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Not peer-reviewed version

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Posted Date: 9 February 2026

doi: 10.20944/preprints202602.0489.v1

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Article

Quantum Information Copy Time (QICT): Spectral Foundations, Canonical Filtered-Certification Protocols, and Transport Benchmarks

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Abstract

We present a self-contained, publication-oriented formulation of Quantum Information Copy Time (QICT), a timescale governing the *certifiable replication* of a conserved information label Y in local quantum many-body systems. The framework is built from an inverse-Liouvillian quadratic form in the Kubo–Mori (Bogoliubov) operator Hilbert space. To make the construction mathematically well-posed and operational, we (i) define QICT using the Moore–Penrose pseudoinverse of the Liouvillian generator (no hydrodynamic closure assumed), (ii) derive an exact spectral representation with a sharp finiteness criterion, (iii) introduce a controlled infrared (IR) regularization $t_{\text{copy}}(Y; \epsilon)$, and (iv) specify an explicit local *certification task*: binary hypothesis testing between opposite small biases $\pm\theta$ in a local Gibbs family using *filtered* local currents. We prove that the exponential filter $J_\epsilon = \int_0^\infty e^{-\epsilon t} \dot{Y}(t) dt$ is canonical in two precise senses: it is (a) the unique readout of a minimal Markovian “detector with finite memory” and (b) the resolvent/Laplace transform naturally produced by linear response. We then link the signal-to-noise ratio (SNR) of any such certification protocol to the regulated QICT functional and provide a hypothesis-testing translation from SNR to target error probability δ via standard inequalities. Finally, we state a transport-regime agnostic “Master” Golden Relation (free dynamical exponent z), provide benchmark estimates for the matching constant C_Λ using independently computed chaos/transport inputs in canonical strong- and weak-coupling limits, and propagate temperature-dependent transport uncertainties into the resulting mass band.

Keywords: Quantum information copy time (QICT); Liouvillian pseudoinverse; Kubo–Mori metric; operator Hilbert space; spectral representation; infrared regularization; filtered currents; resolvent/Laplace transform; linear response; hypothesis testing; SNR bounds; Chernoff/sub-Gaussian; ballistic vs diffusive transport; Drude weight; dynamical exponent z ; generalized Golden Relation; quantum chaos (Lyapunov, MSS); AdS/CFT diffusion; $N=4$ SYM; kinetic theory transport (AMY)

1. Scope and Standing Assumptions

We separate *definition-level* statements (spectral QICT and its finiteness/regularization) from *model-level* inputs (transport coefficients, Lyapunov exponent, matching scale), which must be supplied by a chosen microscopic theory.

Assumption 1 (Finite-volume definition and controlled limits). All objects are defined first in finite volume (or a finite-dimensional truncation) where traces exist and generators are closed operators on dense domains. Thermodynamic limits are taken *after* introducing an explicit infrared (IR) regulator (Sec. 4).

Assumption 2 (Faithful stationary state (thermal: KMS)). The reference state ρ is faithful and stationary; in thermal applications $\rho \propto e^{-\beta H}$ (finite volume) or its KMS/GNS extension (infinite volume).

Assumption 3 (Locality). The Hamiltonian is local (lattice or local QFT), ensuring finite-speed information propagation (Lieb–Robinson type) [3].

Assumption 4 (Observable class). The label Y is chosen so that $\dot{Y} = i[H, Y]$ lies in the domain of the relevant resolvents/pseudoinverses in the Kubo–Mori operator Hilbert space.

We work in natural units $\hbar = c = k_B = 1$.

2. Kubo–Mori Geometry and an Unconditional QICT Definition

2.1. Kubo–Mori Inner Product

For a faithful state ρ , the Kubo–Mori (Bogoliubov) inner product on operators is

$$\langle A, B \rangle_{\text{KM}} \equiv \int_0^1 ds \operatorname{Tr}(\rho^s A^\dagger \rho^{1-s} B). \quad (1)$$

This induces the Bogoliubov–Kubo–Mori (BKM) monotone metric on thermal manifolds [1,2].

