doi:10.20944/preprints202207.0318.v1

Article

Thermal Bridge Modeling According to Time-varying Indoor Temperature for Dynamic Building Energy Simulation Using System Identification

Heegang Kim [1], Jihye Kim [2]* and Myoungsouk Yeo [3]

[1] Institute of Construction and Environmental Engineering, Seoul National University, Seoul 08826, South Korea; etoile9@snu.ac.kr (H. K.); msyeo@snu.ac.kr (M.Y.)
[2] Department of Architectural Engineering, College of Engineering, Kwangwoon University, Seoul 01897, South Korea; jikim@kw.ac.kr
[3] Department of Architecture and Architectural Engineering, Institute of Construction and Environmental Engineering, Seoul National University, Seoul 08826, South Korea; msyeo@snu.ac.kr
* Correspondence: jikim@kw.ac.kr; Tel.:+82-2-940-8458

Abstract: It is not easy to dynamically analyze thermal bridges that require multidimensional analysis in building energy simulations, which are mostly one-dimensional platforms. To solve this problem, many studies have been conducted and, recently, a study was conducted to model the thermal bridge based on the data by approaching this in a similar way to steady-state analysis, showing high accuracy. This was an early-stage study, which is only applicable when the indoor temperature is constant. By extending this study, a thermal bridge model that can be applied even when the indoor temperature changes over time is proposed and validated. Since the governing equation, the heat diffusion equation, is linear, the key idea is to create and apply two thermal bridge transfer function models by expressing the heat flow entering the room as a linear combination of the transfer function for indoor temperature and the transfer function for outdoor temperature. For the proposed thermal bridge model, the NRMSE of the model itself showed a high accuracy of 99.9%, and in the verification through annual simulation using the model, the NRMSE showed an accuracy of 88.8%.

Keywords: thermal bridge; data-driven system modeling; system identification; time-varying indoor temperature; dynamic analysis; building energy simulation

1. Introduction

We live in an era of high oil prices and are facing energy problems. Efforts are being made in terms of energy supply to find various sources of renewable energy, such as solar power and fuel cells, to replace the existing fossil fuels that cause environmental problems such as greenhouse gases and fine dust. In addition, research on energy reductions in terms of energy demand is also a major axis for solving energy problems. Almost all fields are concerned with the issue of energy reduction, and agree that sustainable development is possible with minimal energy consumption.

A building, a space where humans live, must be kept in a comfortable state from external environments such as cold and hot environments, and energy required for heating and cooling is consumed to maintain this. The energy required for a building may increase or decrease depending on the architectural design and materials used in the building [1-3]. In the field of building energy engineering, research on energy reduction technology for each building element, such as the building envelope and window system, is being conducted [4-6]. The building envelope is the part that is in direct contact with the external environment, minimizing heat flow through the building envelope is one of the key factors in reducing building energy [7].

 doi:10.20944/preprints202207.0318.v1

Meanwhile to develop and evaluate building energy reduction technology, experimental methods and computer simulation methods are used. Although the experimental method of making an actual building for each building element or experimental mock-up test is important, the computer simulation method is used because the experimental building scale is large and it is not easy to experiment with various methods under various conditions. Therefore, building energy simulation (BES) programs that can explain various physical phenomena related to buildings are widely used in building energy engineering [8-10].

In the BES, numerous architectural elements, such as building envelopes and windows and building equipment such as HVAC systems, are modeled, and the complex heat-transfer phenomenon that occurs in each component is dynamically calculated to analyze the building energy. In addition, since the simulation period is usually one year, the BES is computationally demanding and takes a lot of time [11]. For this reason, the model for each BES component should be simple, uncomplicated, and low in computation.

To analyze the building envelope, it is necessary to analyze the convection and radiation occurring on the surface of the building envelope, as well as the conduction occurring inside the building envelope. This heat-transfer phenomenon is expressed as a heat diffusion equation, which is a three-dimensional dynamic equation. To solve this heat diffusion equation in the BES, it is simplified with a one-dimensional assumption. This assumption is reasonable in that the building envelope is usually composed of several material layers, which are arranged one-dimensionally. However, this assumption is not suitable for complex geometries that are difficult to analyze in one dimension because materials are broken, such as so-called thermal bridges [12-14].

The thermal bridge (TB) is the region of the building envelope where the insulation is broken, and the thermal performance is weak. Many studies have pointed out the energy loss caused TB [15-18], and technological developments such as thermal breakers have been developed as a solution to TBs [19-21]. In addition, various modeling and analysis methods for TBs in BES have been developed [12].

Most BES programs, such as EnergyPlus [22] and TRNSYS [23] do not analyze TBs that require multidimensional analysis because they are one-dimensional platforms. When the evaluation of TBs is considered in BES, the method of adding the steady-state TB analysis results to the BES results is used, with the linear thermal transmittance reflecting the effect of TBs in the steady state. In addition to these methods, various methods, such as the structure factors method [24], the matrix of transfer functions method [25], the one harmonic method [26], and the identification method [27], have been proposed. However, although, theoretically, many methods have been proposed, they are not being applied to BES because they are not simple enough.

Thermal bridge modeling and the dynamic analysis method have been proposed using the analogy of a steady-state thermal bridge analysis [28]. This method started from the relatively widely used steady-state analysis method for TBs, and is easy to access as it is a data-driven method that has been widely used in many academic fields. The most important idea is to separate the time series heat flow data that enter the room through the entire wall into the time series heat flow data that enter the room through the wall and can be analyzed in one dimension (the clear wall), and the time series heat flow data that enter the room through the thermal bridge region (TB region), the remaining region. Here, to create a model that can explain the time series heat flow data that enter the room through the TB region, the transfer function of the TB using the data is proposed as a data-driven model. The modeling method that considered the TB as a dynamic system and uses the outdoor temperature as the input and the heat flow as the output has a fairly high accuracy, but has a limitation in that all parameters except input/output are assumed to be constants. Assuming that the indoor temperature is constant is sometimes used to find the size of a heating and cooling equipment when the indoor temperature is kept constant at the set temperature, but the actual indoor temperature is not constant and changes with time. For a more accurate building energy simulation, it is necessary to consider the

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 July 2022 doi:10.20944/preprints202207.0318.v1

change in indoor temperature over time. Therefore, it is necessary to examine whether the transfer function model is applicable when the indoor temperature changes over time.

The aim of this study is to establish a TB model that can be applied even when the indoor temperature changes as well as the outdoor temperature, starting from the TB model in the BES that was studied in the previous study [28]. The method of analyzing the entire wall is divided into the heat flow entering the room through the wall, which can be analyzed in one dimension, and the rest of the heat flow, as in the previous study. The heat flow entering the room through a one-dimensional analyzable wall can be analyzed by a model such as conduction transfer function (CTF) or the finite difference method (FDM) based on the first principles (physical laws) and is currently being used well in all building energy simulations. For this reason, we focus on a model that can only explain the rest of the heat flow, using what is called the TB model. This approach is a method of dividing the building envelope by heat flow rather than geometrically dividing the building envelope to account for TB. Therefore, the objective of this study is to establish a TB model from the perspective of heat flow when the indoor temperature changes over time. One of the things to consider in TB model development is the ability to accurately represent the heat flow through the TB when indoor temperature changes over time. This method should be simple and not computationally heavy, so that it can be calculated in a building energy simulation program.

2. Methods

2.1. Building Envelope Analysis and Thermal Bridge Modeling Concept

The overall concept for the building envelope analysis and TB modeling in this paper, which is consistent with the previous study [28], is shown in Figure 1. From the perspective of heat flow, the total heat flow entering the room through the building envelope is divided into the heat flow entering through the building envelope (the clear wall) that can be analyzed in one dimension, and the heat flow entering through the building envelope (TB region), which requires multi-dimensional analysis. In order to analyze the building envelope by dividing it in this way, a model is required that can explain the heat flow entering the room through each path. In this research, that the clear wall model uses the first physical law (fundamentals), and TB region uses an empirical method (data-driven) using data. The model for the clear wall is currently used in most BES in the form of FDM [22] or Transfer Function [22,23]. Therefore, the building envelope can be accurately analyzed by creating a model for the TB region, that is, the TB model and adding it to the BES.

