(ii) the price ratio $\frac{p_{NC}(t)}{p_{GC}(t)}$ is playing a major role in (1), in the next paragraph we discuss its main features.

2.4.1. The "price ratio"

Let's indicate the two relevant markets for trading the tokens as NC and GC, which are available in one, or more than one, exchange node. Hence disregarding transaction fee the ratio, exchange rate, defined as

$$\varphi_{NGC}(t) = e_{NGC}(t) = \begin{cases} \frac{p_{NC}(t)}{p_{GC}(t)} & if\ p_{NC}(t) \neq 0, p_{GC}(t) \neq 0\ or\ both \\ 1 & if\ p_{N\$}(t)\ and\ p_{G\$}(t) = 0 \end{cases} \quad (4)$$

and expressed in terms of $\frac{G}{N}$, represents the number of G units that can be purchased with 1 unit of N in the market, by selling and buying C. Besides such natural interpretation, notice that (4) satisfies both (i) and (ii), provided the market is *well-functioning*, arbitrage-free. That is, the number of G tokens than can be purchased with 1 unit of N tokens is the same if rather than buying and selling C

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one would buy and sell any other currency C', with $C \neq C'$. Indeed, since $p_{NC'}(t) = p_{NC}(t)p_{CC'}(t)$ and $p_{GC'}(t) = p_{GC}(t)p_{CC'}(t)$ it would immediately follow that $\frac{p_{NC}(t)}{p_{GC}(t)} = \frac{p_{NC'}(t)}{p_{GC'}(t)}$.

For this reason, $e_{NGC}(t)$ from now on will be written as $e_{NG}(t)$.

However, it is worth anticipating that informative as it may be, below we shall discuss that $e_{NG}(t)$ could be an incomplete, partial, indicator since it does not take explicitly into account the volumes of tokens exchanged in the market.

It is also important to point out again that $e_{NG}(t)$ does not derive from the quantities of N and G *directly* traded with each other in the market. Indeed, it could be $e_{NG}(t) > 0$ even if no unit of N is effectively exchanged against any unit of G, *via* any currency C. In this case, $e_{NGC}(t)$ should be interpreted as a *hypothetical* price, if a user wanted to sell one token to buy the other token and viceversa.

Hence the following basic, and intuitive, interpretations of (4) can also be made. Broadly speaking, the larger $e_{NG}(t)$ the *stronger*, the more desirable is N compared to G, while the contrary holds the smaller is $e_{NG}(t)$. Moreover, if $e_{NG}(t) < 1$ then one could claim that G is *more desirable* than N, if $e_{NG}(t) > 1$ that N is *more desirable* than G while in the limiting case of $e_{NG}(t) = 1$ that they are *equally desirable*.

It is appropriate to point out that such interpretation certainly gains value when the circulating number of both tokens is sufficiently large, and the markets (in principle) *thick*, that is exhibiting some meaningful volumes of trades. In that case, market prices and traded quantities can be appropriate signals of tokens desirability. Instead, when the circulating quantity of a token is low, in the extreme case just one unit, then care is required in interpreting the price ratio. Later we shall come back to the issue when introducing quantities.

In the NEO blockchain, intuitively, one would expect $e_{NG}(t) > 1$ because of the intrinsic *asymmetric* relationship between the two tokens. Indeed, G is distributed to $N's$ holders for voting participation, without any *out-of-pocket* payment, while the contrary is not true. That is, G holders cannot obtain N unless they pay for them while N holders can obtain G also without explicitly paying for them. It is true that voting participation requires attention, is time consuming and for this reason it bears an *opportunity cost*. However, this is not an *out-of-pocket*, explicit, disbursement of money.

Finally, posing $f(p_{NC}(t), p_{GC}(t)) = f(\frac{p_{NC}(t)}{p_{GC}(t)}, 1) = g(e_{NG}(t))$, implies that choosing $e_{NG}(t)$ as indicator for the relative desirability of the tokens means to choose $g(e_{NG}(t))$ as an identity function $g\big(e_{NG}(t)\big) = e_{NG}(t)$.

An analogous, simple, indicator to (4), still based only on prices, could be the following

$$\varphi_{NGC}(t) = d_{NGC}(t) = p_{NC}(t) - p_{GC}(t) \quad (5)$$

that is the difference between the amount of currency that, respectively, a single unit of N and a single unit of G can buy. However, as compared to (4), indicator (5) does not satisfy (i)-(ii) and its interpretation requires some care, since $p_{NC}(t)$ is expressed in terms of $\frac{C}{N}$ and $p_{GC}(t)$ in terms of $\frac{C}{G}$. Hence, to make sense of $d_{NGC}(t)$ one may assume that $p_{NC}(t)$ is multiplied by one unit of N and $p_{GC}(t)$ by unit of G, so that $d_{NGC}(t)$ is simply expressed in terms of C. In case prices are the same it is $d_{NGC}(t) = 0$, which corresponds to $e_{NG}(t) = 1$ in (4), while $d_{NG}(t) > 0$ corresponds to $e_{NG}(t) > 1$ and $d_{NG}(t) < 0$ to $e_{NG}(t) < 1$. Based on the above discussion, we find $e_{NG}(t)$ the most intuitive index to discuss the economics of the two tokens, and in the rest of the paper we shall focus on it.

Following the above considerations, in general we expect N to be somehow more *attractive* than G hence $e_{NG}(t) > 1$. Yet, the level of $e_{NG}(t)$ can be affected by several factors, some of which we discuss later.

As an example of the above considerations, data from Coinmarketcap indicate that on $16\,July\,2022$ it was $p_{N\$}(t) = 9.24$ and $p_{G\$}(t) = 2.72$, while on $16\,July\,2023$ it was $p_{N\$}(t) = 9.27$ and $p_{G\$}(t) = 2.72$. Therefore, at both dates, the exchange rate was about $e_{NG}(t) = 3.39$ which suggests that over that period absolute and relative prices remained stable. However, in February 2023 there was an increase in both prices which went up to $p_{N\$}(t) = 14.45$ and $p_{G\$}(t) = 3.17$, with the price ratio going up to $e_{NG}(t) = 4.55$. Hence, these empirical observations are consistent with the intuition that $e_{NG}(t) > 1$ and, moreover, show that even if the price ratio was basically stable across the relevant year, within that year it changed its value. This means that the two tokens market prices may not always move synchronically, and even when they do not necessarily by the same extent.

In what follows we shall discuss how the indications provided by the prices can be complemented with quantities, to extract additional information from the data on the attractiveness of N and G.

2.4. The absolute supply-demand ratio of N and G

To gain further insights on the interpretation of $e_{NGC}(t)$, and discuss how quantities could be informative on the desirability of the two tokens, consider the limiting case $e_{NGC}(t) = 1$, that is $p_{NC}(t) = p_{GC}(t)$, which means that with $1C$ it is possible to buy the same number of N and G units. Notice that, because of the arbitrage activity, it will also have to be $p_{NC'}(t) = p_{GC'}(t)$ for any other currency $C' \neq C$.

