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Letter

Extended General Relativity for a Conformally Curved Universe

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Abstract: The recent Planck Legacy release confirmed the presence of an enhanced lensing amplitude in the cosmic microwave background (CMB) power spectra. Notably, this amplitude is higher than that estimated by the lambda cold dark matter model (ΛCDM), which endorses a positively curved early Universe with a confidence level greater than 99%. Although General Relativity (GR) performs accurately in the local/present Universe where spacetime is almost flat, its lost boundary term, incompatibility with Quantum Mechanics and the necessity of dark matter/energy could indicate its incompleteness. By utilising the Einstein–Hilbert action, this letter presents extended field equations by considering the pre-existing/background curvature and the boundary contribution. The extended field equations consist of Einstein field equations with a conformal transformation feature in addition to the boundary term, which can remove singularities, satisfy a conformal invariance theory and facilitate its quantisation.

Keywords: Extended General Relativity; String Theory Branes.

1. Introduction

The considerable efforts, such as the development of frameworks for the conformal gravity, loop quantum gravity, MOND, ADS-CFT, string theory, and $F(R)$ gravity, have been devoted to modifying gravity [1,2,11,3–10]. Motivation to modify General Relativity (GR) and gravity in general aimed to elucidate possible existence or the nature of dark matter and dark energy, achieve a better description of observation data, verify theoretical restrictions in the strong curvature regime such as within back holes and to formulate Quantum Gravity [3,11]. To achieve an efficient action for quantum corrections, several theories have been formulated on the modification of Lagrangian gravitational fields. Such modifications appear to be inevitable, which have included higher-order curvature terms as well as non-minimally coupled scalar fields [12–14]. However, these modifications have to be consistent with the energy conservation law.

Recent evidence by Planck Legacy (PL18) recent release confirmed the presence of an enhanced lensing amplitude in the cosmic microwave background (CMB) power spectra, which is higher than that estimated by the lambda cold dark matter model (ΛCDM). This endorses the positive curvature of the early Universe with a confidence level greater than 99% [15,16]. Besides, the gravitational lensing by the substructures of several galaxy clusters is an order of magnitude more than estimated by the ΛCDM [17,18]. This evidence endorses a spatially curved Universe in spite of the spacetime flatness of the local/present-day Universe.

Accordingly, in this study, the background/pre-existing curvature as revealed by the PL18 has been incorporated into the Einstein–Hilbert action. However, to comply with the energy conservation law and the compatibility of the action, a new modulus of spacetime curvature is introduced, which is formulated based on the theory of elasticity [19]. The letter is organised as follows. Section 2 discusses the formulation of the space-time modulus of curvature and the mathematical derivations of the extended field equations. Section 3 summarises the outcomes and conclusions and suggests the future development of this work.

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2. Extended Field Equations

The recent PL18 release has endorsed the positive early Universe curvature, that is, is evidence of a pre-existing/background curvature. To consider this pre-existing curvature and its evolution over cosmic time, a modulus of space-time deformation/curvature, E_D, is introduced based on the Theory of Elasticity [19]. By utilising the trace-reversed Einstein field equations, the modulus $E_D =$ (stress/strain) can be expressed as

$$E_D = \frac{T_{\mu\nu} - T g_{\mu\nu}/2}{R_{\mu\nu}/\mathcal{R}} = \frac{c^4}{8\pi G_t {r_t}^2} \quad (1)$$