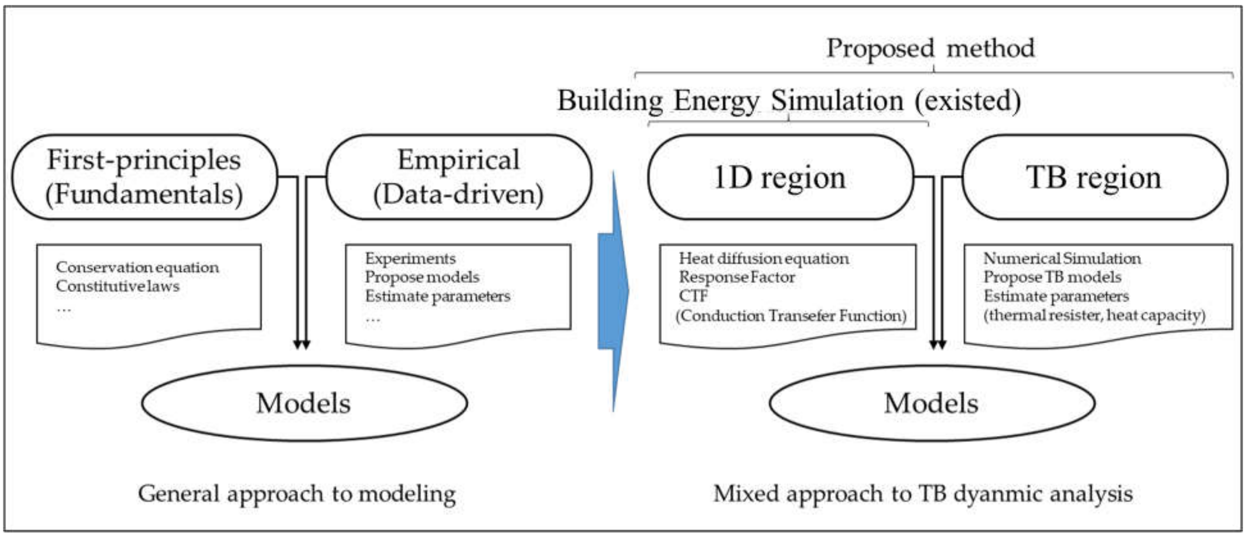


Figure 1. The overall concept for building envelope analysis and thermal bridge modeling from general approach to modeling concept [29].

2.2. Indoor Temperature and Thermal Bridge Modeling

2.2.1. Indoor Temperature : Constant($T_i(t) = T_{\mathrm{i}}$)

In the previous study [28], the TB model was regarded as a Linear Time Invariant system (LTI system), and the modeling method is proposed in the form of a transfer function using system identification, with the outdoor temperature as the input and the heat flow entering the room through the TB region as the output. The study showed very high accuracy. When analyzing the heating and cooling load to maintain a constant, set indoor temperature, it is correct to assume that the indoor temperature does not change over time, but it is predicted that this method will not be valid when the indoor temperature changes over time. In a dynamic situation, although the indoor temperature changes, assuming that the change is small, it is judged that this method is appropriate to some extent. Therefore, in this study, the TB model, the result of the previous study, is applied when the room temperature changes and its effectiveness is validated.

2.2.2. Indoor Temperature : Variable($T_i(t)$)

To model the heat flow entering the room when the indoor temperature changes over time, it is necessary to check the relation between the heat flow and the indoor temperature with the fundamentals (the laws of physics). The heat transfer phenomenon occurring in the building envelope is expressed by the thermal diffusion equation, which is a linear equation with respect to temperature. Linear with respect to temperature means that the effect of temperature is equal to the sum of the separate effects of the temperature. In other words, the heat flow entering the room through the TB region can be expressed by linearly combining the heat flow to the influence of the outdoor temperature and the heat flow to the influence of the indoor temperature. Therefore, paying attention to the linearity of the governing equation, the TB model can be expressed by dividing it into two models, one for the indoor temperature and one for the outdoor temperature. In the previous study, a TB model for the outdoor temperature was studied using the outdoor temperature as input and the heat flow entering the room as output. In this study, a TB model for the indoor temperature was studied in a similar way, using the indoor temperature as the input and the heat flow entering the room as the output.

The thermal bridge modeling procedure according to the indoor temperature change, was performed in the same way as the thermal bridge modeling procedure according to the outdoor temperature change, using the following four steps [28].

1. Step 1: Disaggregation stage
   Determine the dimensional system.
2. Step 2: Dynamic simulation stage
   Perform the dynamic simulation of the entire wall and the clear wall.
3. Step 3: Model construction stage
   Choose the LTI system order of the TB region and construct the TB transfer function.
4. Step 4: System identification stage
   Obtain the parameters of TB transfer function using the system identification process.

In the disaggregation stage, the extent to which the heat flow entering the room through the building envelope is analyzed in one dimension is determined. The concept used to obtain the linear thermal transmittance in the steady-state thermal bridge analysis is borrowed from the dynamic analysis. This step is the same as determining the amount of output used in the system identification stage (Step 4), because it aims to create a TB model that explains the heat flow for the remainder after subtracting the heat flow that can be analyzed in one dimension from the heat flow entering the room are through the building envelope. In the next dynamic simulation stage, input/output data are obtained for the system identification stage. The data are obtained using a dynamic simulation program such as VOLTRA [30], which can dynamically and precisely analyze the heat flow entering the room through the building envelope. In the model construction stage, the form of TB transfer function is determined by analyzing the relationship between the indoor temperature and heat flow. This provides a priori knowledge in System Identification, and helps to select the type and number of model parameters. At this stage, the difference between the model of the outdoor temperature and the model of the indoor temperature arises. Finally, in the System Identification stage, the TB model is estimated using the data obtained in step 2 and the model form obtained in step 3. In this study, this step is performed using the system identification tool box of MATLAB [31].

2.3. Model Construction and System Identification for Thermal Bridge

This study is a follow-up study to the previous study, and the basic methodology is the same as that of the previous study [28]. However, since it aims to create a transfer function that can simulate the heat flow entering the room according to the change in indoor temperature, the difference lies in creating the TB transfer function by considering the relationship between the indoor temperature and the heat flow entering the room. Since the governing equation, the heat diffusion equation [32], is linear with respect to temperature, the relationship between indoor temperature and heat flow can be viewed as a Linear Time-Invariant system (LTI system), and the order of the system can be estimated using the thermal network model. Finally, the TB transfer function according to the indoor temperature is derived through this process.

2.3.1. Linear Time-Invariant system

The Fourier's first law (Equation (1)), which is the law used to calculate the heat flow related to conduction and the heat diffusion equation (Equation (2)) to analyze the building envelope, consists of a differential operator and a gradient operator [32]. All are linear operators, and the relationship between temperature and heat flow is linear.

$$q''(\boldsymbol{x},t) = -k\nabla T(\boldsymbol{x},t) \tag{1}$$

$$\frac{\partial T(\boldsymbol{x},t)}{\partial t} = \alpha \nabla^2 T(\boldsymbol{x},t) \tag{2}$$

where q'' is the heat flux (W/m^2), k is the thermal conductivity (W/mK), T is the temperature (K), t is the time(s), and α is the thermal diffusivity (m^2/s). Based on the linearity, it can be inferred that the relationship between the indoor temperature and the heat flow entering the room can be expressed as a LTI system [33]. This relationship is expressed in the general formula below:

$$a_0 \frac{d^n}{dt^n} q_{in}(t) + a_1 \frac{d^{n-1}}{dt^{n-1}} q_{in}(t) + \cdots + a_{n-1} \frac{d}{dt} q_{in}(t) + a_n q_{in}(t) = b_0 \frac{d^m}{dt^m} T_i(t) + b_1 \frac{d^{m-1}}{dt^{m-1}} T_i(t) + \cdots + b_{m-1} \frac{d}{dt} T_i(t) + b_m T_i(t) \tag{3}$$

where $q_{in}(t)$ is the heat flow into the room through the TB region (the output of the system), $T_i(t)$ is the indoor temperature (the input of the system), and $a_0, a_1, \cdots, a_{n-1}, a_n, b_0, b_1, \cdots, b_{m-1}, b_m$ are the coefficients of the LTI system.

Just as the relationship between outdoor temperature and heat flow is made simple by using the thermal network model, which is a method of wall analysis[34], the relationship between indoor temperature and heat flow can also be verified using the thermal network model (Appendix A) to confirm that the differential order of the indoor temperature (n) and the differential order of the heat flow (m) are the same (Equation (4)).

$$n = m \tag{4}$$

Therefore, using Equation (4), Equation (3) is expressed as Equation (5).

$$a_0 \frac{d^n}{dt^n} q_{in}(t) + a_1 \frac{d^{n-1}}{dt^{n-1}} q_{in}(t) + \cdots + a_{n-1} \frac{d}{dt} q_{in}(t) + a_n q_{in}(t) = b_0 \frac{d^n}{dt^n} T_i(t) + b_1 \frac{d^{n-1}}{dt^{n-1}} T_i(t) + \cdots + b_{n-1} \frac{d}{dt} T_i(t) + b_n T_i(t) \tag{5}$$

2.3.2. Thermal Bridge Transfer Function for the Indoor Temperature

The TB transfer function for explaining the heat flow into the room according to the change in the indoor temperature can be obtained by Laplace transform of Equation (5) (Equation (6)) [33].

$$\frac{\mathcal{L}[output]}{\mathcal{L}[input]} = \frac{q_{in}(s)}{T_i(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1}s + b_n}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n} \quad (6)$$

The number of poles and zeros corresponding to the system order is the same n. To distinguish this from the TB transfer function for outdoor temperature proposed in previous studies, T_i is added after the transfer function. The parameters to be estimated through the system identification process are $a_0, a_1, \cdots, a_{n-1}, a_n$ and $b_0, b_1, \cdots, b_{n-1}, b_n$.

