Article

Not peer-reviewed version

# On the Possible Role of the Planck Length in Fitting the Neutron Lifetime

U. V. Satya Seshavatharam * , T. Gunavardhana Naidu , S. Lakshminarayana

Posted Date: 15 July 2025

doi: 10.20944/preprints202507.0530.v2

Keywords: Planck length; neutron decay & lifetime; nuclear charge radius; 4G model of final unification; weak fermion of rest energy; 584.725 GeV; big G; 4G model of weak interaction range; 4G model of electron neutrino; neutrino rest masses

doi:10.20944/preprints202507.0530.v2



*Article*

# On the Possible Role of the Planck Length in Fitting the Neutron Lifetime

**U. V. Satya Seshavatharam [1,2,*], T. Gunavardhana Naidu [3] and S. Lakshminarayana [4]**

[1] Honorary faculty, I-SERVE, Survey no-42, Hitech city, Hyderabad-500084,Telangana, India
[2] Quality Assurance Dept, Casting, DIP Division, Electrosteel Castings Ltd., Srikalahasthi-517641, AP, India
[3] Dept. of Physics, Aditya Institute of Technology and Management, Tekkali-532201, AP, India
[4] Dept. of Nuclear Physics, Andhra University, Visakhapatnam-530003, AP, India
* Correspondence: seshavatharam.uvs@gmail.com

**Abstract**

Despite decades of effort, expressing the Planck length and Newton's gravitational constant in terms of elementary constants remains a challenge; in this work, we apply our 4G final-unification model to establish a relation tying the distance light travels during the neutron lifetime to nuclear parameters-namely the proton mass, nuclear volume, and neutron-proton mass difference-showing that slight variations in the nuclear charge radius influence neutron lifetime and resolve the beam and bottle method discrepancies (approximately 885 sec Vs 875 sec) through thermodynamic modulation of decay processes. This thermodynamic sensitivity, central to our framework, finds experimental validation in the recent J-PARC pulsed cold neutron beam study, which offers high-precision timing and statistical control, and strengthens the connection between nuclear structure and decay dynamics. We also derive a semi-empirical formula for Newton's gravitational constant based on nuclear observables-such as Fermi's weak coupling constant, which governs the strength of weak interactions in neutron beta decay, thus linking low-energy nuclear phenomena to quantum gravity. Extending the framework, we present a semi-empirical neutrino-mass model anchored on a benchmark electron-neutrino rest mass of $2.45 \times 10^{-11}$ eV / $c^2$, predicting the rest masses of about $3.3 \times 10^{-4}$ eV / $c^2$ for the electron neutrino, $7.7 \times 10^{-3}$ eV / $c^2$ for the muon neutrino, and $5.1 \times 10^{-2}$ eV / $c^2$ for the tau neutrino, with a combined 'neutrino' plus 'antineutrino' mass sum near (2 x 0.059) = 0.118 eV consistent with cosmological limits. These findings imply that laboratory-scale nuclear measurements contain signatures of Planck-scale physics, opening new avenues for experimental tests and theoretical developments in quantum gravity.

**Keywords:** Planck length; neutron decay & lifetime; nuclear charge radius; 4G model of final unification; weak fermion of rest energy; 584.725 GeV; big G; 4G model of weak interaction range; 4G model of electron neutrino; neutrino rest masses

## 1. Introduction

The quest to unify gravity with the quantum realm continues to be the centre piece of modern theoretical physics [1]. Despite the success of quantum field theories in describing electromagnetic, weak, and strong interactions, gravity has resisted integration into this framework. Most unification attempts invoke extrapolated scales or speculative frameworks-string theory [2], loop quantum gravity [3], or higher-dimensional models-yet lack empirical pathways connecting measurable constants to Planck-scale quantities. In this context, we explore the possibility that Planck-scale physics-specifically, the Planck length [4] and the Newton's gravitational constant-may find expression not in the remote energy domains but through the structure of nuclear matter and decay phenomena. In the following section, we introduce the assumptions and simple applications of our 4G model of final unification [5–12]. Readers are encouraged to refer our recent papers for a better understanding [5,6].

## 2. Three Assumptions of 4G Model of Final Unification and Simple Applications

Following our 4G model of final unification [5–12]

1) There exists a characteristic electroweak fermion of rest energy, $M_{wf}c^2 \cong 584.725$ GeV. It can be considered as the zygote of all elementary particles.
2) There exists a nuclear elementary charge in such a way that, $\left(\frac{e}{e_n}\right)^2 \cong \alpha_s \cong 0.1152$ = Strong coupling constant and $e_n \cong 2.9464e$.
3) Each atomic interaction is associated with a characteristic large gravitational coupling constant. Their fitted magnitudes are,

$$G_e \cong \text{Electromgnetic gravitational constant} \cong 2.374335\times10^{37}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$$
$$G_n \cong \text{Nuclear gravitational constant} \cong 3.329561\times10^{28}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$$
$$G_w \cong \text{Electroweak gravitational constant} \cong 2.909745\times10^{22}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$$

It may be noted that,

1) Weak interaction point of view [13,14], following our assumptions, Fermi's weak coupling constant can be fitted with the following relations.

$$\left.\begin{array}{l} G_F \cong \left(\frac{m_e}{m_p}\right)^2 \hbar c R_0^2 \cong G_w M_{wf}^2 R_w^2 \cong 1.44021\times10^{-62}\ \text{J.m}^3 \\ \text{where,} \left\{\begin{array}{l} R_0 \cong \frac{2G_n m_p}{c^2} \cong 1.24\times10^{-15}\ \text{m} \\ R_w \cong \frac{2G_w M_{wf}}{c^2} \cong 6.75\times10^{-19}\ \text{m} \end{array}\right. \end{array}\right\} \quad (1)$$

2) In a unified approach, most important point to be noted is that,

$$\hbar c \equiv G_w M_{wf}^2 \quad (2)$$

Clearly speaking, based on the electroweak interaction, the well believed quantum constant $\hbar c$ seems to have a deep inner meaning. It needs further study with reference to EPR argument [1,10]. String theory [2,3] can be made practical with reference to the three atomic gravitational constants associated with weak, strong and electromagnetic interaction gravitational constants. See Tables 1 and 2. for sample string tensions and energies without any coupling constants.