Suppose, for example, $C = \$$, that $p_{N\$}(t) = 2 = p_{G\$}(t)$ and assume that both prices are equilibrium prices, that is they equalize supply and demand in the $N\$$ and $G\$$ markets. Before proceeding a note on terminology is in order, to point out that, for example, *at the equilibrium price* the market supply $S_{N(\$)}(t)$ of N against \$, in the $N\$$ market, coincides with the market demand of N against \$, in the same market, that is with the supply $S_{\$(N)}(t)$ of \$ in that market. That is, the exchange of those two quantities effectively takes place at the prevailing price. The same holds for the $G\$$ market.

Consider first the $N\$$ market, where $p_{N\$}(t) = \frac{S_{\$(N)}(t)}{S_{N(\$)}(t)}$. As above, if $p_{N\$}(t) = 0$ it follows that $S_{\$(N)}(t) = 0$ while $S_{N(\$)}(t)$ could be any non-negative number.

Then, of course, $p_{N\$}(t) = 2$ can obtain if $S_{N(\$)}(t) = 10$ and $S_{\$(N)}(t) = 20$, so that $p_{N\$}(t) = \frac{S_{\$(N)}(t)}{S_{N(\$)}(t)} = \frac{20}{10} = 2$ or, alternatively, it could be $p_{N\$}(t) = \frac{S_{\$(N)}(t)}{S_{N(\$)}(t)} = \frac{400}{200} = 2$ etc. Namely, the value $p_{N\$}(t) = 2$ can be generated by, possibly, very different supply and demand volumes in the $N\$$ market, having the same proportion. Indeed, in general, any pair $S_{\$(N)}(t)$ and $S_{N(\$)}(t)$ satisfying the equality

$$S_{\$(N)}(t) = 2S_{N(\$)}(t)$$

would generate the same price $p_{N\$}(t) = 2$.

Likewise, also the value $p_{G\$}(t) = 2$ may in principle be generated by any suitable supply-demand pair, in the $G\$$ market. Suppose now, for instance, that $p_{N\$}(t) = \frac{400}{200} = \frac{4}{2} = p_{G\$}(t)$; can one really claim that, in general, N and G are *equally strong*, or *equally desirable*, in the market? Based on the demand-supply *quantities* providing the two prices the answer may be *dubious*. This is because the prices are simple, effectively exchanged, demand-supply *ratios* and, therefore, do not embody information on the *absolute volume* of the transactions executed.

To take account of volumes, in what follows we introduce some simple quantity indicators which, however, as we shall discuss, they are also not exempt from interpretational ambiguities.

To see why consider for example, *basic* indicators such as the quantity ratios

$$Q_{\$}(t) = \frac{S_{\$(N)}(t)}{S_{\$(G)}(t)} = \frac{400}{4} = 100 = \frac{200}{2} = \frac{S_{N(\$)}(t)}{S_{G(\$)}(t)} = Q_{NG}(t) \quad (6)$$

that is the ratio of the supplied \$, N and G volumes, which could be used to argue about the *desirability* of N as compared to G. That is, quite simply, also the absolute volume of transacted currencies may be informative on the two tokens' attractiveness. By considering the ratio $\frac{400}{4}$ in (6) we observe that, at the market equilibrium, the traded volume of \$ against N is hundred times the traded volume of \$ against G, which may be interpreted as a much larger market *willingness to buy*, desirability for, N.

However, at the same time, in (6) the ratio $\frac{200}{2}$ may also be interpreted as a higher *willingness to sell* N, instead of G, against \$, and so of a stronger preference, by the platform users, for keeping G instead of N. But of course, willingness to sell may also be affected by the tokens held by a user, and in general by the number of tokens circulating in the systems, those issued by the platform. We shall defer the discussion of this point until later.

The previous considerations suggest that the interpretation of quantity ratios may be approached from *two perspectives*: the point of view of the *buyers*, who induce demand for the tokens in terms of $\$$, and that of the *sellers*, who provide the supply for N and G against $\$$. Indeed, in the above example, the buyers seem to be more interested in N while the sellers in G. Moreover, since $p_{N\$}(t) = p_{G\$}(t)$ one may also claim that the preferences, N for the buyers and G for the sellers, are of the same extent, degree.

To further develop the above discussion, based on quantities, consider now the case of $p_{N\$}(t) \neq p_{G\$}(t)$. As an example, suppose again $p_{N\$}(t) = \frac{400}{200} = 2$ but $p_{G\$}(t) = \frac{10}{2} = 5$ so that, according to the price ratio $e_{NG}(t) = \frac{2}{5} < 1,$ we would argue that G is stronger, relatively more desirable, than N.

The interpretation based on the quantity ratios

$$q_{\$}(t) = \frac{400}{10} = 40 < 100 = \frac{200}{2} = q_{NG}(t)$$

would be analogous, but not identical, to the previous one. While $q_{\$}(t) = 40$ suggests that the volume of exchanged $\$$ against N is 40 times larger than the one exchanged against G, the number of N supplied against $\$$ is 100 times the number of G supplied against $\$$. Therefore, one may observe that N is preferred by the *buyers*, G by the *sellers*, however with the latter preference being stronger than the former. That is, the quantity ratios may complement (4) with interesting information on which side of the market can explain the value of $e_{NG}(t)$.

Finally, notice that the left-hand side of (6) is a pure number, since is the ratio of $\frac{\$}{\$}$, while the right-hand side is expressed in terms of $\frac{N}{G}$.

2.5. *Arbitrage: direct vs indirect markets for N and G*

Before proceeding it is worth reminding that the price ratio (4), expressed in terms of $\frac{G}{N}$, *cannot* be interpreted as the quantity of N traded against G, since we assumed no *direct* exchange market for that. It only represents the ratio between the two quantities traded in the market against $\$$. Likewise, in the above example, the ratio $\frac{N}{G} = \frac{200}{2} = 100$ *could not* be considered as the number of N tokens exchanged against G tokens. However, if a direct (NG) exchange market exists, then due to arbitrage activity the price $p_{NG}(t)$ could not differ from $e_{NG}(t)$ and so $p_{NG}(t) = e_{NG}(t)$.

Indeed, suppose $p_{NG}(t) = \frac{1}{100} = \frac{G}{N}$ while $e_{NG}(t) = 1$, with $p_{N\$}(t) = \frac{400}{200} = 2 = \frac{4}{2} = p_{G\$}(t)$. Then a user owning $1G$ could sell it in the NG market to obtain 100 units of N tokens. Subsequently, by selling these $100N$ units against $\$$ she would obtain $200\$$ which, in turn, when sold against G tokens would generate $100G$. Therefore, by doing this the user could obtain a very large number of G tokens with an initial single G token. But of course, by replicating the same procedure more than once the supply of G tokens in the NG direct market will increase, possibly also the supply of N will decrease, and the price $p_{NG}(t)$ will tend to increase. Analogous considerations apply for the other two markets, until the equality $p_{NG}(t) = e_{NG}(t)$ would tend to prevail.