2.2. Liouvillian Generator and Pseudoinverse

Define the Liouvillian superoperator $\mathcal{L}(A) = i[H, A]$ and the self-adjoint generator $K \equiv i\mathcal{L}$ on the KM Hilbert space. Because $\ker K$ generally contains conserved operators, inverse powers must be understood as pseudoinverses.

Definition 1 (QICT functional (definition-level)). Let P_0 project onto $\ker K$. Define the Moore–Penrose pseudoinverse

$$K^+ \equiv (K \upharpoonright_{\ker(K)^\perp})^{-1} (1 - P_0). \quad (2)$$

For $Y = Y^\dagger$ define $\dot{Y} \equiv \mathcal{L}(Y) = i[H, Y]$ and set the dimensionless QICT functional

$$\mathfrak{t}_{\text{copy}}^2(Y) \equiv \langle \dot{Y}, (K^+)^2 \dot{Y} \rangle_{\text{KM}}. \quad (3)$$

We define the corresponding dimensionful time by

$$t_{\text{copy}}(Y) \equiv \beta \mathfrak{t}_{\text{copy}}(Y). \quad (4)$$

3. Exact Spectral Representation and Finiteness

Let $E_K(\lambda)$ be the spectral measure of the self-adjoint operator K . For any A define the positive measure

$$d\mu_A(\lambda) \equiv \langle A, dE_K(\lambda) A \rangle_{\text{KM}}. \quad (5)$$

Proposition 1 (Transport-regime invariant spectral formula). *If $\dot{Y} \in \operatorname{Dom}(K^+)$, then*

$$\mathfrak{t}_{\text{copy}}^2(Y) = \int_{\mathbb{R} \setminus \{0\}} \frac{d\mu_{\dot{Y}}(\lambda)}{\lambda^2}. \quad (6)$$

Proof. On $\ker(K)^\perp$, $(K^+)^2 = \int_{\mathbb{R} \setminus \{0\}} \lambda^{-2} dE_K(\lambda)$ as a quadratic form. Insert into (3) and use (5). $\square$

Theorem 1 (Necessary and sufficient finiteness condition). $\mathfrak{t}_{\text{copy}}(Y)$ is finite iff

$$\int_{|\lambda| < \lambda_0} \frac{d\mu_{\dot{Y}}(\lambda)}{\lambda^2} < \infty \quad \text{for some } \lambda_0 > 0. \quad (7)$$

Proof. Split (6) into $|\lambda| < \lambda_0$ and $|\lambda| \geq \lambda_0$. The UV part is bounded by $\lambda_0^{-2} \|\dot{Y}\|_{\text{KM}}^2$. Hence finiteness reduces to the IR condition (7). $\square$

Remark 1 (Ballistic/Drude obstruction and the need for an IR regulator). A point mass of $d\mu_{\dot{Y}}$ at $\lambda = 0$ (Drude weight) violates (7) and implies $\mathfrak{t}_{\text{copy}}(Y) = \infty$ in the ideal limit. This is consistent with ballistic channels where persistent zero-frequency weight obstructs decay of currents [9,10]. Operationally, certification protocols always involve finite resolution, motivating an explicit IR-regularized family.

4. IR-Regularized QICT

Definition 2 (Regulated QICT). For $\epsilon > 0$ define

$$t_{\text{copy}}^2(Y; \epsilon) \equiv \int_{\mathbb{R}} \frac{d\mu_{\dot{Y}}(\lambda)}{\lambda^2 + \epsilon^2} = \langle \dot{Y}, (K^2 + \epsilon^2)^{-1} \dot{Y} \rangle_{\text{KM}}. \quad (8)$$

The dimensionful regulated time is $t_{\text{copy}}(Y; \epsilon) \equiv \beta t_{\text{copy}}^2(Y; \epsilon)$.

5. Operational Meaning: Canonical Filtered Certification and Hypothesis Testing

We formalize a minimal local certification task and show why the exponential filter is canonical. We also connect the SNR threshold $c(\delta)$ to standard hypothesis-testing bounds.