**Table 1.** Charge dependent string tensions and string energies.

| S.No | Interaction | String Tension | String energy |
|---|---|---|---|
| 1 | Weak | $\frac{c^4}{4G_w} \cong 6.94\times10^{10}$ N | $\sqrt{\frac{e^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_w}\right)} \cong 24.975$ GeV |
| 2 | Strong | $\frac{c^4}{4G_n} \cong 6.065\times10^{4}$ N | $\sqrt{\frac{e_n^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_n}\right)} \cong 68.79$ MeV |
| 3 | Electromagnetic | $\frac{c^4}{4G_e} \cong 8.505\times10^{-5}$ N | $\sqrt{\frac{e^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_e}\right)} \cong 874.3$ eV |

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 July 2025 doi:10.20944/preprints202507.0530.v2



**Table 2.** Quantum string tensions and string energies.

| S.No | Interaction | String Tension | String energy |
|---|---|---|---|
| 1 | Weak | $\frac{c^4}{4G_w} \cong 6.94 \times 10^{10}$ N | $\sqrt{\hbar c \left(\frac{c^4}{4G_w}\right)} \cong 292.36$ GeV |
| 2 | Strong | $\frac{c^4}{4G_n} \cong 6.065 \times 10^{4}$ N | $\sqrt{\hbar c \left(\frac{c^4}{4G_n}\right)} \cong 273.3$ MeV |
| 3 | Electromagnetic | $\frac{c^4}{4G_e} \cong 8.505 \times 10^{-5}$ N | $\sqrt{\hbar c \left(\frac{c^4}{4G_e}\right)} \cong 10234.77$ eV |

3) Newtonian gravitational constant can be expressed as [15,16],

$$G_N \cong \frac{G_w^{21} G_e^{10}}{G_n^{30}} \cong 6.679851 \times 10^{-11} \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \tag{3}$$

4) Strong coupling constant can be expressed as [17],

$$\alpha_s \cong \frac{G_w^6 G_e^4}{G_n^{10}} \cong 0.115193455 \tag{4}$$

5) Avogadro like large number can be expressed as [18],

$$\begin{aligned} X &\cong \frac{\text{Product of short range gravitational constants}}{\text{Product of long range gravitational constants}} \\ &\cong \frac{G_n G_w}{G_N G_e} \cong 6.1088144 \times 10^{23} \end{aligned} \tag{5}$$

## 3. Photon Transit Over Neutron Lifetime: An Assumed Fundamental Construct

Consider a photon traveling a distance $S_{tn} \cong ct_n$, where $t_n$ is the free neutron lifetime. We hypothesize that this macroscopic-seeming length can be re-expressed through a combination of nuclear-scale quantities:

$$S_{tn} \cong \left[\frac{m_p}{(m_n - m_p)}\right] \frac{V_0}{R_{pl} R_w} \cong \left[\frac{m_p}{(m_n - m_p)}\right] \frac{4\pi R_0^3}{3 R_{pl} R_w} \tag{6}$$

Here,

$S_{tn} \cong ct_n$ =Distance travelled by photon in the lifetime of neutron $t_n$.

$m_p \cong 938.272 \text{ MeV}/c^2$ = Proton rest mass

$m_n \cong 939.5654 \text{ MeV}/c^2$ =Neutron rest mass

$G_n \cong 3.329561 \times 10^{28} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2}$ = Nuclear gravitational constant

$R_0$ =Nuclear charge radius = $\frac{2G_n m_p}{c^2}$ =1.23929 fermi

$V_0$ = Nuclear volume corresponding to $R_0$

$G_N$ = Newtonian gravitational constant

$R_{pl} = \frac{2G_N M_{pl}}{c^2} \cong 2\sqrt{\frac{G_N \hbar}{c^3}}$ = Schwarzschild radius of the Planck mass, $M_{pl} \cong \sqrt{\frac{\hbar c}{G_N}}$

$G_F$ =Fermi's weak coupling constant.

$R_w$ =Weak interaction range = $\frac{2G_w M_{wf}}{c^2} \cong \sqrt{\frac{G_F}{\hbar c}} \cong 6.75\times10^{-19}$ m

$G_w \cong 2.909745\times10^{22}$ $\text{m}^3\text{kg}^{-1}\text{sec}^{-2}$ = Weak gravitational constant

$M_{wf} \cong 584725$ MeV/$c^2$ = Rest mass of Electroweak fermion

Based on this relation, Planck length can be expressed as,

$$\sqrt{\frac{G_N \hbar}{c^3}} \cong \left[\frac{m_p}{(m_n - m_p)}\right]\frac{2\pi R_0^3}{3R_w c t_n} \tag{7}$$

$$t_n \cong \left[\frac{m_p}{(m_n - m_p)}\right]\frac{2\pi R_0^3}{3R_w}\sqrt{\frac{c}{\hbar G_N}} \cong 884.245 \text{ sec} \tag{8}$$

where $G_N \cong 6.6743\times10^{-11}\,\text{m}^3\text{kg}^{-1}\text{sec}^{-2}$

Newtonian gravitational constant can be expressed as

$$G_N \cong \left[\frac{m_p}{(m_n - m_p)}\right]^2 \frac{4\pi^2 R_0^6 c}{9R_w^2 t_n^2 \hbar} \tag{9}$$

$$\hbar G_N \cong \left[\frac{m_p}{(m_n - m_p)}\right]^2 \frac{4\pi^2 R_0^6 c}{9R_w^2 t_n^2} \tag{10}$$

## 4. Neutron Lifetime Dependence on Nuclear Charge Radius

Based on relation (8), it is possible to show that,

$$t_n \propto \frac{R_0^3}{R_w} \tag{11}$$

What is particularly striking is the **sensitivity** of neutron lifetime to small variations in the nuclear volume. It may be noticed that 0.01 fm reduction in nuclear charge radius can lead to noticeable changes in the neutron lifetime [21–25]. For example, considering a nuclear charge radius of $R_0 \cong (1.23 \text{ to } 1.24)$ fm, obtained $t_n \cong (875.2 \mp 10.6)$ sec. It seems that, there exists an unknown interaction between neutron decay phenomenon, nuclear volume and the Planck scale. It needs further study.

Interesting point to be noted is that, Planck scale [26,27] seems to be a kind of reference scale for the elementary particles independent of the generally believed concept of 'higher energy limit'. Proceeding further, Planck scale can also be viewed as 'a hidden and unknown component of quantum gravity' linked with nuclear and atomic structures.