In case a direct market is introduced, with the arbitrage activity inducing

$$p_{NG}(t) = e_{NG}(t)$$

then this *non-arbitrage* equation poses some condition on the traded relevant quantities.

For completeness, in what follows we illustrate the point. Consider the three markets 1) $N\$$, 2) $G\$$ and 3) NG, and indicate with $\$_i, G_i, N_i$, with $i = 1,2,3,$ the quantities of the three currencies exchanged in the three markets, where $G_1 = N_2 = \$_3 = 0$. Finally, suppose $\$_T = \$_1 + \$_2$; $G_T = G_2 + G_3$; $N_T = N_1 + N_3$ are the total quantities of the three currencies exchanged in the three markets as, moreover, for the time being we assume the two tokens are not traded in other markets. Then $p_{NG}(t) = e_{NG}(t)$ implies

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$$p_{NG}(t) = \frac{(G_T - G_2)}{(N_T - N_1)} = \frac{G_3}{N_3} = \frac{\frac{\$_1}{N_1}}{\frac{\$_2}{G_2}} = \frac{\$_1}{N_1}\frac{G_2}{\$_2} = e_{NG}(t) \quad (7)$$

Equation (7) includes many variables so that none of them, alone, could be fully determined unless we fix all the others. Therefore, there could be several, in fact unlimited, combinations of the relevant quantities which can satisfy (7). To gain some insights, below we take as given $r = \frac{\$_1}{\$_2}$, N_T and G_T to investigate the relationship between N_1 and G_2. Indeed, after appropriate rearrangement (7) can be written as

$$N_1 = \frac{rG_2N_T}{(G_T - G_2(1-r))} \quad (8)$$

In absence of arbitrage possibilities, the above expression (8) provides some interesting indications on N_1. First, for any $r > 0$, it is increasing in G_2 and as $G_2 \to G_T$ then $N_1 \to N_T$, while as $G_2 \to 0$ then also $N_1 \to 0$. Additionally, it is increasing in r, converging to N_T as r goes to infinity, and N_T but decreasing in G_T.

Notice that, for given N_1, N_T, G_2 and G_T, in (8) the value of r is the same for any currency C. Indeed, since N_1, N_T, G_2 and G_T are uniquely determined quantities in the market, independently of the third currency, it follows that the ratio r must be the same for any C. For instance, if rather than \$ we would consider € then the ratio $r' = \frac{€_1}{€_2}$ will be such that $r' = \frac{p_{\$€}(t)\$_1}{p_{\$€}(t)\$_2} = r$, where $p_{\$€}(t)$, expressed in terms of $\frac{€}{\$}$, is the price, exchange rate, of € in terms of \$.

As a simple numerical illustration, suppose $r = 1$, $N_T = 1000$ and $G_T = 100$; then (8) would lead to $N_1 = 10G_2$, regardless of the absolute size of $\$_1$ and $\$_2$, since what it counts in (7) is their ratio only. Hence, in this case

$$p_{NG}(t) = \frac{(100 - G_2)}{(1000 - 10G_2)} = \frac{1}{10} \quad (9)$$

Expression (8) is of course an identity which endows $G_2 > 0$ with the freedom to take any value no larger than G_T, leaving indeterminate also the absolute levels of $p_{N\$}(t)$ and $p_{G\$}(t)$.

If $G_2 = 1$ then $G_3 = 99, N_1 = 10$ and $N_3 = 990$. Since $r = 1$ then $\$_1 = \$_2$, so that if $\$_1 = \$_2 = 100$ it follows that $p_{N\$}(t) = 10$ and $p_{G\$}(t) = 100$ while if $\$_1 = \$_2 = 1000$ then $p_{N\$}(t) = 100$ and $p_{G\$}(t) = 1000$. Therefore, the amount of \$ determines the absolute level of the two prices while the arbitrage activity their ratio, which indeed could now inform on the number of G tokens exchanged against N tokens, in the direct market.

To conclude, we extend to more than one currency the above analysis. Suppose there are M currencies, with which the two tokens could be directly exchanged. Hence markets $1,..,M$ are for N against the currencies, markets $(M+1),\dots,2M$ are for G against the same currencies and the $(2M+1)th$ market is the NG market. In particular C, N_C and G_C stand, respectively, for the currency units, the N units and the G units exchanged in market C.

Moreover, for any market $C = 1,,..,M$ where N is traded against the relevant currency, there is a corresponding market $(C+M)$ where G is traded against the same currency.

So, in total, there would be $2M+1$ markets and, assuming non-arbitrage, the following conditions must hold

$$\frac{G_{2M+1}}{N_{2M+1}} = \frac{\frac{C}{N_C}}{(\frac{(C+M)}{G_{C+M}})} \quad with\ C = 1,..,M \ (10)$$

Since the left-hand side of (10) must be the same for all markets it follows that

$$\frac{\frac{C}{N_C}}{(\frac{C+M}{G_{C+M}})} = \frac{\frac{C'}{N_{C'}}}{(\frac{C'+M}{G_{C'+M}})} \quad (11)$$

for any pair of markets $C \neq C'$. As well as for (8), equation (11) suggests that the exchanged quantities are not free to take any value, since they must comply with the proportions *dictated* by the non-arbitrage condition.

2.6. The relative supply-demand ratio of N and G

The tokens' market price, being defined as the ratio between the absolute levels of supply and demand, does not consider the number of circulating tokens. For example, suppose again that in the $N\$$ market prices are $p_{N\$}(t) = \frac{400}{200} = 2 = \frac{4}{2} = p_{G\$}(t)$. Of course, the number of traded N tokens, that is 200, being much larger than the number of traded G tokens, may induce to think that N is more attractive than G for the buyers, and the contrary for the sellers.