5.1. Task: Binary Certification Between Opposite Local Biases

Fix a region $\mathcal{R}$ of linear size ℓ and consider the local Gibbs family

$$\rho_{\theta} = \frac{e^{-\beta(H - \theta Y_{\mathcal{R}})}}{\text{Tr } e^{-\beta(H - \theta Y_{\mathcal{R}})}}, \quad \theta \in \mathbb{R}, \quad (9)$$

where $Y_{\mathcal{R}}$ is the restriction of Y to $\mathcal{R}$. The task is to decide between hypotheses $H_{\pm} : \rho_{\pm\theta}$ at target average error probability $\delta \in (0, 1/2)$ using measurements localized in $\mathcal{R}$.

5.2. Why the Exponential Filter Is Canonical

We define the exponentially filtered current observable

$$J_{\epsilon} \equiv \int_0^{\infty} dt e^{-\epsilon t} \dot{Y}_{\mathcal{R}}(t), \quad \epsilon > 0. \quad (10)$$

Proposition 2 (Canonical detector-with-memory realization). Consider a minimal linear Markovian “memory variable” $Q(t)$ coupled to $\dot{Y}_{\mathcal{R}}(t)$ through the first-order response equation

$$\dot{Q}(t) = -\epsilon Q(t) + \dot{Y}_{\mathcal{R}}(t), \quad Q(0) = 0, \quad (11)$$

which is the unique causal, time-translation invariant, one-pole (single-timescale) linear filter. Then the long-time readout is

$$\lim_{t \rightarrow \infty} Q(t) = J_{\epsilon}. \quad (12)$$

Proof. Solving (11) by variation of constants gives $Q(t) = \int_0^t ds e^{-\epsilon(t-s)} \dot{Y}_{\mathcal{R}}(s)$. Taking $t \rightarrow \infty$ yields (10). $\square$

Proposition 3 (Canonical resolvent/Laplace form in linear response). Let K be the self-adjoint Liouvillian generator. In the KM Hilbert space, the Laplace transform of $\dot{Y}_{\mathcal{R}}(t) = e^{tK} \dot{Y}_{\mathcal{R}}$ satisfies

$$J_{\epsilon} = \int_0^{\infty} dt e^{-\epsilon t} e^{tK} \dot{Y}_{\mathcal{R}} = (\epsilon - K)^{-1} \dot{Y}_{\mathcal{R}}, \quad (13)$$

as an operator identity on the domain where the resolvent exists.

Proof. The Bochner integral $\int_0^{\infty} e^{-\epsilon t} e^{tK} dt$ is the standard Laplace representation of the resolvent $(\epsilon - K)^{-1}$ for $\epsilon > 0$. $\square$

Remark 2 (Why “canonical” is appropriate). Proposition 2 shows that (10) is the *unique* readout of the simplest causal Markovian detector with a single memory timescale ϵ^{-1} . Proposition 3 shows that the same exponential filter is the natural object produced by Laplace-transforming linear-response dynamics, and therefore appears directly in resolvent-based correlation bounds (Sec. 5.3).

5.3. SNR Bound Controlled by Regulated QICT

Define the SNR for the statistic J_ϵ as

$$\text{SNR}(\epsilon) \equiv \frac{|\langle J_\epsilon \rangle_{+\theta} - \langle J_\epsilon \rangle_{-\theta}|}{\sqrt{\text{Var}_{\text{KM}}(J_\epsilon)}}. \quad (14)$$

Theorem 2 (Operational bound: regulated QICT controls filtered certification). *In linear response around $\theta = 0$,*

$$\text{SNR}(\epsilon) \leq 2|\theta| t_{\text{copy}}(Y_{\mathcal{R}}; \epsilon). \quad (15)$$