## 5. Beam–Bottle Methods and the Thermodynamic Context

Experimental discrepancies in neutron lifetime measurements using beam and bottle methods [21–25] offer a natural testing ground for this framework. The beam method typically results in lifetime of 885 seconds, while the bottle approach reports a lifetime of 878 seconds. We propose that this divergence reflects subtle thermodynamic influences:

a) The **bottle method**, involving ultracold environments and magnetic confinement, may influence quantum tunneling and energy levels of neutron decay.
b) The **beam method**, operating under different vacuum and interaction conditions, may alter decay probabilities.

In this light, neutron decay ceases to be an immutable constant and instead becomes a **thermodynamic variable,** contingent on environmental factors such as energy distribution, boundary conditions, and quantum state configuration. Recently, improved measurements using pulsed cold neutron beams [21] at J-PARC yielded a neutron lifetime of approximately 877 seconds - consistent with bottle method results! This convergence supports the hypothesis that neutron decay rates are sensitive to environmental and detection conditions. Such consistency across distinct experimental setups strengthens our argument for a thermodynamically modulated quantum decay framework.

## 6. Newton's Gravitational Constant from Nuclear Metrics

Based on relation (9), Newtonian gravitational constant [15,16,28] can be expressed as,

$$G_N \propto \left(\frac{R_0^6}{t_n^2 R_w^2}\right) \tag{12}$$

In a unified approach, this relation seems to show a path for estimating the currently believed 'big G' in a semi empirical approach connected with nuclear physical constants. Here it seems important to highlight our two observed or defined relations for understanding the relationship between the nuclear scale and the Planck scale [6].

$$\frac{R_0}{R_{pl}} \cong \frac{G_n m_p}{G_N M_{pl}} \cong \frac{G_n m_p}{\sqrt{\hbar c G_N}} \cong \left(\frac{m_p}{m_e}\right)^6 \\ \cong (\text{'Proton and Electron' mass ratio})^6 \tag{13}$$

$$\frac{G_w}{G_N} \cong \left(\frac{m_p}{m_e}\right)^{10} \cong (\text{'Proton and Electron' mass ratio})^{10} \tag{14}$$

These two relations will certainly help in exploring the secrets of microscopic quantum gravity.

## 7. Connecting the Newton's Gravitational Constant and the Fermi's Weak Coupling Constant

Considering the 4G model of our weak interaction range, Newtonian gravitational constant and the Fermi's weak coupling constant can be related in the following way. Writing our 4G model of weak interaction range as, $R_w \cong \frac{2G_w M_{wf}}{c^2} \cong \sqrt{\frac{G_F}{\hbar c}}$, Planck length can be expressed as shown.

$$\sqrt{\frac{G_N \hbar}{c^3}} \cong \left[\frac{m_p}{(m_n - m_p)}\right] \frac{2\pi R_0^3}{3ct_n} \sqrt{\frac{\hbar c}{G_F}} \tag{15}$$

Thus,

 doi:10.20944/preprints202507.0530.v2



$$G_F \cong \left[\frac{m_p}{(m_n - m_p)}\right]^2 \frac{4\pi^2 R_0^6 c^2}{9t_n^2 G_N} \quad (16)$$

$$G_N \cong \left[\frac{m_p}{(m_n - m_p)}\right]^2 \frac{4\pi^2 R_0^6 c^2}{9t_n^2 G_F} \quad (17)$$

$$t_n \cong \left[\frac{m_p}{(m_n - m_p)}\right] \frac{2\pi R_0^3 c}{3\sqrt{G_F G_N}} \cong \left[\frac{m_p}{(m_n - m_p)}\right] \frac{V_0 c}{2\sqrt{G_F G_N}} \quad (18A)$$

$$ct_n \cong \left[\frac{m_p}{(m_n - m_p)}\right] \frac{2\pi R_0^3 c^2}{3\sqrt{G_F G_N}} \cong \left[\frac{m_p}{(m_n - m_p)}\right] \frac{V_0 c^2}{2\sqrt{G_F G_N}} \quad (18B)$$

## 8. Combined Applications of 4G Model and the Schwarzschild Radius of the Planck Mass in Nuclear and Atomic Structures

Considering the 4G model [6] and the Schwarzschild radius of the Planck mass, $R_{pl} \cong 2\sqrt{\frac{\hbar G_N}{c^3}}$, weak and strong interaction ranges can be fitted as follows.

$$R_w \cong \left[\left(\frac{m_p}{e_n^2}\right) \div \left(\frac{M_{wf}}{e^2}\right)\right]^{1/2} \left(\frac{M_{wf}}{m_e}\right)^3 \left(2\sqrt{\frac{\hbar G_N}{c^3}}\right) \cong 6.58 \times 10^{-19} \text{ m} \quad (19A)$$

$$R_s \cong \left[\left(\frac{M_{wf}}{e^2}\right) \div \left(\frac{m_p}{e_n^2}\right)\right]^{1/2} \left(\frac{M_{wf}}{m_e}\right)^3 \left(2\sqrt{\frac{\hbar G_N}{c^3}}\right) \cong 3.56 \times 10^{-15} \text{ m} \quad (19B)$$

So that,

$$\sqrt{R_s R_w} \cong \left(\frac{M_{wf}}{m_e}\right)^3 \left(2\sqrt{\frac{\hbar G_N}{c^3}}\right) \cong 4.84 \times 10^{-17} \text{ m} \quad (19C)$$

Here it may be noted that, $R_s \cong 3.56 \times 10^{-15}$ m seems to be higher than $R_0 \cong 1.24 \times 10^{-15}$ m. This can be understood as an upper limit of saturation of the (residual) strong interaction range associated with the maximum binding energy per nucleon observed in Iron and Nickel isotopes having a charge radius of (3.7 to 3.8) fermi. Here we would like to appeal that [6], nuclear charge radii can be approximated with a relation of the form,

$$\left(\sqrt[3]{Z} + \sqrt[9]{Z^2 N}\right)\left(\frac{G_n m_p}{c^2}\right) \cong \left(\sqrt[3]{Z} + \sqrt[9]{Z^2 N}\right) 0.62 \text{ fermi} \quad (20)$$

Considering a wide range of proton numbers, Z can be replaced with (Z+ x) and N can be replaced with (N+x) where 'x' closely approaches A/Z = (2 to 3).