However, as for the two tokens' market attractiveness is concerned, such direct comparison between absolute quantities may be deceiving. Indeed, what may be more interesting/informative to consider is the proportion between traded and circulating tokens, where by circulating we meant the total number of token issued by the platform. Therefore if at time t, for instance, the number of N circulating tokens is $N_c(t) = 400000$ and the number of circulating G tokens is $G_c(t) = 200$ then

$$s_{N(\$)}(t) = \frac{S_{N(\$)}(t)}{N_c(t)} = \frac{200}{400000} = \frac{1}{2000} < \frac{2}{200} = \frac{1}{100} = \frac{S_{G(\$)}(t)}{G_c(t)} = s_{G(\$)}(t) \quad (12)$$

That is, the relative number of supplied G tokens $s_{G(\$)}(t)$ would be higher than the relative number of supplied N tokens $s_{N(\$)}(t)$, and the ratio of these two relative quantities equal to

$$q_{NG\$}(t) = \frac{s_{G(\$)}(t)}{s_{N(\$)}(t)} = \frac{0.01}{0.002} = 5 \qquad (13)$$

Notice that such ratio would be a pure number, that is independent of the measurement units, as well as the ratio $q_{\$}(t) = \frac{400}{4} = 100$ between the traded dollars. However relative trades, concerning different currencies, in general differ. Therefore, comparing now the two relative-quantity ratios we observe that $q_{\$}(t) = 100 > 5 = q_{NG\$}(t)$ and so that, despite the price ratio being equal, it seems to suggest that in fact N is more desirable, for both the *buyers* and the *sellers*, than G since it is *relatively* less traded. The relative supplies will be used latter to build combined, price-quality, indicators for the tokens attractiveness. However, prior to doing so it is worth mentioning an additional notion of price as well as discussing how a notion of optimal $\varphi_{NGC}(t)$ may be introduced.

2.7. The "virtual" price of G and N

In the above discussion we took as reference for the economic value of the two tokens their prices against $\$$, when considering indirect exchanges with respect to a generic currency, or the price of N against G in a direct market. Then, the arbitrage activity led to

$$p_{NG}(t) = \frac{p_{N\$}(t)}{p_{G\$}(t)} = e_{NG}(t)$$

The relevant prices $p_{NG}(t), p_{N\$}(t), p_{G\$}(t)$ are all computed in the three *bilateral* markets $NG, N\$, G\$$ on the basis of the demand and supply, hence quantities exchanged, in those markets. However, whether or not a direct GN market exists, it is always possible to compute a ratio between the total number of $N, S_N(t)$, exchanged against all currencies and the total number of G, $S_G(t)$, traded against all currencies. That is, the ratio $v_{NG}(t)$ defined as

$$v_{NG}(t) = \frac{S_G(t)}{S_N(t)}$$

which we call a *virtual price* since, typically, it is not explicitly computed and yet it may also be a useful indicator to evaluate the relative desirability of the two tokens.

To see how informative it may be, as compared to the previous indicators, consider the following very simple example. Suppose there are only two currencies to trade the two tokens with: $\$$ and €. Moreover, assume $p_{N\$}(t) = \frac{S_{\$(N)}(t)}{S_{N(\$)}(t)} = \frac{400}{200} = 2$, $p_{G\$}(t) = \frac{S_{\$(G)}(t)}{S_{G(\$)}(t)} = \frac{4}{2} = 2$ so that $e_{NG}(t) = \frac{p_{N\$}(t)}{p_{G\$}(t)} = \frac{2}{2} = 1 = p_{NG}(t)$. Furthermore, suppose that $p_{N€}(t) = \frac{S_{€(N)}(t)}{S_{N(€)}(t)} = \frac{100}{20} = 5$, $p_{G€}(t) = \frac{S_{€(G)}(t)}{S_{G(€)}(t)} = \frac{20}{4} = 5$ so that

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$e_{NG}(t) = \frac{p_{N€}(t)}{p_{G€}(t)} = \frac{5}{5} = 1 = p_{NG}(t)$. So according to (4), and considering arbitrage activity, the two tokens are equally desirable by the market.

However, computing the *virtual* price we obtain $v_{NG}(t) = \frac{S_G(t)}{S_N(t)} = \frac{2+4}{200+20} = \frac{6}{220} \sim 0.03$ suggesting that G is a stronger token than N, because the total number of traded Gs is much lower. Again, $v_{NG}(t)$ is not a *proper* price, since quantities are supplied and demanded in separate markets and not in a single, global, market. Hence it can only be interpreted as an *hypothetical* price in the following way: if the total traded quantities were exchanged as a whole, rather than on bilateral markets, then $v_{NG}(t)$ would be the equilibrium price. Though not computed in practice, $v_{NG}(t)$ may be informative as a ratio of total quantities traded on the market. The example shows a major difference between the indicators based on bilateral markets and the virtual price. Again, this may be because in $v_{NG}(t)$ we considered absolute instead of relative, to the circulating quantities, exchanged volumes. With relative quantities we may expect a reduction of the difference, as compared to bilateral markets, yet there is no *a-priori* reason to believe that such difference would be completely eliminated.

2.8. The "optimal" level of $\varphi_{GC}(t)$

Upon having defined $\varphi_{GC}(t)$ an additional, interesting, question to ask is whether there is an optimal level of $\varphi_{NGC}(t)$ for the platform to target. This is what we discuss in the section. The starting point is to take a user with one unit of a fiat currency, say $, considering the possibility to buy N tokens or G tokens, or perhaps both. In the first case she would receive $p_{\$N}(t)$ units of N tokens plus $p_{\$N}(t)\rho$ units of G tokens, where ρ, expressed in terms of $\frac{G}{N}$ is the number of G tokens obtained by the user, in a time period, by holding $p_{\$N}(t)$ units of N tokens and participating to governance and voting sessions. If $U(N,G)$ is the user's utility obtained by holding N and G tokens, then in this case her utility would be $U(p_{\$N}(t), p_{\$N}(t)\rho)$. In the second case the user's utility level would be $U(0, p_{\$G}(t))$. It follows that if $U(p_{\$N}(t), p_{\$N}(t)\rho) > U(0, p_{\$G}(t))$ then the user would prefer to buy N.

In particular, assuming $\varphi_{NG}(t) = e_{NG}(t)$, as well as the utility function to be increasing in both arguments, if $p_{\$N}(t)\rho > p_{\$G}(t)$ then purchasing N would be a *dominant* action for the user, that is, if $\rho > e_{NG}(t)$.

Instead, if $U(p_{\$N}(t), p_{\$N}(t)\rho) < U(0, p_{\$G}(t))$ the user will purchase G while if $U(p_{\$N}(t), p_{\$N}(t)\rho) = U(0, p_{\$G}(t))$ she would be indifferent on how to allocate the dollar between the two tokens. Therefore, broadly speaking, if the above represents an *average* user, a representative agent, for the two markets $N\$$ and $G\$$ to be up and running, the condition $U(p_{\$N}(t), p_{\$N}(t)\rho) = U(0, p_{\$G}(t))$ would be likely to prevail. If this is acceptable then, for any given ρ, one hopes to be able to solve the above equality to obtain a relationship between $\varphi_{GC}(t)$ and ρ. Admittedly, this may be a demanding task: yet, conceptually should be the way the follow for a platform and, in some circumstances, an explicit form could be found in a relatively simple way. For example, with a particularly simple form as $U(p_{\$N}(t), p_{\$N}(t)\rho) = p_{\$N}(t) + p_{\$N}(t)\rho$ and $U(0, p_{\$G}(t)) = p_{\$G}(t)$ then, in analogy with the above discussion, it will have to be

$$e_{NG}(t) = 1 + \rho \quad (14)$$

It follows, that once ρ is fixed, if the goal of the platform is to have both markets functioning, then $e_{NG}(t)$ as in (14) would be optimal and market policies should be pursued to target that value. It is clear that a crucial role is played by the users' preferences, and utility functions may be easily more complex than the one above. For example, if

$$U(p_{\$N}(t), p_{\$N}(t)\rho) = \sqrt{p_{\$N}(t) + p_{\$N}(t)\rho}$$

that is when the user would be risk averse with respect to $p_{\$N}(t)$, then it is immediate to verify that the optimal level of $e_{NG}(t)$ depends not only ρ but also on the prices, and it is a much more involved expression than (14).