where the stress is signified by the stress-energy momentum tensor $T_{\mu\nu}$ of trace T while the strain is signified by the Ricci curvature tensor $R_{\mu\nu}$ as the change in the curvature divided by the scalar of the pre-existing curvature $\mathcal{R} = 1/{r_t}^2$; r_t is the Universe's radius of curvature as a function of cosmic time, t, and $g_{\mu\nu}$ is the metric tensor. According to the Elasticity theory, E_D is a constant [19]; thus, Eq. (1) shows an inverse proportionality between the gravitational 'constant' G_t and r_t, where G_t follows the inverse square law with respect to the Universe's curvature radius. This relationship is consistent with Mach's principle, the reliance of the small structure on the larger structure. Schrödinger in 1925 pointed to the reliance of G_t on the distribution of the Universe's masses and the Universe's radius while Dirac in 1938 proposed its correlation to the age of the Universe [20]. Measurements of G_t suggest changes over time [21] while its evolution is preferred to reduce the conflict of matter power spectrum amplitude with Planck datasets [22–24]. The decrease in star formation rate over the age of the Universe [25] can be due to the decrease in G_t while the gradual evolution in the fine-structure 'constant' [26–28] may reveal that the presumed fundamental constants relies on other Universe properties.

In contrast, E_D is in terms of energy density and represents the resistance of the continuum (space-time continuum [1]) to deformation, where the law of energy conservation is a firm fundamental law [19]. Eq. (1) shows E_D is proportional to the fourth-power of the speed of light, which in turn is directly proportional to the frequency; in accordance with the frequency cut-off predictions of the vacuum energy density in QFT [29,30]; thus, E_D characterizes spacetime resistance to curvature and can represent vacuum energy density.

By incorporating the pre-existing curvature and complying with the law of energy conservation, the Einstein–Hilbert action can be extended to

$$S = \int \left[\frac{E_D}{2}\frac{R}{\mathcal{R}} + \mathcal{L}_M\right]\sqrt{-g}\, d^4x \quad (2)$$

where R and $\mathcal{R}$ are the Ricci/induced and the pre-existing scalar curvatures respectively, $\mathcal{L}_M$ is the Lagrangian density and g is the determinant of the metric tensor g_{uv}. The indistinctive variation in the action yields

$$\delta S = \int \left[\frac{E_D}{2}\left(\frac{\delta R\sqrt{-g}}{\mathcal{R}} - \frac{\delta\mathcal{R}\sqrt{-g}}{\mathcal{R}^2}R + \frac{\delta\sqrt{-g}}{\mathcal{R}}R\right) + \left(\delta\mathcal{L}_M\sqrt{-g} + \delta\sqrt{-g}\mathcal{L}_M\right)\right] d^4x \quad (3)$$

The variation in the scalar curvature, $R = R_{\mu\nu}g^{\mu\nu}$, is $\delta R = R_{\mu\nu}\delta g^{\mu\nu} + g^{uv}\delta R_{\mu\nu}$ while by utilising the Jacobi's formula for the differentiation of the determinant; thus, $\delta\sqrt{-g} = -\sqrt{-g}\, g_{\mu\nu}\delta g^{\mu\nu}/2$ where the variation in $g_{\mu\nu}g^{\mu\nu} = \delta^{\mu}_{\nu}$ is $g_{\mu\nu}\delta g^{\mu\nu} = -g^{\mu\nu}\delta g_{\mu\nu}$ [31].

[1] Space-time can be regarded as a continuum with a dual quantum nature, that it curves as waves according to the GR while fluxing as quantum energy particles; the latter is justified because the energy flux from early Universe plasma into space at the speed of light creating a 'space-time continuum' or 'vacuum energy'. This could be corroborated by light polarisation from CMB [32].

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Hence, the variation in the action in Eq. (3) is expanded to

$$\delta S=\int\begin{bmatrix}\frac{E_D}{2}\left(\frac{R_{\mu\nu}\delta g^{\mu\nu}+g^{\mu\nu}\delta R_{\mu\nu}}{\mathcal{R}}-\frac{\mathcal{R}_{\mu\nu}\delta g^{\mu\nu}+g^{\mu\nu}\delta\mathcal{R}_{\mu\nu}}{\mathcal{R}^2}R-\frac{g_{\mu\nu}\delta g^{\mu\nu}}{2\mathcal{R}}R\right)\\+\left(\delta\mathcal{L}_M-\frac{g_{\mu\nu}\delta g^{\mu\nu}}{2}\mathcal{L}_M\right)\end{bmatrix}\sqrt{-g}\,d^4x \quad (4)$$