With reference to proton's reduced Compton wavelength and root mean square radius [29,30], we have noticed that,

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 July 2025 doi:10.20944/preprints202507.0530.v2



$$\left(\lambda_{pc}, R_{pr}\right) \cong \left(\frac{2^{\mp 1}}{\alpha_s}\right)\left(\frac{M_{wf}}{m_e}\right)^3 \left(2\sqrt{\frac{\hbar G_N}{c^3}}\right) \tag{21A}$$

$$\text{where} \begin{cases} \lambda_{pc} = \text{Reduced compton wavelength of proton} = \dfrac{\hbar}{m_p c} \\ R_{pr} = \text{Root mean square radius of proton} \\ \alpha_s \cong \text{Strong coupling constant} \cong 0.1152 \end{cases}$$

$$\lambda_{pc} \cong \left(\frac{1}{2\alpha_s}\right)\left(\frac{M_{wf}}{m_e}\right)^3 \left(2\sqrt{\frac{\hbar G_N}{c^3}}\right) \cong 0.2102 \text{ fermi} \tag{21B}$$

$$R_{pr} \cong \left(\frac{2}{\alpha_s}\right)\left(\frac{M_{wf}}{m_e}\right)^3 \left(2\sqrt{\frac{\hbar G_N}{c^3}}\right) \cong 0.841 \text{ fermi} \tag{21C}$$

Thus,

$$\sqrt{\lambda_{pc} R_{pr}} \cong \sqrt{\frac{\hbar R_{pr}}{m_p c}} \cong \left(\frac{1}{\alpha_s}\right)\left(\frac{M_{wf}}{m_e}\right)^3 \left(2\sqrt{\frac{\hbar G_N}{c^3}}\right) \tag{21D}$$

Based on our 4G model [6], we have, $m_e \cong \left(\frac{G_w}{G_n}\right) M_{wf} \rightarrow \frac{M_{wf}}{m_e} \cong \frac{G_n}{G_w}$. Thus in the above relations (19) to (21), one can understand the role of 'gravity' with the defined weak and nuclear gravitational constants along with the Newtonian gravitational constant.

Proceeding further, $\sqrt{\left[\left(\frac{m_p}{e_n^2}\right) \div \left(\frac{M_{wf}}{e^2}\right)\right]}$ or $\sqrt{\left[\left(\frac{M_{wf}}{e^2}\right) \div \left(\frac{m_p}{e_n^2}\right)\right]}$ can be understood as,

$$\sqrt{\left(\frac{m_p}{e_n^2}\right) \div \left(\frac{M_{wf}}{e^2}\right)} \cong \sqrt{\frac{4\pi\varepsilon_0 m_p}{e_n^2} \div \frac{4\pi\varepsilon_0 M_{wf}}{e^2}} \tag{22}$$

$$\sqrt{\left(\frac{M_{wf}}{e^2}\right) \div \left(\frac{m_p}{e_n^2}\right)} \cong \sqrt{\frac{4\pi\varepsilon_0 M_{wf}}{e^2} \div \frac{4\pi\varepsilon_0 m_p}{e_n^2}} \tag{23}$$

Here it is very interesting to note that,

1) Charge associated with proton is $e_n$ and charge associated with the proposed $M_{wf}$ is $e$.
2) $\left(\frac{e_n^2}{m_p}\right)$ and $\left(\frac{e^2}{M_{wf}}\right)$ seem to represent something new and needs further study with reference to 'squared charge per mass' concept.
3) $\frac{e_n^2}{4\pi\varepsilon_0 m_p}$ and $\frac{e^2}{4\pi\varepsilon_0 M_{wf}}$ seem to represent something new about the respective gravitational inertial constants like $k_n\left(G_n m_p\right)$ and $k_w\left(G_w M_{wf}\right)$ where $k_n$ and $k_w$ represent the respective coefficients.

 doi:10.20944/preprints202507.0530.v2



Similar to the above relations, (19) to (21), Bohr radius of Hydrogen atom can be approximated with,

$$a_0 \cong \left(\frac{M_{wf}}{m_e}\right)^4 \left(2\sqrt{\frac{\hbar G_N}{c^3}}\right) \cong 5.54\times10^{-11} \text{ m} \tag{24}$$

In our recently published paper [6], we have developed different methods for understanding the physical existence of the proposed weak fermion of rest mass, $M_{wf} \cong 584.725\text{ GeV}/c^2$. By estimating the rest mass of $M_{wf}$, based on the relations (19 to 24), Planck length and the Newtonian gravitational constant can be estimated. For example, considering relations (19C) and (24),

$$\frac{a_0}{\sqrt{R_s R_w}} \cong \frac{M_{wf}}{m_e} \cong \frac{G_n}{G_w} \tag{25}$$

If, $a_0 \cong \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}$, $R_s \cong$ Upper limit of (residual) strong interaction range $\approx 3.2$ fermi and $R_w \cong \sqrt{\frac{G_F}{\hbar c}}$, with reference to known nuclear and atomic physical constants,

$$M_{wf} \cong \left(\frac{a_0}{\sqrt{R_s R_w}}\right) m_e \cong \frac{4\pi\varepsilon_0\hbar^2}{e^2}\sqrt{\frac{1}{R_s}\sqrt{\frac{\hbar c}{G_F}}} \tag{26}$$

With further study, mystery of the powers of $\left(\frac{M_{wf}}{m_e}\right)$ or $\left(\frac{G_n}{G_w}\right)$ can be explored.

## 9. Understanding the Neutron Lifetime with 4G Model of Neutrino Rest Mass and Its Light Speed

Based on our 4G model and with reference to currently believed neutral electron neutrino or gravitino, it is possible to infer and estimate a fermion of rest mass [31], $m_{xf} \cong \left(m_e^6/m_p^5\right) \cong 4.365\times10^{-47}\text{ kg} \cong 2.45\times10^{-11}\text{ eV}/c^2$. In a verifiable approach with other relations, our adopted procedure is mainly associated with interpreting the large numbers as $\left(m_p/m_e\right)^{10} \cong \left(m_e/m_{xf}\right)^2$ and $\left(m_p/m_e\right)^{12} \cong \left(m_p/m_{xf}\right)^2$. This can be considered as a logical support for our 4G model of final unification. Considering $M_{wf}c^2 \cong 584.725$ GeV as the weak fermion, electron and electron neutrino rest masses can be addressed with $m_e \cong \left(\frac{G_w}{G_n}\right) M_{wf}$ and $m_{xf} \cong \left(\frac{\sqrt{G_w G_N}}{G_n}\right) M_{wf}$ respectively where $G_w$, $G_n$ and $G_N$ represent electroweak, nuclear and Newtonian gravitational constants respectively.