2. The relative desirability of N and G as a combination of prices and quantities

In the previous sections we discussed some alternative criteria to evaluate the relative attractiveness of the two tokens, based on price and quantity market data, on a separate basis. We have also seen that the suggestions emerging from different criteria may sometimes be consistent, while on other circumstances they could differ. Based on this, in the section we propose some composed indicators, combining prices and quantities to embody the above considerations. We shall then compare such indicators to $e_{NG}(t)$.

Prior to entering the discussion it is useful to introduce the following quantities

$$\pi_{NC} = \begin{cases} \dfrac{s_{N(C)}(t)}{s_{N(C)}(t) + s_{G(C)}(t)} & if\ s_{N(C)}(t) \neq 0, s_{G(C)}(t) \neq 0\ or\ both \\ 1 & if\ s_{N(C)}(t), s_{G(C)}(t) = 0 \end{cases} \quad (15)$$

and

$$\pi_{GC} = \begin{cases} 1 - \pi_{NC} = \dfrac{s_{G(C)}(t)}{s_{N(C)}(t) + s_{G(C)}(t)} & if\ s_{N(C)}(t) \neq 0, s_{G(C)}(t) \neq 0\ or\ both \\ 1 & if\ s_{N(C)}(t), s_{G(C)}(t) = 0 \end{cases} \quad (16)$$

The above, (15) and (16), expressions inform on the relative proportion of traded tokens, against currency C. For this reason, and also because they are pure numbers, units of measurement free, they could be conveniently used as weights in indicators evaluating the relative attractiveness of the two tokens. For completeness, it is appropriate to point out that we should have written $\pi_{N\$}$ as $\pi_{N\$}(t)$ and $\pi_{G\$}$ as $\pi_{G\$}(t)$, since both of them are time dependent. However, to save on notation we omitted the time index, although we should keep in mind that (15) and (16), as well as the quantities, vary with time. Notice also that, in general, $\pi_{NC} \neq \pi_{NC'}$ and $\pi_{GC} \neq \pi_{GC'}$ with $C \neq C'$ which implies that as weights they are currency specific. Hence, one simple way to obtain a weight which is currency independent could be to take

$$\pi_N = \frac{\sum_{C=1}^{M} \pi_{NC}}{M};\ \pi_G = 1 - \pi_N = \frac{\sum_{C=1}^{M} \pi_{GC}}{M} \quad (17)$$

Considering the above weights, perhaps an extended class of indicators may be indicated as

$$\varphi_{NG}(t) = f(p_{NC}(t), p_{GC}(t), \pi_N, \pi_G) \quad (18)$$

which would combine prices and quantities to inform on the relative attractiveness of the two tokens.

One may also require (18) to satisfy properties

iii) $(Equal\ Desirability\ of\ Tokens)$ $\varphi_{NGC}(t) = 1$, if $p_{NC}(t) = p_{GC}(t)$ for any C.

iv) $(Currency\ Independence\ of\ the\ Indicator)$ $\varphi_{NGC}(t) = \varphi_{NGC'}(t)$ for any $C \neq C'$

Since π_N, π_G are currency independent, also in this case (iv) could be re-written as

(iva) $f(kp_{NC}(t), kp_{GC}(t), \pi_N, \pi_G) = f(p_{NC}(t), p_{GC}(t), \pi_N, \pi_G) = f(e_{NG}(t), 1, \pi_N, \pi_G)$ for all $k > 0$

An additional, desirable, property may be the following

(v) $(Quantity\ Independence) f(p_{NC}(t), p_{GC}(t), \pi_N = \pi = \pi_G) = f(p_{NC}(t), p_{GC}(t))$

While we already commented on (iii)-(iv), property (v) is new. It requires that quantity does not affect the value of the indicator only if the weights π_N, π_G, based on relative trades, are equal.

Before proceeding it is appropriate to anticipate that, with quantities, indicators could take a dual perspective: the one of the sellers, suppliers, of tokens and the one of the buyers who demand

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tokens. The supplier's perspective represents the tokens' owner desirability; that is, how much she's willing to keep or get rid of the tokens. Buyers' instead represent the non-owners tokens' desirability. For this reason, it seems intuitive for a proper discussion of the issue to consider both perspectives. In what follows we shall start with the sellers, assuming $\pi_N, \pi_G > 0$

3.1. Combined price-quantity indicators: the sellers' perspective

It is immediate to envisage that, in principle, one could conceive an infinite number of indicators, combining prices and quantities, to evaluate the two tokens' relative attractiveness. However, since such indicators should represent the platform's view on the tokens' desirability, in principle they could be built, by the blockchain, comparing prices and quantities in the following ways. For example, a blockchain may envisage the pair of market prices $p_{N\$}(t) = 2, p_{G\$}(t) = 1$, together with the pair of traded quantities $\pi_N = \frac{2}{3}, \pi_G = \frac{1}{3}$, so that $e_{NG}(t) = 2$ and $\frac{\pi_G}{\pi_N} = \frac{1}{2}$, as a situation for which the sellers consider the two tokens equally attractive. However, the same blockchain may not consider the situation $\pi_N = \frac{3}{4}, \pi_G = \frac{1}{4}$, $e_{NG}(t) = 3$ and $\frac{\pi_G}{\pi_N} = \frac{1}{3}$ as indicating equally attractive tokens. Other blockchains in turn may have different views. That is, it would be possible to evince platforms' view on tokens relative desirability by comparing alternative situations, as in the above example. Such views could be conveniently summarised by numerical indicators, which would define the price-quantity situations for which tokens may, or may not, considered as equally desirable.

Indicators could be useful tools for the blockchain policy making. For example, suppose an indicator suggests that the sellers are having a much higher preference for the N tokens, as compared to the G tokens. This situation may be considered inappropriate by the platform being a possible sign of power concentration in governance, as well as a limited interest by the sellers for operating on the blockchain. In this case, the platform may intervene on the markets, for example by increasing the supply of N, in so doing lowering its market price, possibly the indicator value and mitigating the risk of power concentration. The platform may also react by increasing the range and quality of the services provided on the blockchain, this way trying to reduce the sales of G.