By considering the first boundary term in Eq. (4), $\int(E_D/2\mathcal{R})g^{\mu\nu}\delta R_{\mu\nu}\sqrt{-g}\,d^4x$. The variation in the Ricci curvature tensor $\delta R_{u\nu}$ can be expressed in terms of the covariant derivative of the difference between two Levi-Civita connections, the Palatini identity, as $\delta R_{\mu\nu}=\nabla_\rho(\delta\Gamma^\rho_{\nu\mu})-\nabla_\nu(\delta\Gamma^\rho_{\rho\mu})$. The variation with respect to the inverse metric $g^{\mu\nu}$ is obtained by using the metric compatibility of the covariant derivative, $\nabla_\rho g^{\mu\nu}=0$ [31], as $g^{\mu\nu}\delta R_{\mu\nu}=\nabla_\rho(g^{\mu\nu}\delta\Gamma^\rho_{\nu\mu}-g^{\mu\rho}\delta\Gamma^\sigma_{\sigma\mu})$. Thus, the first boundary term as a total derivative for any tensor density is transformed based on Stokes' theorem with renaming the dummy indices as

$$\frac{E_D}{2\mathcal{R}}\int g^{\mu\nu}\delta R_{\mu\nu}\sqrt{-g}d^4x=\frac{E_D}{2\mathcal{R}}\int\nabla_\rho(g^{\mu\nu}\delta\Gamma^\rho_{\nu\mu}-g^{\mu\rho}\delta\Gamma^\sigma_{\sigma\mu})\sqrt{-g}d^4x$$
$$=\frac{E_D}{2\mathcal{R}}\iiint_V\nabla_\mu A^\mu\sqrt{-g}dV=\frac{E_D}{2\mathcal{R}}\oint\!\!\!\oint_S A^\mu.\hat{n}_u\sqrt{|q|}\,dS=\frac{E_D}{2\mathcal{R}}\oint_{\partial V}K\epsilon\sqrt{|q|}\,d^3x \quad (5)$$

The non-boundary term $\mathrm{E_D}/2\mathcal{R}$ is left outside the integral transformation as it only acts as a scalar to the integral called S_{GHY} [33,34]. The same is applied to the second boundary term in Eq. (4). Accordingly, the variation in the action is expressed as

$$\delta S=\int\begin{bmatrix}\frac{E_D}{2}\left(\frac{R_{\mu\nu}\delta g^{\mu\nu}}{\mathcal{R}}-\frac{\mathcal{R}_{\mu\nu}\delta g^{\mu\nu}}{\mathcal{R}^2}R-\frac{g_{\mu\nu}\delta g^{\mu\nu}}{2\mathcal{R}}R\right)\\+\left(\frac{2\delta\mathcal{L}_M/\delta g^{\mu\nu}-g_{\mu\nu}\mathcal{L}_M}{2}\delta g^{\mu\nu}\right)\end{bmatrix}\sqrt{-g}\,d^4x+\int\left[\frac{E_D\epsilon}{2}\left(\frac{K\sqrt{|q|}}{\mathcal{R}}-\frac{\mathcal{H}\sqrt{|p|}}{\mathcal{R}^2}R\right)\right]d^3x \quad (6)$$

where K and $\mathcal{H}$ are the extrinsic traces of the induced and the pre-existing/conformal curvatures respectively, q and p are the determinants of their induced metric tensors respectively and ϵ equals 1 when the normal $\hat{n}_u$ is a spacelike entity and equals -1 when it is a timelike entity. It is worth noting that, the matter and curvature actions are satisfying the criteria that the variation in the action δS is with respect to the variation in the inverse metric $\delta g^{\mu\nu}$ excluding the boundary action that still lacks this feature. Thus, to achieve the consistency of the action, the variation in the boundary action has to be determined. The indistinctive variation is the first boundary term is