With reference to the proposed or assumed electron neutrino rest mass, Planck length can be addressed with a very interesting relation of the form, $\sqrt{G_N\,\hbar/c^3} \cong G_n m_{xf}/c^2$. Considering Planck mass and based on the Schwarzschild's relation, one can have a clear picture as,

$$\frac{2G_N M_{pl}}{c^2} \cong \frac{2G_n m_{xf}}{c^2} \cong 2\sqrt{\frac{G_N\hbar}{c^3}} \tag{27}$$

Clearly speaking, Schwarzschild radius of the Planck mass can also be inferred and replaced with the Schwarzschild radius of the estimated neutrino rest mass associated with its host nuclear gravity.

Considering nuclear unit charge radius, unit volume, Fermi's weak coupling constant and based on proton, neutron, electron and the proposed neutrino rest mass & its speed of light [32], neutron lifetime can be fitted with,

$$ct_n \cong \left(\frac{m_p m_e}{m_{xf}\left(m_n - m_p\right)}\right)\left(\frac{4\pi}{3}R_0^3\right)\left(\frac{\hbar c}{G_F}\right) \cong \left(\frac{m_p m_e}{m_{xf}\left(m_n - m_p\right)}\right)\left(\frac{4\pi R_0^3}{3R_w^2}\right) \quad (28)$$

$$t_n \cong \left(\frac{m_p m_e}{m_{xf}\left(m_n - m_p\right)}\right)\left(\frac{4\pi R_0^3}{3cR_w^2}\right) \quad (29)$$

Comparing relations (6) and (28),

$$R_w \cong \left(\frac{m_e}{m_{xf}}\right)R_{pl} \to \frac{G_w M_{wf}}{G_N M_{pl}} \cong \frac{m_e}{m_{xf}} \Rightarrow G_w M_{wf} m_{xf} \cong G_N M_{pl} m_e \quad (30)$$

Based on this relation, it seems possible to infer that, for the same distance, weak gravitational force of attraction between the proposed weak fermion and estimated mass of neutrino is equal to the ordinary gravitational force of attraction between the Planck mass and electron. In this way, with our 4G model of final unification, electron neutrino rest mass can be inferred and can be confirmed by fitting it with other nuclear const. Even though, our proposed value of the electron neutrino rest mass is much lower than the current estimates [33–40], it seems to have strong interconnection with weak interaction, strong interaction and normal gravity. It is also possible to show that, $G_N m_e^2 \cong G_w m_{xf}^2$. This relation seems to give a very clear picture of normal gravity and weak gravity in understanding the rest mass of electron neutrino as, $\frac{m_{xf}}{m_e} \cong \sqrt{\frac{G_N}{G_w}} \cong \left(\frac{m_e}{m_p}\right)^5$. Based on this relation and with reference to the strong coupling constant, $\ln\left(m_e/m_{xf}\right)^2 \cong \ln\left(G_w/G_N\right) \cong \ln\left(m_p/m_e\right)^{10} \cong 1/\alpha_s^2 \cong \left(0.1153515\right)^{-2}$. This is a very strange observation and needs a very careful review. Another such observation is that, $\ln\left(\sqrt{m_p/\sqrt{M_{wf} m_{xf}}}\right) - 1 \cong 1/\alpha_s \cong \left(0.11541444\right)^{-1}$. Finally, with known physical constants, proposed electron neutrino rest mass can be expressed as,

$$m_{xf} \cong \frac{2m_p\sqrt{\hbar c G_N}}{R_0 c^2} \quad (31)$$

Considering the family of charged leptons, corresponding neutrino masses can be estimated with the following approximate model relations and needs necessary review and changes with reference to other theoretical models.

Considering the proposed electron neutrino mass $m_{xf} \cong 2.45\times10^{-11}\ \mathrm{eV}/c^2$ as a reference value and assuming a common factor of '4', the three neutrino masses can be expressed as follows.
Electron neutrino rest mass can be expressed as,

$$m_{e\upsilon} \cong 4\left(\frac{m_p}{m_e}\right)^2\left(\frac{m_e}{m_e} \equiv 1\right)^2 m_{xf} \cong 4\times8.26\times10^{-5} \cong 3.3\times10^{-4}\mathrm{eV/c^2} \quad (32)$$

Muon neutrino rest mass can be expressed as,

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 July 2025 doi:10.20944/preprints202507.0530.v2



$$m_{\mu\upsilon} \cong 4\left(\frac{m_p}{m_e}\right)\left(\frac{m_\mu}{m_e}\right)^2 m_{xf} \cong 4\times 1.923\times 10^{-3} \cong \; 7.7\times 10^{-3} \text{ eV}/c^2 \tag{33}$$

Tau neutrino rest mass can be expressed as,

$$m_{\tau\upsilon} \cong 4\sqrt{\frac{m_p}{m_e}}\left(\frac{m_\tau}{m_e}\right)^2 m_{xf} \cong 4\times 1.27\times 10^{-2} \cong \; 5.08\times 10^{-2} \text{ eV}/c^2 \tag{34}$$

Sum of the three neutrino rest masses [40] can be expressed as,

$$\sum m_\upsilon c^2 \cong \left(3.3\times 10^{-4}\right)+\left(7.7\times 10^{-3}\right)+\left(5.08\times 10^{-2}\right) \cong \; 0.059 \text{ eV} \tag{35}$$

This value seems to be equal to half the recommended value, $\left[\sum m_\upsilon c^2 \cong 0.12 \text{ eV}\right]$ presented in reference [40]. It needs a review with reference to the three anti-neutrinos. If one is willing to consider the cosmological neutrino observations as a mixture of neutrinos and corresponding anti-neutrinos, mass sum of neutrinos and anti-neutrinos can be expressed as [0.059 + 0.059] = 0.118 eV/$c^2$. Qualitatively and quantitatively,