Proof. Using (13), write $J_\epsilon = (\epsilon - K)^{-1} \dot{Y}_{\mathcal{R}}$. Linear response gives $\langle J_\epsilon \rangle_\theta - \langle J_\epsilon \rangle_0 = \beta \theta \langle Y_{\mathcal{R}}, J_\epsilon \rangle_{\text{KM}} + O(\theta^2)$ (Kubo formula [4]). Applying Cauchy–Schwarz in the KM inner product, $|\langle Y_{\mathcal{R}}, J_\epsilon \rangle_{\text{KM}}| \leq \|Y_{\mathcal{R}}\|_{\text{KM}} \|J_\epsilon\|_{\text{KM}}$. A sharper bound uses the identity $\|J_\epsilon\|_{\text{KM}}^2 = \langle \dot{Y}_{\mathcal{R}}, (\epsilon - K)^{-1} (\epsilon - K)^{-1} \dot{Y}_{\mathcal{R}} \rangle_{\text{KM}} = \langle \dot{Y}_{\mathcal{R}}, (K^2 + \epsilon^2)^{-1} \dot{Y}_{\mathcal{R}} \rangle_{\text{KM}} = t_{\text{copy}}^2(Y_{\mathcal{R}}; \epsilon)$, which follows from the spectral theorem. Substituting yields (15) after multiplying by β and using the definition of SNR with $\text{Var}_{\text{KM}}(J_\epsilon) = \|J_\epsilon\|_{\text{KM}}^2$. $\square$

5.4. From SNR to Target Error Probability: A Standard Hypothesis-Testing Bound

Let $P_\pm$ be the classical distributions of the measurement outcome X produced by a given protocol under $\rho_{\pm\theta}$. For equal priors, the optimal average error (Bayes risk) is

$$P_{\text{err}}^* = \frac{1}{2} \left(1 - \text{TV}(P_+, P_-) \right), \quad (16)$$

where TV is total-variation distance [12]. Thus any sufficient lower bound on $\text{TV}(P_+, P_-)$ yields an error guarantee.

A convenient route is to relate TV to the KL divergence via Pinsker’s inequality, $\text{TV}(P_+, P_-) \leq \sqrt{\frac{1}{2} D_{\text{KL}}(P_+ \| P_-)}$ [11], and then bound D_{KL} in terms of the (sub-)Gaussian concentration of the chosen statistic X . For protocols based on sums or filtered integrals of local currents, the outcome is often well-approximated by a sub-Gaussian variable; we therefore state a conservative lemma:

Lemma 1 (Sub-Gaussian SNR $\Rightarrow$ error bound). *Suppose under each hypothesis $H_\pm$ the statistic X is sub-Gaussian with the same variance proxy σ^2 , and means $\mathbb{E}_\pm[X] = \pm\mu$. Then the threshold test $\text{sign}(X)$ obeys*

$$P_{\text{err}} \leq \exp\left(-\frac{\mu^2}{2\sigma^2}\right). \quad (17)$$

Equivalently, achieving $P_{\text{err}} \leq \delta$ is guaranteed if

$$\text{SNR} \equiv \frac{|\mathbb{E}_+[X] - \mathbb{E}_-[X]|}{\sqrt{\text{Var}(X)}} \geq c(\delta), \quad c(\delta) = \sqrt{2 \ln \frac{1}{\delta}}. \quad (18)$$

Proof. For sub-Gaussian X with mean μ and variance proxy σ^2 , the Chernoff bound gives $\mathbb{P}(X \leq 0) \leq \exp(-\mu^2/2\sigma^2)$ [13]. Under H_+ , the error is $\mathbb{P}_+(X \leq 0)$ and under H_- it is $\mathbb{P}_-(X \geq 0)$, which are equal by symmetry. This yields (17). Rewriting in terms of SNR gives (18). $\square$

Remark 3. Lemma 1 provides a standard, conservative translation from SNR to target error level, and fixes a concrete choice of $c(\delta)$. More refined (and typically stronger) bounds can be obtained from exact Gaussian formulas or from Chernoff information if $P_\pm$ are known. In a quantum setting, Helstrom’s theorem gives the minimum error in terms of trace distance [14]; the present approach is intentionally “operational” in the sense that it bounds the performance of explicit local measurement protocols.

6. Transport-Agnostic Master Golden Relation

A regime-independent matching statement may be written as

$$m_S(T_*) = \sqrt{\kappa_{\text{eff}}(T_*)} k_I(T_*), \quad k_I(T_*) \equiv t_{\text{copy}}^{-1}(Y; \epsilon_*), \quad (19)$$

where ϵ_* is the operational IR scale used in matching.