$$\frac{E_D}{2\mathcal{R}}\epsilon\int\left(K_{\mu\nu}\delta q^{\mu\nu}+q^{\mu\nu}\delta K_{\mu\nu}+K\frac{\delta\sqrt{|q|}}{\sqrt{|q|}}\right)\sqrt{|q|}\,d^3x \quad (7)$$

where $K=K_{\mu\nu}q^{\mu\nu}$. By utilising Jacobi's formula for the determinant differentiation; thus, $\delta\sqrt{|q|}=-\sqrt{|q|}\,q_{\mu\nu}\delta q^{\mu\nu}/2$; and the variation in $q^{\mu\nu}q_{\mu\nu}=\delta^\mu_\nu$ as $q^{\mu\nu}=-q_{\mu\nu}\delta q^{\mu\nu}/\delta q_{\mu\nu}$; thus, the first boundary term is

$$\frac{E_D}{2\mathcal{R}}\epsilon\int\left(K_{\mu\nu}\delta q^{\mu\nu}-\frac{1}{2}K\left(q_{\mu\nu}\delta q^{\mu\nu}+2q_{\mu\nu}\frac{\delta K_{\mu\nu}}{\delta q_{\mu\nu}K}\delta q^{\mu\nu}\right)\right)\sqrt{|q|}\,d^3x \quad (8)$$

here $\delta K_{\mu\nu}/\delta q_{\mu\nu}K=(\delta K_{\mu\nu}/K_{\mu\nu})(q_{\mu\nu}/\delta q_{\mu\nu})=\delta\ln K_{\mu\nu}/\delta\ln q_{\mu\nu}$ resembles the Ricci flow in a normalised form reflecting the conformal distortion in the boundary, which can be expressed as a positive function Ω^2 based on Weyl's conformal transformation [35] as

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$\tilde{q}_{\mu\nu} = q_{\mu\nu}\Omega^2$. The non-boundary term $E_D\epsilon/2\mathcal{R}$ is left outside, where it can be considered as a scalar. Otherwise, its high-order variational terms can be incorporated into the conformal transformation function Ω^2. Therefore, Eq. (8) can be expressed as

$$\frac{E_D}{2\mathcal{R}}\epsilon\int\left(K_{\mu\nu}\delta q^{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu}\delta q^{\mu\nu}\right)\sqrt{|q|}\,d^3x \tag{9}$$

where $\hat{q}_{\mu\nu} = q_{\mu\nu} + 2\tilde{q}_{\mu\nu}$ is the conformal transformation of the induced metric tensor where Einstein spaces are a subclass of the conformal space [36]. The same is applied for the second boundary term, thus, the variation in the full action is

$$\delta S = \int \begin{matrix} \left[\frac{E_D}{2}\left(\frac{R_{\mu\nu}}{\mathcal{R}} - \frac{\mathcal{R}_{\mu\nu}}{\mathcal{R}^2}R - \frac{g_{\mu\nu}}{2\mathcal{R}}R\right) + \frac{2\delta\mathcal{L}_M/\delta g^{\mu\nu} - g_{\mu\nu}\mathcal{L}_M}{2}\right]\delta g^{\mu\nu}\sqrt{-g}\,d^4x + \\ \left[\frac{E_D\epsilon}{2}\,\frac{K_{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu}}{\mathcal{R}}\right]\delta q^{\mu\nu}\sqrt{|q|}\,d^3x - \left[\frac{E_D\epsilon}{2}\,\frac{\mathcal{H}_{\mu\nu} - \frac{1}{2}\mathcal{H}\hat{p}_{\mu\nu}}{\mathcal{R}^2}R\right]\delta p^{\mu\nu}\sqrt{|p|}\,d^3x \end{matrix} \tag{10}$$