As an example, two simple indicators combining prices and quantities that may perhaps capture some platform's view on the tokens relative desirability are the following

(*Linearly Weighted Exchange Rate,* $LWE_{NGCs}(t)$; *the sellers' perspective*)

$$LWE_{NGCs}(t) = \left(\frac{\pi_G}{\pi_N}\right)\left(\frac{p_{N\$}(t)}{p_{G\$}(t)}\right) = \left(\frac{\pi_G}{\pi_N}\right) e_{NG}(t) \quad (19)$$

and

(*Exponentially Weighted Exchange Rate,* $EW_{NGs}(t)$; *the sellers' perspective*)

$$EWE_{NGCs}(t) = \begin{cases} e_{NG}(t)^{\left(\frac{\pi_G}{\pi_N}\right)} & if\ e_{NG}(t) \geq 1 \\ e_{NG}(t)^{\left(\frac{\pi_N}{\pi_G}\right)} & if\ e_{NG}(t) < 1 \end{cases} \quad (20)$$

The above indicators embody the same information, however composed differently to put different emphasis on the role of prices and relative traded quantities, which in fact reflect their different roles according to the platform's view. While market prices reflect the token holders' relation with currencies *outside* the platform, the relative traded quantities reflect the token holders relation *within* the platform, namely with respect to the circulating stock of domestic currencies. For these reasons they inform the blockchain on the *external* and *internal* tokens' desirability. Notice however that external and internal levels are not independent of each other, since they linked by the quantity of token traded against a currency, which indeed appears in both expressions.

Prior to considering the essential features of (19) and (20) it is worth stressing a point which they share. Namely that both indicators increase with π_G and decrease with π_N. The intuition is simple; for the sellers, N can be relatively more attractive than G not only if $p_{N\$}(t) > p_{G\$}(t)$ but also if G

is relatively more traded than N. Indeed, this means that tokens' holders prefer to sell a larger share of circulating G tokens, rather than of N tokens.

Below we are going to discuss some of the main differences between (19) and (20).

As for (19), property (iii) is, in general, not satisfied since $LWE_{NG\$s}(t)$ could differ from 1, even when $e_{NG}(t) = 1$, unless $\pi_N = \pi_G$. Hence, since when both prices and traded quantities are symmetric it is $LWE_{NG\$s}(t) = 1$, also in this case it may be natural for the platform to take the unit value of the indicator as the level signalling equal desirability of the two tokens. However, as compared to the case in which prices only are considered, $e_{NG}(t) = 1$ is neither a necessary nor a sufficient condition for $LWE_{NG\$s}(t) = 1$. Indeed, any combination of quantity and price ratios satisfying

$$\left(\frac{\pi_G}{\pi_N}\right) = \frac{1}{e_{NG}(t)}$$

will testify the same attractiveness, for the platform, of the two tokens. Hence, for example, if $e_{NG}(t) = 2$ and $\frac{\pi_G}{\pi_N} = \frac{1}{2}$ then the fact that one unit of N can be exchanged with two Gs is counterbalanced by the fact that N is traded twice as much as G, which means that, in terms of (relative) sales, token owners preferred to sell N than G. As a follow up to the above considerations, we interpret $LWE_{NG\$s}(t) > 1$ as the sellers' finding N more attractive than G and the opposite for $LWE_{NG\$s}(t) < 1$.

Before proceeding it is interesting to point out that, for example, values such as $e_{NG}(t) = 2$ and $\frac{\pi_G}{\pi_N} = \frac{1}{2}$, that is with a high $e_{NG}(t)$ and a low $\frac{\pi_G}{\pi_N}$, and the opposite, providing $LWE_{NG\$s}(t) = 1$, can effectively take place, because they refer to different situations. Indeed suppose, for simplicity, there is just one currency $C = \$$ to exchange the tokens with, that $p_{N\$}(t) = \frac{S_{\$(N)}(t)}{S_{N(\$)}(t)} = \frac{180}{90} = 2$, $p_{G\$}(t) = \frac{S_{\$(G)}(t)}{S_{G(\$)}(t)} = \frac{10}{10} = 1$ so that $e_{NG}(t) = 2$. Additionally, assume $N_c(t) = 450$ and $G_c(t) = 100$ so that $s_{N(\$)}(t) = \frac{90}{450} = \frac{1}{5}$ and $s_{G(\$)}(t) = \frac{10}{100} = \frac{1}{10}$ which implies $\frac{\pi_G}{\pi_N} = \frac{1}{2}$.

In general, for any given level of $LWE_{NG\$s}(t) = L > 0$ equations of the type

$$\left(\frac{\pi_G}{\pi_N}\right) = \frac{L}{e_{NG}(t)} \quad (21)$$

represent the so called *iso-score* curves in the two-dimensional space $\left(\frac{\pi_G}{\pi_N}\right), e_{NG}(t)$, that is the set of pairs providing the same level of score L.

To summarise, (19) reflects the view of a platform for which prices are as important as quantities to establish the tokens attractiveness.

Instead, $EWE_{NGCs}(t)$ satisfies all the properties (iii)-(iv) and (v). For this reason $EWE_{NGCs}(t) = 1$ if and only if $e_{NG}(t) = 1$ regardless of the ratio $\frac{\pi_G}{\pi_N} > 0$. In this case, the *iso-score* curves $EWE_{NGCs}(t) = E$ would be given by the expression

$$\left(\frac{\pi_G}{\pi_N}\right) = \begin{cases} \frac{\log E}{\log e_{NG}(t)} & \text{if } e_{NG}(t) > 1 \\ (0, \infty) & \text{if } e_{NG}(t) = 1 \\ \frac{\log e_{NG}(t)}{\log E} & \text{if } e_{NG}(t) < 1 \end{cases} \quad (22)$$

where (22), with $e_{NG}(t) < 1$, is indeed positive since in this case also $E < 1$. Therefore, a main difference between (19) and (20) is that, unlike $EWE_{NGCs}(t)$, $LWE_{NGCs}(t)$ can take any value for any level $e_{NG}(t)$, as long as $\left(\frac{\pi_G}{\pi_N}\right)$ compensates appropriately, while $EWE_{NGCs}(t) > 1$ if and only $e_{NG}(t) > 1$, $EWE_{NGCs}(t) = 1$ if and only $e_{NG}(t) = 1$ and $EWE_{NGCs}(t) < 1$ if and only $e_{NG}(t) < 1$. So, if also $EWE_{NGCs}(t) = 1$ is taken as a threshold for equal attractiveness, with $EWE_{NGCs}(t) > 1$ indicating that N is more attractive than G, and the opposite for $EWE_{NGCs}(t) < 1$, then it follows that tokens' attractiveness is only determined by the value of $e_{NG}(t)$, with the level of $\left(\frac{\pi_G}{\pi_N}\right)$ affecting only the degree of desirability, but not its direction.