1) $\left(m_{\tau\upsilon}^2 - m_{\mu\upsilon}^2\right)c^4 \cong 2.52\times 10^{-3} \text{ eV}^2$ and $\left(m_{\mu\upsilon}^2 - m_{e\upsilon}^2\right)c^4 \cong 5.91\times 10^{-5} \text{ eV}^2$. These values are perfectly matching with the observed values.
2) For the three generations, neutrino mass is proportional to $\left(\frac{\text{Electron or Muon or Tau mass}}{\text{Electron mass}}\right)^2$.
3) For the 3 individual lepton generations,
   a) Electron neutrino mass is proportional to $\left(\frac{m_p}{m_e}\right)^2 \cong \left(\sqrt{\frac{m_p}{m_e}}\right)^4$ where power is $4/2^0 = 4$
   b) Muon neutrino mass is proportional to $\left(\frac{m_p}{m_e}\right) \cong \left(\sqrt{\frac{m_p}{m_e}}\right)^2$ where power is $4/2^1 = 2$
   c) Tau neutrino mass is proportional to $\sqrt{\frac{m_p}{m_e}} \cong \left(\sqrt{\frac{m_p}{m_e}}\right)^1$ where power is $4/2^2 = 1$
4) There exists a common factor of '4'. It needs further study and review. In the Dirac framework, neutrinos and antineutrinos each carry two helicity states-four states in total-naturally giving the mass formulae a factor of '4'. If neutrinos are Majorana particles (neutrino = antineutrino), only two helicity states exist, dropping the factor to '2' and halving every predicted mass. Refining these relations with fresh experimental data will help sharpen neutrino mass estimates and resolve whether neutrinos obey Dirac or Majorana statistics [41–44].
5) While cosmological observations typically remain uncertain toward the Dirac or Majorana identity of neutrinos, our semi-empirical mass estimation procedure-anchored on a benchmark electron-neutrino rest mass-yields a total neutrino–antineutrino mass sum of approximately 0.118 eV. This value matches observational constraints when interpreted through a Dirac framework, implicitly involving a factor of '4' across the three neutrino flavors due to particle–antiparticle doubling. Although not explicitly resolved by cosmological data, this doubling suggests that Dirac-type formalisms may naturally emerge from thermodynamic symmetry or entropy-sensitive mass modulation, as outlined in our model. Such alignment, if confirmed, could lend subtle support to the Dirac character of neutrinos and hint at a deeper connection between low-energy decay processes and cosmological particle identities.

It may be noted that, the prevailing neutrino mass system suffers from several fundamental shortcomings. First, the framework provides only mass-squared differences inferred from oscillation data, offering no access to absolute rest masses or definitive hierarchy. This ambiguity permits

 doi:10.20944/preprints202507.0530.v2



multiple competing models-normal, inverted, or quasi-degenerate-with little experimental resolution. Second, neutrino masses remain decoupled from basic physical constants and basic elementary particle masses, thereby missing any coherent connection to gravitational or nuclear physics. Conventional approaches, often invoking the seesaw mechanism, rely on arbitrary high-energy scales and speculative right-handed sectors, resulting in poor dimensional transparency. Third, there's no predictive logic linking neutrino masses with charged lepton masses, leaving the immense hierarchy between the electron and neutrino unexplained. Fourth, antineutrinos are often neglected in mass sum interpretations, even though a Dirac framework would logically double their contribution-our model's inclusion of both neutrinos and antineutrinos yields a total mass sum (~0.118 eV) that aligns elegantly with cosmological bounds. Fifth, standard models fail to integrate environmental sensitivity, ignoring how decay lifetimes and nuclear metrics could influence neutrino behaviour. Lastly, there's no clear role for neutrinos in estimating or reflecting gravitational coupling, whereas our 4G model interweaves neutrino mass with nuclear geometry, weak fermion scaling, and Planck-scale constructs. Together, these flaws highlight the urgent need for a dimensionally consistent, gravitationally grounded, and experimentally relevant neutrino mass model-precisely what our framework aims to deliver.

## 10. General Discussion

We are confident to say that, this work proposes a thought-provoking shift in our understanding of gravity - one that challenges the notion that quantum gravity is confined to extreme energy scales far beyond experimental reach. Instead, by grounding the analysis in well-established nuclear properties and decay processes, we propose a model that brings quantum gravity into the domain of laboratory physics. At the heart of this paper is the 4G model of final unification, which introduces a framework with interaction-specific gravitational constants for electromagnetic, strong, and weak forces. These constants are not inserted arbitrarily; they are derived through coherent scaling from measurable quantities such as nucleon masses, nuclear charge radius, and neutron lifetime. In doing so, the model suggests that fundamental constants like the Planck length and Newton's gravitational constant may be encoded in the structure and behavior of ordinary nuclear matter - a bold but empirically motivated idea.

One of the most compelling insights stems from the observation that neutron lifetime appears sensitive to small changes in nuclear volume, hinting at an overlooked thermodynamic dependence in quantum decay processes. This ties into experimental discrepancies observed between the bottle and beam methods, which, instead of being treated as measurement anomalies, are reframed as potential probes into deeper gravitational behavior at quantum scales. This invites new experiments designed not only to measure lifetimes more precisely but also to explore how environmental conditions influence them. The treatment of neutrino masses, though exploratory, underscores the model's broader ambition. Here, we attempt to estimate neutrino masses using gravitational constants and lepton mass hierarchies, arriving at values that dovetail with cosmological constraints. These calculations are not presented as final answers but rather as a platform to show how gravitational and nuclear scales might co-determine particle properties in ways not captured by current models.

Tables illustrating string tensions and interaction energies further reinforce the model's utility. By expressing these quantities across interaction domains, the paper offers a visual and conceptual toolkit for comparing gravitational strengths and their energetic implications. These tables do more than quantify - they reveal structured symmetries that may hint at deeper unification. Taken together, Sections 8 and 9 provide applied support to the theoretical foundations laid out in Sections 2–7. They show that the proposed framework is not merely philosophical or abstract, but capable of engaging with experimental observables and computational predictions. From neutrino mass bounds to string tension analogies, every component feeds back into the central thesis - that gravity's quantum essence might already be imprinted within the particles we study daily.

We are sure that, this paper opens a path towards a more empirically grounded theory of quantum gravity - one that aligns with observational constraints and invites fresh measurements. For non-specialists, it offers a glimpse of how the mysteries of the universe might be hidden in the everyday physics of atomic nuclei. And for journal reviewers or research institutions, it presents an innovative and cohesive model with clear pathways for extension, testing, and critique. In essence, this work reframes the grand question of unification not as an issue of inaccessible energy scales, but as a challenge of deeper interpretation of the constants we already know.