To express Equation (6) more simply, it can be expressed as Equation (7) by dividing it by the parameter of highest order of the denominator (a_0).

$$\text{TB Transfer Function } (T_i) = \frac{\beta_n s^n + \beta_{n-1} s^{n-1} + \cdots + \beta_1 s + \beta_0}{s^n + \alpha_{n-1} s^{n-1} + \cdots + \alpha_1 s + \alpha_0} \quad (7)$$

Therefore, the number of parameters to be estimated is $2n+1$ total, n parameters in the denominator and $n+1$ parameters in the numerator. The higher the system order, the more accurately the model can be estimated, but it will take a little longer to calculate the parameters. Considering that the improvement in accuracy is not evident in the case of the 3rd order or higher [28], in this study, the system identification is performed by increasing the order from the 1st to the 3rd order. Figure 2 shows the results of model construction and the concept of the system identification process.

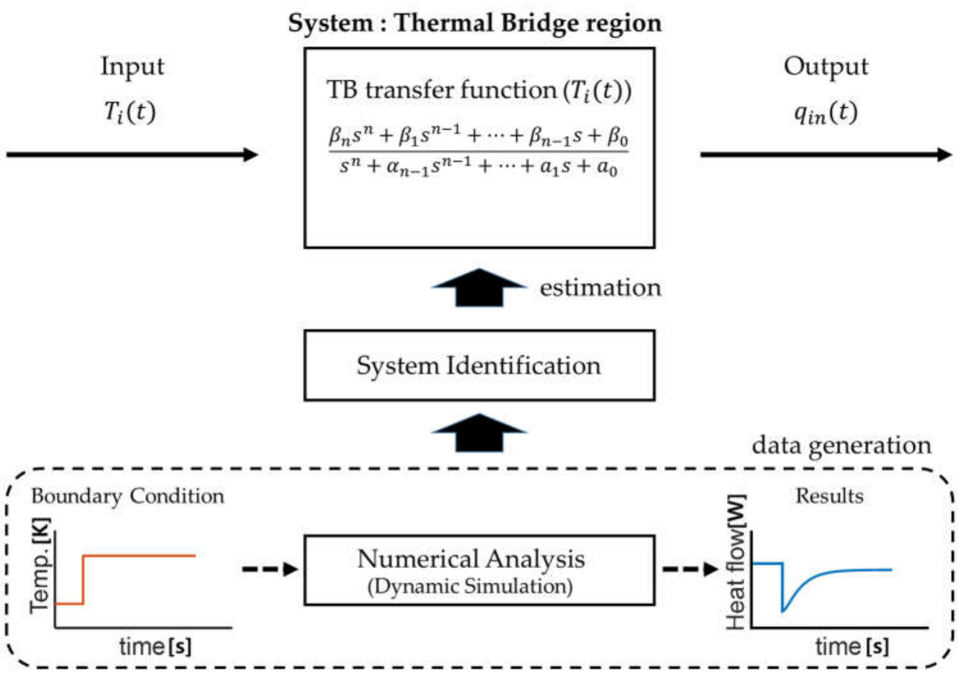


Figure 2. The concept of the system identification process and TB transfer function form according to the indoor temperature.

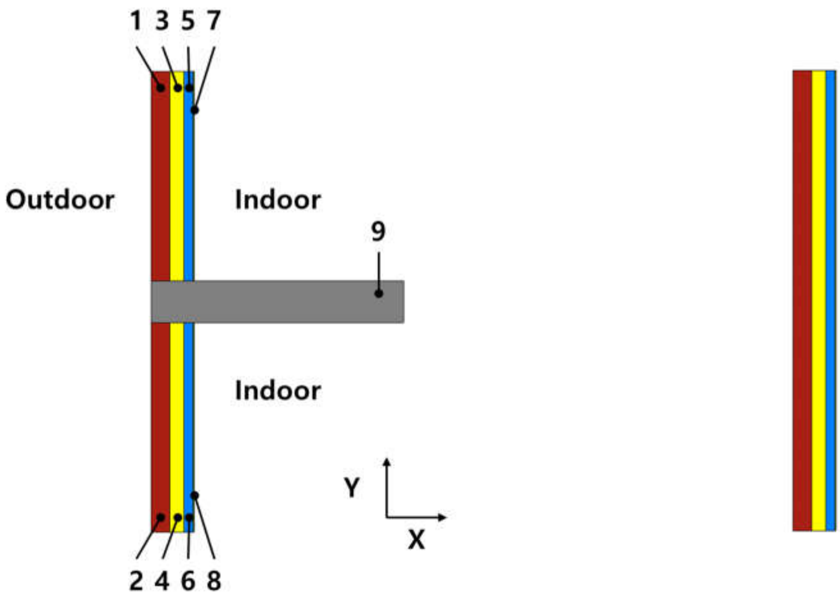
3. Explanatory Example

3.1. Geometry and Materials

To validate the method, which is TB modeling according to time-varying indoor temperature, a simple TB model is analyzed. Since the method linearly combines the transfer function for indoor temperature and the transfer function for outdoor temperature, the target building envelope with the same geometry and materials as in the previous study

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 July 2022 doi:10.20944/preprints202207.0318.v1

[28] is selected so that the transfer function for outdoor temperature studied in the previous study could be used as is. The target wall is the envelope of a typical residential building and is the same as the building envelope described in the research paper [28, 35]. The model geometry is shown in Figure 3a, and the material dimensions and thermal properties are shown in Table 1. The first step (disaggregation stage) of the system identification process is used to determine the dimension system. To use the transfer function for the outdoor temperature studied in the previous study, the dimension system must also be the same dimension system as in the previous study; therefore, in this example, the external dimension system is determined and, accordingly, the clear wall is shown in Figure 3b. This is the same as the BES calculating the building envelope where the TB occurs, as shown in Figure. 3a, by simplifying it as shown in Figure. 3b.



(**a**) (**b**)

Figure 3. Geometry of Explanatory Example: (**a**) the entire wall (1~9 is the material number in Table 1); (**b**) the clear wall; Red: brick, Yellow: insulation (Extruded polystyrene), Blue: air gap, Black: plasterboard, and Grey: concrete [28].

Table 1. The material dimensions and thermal properties [28].

# [1]	Material	Lx [2] (mm)	Ly [3] (mm)	*k* [4] (W/mK)	*ρ* [5] (kg/m³)	*c* [6] (J/kgK)
1	Brick	135	1500	0.700	1600.0	850.0
2		135	1500	0.700	1600.0	850.0
3	Extruded polystyrene	100	1500	0.035	25.0	1470.0
4		100	1500	0.035	25.0	1470.0
5	Air gap	65	1500	0.560	1.185	1004.4
6		65	1500	0.560	1.185	1004.4
7	Plasterboard	10	1500	0.500	1300.0	840.0
8		10	1500	0.500	1300.0	840.0
9	Concrete	1810	300	2.600	2300.0	930.0

[1] #: number. [2] Lx: length(x-direction). [3] Ly: length(y-direction). [4] *k*: thermal conductivity. [5] *ρ*: density. [6] *c*: specific heat.

The target wall should be able to be explained with the proposed thermal bridge model with the steady-state analysis results, as shown in Table 2. Since the geometry and material of the target wall are the same as those of the previous study, the steady-state analysis results are also the same. The steady-state analysis results are simulated using TRISCO [36], a commercial software.

Table 2. Steady-state analysis results (grid size: 20 mm) [28].

Dimensional System	Thermal Transmittance			Heat Flow (ΔT = 20 °C)		
	Entire Wall (W/m²K)	Clear Wall (W/m²K)	TB Region (W/mK)	Entire Wall (W)	Clear Wall (W)	TB Region (W)
External	0.6945	0.2980	1.3086	45.8376	19.6657	26.1719

3.2. System Identification and Validation Process

Input/output data for system identification, which correspond to step 3 of the thermal bridge modeling procedure, are needed to implement the proposed TB model. In order to obtain these data, it is necessary to perform a precise dynamic simulation of the target wall. In this study, the dynamic simulation is performed using VOLTRA [30], which can perform three-dimensional dynamic analysis. Input data are the time series data of indoor temperature and outdoor temperature corresponding to the simulation boundary condition, and output data are the time series data of the heat flow entering the room corresponding to the simulation result. Since this study is to obtain the transfer function for the indoor temperature, as shown in Table 3 and Figure 4, the outdoor temperature is kept constant at 0°C and only the indoor temperature is given as the step function from 0 °C to 20 °C after one day to obtain the data. The simulation duration is set to 20 days, which is a sufficient time to reach a steady state and the time step is set as 60 s.

Table 3. Simulation configuration.