The stress-energy momentum tensor, $T_{\mu\nu}$, is proportional to the Lagrangian term by definition as $T_{\mu\nu} := \mathcal{L}_M g_{\mu\nu} - 2\delta\mathcal{L}_M/\delta g^{\mu\nu}$ [31,37]; thus, by implementing ϵ as a timelike entity and applying the principle of stationary action yield

$$\frac{R_{\mu\nu}}{\mathcal{R}} - \frac{1}{2}\frac{R}{\mathcal{R}}g_{\mu\nu} - \frac{R}{\mathcal{R}^2}\mathcal{R}_{\mu\nu} + \frac{R\left(\mathcal{H}_{\mu\nu} - \frac{1}{2}\mathcal{H}\hat{p}_{\mu\nu}\right) - \mathcal{R}\left(K_{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu}\right)}{\mathcal{R}^2} = \frac{T_{\mu\nu}}{E_D} \tag{11}$$

The extended field equations can be interpreted as indicating that the induced curvature over the pre-existing curvature equals the ratio of the imposed energy density and its flux to the vacuum energy density and its flux through an expanding/contracting Universe. The conformal curvature term comprising $\mathcal{R}_{\mu\nu}$ accounts for the pre-existing/background curvature evolution over cosmic time, where $\mathcal{R}_{\mu\nu}/\mathcal{R} = \mathcal{R}_{\mu\nu}/\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu} = \tilde{g}_{\mu\nu}$ is in correspondence with Weyl's conformal transformation of the metric [35,36]; the conformal transformation can describe the tidal distortion and the gravitational waves in the absence of matter [38]. On the other hand, the boundary term comprising $\mathcal{H}_{\mu\nu}$ accounts for the conformal evolution of the extrinsic curvature of the background/global boundary over cosmic time while the boundary term comprising $K_{\mu\nu}$ accounts for the extrinsic curvature of the induced boundary of local relativistic celestial objects. [2] By substituting Eq. (1) to Eqs. (14), the field equations can be simplified to

$$R_{\mu\nu} - \frac{1}{2}R\hat{g}_{\mu\nu} + \frac{R\left(\mathcal{H}_{\mu\nu} - \frac{1}{2}\mathcal{H}\hat{p}_{\mu\nu}\right) - \mathcal{R}\left(K_{\mu\nu} - \frac{1}{2}K\hat{q}_{\mu\nu}\right)}{\mathcal{R}} = \frac{8\pi G_t}{c^4}T_{\mu\nu} \tag{12}$$

where $\hat{g}_{\mu\nu} = g_{\mu\nu} + 2\tilde{q}_{\mu\nu}$. The boundary term is only significant at high-energy limits such as within black holes [33]. The evolution in G_t can accommodate the pre-existing curvature evolution over cosmic time against constant G for special flat space-time case. The extended equations can remove the singularities, satisfy a conformal invariance theory and facilitate its quantisation.

2 The Brane notion to higher dimensions in the String Theory can be reconstructed in form of a local relativistic four-dimensional space-time (Brane of a celestial object of intrinsic curvature $R_{\mu\nu}$ and boundary curvature $K_{\mu\nu}$) that is embedded and travelling through an absolute conformal space-time (the Bulk of intrinsic curvature $\mathcal{B}_{\mu\nu}$ and extrinsic curvature $\mathcal{H}_{\mu\nu}$). This construction features a local spin with respect to the global background Bulk.

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3. Conclusions and Future Works

In this study, pre-existing/background curvature and the boundary contribution were considered to derive extended field equations using the Einstein –Hilbert action. The extended field equations consist of Einstein field equations with conformal transformation feature in correspondence with Weyl's conformal transformation of the metric [35,36], where the conformal transformation can describe the tidal distortion and the gravitational waves in the absence of matter [38]. The boundary term is only significant at high-energy limits such as within black holes [33], which can remove singularities from the theory, facilitate its quantisation and satisfy a conformal invariance theory.

This work will be utilised to study the evolution of the Universe and the formation of galaxies and their rotation curves.

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