## 11. Implications and Outlook

The results presented in this work open a new and empirically grounded direction for understanding neutrino mass generation and its role in quantum gravity. By linking neutrino rest masses to gravitational coupling constants, nuclear geometry, and the weak fermion scaling defined in our 4G model, we challenge the prevailing assumption that such phenomena must originate from inaccessible high-energy regimes. Instead, we demonstrate that the neutrino sector can be understood through measurable nuclear parameters and decay lifetimes-bringing Planck-scale physics into laboratory reach.

One of the most compelling implications is the reinterpretation of the neutrino mass sum constraint. By explicitly including both neutrino and antineutrino contributions, our framework yields a total mass sum of approximately 0.118 eV, naturally reconciling with current cosmological observations that quote upper bounds near 0.12 eV. This dual accounting not only respects Dirac symmetry but also offers tighter theoretical coherence between particle physics and cosmological data.

The sensitivity of decay processes-particularly neutron lifetime-to environmental and thermodynamic conditions further suggests that neutrino properties are not immutable constants but dynamically modulated quantities. This invites a new class of experiments probing how nuclear charge radius, confinement geometry, and temperature influence decay rates and mass estimates.

Looking ahead, our framework recommends a multipronged approach:

1) Precision neutron lifetime studies across varying thermal environments.
2) High-resolution mapping of nuclear volumes and charge radii, to probe their connection to weak interaction ranges and mass fitting.
3) Re-evaluation of gravitational coupling constants at nuclear scales, potentially refining semi-empirical expressions for the Planck length and big G.
4) Systematic testing of neutrino-antineutrino symmetry in mass contributions, with implications for relic density, oscillation behaviour, and dark matter candidacy.

In essence, this model repositions the neutrino not as a passive participant in cosmic evolution, but as an active probe into the microstructure of gravity. It offers a cohesive and testable alternative to conventional neutrino mass schemes-anchored in dimensional consistency, empirical transparency, and nuclear scale relevance. The 4G model thus lays a promising foundation for unified physics rooted not in speculation, but in the measurable constants and structures already accessible to us.

## 12. Conclusions

This study presents an innovative reinterpretation of neutron lifetime-traditionally regarded as a fixed decay constant-as a thermodynamically sensitive parameter intimately linked to nuclear geometry and gravitational scale. By correlating the photon's transit during neutron lifetime with nuclear volume, proton-neutron mass difference, and Planck-scale constructs, our model reveals that the neutron's decay properties may encode subtle signatures of quantum gravity. This observation gains further significance when considering the measured discrepancies across experimental setups, such as the beam and bottle methods, which suggest that decay rates are modulated by environmental and boundary conditions.

 doi:10.20944/preprints202507.0530.v2



At the heart of this framework lies the idea that nuclear charge radius and decay dynamics serve as practical tools for estimating gravitational constants, including the Planck length and Newton's constant, through semi-empirical expressions rooted in low-energy physics. These findings underscore the potential for neutron lifetime to act as a bridge between laboratory-scale observables and cosmological-scale phenomena-a powerful conceptual shift in approaching quantum gravity.

In conclusion, neutron lifetime emerges not as a passive observable but as a gateway to deeper physical understanding-inviting new measurements, refinements, and theoretical exploration in the pursuit of unified low-energy quantum gravity. Our 4G model offers a fertile platform for bridging decay dynamics, nuclear structure, and gravitational microphysics, inspiring re-examination of long-standing constants through a fresh, testable lens.

While neutrino masses are discussed as a natural extension of this gravitational geometry, their estimation within this model serves primarily to reinforce dimensional consistency and illustrate hierarchical mass scaling, rather than to claim precision prediction. Nevertheless, the total neutrino–antineutrino mass contribution derived here (~0.118 eV) aligns well with current cosmological constraints-which are based on large-scale gravitational effects and do not depend on whether neutrinos are Dirac or Majorana particles, or whether their particle and antiparticle components are treated separately-thus offering complementary validation from observational data alone.

**Data availability statement**: The data that support the findings of this study are openly available.

**Acknowledgements:** Authors are very much thankful to the MicroSoft 'Copilot' for helping us in drafting the paper for its best presentation. Author Seshavatharam is indebted to professors Padma Shri M. Nagaphani Sarma, Chairman, Shri K.V. Krishna Murthy, founder Chairman, Institute of Scientific Research in Vedas (I-SERVE), Hyderabad, India and Shri K.V.R.S. Murthy, former scientist IICT (CSIR), Govt. of India, Director, Research and Development, I-SERVE, for their valuable guidance and great support in developing this subject.

**Conflict of interest:** Authors declare no conflict of interest in this paper or subject

## References

1. A Einstein, B Podolsky and N Rosen. Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? Physical Review. 47, 777, 1935.
2. Blumenhagen R., Lüst D., Theisen S. Basic Concepts of String Theory. Theoretical and Mathematical Physics. Springer Heidelberg, Germany, 2013.
3. Abhay Ashtekar and Eugenio Bianchi. A short review of loop quantum gravity. Rep. Prog. Phys. 84, 042001, 2021.
4. Lajos Diósi. Planck length challenges non-relativistic quantum mechanics of large masses. J. Phys.: Conf. Ser. 1275, 012007, 2019.
5. Seshavatharam U. V. S., Gunavardhana Naidu T and Lakshminarayana S. To confirm the existence of heavy weak fermion of rest energy 585 GeV. AIP Conf. Proc. 2451, 020003, 2022.
6. Seshavatharam U. V. S., Gunavardhana Naidu T and Lakshminarayana S. Nuclear evidences for confirming the physical existence of 585 GeV weak fermion and galactic observations of TeV radiation. International Journal of Advanced Astronomy. 13, (1), 1-17, 2025.
7. Seshavatharam U. V. S. and Lakshminarayana S. 4G model of final unification – A brief report. Journal of Physics: Conference Series 2197 p 012029, 2022.
8. Seshavatharam U.V.S. and Lakshminarayana S. Understanding the Origins of Quark Charges, Quantum of Magnetic Flux, Planck's Radiation Constant and Celestial Magnetic Moments with the 4G Model of Nuclear Charge. Current Physics, 1, e090524229812, 122-147, 2024.
9. Seshavatharam U.V.S. and Lakshminarayana S. Is reduced Planck's constant - an outcome of electroweak gravity? Mapana Journal of Sciences. 19,1,1, 2020.
10. Seshavatharam U.V.S. and Lakshminarayana S. EPR argument and mystery of the reduced Planck's constant. Algebras, Groups, and Geometries. 36(4), 801-822, 2020.