Time Step	Duration	Initial Condition	Boundary Condition
60 s	1,728,000 s (20 days)	All structures and $T_o = 0$ °C	$T_i(t) = 0$ °C, $t < 86{,}400$ s (1 day) $T_i(t) = 20$ °C, $t \geq 86{,}400$ s (1 day)

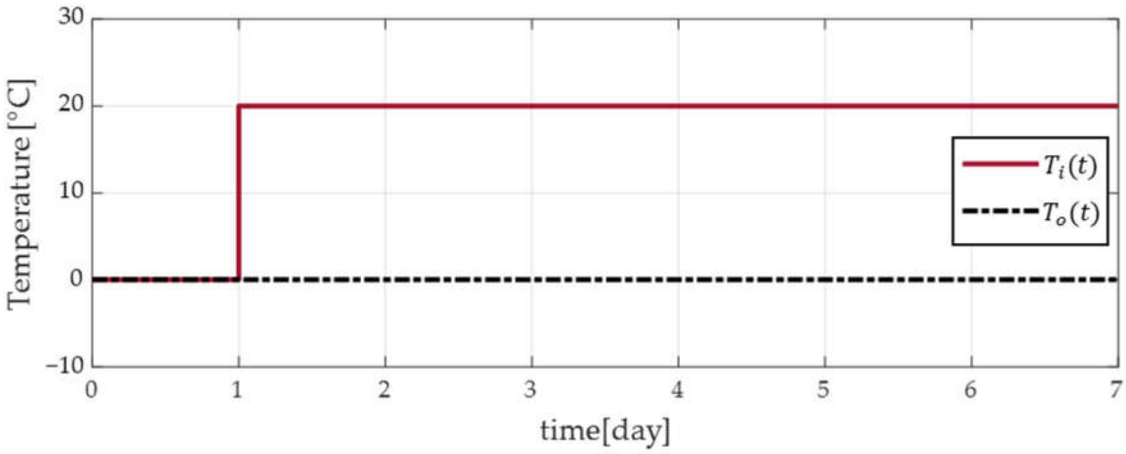


Figure 4. Boundary conditions (the indoor temperature and the outdoor temperature) in the dynamic simulation.

The heat flow entering the room, which is explained using the TB model, is the amount obtained by subtracting the heat flow entering the room through the clear wall from the heat flow entering the room through the entire wall. Therefore, when obtaining the time series data of the heat flow entering the room corresponding to the output data, the simulation result for Figure 3a and the simulation result for Figure 3b must be subtracted, so two precise dynamic simulations should be performed (Figure 5).

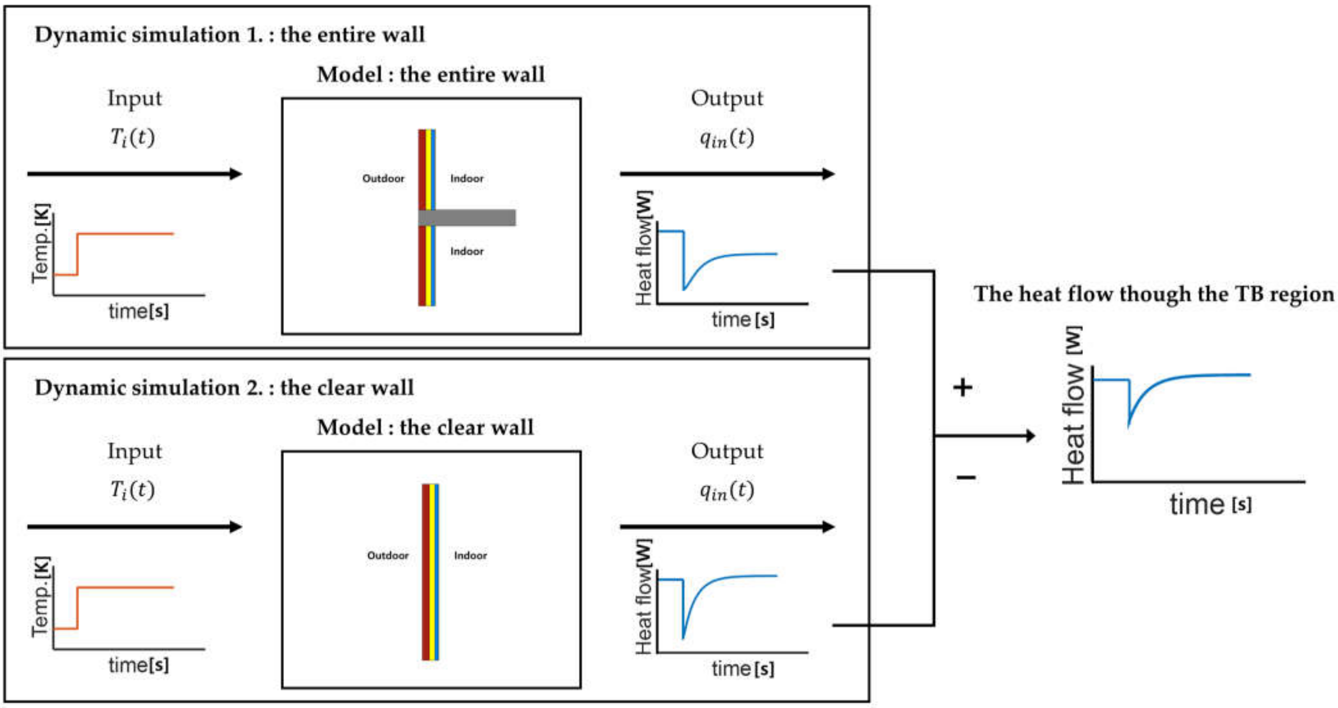


Figure 5. The conceptual process for obtaining the heat flow through the TB region.

The parameters($\beta_n, \beta_{n-1}, \ldots \beta_1, \beta_0$ and $\alpha_{n-1}, \ldots \alpha_1, \alpha_0$) are estimated using the system identification toolbox in MATLAB using the input/output data obtained as a result of performing dynamic simulation and the form of the TB transfer function obtained through model construction (Equation (7)). Since the TB model is proposed in the form of a transfer function, Transfer Function ESTimation ("tfest"), which is a function in MATLAB, is used [31]. The higher the system order (the number of poles and the number of zeros), the higher the precision, but this is complicated and takes a lot of time. Therefore, in this study, system identification is performed by limiting the system order from the 1st to the 3rd order. The last step, the system identification stage, is performed to obtain the transfer function for the indoor temperature.

To validate the method proposed in this study, a two-step validation process is performed. As data-driven modeling methods can sometimes struggle to explain given data, the first step is validation of the model itself. This step validates the step response. Since the model is estimated through the system identification process using FDM result data, the first thing to check is whether the proposed model can explain the given data well. The second step is an annual validation using weather data for one year using both the proposed model (TB transfer function (T_i)) and the transfer function for outdoor temperature, studied in previous studies. After validation is completed with the data for making the model (validation of the model itself in the first step), it is necessary to validate whether it can be explained well even when other data are given by using the model. Since the purpose of this study is to enable the proposed TB model to be implemented in the BES, it should be validated using the outdoor temperature used in the BES, that is, weather data for one year. In this step, the indoor temperature should be a variable that changes with time, not a constant.

Figure 6 shows the outdoor and indoor temperatures for annual simulation. The outdoor temperature is the weather data in Seoul, South Korea, which are used in BES, and there are 8760 annual data given every hour (max.: 31.3 °C; min.: −10.6 °C). The indoor temperature can be assumed to be an arbitrary value that changes with time. In this study, it is expressed as the sum of a daily sin function with an amplitude of 2 °C and an annual sin function with an amplitude of 2.5 °C based on 22.5 °C. The reason for this arbitrarily selection is to roughly represent the indoor temperature change over one day and the indoor temperature change over one year. In addition, noise with an amplitude of 0.5 °C for each hour is randomly added to simulate an uncertain indoor temperature.

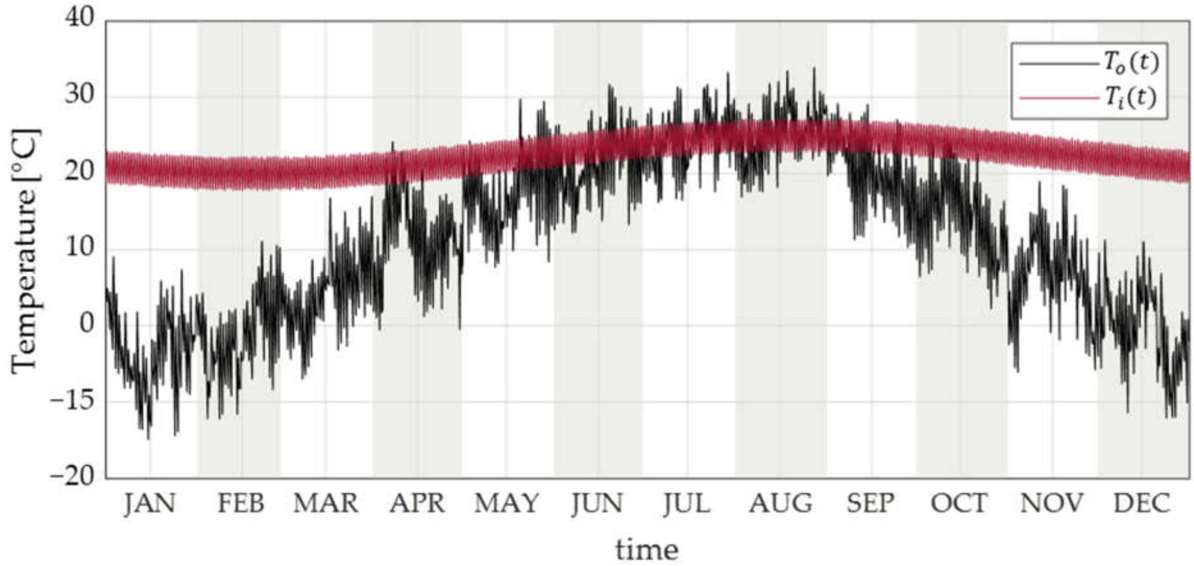


Figure 6. Boundary conditions (the indoor temperature and the outdoor temperature) for annual simulation.