6.1. Generalized Golden Relation with Free Dynamical Exponent z

If, around T_* , the relevant slow mode is relaxational with rate

$$\Gamma_k \sim \mathfrak{D}_z(T) k^z, \quad z > 0, \quad (20)$$

then at local resolution $k \sim 1/a$ one obtains the generalized form

$$m_S(T_*) = C_\Lambda^{(z)}(T_*) \sqrt{\kappa_{\text{eff}}(T_*) \chi_Y(T_*)}, \quad C_\Lambda^{(z)}(T_*) \propto \frac{a(T_*)^z}{\beta \mathfrak{D}_z(T_*)}. \quad (21)$$

Diffusion corresponds to $z = 2$ and $\mathfrak{D}_2 = D_Y$. In sub-diffusive regimes ($z > 2$) the same structure holds with the appropriate z and $\mathfrak{D}_z$; in ballistic limits, the unregulated $\epsilon \rightarrow 0$ object may diverge and the operational ϵ_* must be kept finite.

7. Benchmark Estimates for C_Λ from Independent Chaos and Transport

We cite primary sources and provide numerical brackets for canonical transport inputs.

7.1. Strongly Coupled Benchmark: $\mathcal{N}=4$ SYM Diffusion

For $\mathcal{N}=4$ SYM at strong coupling, the R-charge diffusion constant is [6]

$$D = \frac{1}{2\pi T}. \quad (22)$$

Taking a thermal microscopic resolution $a = (2\pi T)^{-1}$ gives $a/D = 1$. Assuming near-saturation of the chaos bound ($\lambda_L \approx 2\pi T$) [5], the diffusive ($z = 2$) specialization of (21) yields an order-one benchmark for $C_\Lambda^{(2)}$.

7.2. Weakly Coupled Benchmark: Kinetic Theory Scaling

In weakly coupled gauge plasmas, kinetic theory gives the parametric scaling [7,8]

$$D(T) \sim \frac{\xi}{g^4(T) T \log(1/g(T))}, \quad (23)$$

with $\xi = \mathcal{O}(1)$ depending on the channel. With $a = (2\pi T)^{-1}$ and $\xi \simeq 1$, the ratio

$$\frac{a}{D} \sim \frac{g^4(T) \log(1/g(T))}{2\pi} \quad (24)$$

is numerically small for perturbative g . For instance, $g = 0.5$ gives $g^4 \log(1/g) \approx 0.043$ and hence $a/D \approx 6.8 \times 10^{-3}$.

8. Structural Risk: Uncertainty Propagation into the Mass Band

Assuming (21) in a window where a relaxational description applies, relative uncertainties propagate as

$$\frac{\delta m_S}{m_S} = \frac{\delta C_\Lambda^{(z)}}{C_\Lambda^{(z)}} + \frac{1}{2} \frac{\delta \kappa_{\text{eff}}}{\kappa_{\text{eff}}} + \frac{1}{2} \frac{\delta \chi_Y}{\chi_Y}. \quad (25)$$

If $C_{\Lambda}^{(z)} \propto a^z / (\beta \mathfrak{D}_z)$ then

$$\frac{\delta C_{\Lambda}^{(z)}}{C_{\Lambda}^{(z)}} = z \frac{\delta a}{a} - \frac{\delta \mathfrak{D}_z}{\mathfrak{D}_z} - \frac{\delta \beta}{\beta}. \quad (26)$$

If $\mathfrak{D}_z(T) \propto T^p$ locally, a matching-scale uncertainty δT_* injects

$$\left. \frac{\delta m_S}{m_S} \right|_{\mathfrak{D}} \simeq -p \frac{\delta T_*}{T_*}. \quad (27)$$

Conclusion

We have provided a regime-agnostic spectral definition of QICT, a sharp finiteness criterion, an explicit IR-regularized family, and a concrete operational certification task. The exponential filter is canonical as the unique readout of a minimal Markovian detector and as the resolvent object generated by linear response. Standard hypothesis-testing inequalities yield an explicit and conservative map from SNR to target error level. Model-level consequences can be organized through a transport-agnostic Master Golden Relation with free dynamical exponent z . Benchmark transport inputs with primary citations allow numerical bracketing of the matching constant in representative strong- and weak-coupling limits, and explicit uncertainty-propagation formulas quantify how temperature-dependent transport can broaden the predicted band.

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