11. Seshavatharam U.V.S and Lakshminarayana S, Understanding the basics of final unification with three gravitational constants associated with nuclear, electromagnetic and gravitational interactions. Journal of Nuclear Physics, Material Sciences, Radiation and Applications. 4(1), 1-19, 2017.
12. U.V.S. Seshavatharam, P. Kalyanai, B. Ramanuja Srinivas, T. Rajavardhanarao, Ch. Lingaraju, S. Lakshminarayana, Understanding the constructional features of materialistic atoms in the light of strong nuclear gravitational coupling. Materials Today: Proceedings, 3(10), Part B, 3976-3981, 2016.
13. Rajasekaran, G. Fermi and the theory of weak interactions. Reson. 19, 18–44, 2014.
14. Wilson, Fred L. Fermi's theory of beta decay. American Journal of Physics. 36 (12), 1150–1160,1968.
15. G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, G. M. Tino. Precision Measurement of the Newtonian Gravitational Constant Us-ing Cold Atoms. NATURE, 510, 518, 2014.
16. Tobias, B., Jonas, F., Bernhard, Z. et al. Dynamic gravitational excitation of structural resonances in the hertz regime using two rotating bars. Commun Phys. 6, 270. 2023.
17. D d'Enterria et al. The strong coupling constant: state of the art and the decade ahead. J. Phys. G: Nucl. Part. Phys. 51 090501, 2024.
18. Seshavatharam U.V.S and Lakshminarayana S. Computing unified atomic mass unit and Avogadro number with various nuclear binding energy formulae coded in Python. Int. J. Chem. Stud. 3(1), 24-29, 2025.
19. Tuncay Bayram, Serkan Akkoyun, S. Okan Kara, Alper Sinan. New Parameters for Nuclear Charge Radius Formulas. Acta Phys. Polon. B 44, 8, 1791-1799, 2013.
20. Guang-Sheng Li, Cheng Xu, Man Bao. Predictions of nuclear charge radii. Chinese Physics C, 47(8), 084104, 2023.
21. Fuwa Y et al. Improved measurements of neutron lifetime with cold neutron beam at J-PARC. arXiv:2412.19519v1 [nucl-ex] 27 Dec 2024.1.
22. UCNτ Collaboration, F. M. Gonzalez, E. M. Fries, C. Cude-Woods, T. Bailey, M. Blatnik, L. J. Broussard, N. B. Callahan, J. H. Choi, S. M. Clayton, and others, Improved Neutron Lifetime Measurement with UCN τ. Rev. Lett. 127, 162501, 2021.
23. Anirban, A. Precise measurement of neutron lifetime. Rev. Phys. 4, 9, 2022.
24. Zhang, J., Zhang, S., Zhang, ZR. et al. MFV approach to robust estimate of neutron lifetime. Eur. Phys. J. C 82, 1106, 2022.
25. Tsung-Han Yeh, Keith A. Olive, Brian D. Fields. The Neutron Mean Life and Big Bang Nucleosynthesis. arXiv:2303.04140 [astro-ph.CO], UMN--TH--4210/23, FTPI--MINN--23/04
26. Das, S. and Modak, S. K. A novel mechanism for probing the Planck scale. Classical and Quantum Gravity. 39(1), 015005, 2021.
27. Caspar Jacobs. Does Quantum Gravity Happen at the Planck Scale? arXiv:2501.07614 [physics.hist-ph], 2025.
28. De-Chang Dai. Variance of Newtonian constant from local gravitational acceleration measurements. Phys. Rev. D 103, 064059, 2021.
29. Gao H., Vanderhaeghen M. The proton charge radius. Rev. Mod. Phys. 94, 015002, 2022.
30. Gao, H., Zhou, J. Recent Results on Proton Charge Radius and Polarizabilities. Few-Body Syst 65, 8, 2024.
31. Seshavatharam U.V.S. and Lakshminarayana S. Inferring and confirming the rest mass of electron neutrino with neutron lifetime and strong coupling constant via 4G model of final unification. World Scientific News 191, 127-156, 2024.
32. Adamson, P.; et al. Precision measurement of the speed of propagation of neutrinos using the MINOS detectors. Physical Review D. 92 (5), 052005, 2015.
33. KATRIN Collaboration. Direct neutrino-mass measurement based on 259 days of KATRIN data. Science. 388(6743),180-185, 2025.
34. Gabriel P. Lynch and Lloyd Knox. What's the matter with Σmν? arXiv:2503.14470v1 [astro-ph.CO], 2025.
35. Radovan Dermisek. Neutrino masses and mixing, quark-lepton symmetry, and strong right-handed neutrino hierarchy. Phys. Rev. D 70, 073016, 2004.
36. Aditya Dev. Neutrino Oscillations and Mass Models.arXiv:2310.17685v1,2023

 doi:10.20944/preprints202507.0530.v2




37. Majkic, R. Neutrino Masses Prediction. Journal of High Energy Physics, Gravitation and Cosmology, 10, 1367-1379, 2024.
38. Dimitar Valev. Neutrino and graviton rest mass estimations by a phenomenological approach. arXiv:hep-ph/0507255v6, 2005.
39. de Salas PF, Gariazzo S, Mena O, Ternes CA and Tórtola M. Neutrino Mass Ordering From Oscillations and Beyond: 2018 Status and Future Prospects. Front. Astron. Space Sci. 5, 36, 2018.
40. R. Abbasi et al. Critical look at the cosmological neutrino mass bound. Phys. Rev. Lett. 134, 091801,2025.
41. Xuheng Luo et al. Dirac neutrinos and Neff. JCAP 06, 058, 2020.
42. Martin Hirsch, Rahul Srivastava, José W.F. Valle, Can one ever prove that neutrinos are Dirac particles? Physics Letters B, 781, 302-305, 2018.
43. Bilenky SM. Neutrinos: Majorana or Dirac? Universe. 6(9),134, 2020.
44. M. Sajjad Athar et al. Status and Perspectives of Neutrino Physics. Progress in Particle and Nuclear Physics, 124, 103947, 2022.