For the above reasons, (20) would reflect the view of a platform which considers the exchange rate as the main source of information to establish which token is more attractive, while the degree of desirability is established by the traded quantities.

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Obviously, in general, neither $LWE_{NG\$s}(t)$ nor $EWE_{NG\$s}(t)$ could not be interpreted in terms of number of G exchanged against N but rather as a function of it. As a matter of fact, the difference between the indicators and the exchange rate could be thought of as the additional contribution of the quantities, to the exchange rate, in forming the indicator's value.

In particular, one can rewrite them as

$$LWE_{NG\$s}(t) = [LWE_{NG\$s}(t) - e_{NG}(t)] + e_{NG}(t) \; and \; EWE_{NG\$s}(t) = [EWE_{NG\$s}(t) - e_{NG}(t)] + e_{NG}(t)$$

where the above squared brackets contain the additional contribution of the traded quantities, to the indicator, on top of the prices contribution as formalized by the exchange rate.

For example, suppose $e_{NG}(t) = 16$ and $\left(\frac{\pi_G}{\pi_N}\right) = \frac{1}{2}$. Then $LWE_{NG\$s}(t) = 8$ and $EWE_{NG\$s}(t) = 4$. Therefore, as for $LWE_{NG\$s}(t)$ the quantities contribution is -8 while for $EWE_{NG\$s}(t) = -12$, that is in both indicators the higher (proportional) trades of N compensated for the price ratio, although to a different degree. Which between (19) and (20), and possibly other indicators, is chosen by the platform to quantify the tokens relative desirability depends on the blockchain, its preferences and policy targets.

There could certainly be other ways to combine prices and quantities for representing the platform views. For example,

$$f(p_{NC}(t), p_{GC}(t), \pi_N, \pi_G) = e_{NG}(t)^{(\pi_G - \pi_N)} \quad (23)$$

satisfies (iii)-(iv), and for this reason may look a promising candidate as an indicator. However, for $\pi_G = \pi_N$ would become equal to 1, regardless of the exchange rate value. That is, for such platform equal attractiveness could also be due to quantities only, irrespectively of the prices. Therefore. (23) could be adopted by a blockchain putting additional emphasis on quantities, as compared to (20) .

3.2. Combined price-quantity indicators: the buyers' perspective

As well as for the suppliers, below we define the indicators formalising the buyers' perspective on the relative attractiveness of the two tokens. For example, indicators (19) and (20) become, respectively,

(*Linearly Weighted Exchange Rate, $LWE_{NGCb}(t)$; the buyers' perspective*)

$$LWE_{NGCb}(t) = \left(\frac{\pi_N}{\pi_G}\right) e_{NG}(t) \quad (24)$$

and

(*Exponentially Weighted Exchange Rate, $EWE_{NGb}(t)$; the buyers' perspective*)

$$EWE_{NGCb}(t) = \begin{cases} e_{NG}(t)^{\left(\frac{\pi_N}{\pi_G}\right)} & if \; e_{NG}(t) \geq 1 \\ e_{NG}(t)^{\left(\frac{\pi_G}{\pi_N}\right)} & if \; e_{NG}(t) < 1 \end{cases} \quad (25)$$

That is, to capture the buyers' perspective we simply switch the quantity weights in the sellers' perspectives.

It follows that, for both indicators, the sellers' perspective would prevail if $\left(\frac{\pi_G}{\pi_N}\right) > \left(\frac{\pi_N}{\pi_G}\right)$ and the opposite for the buyers' perspective. In case $\left(\frac{\pi_G}{\pi_N}\right) = \left(\frac{\pi_N}{\pi_G}\right)$ the prevailing perspective will be determined by the exchange rate only.

The above considerations imply some additional, interesting, consequences. In particular if $LWE_{NGCs}(t) = 1$, that is the sellers find the two tokens equally attractive, then $LWE_{NGCb}(t) \neq 1$, unless $\left(\frac{\pi_G}{\pi_N}\right) = \left(\frac{\pi_N}{\pi_G}\right)$. Considering again $e_{NG}(t) = 2$ and $\frac{\pi_G}{\pi_N} = \frac{1}{2}$ we saw that $LWE_{NGCs}(t) = 1$ but $LWE_{NGb}(t) = 4$, that is the buyers are more attracted by N.

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Taking the same numerical example we obtain $EWE_{NGCs}(t) = \sqrt{2}$ and $EWE_{NGCb}(t) = 4$ which, as we saw, implies that both the buyers and the sellers are more attracted by N, although to a different extent. This means that depending upon the platform's view, there would be a variety of ways to formalize the tokens' relative attractiveness.

The previous observations suggest that both $LWE_{NGC}(t)$ and $EWE_{NGC}(t)$ are not symmetric, in the sense that if they indicate one of the two tokens to be more attractive for the sellers-buyers, it does not follow that buyers-sellers would be more attracted by the other token. Indeed, the indicator $EWE_{NGC}(t)$ captures a platform's view for which sellers and buyers *must* have the same preferences about the two tokens, though to a different extent, which in general is not the case for $LWE_{NGC}(t)$.

3.3. Combined price-quantity currency-dependent indicators: the sellers' perspective

We conclude by considering an indicator, which may be more flexible than the previous ones, but whose value could differ across different currencies. Then, if of interest, a unique indicator could be obtained by aggregating across currencies the single indicators.

Suppose, for the sake of exposition, that $C = \$$, $p_{N\$}(t), p_{G\$}(t) > 0$ and $\pi_{N\$}, \pi_{G\$} > 0$: then the indicator

(Exponentially Weighted Prices, $EWP_{NG\$s}(t)$; the sellers' perspective)

$$EWP_{NG\$s}(t) = \frac{[p_{N\$}(t)]^{\pi_{G\$}}}{[p_{G\$}(t)]^{\pi_{N\$}}} \quad (26)$$

Is inspired by (20) where, however, in the ratio prices have different weights. Since, as well as for $LWE_{NGCs}(t) = 1$, also $EWP_{NG\$s}(t) = 1$ when both $p_{N\$}(t) = p_{G\$}(t)$ then $\pi_{N\$} = \pi_{G\$}$, it makes sense to take $EWP_{NG\$s}(t) = 1$ as the value for equal attractiveness, with $EWP_{NG\$s}(t) > 1$ indicating a preference for N while $EWP_{NG\$s}(t) < 1$ a preference for G. Though similar to (20), (26) does not embody the same prominent role played by $e_{NG}(t)$, in particular preventing $e_{NG}(t) = 1$ to become the critical threshold for both the sellers and the buyers' preferences.

Indeed, as well as (19) it could be $EWP_{NG\$s}(t) = 1$ also for $p_{N\$}(t) \neq p_{G\$}(t)$, as long as the value of $\pi_{N\$}$ appropriately compensates for the price difference.