The true value used for validation is the result of precise dynamic simulation. Since the dynamic simulation is the result of numerical analysis of the governing equation using the finite difference method (FDM), this is judged to be the basis for accurately describing the physical phenomenon. Based on this FDM model, a total of five models are selected as a comparative model of the annual simulation, as shown in Table 4.

Table 4. Comparative models for the annual simulation.

Model #	Description
Model 0	FDM Model as exact solution
Model 1	Steady-state model ($\psi\Delta T$)
Model 2	only 3rd order TB transfer function T_o
Model 3	only 3rd order TF To with arithmetic correction of T_i
Model 4	3rd order TB transfer function T_o + 1st order TB transfer function T_i
Model 5	3rd order TB transfer function T_o + 2nd order TB transfer function T_i
Model 6	3rd order TB transfer function T_o + 3rd order TB transfer function T_i

Model 1 is a steady-state model, and is a method of calculating the value obtained by multiplying the linear thermal transmittance (ψ), which is the result of the steady-state analysis, according to the difference between indoor and outdoor temperatures (ΔT). This is the simplest method considering TB in BES. Model 2 is a transfer function model for outdoor temperature in previous study [28]. The TB model, assuming that the indoor temperature is constant, is selected to confirm whether this is applicable even when the indoor temperature changes. Model 3 is a model in which the indoor temperature change is arithmetically corrected by using the TB model for the outdoor temperature from the previous study [28]. This approach corrects the results of the previous study by assuming that the indoor temperature changes over time but is constant within a time step. Models 4 to 6 linearly combine the transfer function for outdoor temperature and the transfer function for indoor temperature proposed in this study. The system order of the transfer function for the outdoor temperature is 3rd order, and the system order of the transfer function for

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 July 2022 doi:10.20944/preprints202207.0318.v1

the indoor temperature is from 1st to 3rd order. These models check the accuracy of the system order of the transfer function for the indoor temperature. Model 0 is the result of using VOLTRA, and the rest of the models are simulated using the Linear SIMulation ("lsim") function in MATLAB [31].

4. System Identification Results and Step Response Validation

The output data for system identification (Step 2 of the thermal bridge modeling procedure) are shown in Figure 7. These data are the result of performing a precise dynamic simulation. These data are the result of performing precise dynamic simulation of the entire wall and the clear wall, and the TB region shows the difference between the two results. Since all the indoor and outdoor temperatures are 0 °C before 1 day, and only the indoor temperature rises to 20 °C after 1 day, the heat flow entering the room has a negative value. That is, the direction of heat flow is from the indoors to the outdoors.

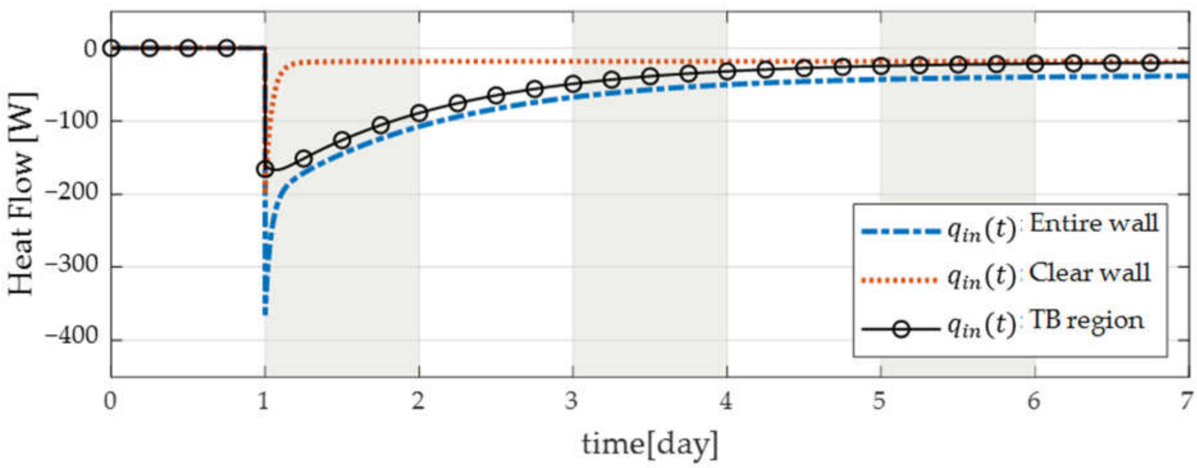


Figure 7. The output data for system identification (dynamic simulation results)

Table 5 shows the results of system identification using the indoor temperature (input) in Figure 4 and the heat flow (output) entering the room through the TB in Figure 7. In addition to the estimated parameters, the normalized root mean square error (NRMSE or CVRMSE), final prediction error (FPE), and mean square error (MSE) indicators, which can determine the accuracy of the model, are also shown. Looking at NRMSE, all three models show high accuracy. In particular, the model estimated as the 3rd-order system has an NRMSE of 99.9%. In the case of the 4th- or 5th-order system, a higher accuracy will be revealed, but considering that it is a model using data, more than 99.9% does not seem to have much meaning. The estimated transfer function of heat flow with respect to indoor temperature becomes a TB model according to the time-varying indoor temperature.

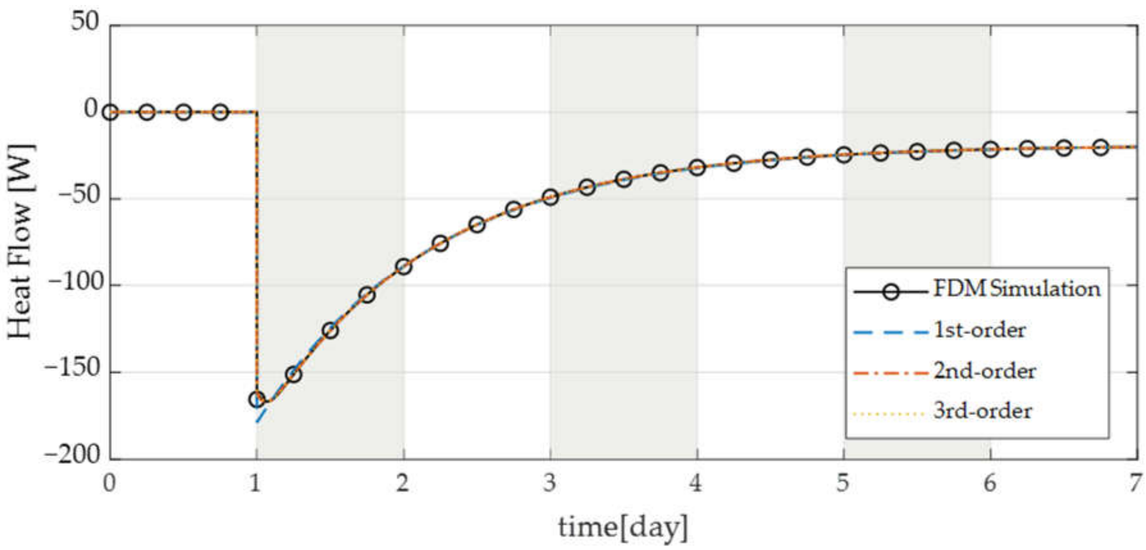
Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 21 July 2022 doi:10.20944/preprints202207.0318.v1

Table 5. Estimated parameters for thermal bridge transfer functions (system identification results).

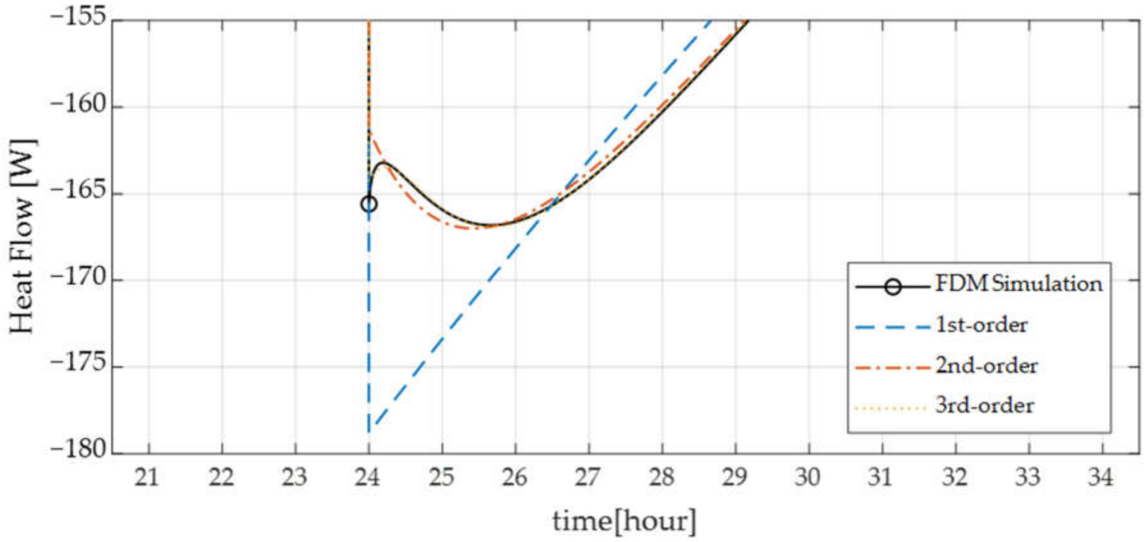
System Order		First-Order	Second-Order	Third-Order
Transfer Function		$\frac{\beta_1 s + \beta_0}{s + \alpha_0}$	$\frac{\beta_2 s^2 + \beta_1 s + \beta_0}{s^2 + \alpha_1 s + \alpha_0}$	$\frac{\beta_3 s^3 + \beta_2 s^2 + \beta_1 s + \beta_0}{s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0}$
# [1] of Poles		1	2	3
# [1] of Zeros		1	2	3
β	β_0	−9.1193 × 10⁻⁶	−1.8796 × 10⁻⁹	−1.7402 × 10⁻¹²
	β_1	−8.9426	−1.8304 × 10⁻³	−1.6960 × 10⁻⁶
	β_2	-	−8.0620	−8.0498 × 10⁻³
	β_3	-	-	−8.2158
α	α_0	9.6066 × 10⁻⁶	1.9727 × 10⁻⁹	1.8266 × 10⁻¹²
	α_1	-	2.1032 × 10⁻⁴	1.9551 × 10⁻⁷
	α_2	-	-	9.9425 × 10⁻⁴
NRMSE [2]		97.46 %	99.76 %	99.92 %
FPE [3]		0.4573	0.0041	3.5690 × 10⁻⁴
MSE [4]		0.4572	0.0041	3.5665 × 10⁻⁴

[1] #: number. [2] NRMSE: Normalized root mean square error. [3] FPE: Final prediction error for the model. [4] MSE: Mean square error.

Although the index of the accuracy of the model is checked in Table 5, the step response results for each estimated transfer function are checked to confirm this as a graph (Figure 8). In a graph of the same scale as the data for system identification (Figure 7), all three systems agree with the FDM simulation result to the extent that it cannot be confirmed with the naked eye (Figure 8a). However, when zooming in on the section where the indoor temperature suddenly changes (near one day), it can be seen that the first-order system changes slightly excessively (Figure 8b). The third-order system is not easy to distinguish from the FDM simulation results, even if it is enlarged. Taken together, system identification is properly validated from the 1st system to the 3rd system. In other words, we can say that we have created a transfer function that can describe a given input/output well. The accuracy increases as the order of the system increases, but the 3rd-order or higher system shows a high accuracy of 99.9% or more, making it sufficient to model as a 3rd-order system.



(a)

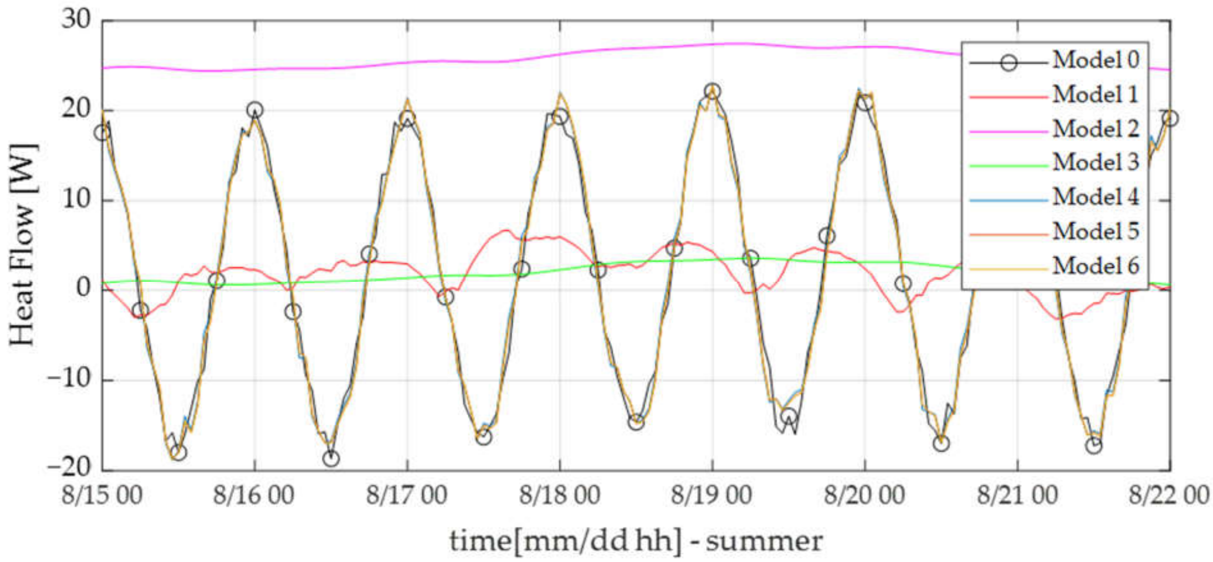


(b)

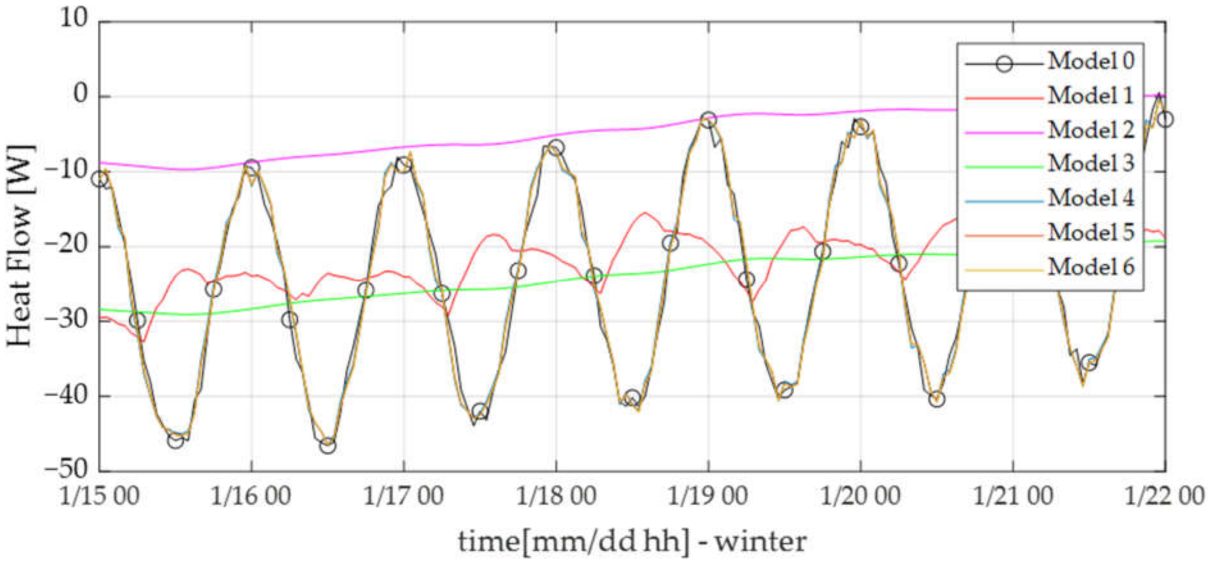
Figure 8. Step response of the estimated thermal bridge transfer functions for validation: (a) overall time scale; (b) scaled zoom in near 24 h.

5. Annual Simulation Validation

Since this study aims to create a TB model that can be implemented in BES where indoor and outdoor temperatures change over time, system identification validation alone is insufficient. Similar to performing simulations in BES, it is necessary to validate whether the proposed model is appropriate even in a situation where the outdoor temperature uses weather data and the indoor temperature changes over time in the vicinity of the set temperature. Figure 9 shows the results of the annual simulation validation, divided into summer and winter. Indices of accuracy were selected as root meat square error (RMSE), NRMSE, and R-square, and the results are shown in Table 6. Model 0 is an FDM model, obtained by performing precise dynamic simulation, and is considered to be the true value in this study.



(a)



(**b**)

Figure 9. Heat flow entering the room through the TB region (annual simulation of comparative models): (a) summer; (b) winter.

Table 6. Accuracy of comparative model based on Model 0.

	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**
RMSE	13.3204	24.9975	12.8481	1.8147	1.6967	1.6951
NRMSE	12.28%	-64.24%	15.39%	88.05%	88.83%	88.84%
R^2	0.2396	0.2996	0.2845	0.9857	0.9875	0.9875

Model 1 is the easiest way to model the TB by multiplying the linear thermal transmittance using the indoor/outdoor temperature difference, but differs from the result of the FDM Model. The reason for this is that, in the actual simulation, since the indoor and outdoor temperatures are dynamic situations that change over time, the values calculated at each time in the steady state do not reflect the time delay effect. This cause can be confirmed by the difference between the daily maximum value of Model 1 and the daily maximum value of the FDM Model in Figure 9.