For example, suppose $p_{N\$}(t) = 10$ and $p_{G\$}(t) = 2$; then if $\pi_{N\$} = \frac{lnp_{N\$}}{lnp_{G\$}+lnp_{N\$}} \sim 0.77$ it is $EWP_{NG\$s}(t) = 1$, that is if tokens N are relatively more traded than G. So, with the above values, $e_{NG}(t) = 5$ would suggest that N is more desirable than G while $EWP_{NG\$s}(t) = 1$ that they are equally desirable, from the sellers' perspective.

Therefore, $EWP_{NG\$s}(t) \geq 1$ if

$$p_{N\$}(t) \geq [p_{G\$}(t)]^{\left(\frac{\pi_{N\$}}{\pi_{G\$}}\right)} \quad (27)$$

where in (27) the expression $[p_{G\$}(t)]^{\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)}$ is linear in $p_{G\$}(t)$ for $\pi_{G\$} = \frac{1}{2}$, convex if $\pi_{G\$} > \frac{1}{2}$ and concave if $\pi_{G\$} < \frac{1}{2}$.

Likewise, we can define

(Exponentially Weighted Prices, $EWP_{NG\$b}(t)$; the buyers' perspective)

$$EWP_{NG\$b}(t) = \frac{[p_{N\$}(t)]^{\pi_{N\$}}}{[p_{G\$}(t)]^{\pi_{G\$}}} \quad (28)$$

and so $EWP_{NG\$b}(t) \geq 1$ if

$$p_{N\$}(t) \geq [p_{G\$}(t)]^{\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)} \quad (29)$$

Therefore, the following holds

Proposition. *Both buyers and sellers find N at least as attractive as G if $p_{N\$}(t) \geq max[p_{G\$}(t)^{\left(\frac{\pi_{N\$}}{\pi_{G\$}}\right)}; p_{G\$}(t)^{\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)}]$, find N no more attractive than G if $p_{N\$}(t) \leq$*

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$min[p_{G\$}(t)^{\left(\frac{\pi_{N\$}}{\pi_{G\$}}\right)}; p_{G\$}(t)^{\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)}]$. *If* $p_{G\$}(t) \leq 1$ *and* $min[p_{G\$}(t)^{\left(\frac{\pi_{N\$}}{\pi_{G\$}}\right)}; p_{G\$}(t)^{\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)}] \leq p_{N\$}(t) \leq max[p_{G\$}(t)^{\left(\frac{\pi_{N\$}}{\pi_{G\$}}\right)}; p_{G\$}(t)^{\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)}]$ *then sellers(buyers) would prefer* N *and buyers(sellers)* G *while the opposite is true if* $p_{G\$}(t) > 1$.

So, although $EWP_{NGC}(t)$ resembles $EWE_{NGC}(t)$, it is more flexible since it allows all possible combinations of preferences, towards N and G, across buyers and sellers, depending on the value of $\left(\frac{\pi_{G\$}}{\pi_{N\$}}\right)$.

Finally, it is worth noticing that $EWP_{NG\$s}(t)$, in (26) has been defined considering $p_{N\$}(t)$ and $p_{G\$}(t)$, that is referring to the indirect markets $N\$$ and $G\$$, rather than to the direct market NG, that is to the price $p_{NG}(t)$. However, in principle it would make perfect sense to consider $p_{NG}(t)$ as a reference for the combined price-quantity indicator for the values of N and G. Below we briefly discuss how $EWP_{NG\$s}(t)$ relates to $p_{NG}(t)$, under the $p_{NG}(t) = e_{NG}(t)$ non-arbitrage condition. Indeed,

$$EWP_{NG\$s}(t) = \frac{[p_{N\$}(t)]^{\pi_{G\$}-\pi_{N\$}+\pi_{N\$}}}{[p_{G\$}(t)]^{\pi_{N\$}}} = \frac{[p_{N\$}(t)]^{\pi_{N\$}}}{[p_{G\$}(t)]^{\pi_{N\$}}}[p_{N\$}(t)]^{\pi_{G\$}-\pi_{N\$}} = [p_{NG}(t)]^{\pi_{N\$}}[p_{N\$}(t)]^{\pi_{G\$}-\pi_{N\$}} \quad (30)$$

Namely, $EWP_{NG\$s}(t)$ is positively related to $p_{NG}(t)$, according to the function $[p_{NG}(t)]^{\pi_{N\$}}$, scaled by the quantity $[p_{N\$}(t)]^{\pi_{G\$}-\pi_{N\$}}$.

It follows that it is also $EWP_{NG\$s}(t) = [e_{NG}(t)]^{\pi_{N\$}}[p_{N\$}(t)]^{\pi_{G\$}-\pi_{N\$}}$, which means that it does not depend on prices only through $e_{NG}(t)$, except for when $\pi_{G\$} = \frac{1}{2} = \pi_{N\$}$, in which case

$$EWP_{NG\$s}(t) = \sqrt{p_{NG}(t)} = \sqrt{e_{NG}(t)}$$

Likewise, from the buyers' perspective we now obtain

$$EWP_{NG\$b}(t) = \frac{[p_{N\$}(t)]^{\pi_{N\$}-\pi_{G\$}+\pi_{G\$}}}{[p_{G\$}(t)]^{\pi_{G\$}}} = \frac{[p_{N\$}(t)]^{\pi_{G\$}}}{[p_{G\$}(t)]^{\pi_{G\$}}}[p_{N\$}(t)]^{\pi_{N\$}-\pi_{G\$}} = [p_{NG}(t)]^{\pi_{G\$}}[p_{N\$}(t)]^{\pi_{N\$}-\pi_{G\$}}$$

and so that $EWP_{NG\$b}(t)$ depends on $[p_{NG}(t)]^{\pi_{G\$}}$, unless $\pi_{G\$} = \frac{1}{2} = \pi_{N\$}$.

3. Conclusions

In the paper, to our knowledge, for the first time we addressed a discussion on the economic fundamentals of a TTE. In particular, we have introduced a number of economic indicators that might be considered to define absolute and relative economic values for the tokens, as well as for the whole platform. To construct such indicators we used market prices, traded and circulating quantities of the tokens. These are only a subset of the possible metrics that one could consider, and for this reason our proposed indicators are by no means the only ones. Indeed, we mentioned the number of transactions and their average monetary size, the block size and others, could also be informative variables to consider.

The analysis on traded quantities suggested the introduction of two different perspectives, the *buyers*' and the *sellers*', for the combined price-quantity indicators. We conceive the proposed indicators, computed in real time, as composing a *dashboard* for the blockchain policy makers, that can enjoy the continuous observation of the absolute and relative economic values of the tokens, as well as of the platform. As simple as they may be, we believe that the indicators can convey interesting, and effective, information to the platform decision makers.

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