Model 2 assumes that the indoor temperature is constant and only considers the change in outdoor temperature. Changes in indoor temperature are not reflected. An error occurs due to the difference between the indoor temperature (set temperature) value, which is used as a reference when TB modeling for the outdoor temperature, and the indoor temperature in the simulation. The larger this difference, the larger the error.

Model 3 is a supplementary model of Model 2, and is used to properly compensate for the indoor temperature as a constant within one time-step. The result of this method showed the daily average value of the FDM result, and it also seems difficult to implement in BES.

Models ranging from Model 4 to Model 6 are all the methods proposed in this study, but the system order is the 1st-, 2nd-, and 3rd-order, respectively. The results of these models are similar to the FDM results. System order accuracy is at a similar level, and although subtle, it can be seen that the higher the order, the more accurate it becomes. The NRMSE level is 88.8%, and the R-square value is 0.99. This level is sufficient to implement the proposed method in BES. In annual simulation, a random noise (amplitude is 0.5 °C) is given to the indoor temperature, but the accuracy was somewhat lower than the step response validation because the indoor temperature change is large due to the noise that occurs within a short time.

6. Discussion

Thermal bridges The thermal bridge modeling according to the time-varying indoor temperature proposed in this study is methodologically valid and the results are also verified. It is found that the system order or the number of poles and zeros of the TB transfer function affects the model accuracy, and that even the lowest order is reasonable to some extent. It takes a lot of time to simultaneously perform precise dynamic simulation in BES, but applying the proposed method to additionally analyze one TB is the same as analyzing one additional one-dimensional wall, so it does not take long to simulate in BES. However, it takes time to implement the thermal bridge model because precise dynamic simulations must be performed. Minimizing the simulation period of the precise dynamic simulation to a period that can generate enough data for system identification will save overall time. Minimizing the simulation period of the precise dynamic simulation to a period that can generate enough data for system identification will be a way to save overall time. The same issue occurs when implementing a thermal bridge model for outdoor temperature: the previous study recommended about 10 days and said that it should be at least 5 days [28]. In this study, looking at Figure 7, it is necessary to perform a precise dynamic simulation for at least 5 days, because steady state is reached in about 5 days.

A discussion follows regarding the meaning of the methodology. A method of adding the transfer function for outdoor temperature and transfer function for indoor temperature by creating a data-driven model using the system identification method has been proposed. This is because the governing equation is linear, so the heat flow is divided as in Equation (8)

$$q_{in}(t) = q_{in,T_o}(t) + q_{in,T_i}(t) \tag{8}$$

where $q_{in}(t)$ is the heat flow through the TB region, $q_{in,T_o}(t)$ is the heat flow through the TB region according to the outdoor temperature, and $q_{in,T_i}(t)$ is the heat flow through the TB region according to the indoor temperature. Only the outdoor temperature and the outdoor temperature are considered influence each heat flow. The Laplace Transform of the right term of Equation (8) can be expressed by Equation (9).

$$\frac{q_{in,T_o}(s)}{T_o(s)} + \frac{q_{in,T_i}(s)}{T_i(s)} \tag{9}$$

In the previous study, it was proposed that the right term of Equation (9) is regarded as a constant and the left term is estimated using the data. On the other hand, in this study, the right term of Equation (9) is estimated using the data and added to the left term. This is not different from the Response Factor method (equation), which is a method of analyzing a wall (Equation (10)) [22,37].

$$q_{i,t} = \sum_{j=1}^{\infty} T_{o,t-j+1} Y_j + \sum_{j=1}^{\infty} T_{i,t-j+1} Z_j \tag{10}$$

The difference is that, in the response factor method, the coefficients of Y_j and Z_j are calculated by approaching the fundamental law and, in this study, the transfer function parameters are estimated by approaching the data. Calculation of the continuous transfer function by converting it to a discrete transfer function is expressed as Equation (10). Additionally, in this study, j in Equation (10) is not viewed as infinite, but simplified to a finite value.

7. Conclusions

Thermal bridges that require multidimensional analysis are not easy to analyze in the building energy simulation program, which is a one-dimensional platform. Many studies have attempted to solve this problem, and among them, one study focuses on how to make the thermal bridge model and analysis easier on a one-dimensional platform by using data (system identification) in a similar method to that of steady-state analysis. This study proposed a method of estimating the thermal bridge model in the form of a transfer function when the indoor temperature is constant and the outdoor temperature is changed. Based on this study, a method for modeling a thermal bridge was proposed and validated when both indoor and outdoor temperatures change over time. The thermal bridge modeling method proposed in this study is basically the same as that of the previous study.

The thermal bridge model introduced in this study is as follows.

- In the same way as in steady-state thermal bridge analysis, the thermal bridge model explains the remaining heat flow after subtracting the heat flow that enters the room through the clear wall that can be analyzed in one dimension from the heat flow that enters the room through the entire building envelope.
- The heat flow entering the room is divided into the heat flow according to the indoor temperature and the heat flow according to the outdoor temperature, and is obtained by adding these.
- The thermal bridge model appears in the form of a transfer function, and is divided into two types: a transfer function for indoor temperature and a transfer function for outdoor temperature.
- Each thermal bridge model is estimated through system identification using data. At this time, the data are obtained by performing a precise dynamic analysis program and the transfer function form is determined using the number of poles and zeros by analyzing the thermal network model, considering the relationship between input (indoor temperature and outdoor temperature) and output (heat flow entering the room) as a linear, time-invariant system.

The proposed thermal bridge model according to time-varying indoor temperature was verified in the following two steps

- The first step: the validation of the model itself.
  - Validation of whether the thermal bridge model can explain the data used for system identification.
- The second step: the validation of the annual simulation.
  - Validation of whether the thermal bridge model can explain the random annual data.

As a result of the validation in the first step, NRMSE showed an accuracy of 99.9%, and as a result of validation in the second step, NRMSE showed an accuracy of 88.8%, indicating that the methodology proposed in this study is valid.

In previous studies, the third-order system of the thermal bridge transfer function for the outdoor temperature was recommended. In this study, it was also recommended that, if the system order of the thermal bridge transfer function for the indoor temperature is of the 3rd order, it is sufficiently accurate, and time can be reduced.

Further studies will focus on how to implement the proposed TB model in BES. After selecting one BES program, such as EnergyPlus, to actually implement a TB model, it is necessary to study the BES program alone to analyze building energy, including TB. In addition, performing BES with various TB geometries and materials and examining how much influence they have on building energy will be the subject of an additional study. In terms of thermal bridge modeling, to date, it has been assumed that factors other than indoor and outdoor temperatures are constants, but in reality they are variables that change over time. It is necessary to study how other variables that change with time, such as convective heat transfer coefficient, should be reflected in thermal bridge modeling.

Author Contributions: Conceptualization, data curation, formal analysis, investigation, methodology, software, validation, visualization, writing—original draft preparation, writing—review & editing, H.K.; Conceptualization, data curation, formal analysis, funding acquisition, methodology, project administration, supervision, validation, writing—original draft preparation, writing—re-

view & editing, J.K.; Conceptualization, data curation, methodology, resources, software, supervision, writing—review & editing, M.Y.. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIT) (No. 2021R1G1A1094364). Also, the present research has been conducted by the Research Grant of Kwangwoon University in 2021.

Data Availability Statement: Not applicable.

Acknowledgments: In this section, you can acknowledge any support given which is not covered by the author contribution or funding sections. This may include administrative and technical support, or donations in kind (e.g., materials used for experiments).

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A Thermal Network Model and Transfer Function

The thermal network model is a traditional model that uses virtual heat resistance (R) and heat capacity (C) to analyze the heat transfer phenomenon through an electrical analogue [28]. This model has been applied to model the walls of buildings, whole buildings, and various components in the BES. The thermal network model can also be converted to an LTI system (via the LTI differential equation). Furthermore, the LTI system can be expressed as a transfer function. Here, the process for converting the 2R1C model into an LTI system and expressing it as a transfer function is briefly described.

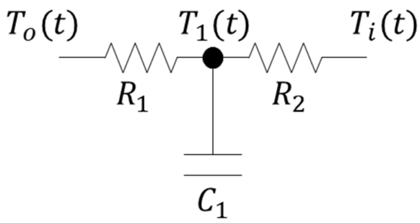


Figure A1. The 2R1C model.

- 2R1C model

  The governing equations of the 2R1C model (Figure A1) are

$$C_1\frac{dT_1(t)}{dt} = \frac{T_o(t) - T_1(t)}{R_1} + \frac{T_i(t) - T_1(t)}{R_2} \tag{A1}$$

$$q_{in}(t) = \frac{T_1(t) - T_i(t)}{R_2} \tag{A2}$$

Equation (A1) describes the state of the model, and Equation (A2) describes the output of the model. When Equation (A2) is expressed with respect to $T_1(t)$, it is the same as Equation (A3).

$$T_1(t) = R_2 q_{in}(t) + T_i(t) \tag{A3}$$

In Equation (A1), by replacing Equation (A3) instead of $T_1(t)$, it is the same as Equation (A4).

$$\frac{d}{dt}q_{in}(t) + \frac{(R_1 + R_2)}{R_1R_2C_1}q_{in}(t) = \frac{1}{R_1R_2C_1}T_o(t) - \left(\frac{1}{R_2}\frac{d}{dt}T_i(t) + \frac{1}{R_1R_2C_1}T_i(t)\right) \tag{A4}$$

Then, the LTI system for the 2R1C model can be provided as Equation (A4). Finally, Equation (A4) is expressed as a transfer function by Laplace transform.

$$\frac{q_{in}(s)}{T_i(s)} = -\frac{\frac{1}{R_2}s + \frac{1}{R_1R_2C_1}}{s + \frac{(R_1 + R_2)}{R_1R_2C_1}} \tag{A5}$$

$$\frac{q_{in}(s)}{T_o(s)} = \frac{\frac{1}{R_1R_2C_1}}{s + \frac{(R_1 + R_2)}{R_1R_2C_1}} \tag{A6}$$

Equation (A5) is the transfer function for the indoor temperature and Equation (A6) is the transfer function for the outdoor temperature.

References

1. Kamel, E.; Memari, A.M. Residential Building Envelope Energy Retrofit Methods, Simulation Tools, and Example Projects: A Review of the Literature. *Buildings* **2022**, *12*, 954. https://doi.org/10.3390/buildings12070954.
2. Otaegi, J.; Hernández, R.J.; Oregi, X.; Martín-Garín, A.; Rodríguez-Vidal, I. Comparative Analysis of the Effect of the Evolution of Energy Saving Regulations on the Indoor Summer Comfort of Five Homes on the Coast of the Basque Country. *Buildings* **2022**, *12*, 1047. https://doi.org/10.3390/buildings12071047
3. Emanuele, N.; Antomio, M.; Yi, Z.; Furio, B. Defining The Energy Saving Potential of Architectural Design. *Energy Procedia* **2015**, 83, pp. 140–146.
4. Ionescu, C.; Baracu, T.; Vlad, G. E.; Necula, H.; Badea, A. The historical evolution of the energy efficient buildings. *Renewable and Sustainable Energy Reviews* **2015**, *49*, pp. 243-253.
5. Li, C. Z.; Zhang, L.; Liang, X.; Xiao, B.; Tam, V. W.; Lai, X.; Chen, Z. Advances in the research of building energy saving. *Energy and Buildings* **2022**, *254*, 111556.
6. Charde, M.; Gupta, R. Effect of energy efficient building elements on summer cooling of buildings. *Energy and buildings* **2013**, *67*, pp. 616-623.
7. Aslani, A.; Bakhtiar, A.; Akbarzadeh, M. H. Energy-efficiency technologies in the building envelope: Life cycle and adaptation assessment. *Journal of Building Engineering* **2019**, *21*, pp. 55-63.
8. Crawley, D.B.; Hand, J.W.; Kummert, M.; Griffith, B.T. Contrasting the capabilities of building energy performance simulation programs. *Building and environment* **2008**, 43, pp. 661–673.
9. Harish, V.; Kumar, A. A review on modeling and simulation of building energy systems. *Renewable and sustainable energy reviews* **2016**, 56, pp. 1272–1292.
10. Foucquier, A.; Robert, S.; Suard, F.; Stéphan, L.; Jay, A. State of the art in building modelling and energy performances prediction: A review. *Renewable and Sustainable Energy Reviews* **2013**, 23, pp. 272–288.
11. Agdas, D.; Srinivasan, R. S. Building energy simulation and parallel computing: Opportunities and challenges. In *Proceedings of the Winter Simulation Conference 2014*, pp. 3167-3175
12. Quinten, J.; Feldheim, V. Dynamic modelling of multidimensional thermal bridges in building envelopes: Review of existing methods, application and new mixed method. *Energy and Buildings* **2016**, *110*, pp. 284-293.
13. Ge, H.; Baba, F. Effect of dynamic modeling of thermal bridges on the energy performance of residential buildings with high thermal mass for cold climates. *Sustainable Cities and Society* **2017**, *34*, pp. 250-263.
14. Martin, K.; Erkoreka, A.; Flores, I., Odriozola, M.; Sala, J. M. Problems in the calculation of thermal bridges in dynamic conditions. *Energy and Buildings* **2011**, *43*(2-3), pp. 529-535.
15. Martin, K.; Campos-Celador, A.; Escudero, C.; Gómez, I.; Sala, J. M. Analysis of a thermal bridge in a guarded hot box testing facility. *Energy and Buildings* **2012**, *50*, pp. 139-149.
16. Garay, R.; Uriarte, A.; Apraiz, I. Performance assessment of thermal bridge elements into a full scale experimental study of a building façade. *Energy and buildings* **2014**, *85*, pp. 579-591.
17. Berggren, B.; Wall, M. State of knowledge of thermal bridges—A follow up in Sweden and a review of recent research. *Buildings* **2018**, 8(11), 154.
18. Zhang, X.; Jung, G.-J.; Rhee, K.-N. Performance Evaluation of Thermal Bridge Reduction Method for Balcony in Apartment Buildings. *Buildings* **2022**, *12*, 63. https://doi.org/10.3390/buildings12010063
19. Kim, M. Y.; Kim, H. G.; Kim, J. S.; Hong, G. Investigation of Thermal and Energy Performance of the Thermal Bridge Breaker for Reinforced Concrete Residential Buildings. *Energies* **2022**, *15*(8), 2854.
20. Kim, Y. H.; Kim, H. J.; Lee, H. Y. Investigation and analysis of patents for the thermal bridge breaker in green buildings. *Journal of The Korean Digital Architecture Interior Association* **2013**, *13*(2), 35-43.
21. Aghasizadeh, S.; Kari, B. M.; Fayaz, R. Thermal performance of balcony thermal bridge solutions in reinforced concrete and steel frame structures. *Journal of Building Engineering* **2022**, *48*, 103984.
22. DOE. EnergyPlus Version 22.1.0 Documentation: EnergyPlus Engineering Reference; US Department of Energy: Washington, DC, USA, 2016.

 doi:10.20944/preprints202207.0318.v1

23. Fiksel, A.; Thornton, J.; Klein, S.; Beckman, W. Developments to the TRNSYS simulation program. *Journal of solar energy engineering* **1995**, *117*(2), pp. 123-127.
24. Kossecka, E.; Kosny, J. Equivalent wall as a dynamic model of a complex thermal structure. *Journal of Thermal Insulation and Building Envelopes* **1997**, 20(3), pp. 249–268.
25. Nagata, A. A simple method to incorporate thermal bridge effects into dynamic heat load calculation programs. In Proceedings of the International IBPSA Conference, Montreal, QC, Canada, 15–18 August 2005; pp. 817–822.
26. Xiaona, X.; Yi, J. Equivalent slabs approach to simulate the thermal performance of thermal bridges in building constructions. *IbpsaProc. Build Simulation* **2007**, pp. 287-293.
27. Martin, K.; Escudero, C.; Erkoreka, A.; Flores, I.; Sala, J. M. Equivalent wall method for dynamic characterisation of thermal bridges. *Energy and Buildings* **2012**, 55, pp. 704-714.
28. Kim, H.; Yeo, M. Thermal Bridge Modeling and a Dynamic Analysis Method Using the Analogy of a Steady-State Thermal Bridge Analysis and System Identification Process for Building Energy Simulation: Methodology and Validation. *Energies* **2020**, *13*, 4422. https://doi.org/10.3390/en13174422
29. Tangirala, A. K. (2018). *Principles of system identification: theory and practice*. CRC Press. 2014; p. 123
30. VOLTRA v6.0w Manual; PHYSIBEL: Gent, Belgium, 2006.
31. Matlab, M. MATLAB R2021b; Mathworks: Natick, MA, USA, 2021.
32. Bergman, T.L.; Incropera, F.P.; DeWitt, D.P.; Lavine, A.S. Fundamentals of Heat and Mass Transfer; John Wiley & Sons: Hobokene, NJ, USA, 2011.
33. Ogata, K. System Dynamics; Pearson/Prentice Hall Englewood Cliffs: Upper Saddle River, NJ, USA, 2004; Volume 13.
34. Xu, X.; Wang, S. A simplified dynamic model for existing buildings using CTF and thermal network models. *International Journal of Thermal Sciences* **2008**, 47(9), pp. 1249-1262.
35. Martinez, R. G.; Riverola, A.; Chemisana, D. Disaggregation process for dynamic multidimensional heat flux in building simulation. *Energy and Buildings* **2017**, 148, pp. 298-310.
36. TRISCO v11.0w Manual; PHYSIBEL: Gent, Belgium, 2005.
37. Li, X. Q.; Chen, Y.; Spitler, J. D.; Fisher, D. Applicability of calculation methods for conduction transfer function of building constructions. *International Journal of Thermal Sciences* **2009**, 48(7), pp. 1441-1451.