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Mathematical Modeling Regarding Growth Energy Efficiency at Soil Processing with Disc Harrow

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Abstract

To increase the energy efficiency of soil tillage with a disc harrow, the interaction of the working elements (discs) with the soil was studied through mathematical modeling, with the aim of finding a local optimum. This involves a systemic approach, considering the agrotechnical, agrobiological aspects and the characteristics of the soil work process, and from the point of view of engineering sciences, studying the inputs and outputs of the system (the desired final state of the soil, correlated with the initial state). The present paper deals with problems related to the disc harrow aggregate, studying the phenomena of the dynamics of the interaction between the working elements (discs) and the soil, only from the point of view of energy aspects (to work with minimal specific energy consumption, relative to the soil surface worked), the other facets being touched upon only tangentially, where necessary and possible. Knowing the initial and final state of the soil, how these changes can be achieved is defined, a state that must be maintained over a longer term, in correlation with the requirements of the plants to be cultivated, the pedo-climatic conditions, the requirements for maintaining soil fertility, and slowing down the process of soil destruction.

Keywords: disc harrow; energy efficiency; disc-soil interaction

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1. Introduction

The research of phenomena related to soil work, in correlation with the development/design of the equipment/organs necessary for their performance, resembles more an art than a science, in that it is based primarily on intuition. The methods used (even quantitative) are empirical and represent a materialization of optimization processes through successive trials. Through trials, certain parameters of the working organ are modified, accepting a local optimum if the initial requirements are satisfied, and the qualitative criteria regarding the soil condition reach appropriate levels. This approach also has a scientific foundation; however, the use of these methods rarely leads to optimal solutions, although the intended result is finding a local optimum.

First of all, researchers in the field consider the agrotechnical and agrobiological aspects of soil work systematically, regarding the process itself as a "black box", studying the inputs and outputs of the system [1]. A more complex approach by researchers must also take into account the characteristics of the soil processing process, also from the point of view of engineering sciences. Today, research and development in the field studies what happens in these processes, but without controlling them. This situation will not be able to change until a coherent system is developed that quantitatively and qualitatively describes the processes.

Any change in the state of the soil can be considered soil work, which is usually performed annually, using mechanical methods of soil manipulation, modifying its physical, chemical, and

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biological properties, among other things, depending on the requirements of the plants being cultivated.

There are two very important aspects of the study of soil work:

1. defining the optimal state of the soil for a given purpose (usually, the cultivation of a certain crop);

2. determining the methods, technologies, and equipment that best correspond to the achievement of this local optimum.

The present work will deal with some problems related to the second aspect by considering the disc harrow equipment/aggregate, studying the phenomena of the dynamics of the interaction between the working organs (stellated discs) and the soil, only from the point of view of energy aspects, the other facets of the problem being touched upon only tangentially, and strictly necessary, respectively possible.

There are two approaches to studying these problems from two different angles, usually. The first is the technical, engineering way of thinking, which tries to improve the financial and constructive efficiency of their working organs, while the second is the way in which agricultural technicians directly look at the effects on crop plants. Both approaches are unilateral and, to some extent, empirical, if not studied in a correlated manner and through their results, offer only partial solutions, if they are not integrated into a systemic way of thinking [2].

Such a systemic approach requires research that, in addition to studying the processes from an energetic point of view, also investigates the effect of the working organs of soil-working machines and/or various technologies on the soil, both in the short term and in the medium or long term.

The study of dynamic effects in the soil-working tool system is important primarily because of the analysis of the energy consumption of the respective processes, but the intended purpose must also be considered, i.e., the desired final state of the soil, correlated with the initial state. As shown by Young et al. [3], the main purpose of soil works is to bring the soil from an existing initial state to a desired final state, obviously from a technical point of view.

Soil, which is one of the most important natural resources, has been and continues to be subject to a process of degradation due to erosion caused by natural forces and/or agricultural practices [4]. Maintaining a productive and profitable agriculture in the long term requires improving soil management methods, more precisely, its fertility and physical condition, which must be maintained within a range in which the soil can regenerate. This is possible, recently, by increasing the share of new soil tillage technologies in agriculture, so that the soil is brought to that state to which, under given environmental conditions, it improves the general condition and fertility of the soil, or at least does not worsen them [5], and achieves high yields. Negi et al. [6] studied the complex relationship between the productions achieved, the soil tillage system, and its settlement due to the traffic of agricultural aggregates. Four systems were compared, namely: no tillage, i.e., direct sowing in the field, ploughing with a plough with a furrow, loosening with a chisel cultivator, and loosening with a subsoiler. The most important conclusion was that where the degree of soil compaction reaches significant values, significantly higher yields are achieved by loosening it.

A common idea among as many researchers as possible is that the point of view according to which the quality of soil processing is determined exclusively by the degree of fragmentation, loosening, and the vertical structure of the soil must be challenged. Singh et al. [4] propose a so-called index of the degree of processing, with values varying between zero (conditions unusable by plants) and one (conditions that do not limit plant growth). This index was calculated as a multiplicative combination of indices determined for apparent density, penetration resistance (cone index, according to ASAE Standard, Anonymous [7]), lump size uniformity coefficient, organic matter content, and plasticity index. Based on experiments, the following characteristic ranges were proposed:

- 0…0.5, for poor growing conditions;
- 0.5…0.8 - average cultivation conditions;
- i 0.8…1, in for optimal plant cultivation conditions.

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Based on field tests, a good correlation was found between the index of the degree of processing and the production per hectare. The correlation coefficient takes values between 0.15 and 0.86, and in 10 out of 16 cases, it is higher than 0.60, meaning that the correlation level is appropriate.

Based on previous research (Colvin et al. [8] and Singh et al. [9]), it can be argued that a synthetic value can be defined, which can globally characterize the state of processing of a soil and its fertility. Once such an index is defined, the notions of prescribed soil works [10] or precision soil works [11] become operative and can help increase the energy efficiency of agricultural production.

In addition to the attention paid to reducing energy consumption and the interest in using soil conservation technologies (conscious and continuous concerns), administrative measures have been taken to encourage farmers to use technologies that slow down the degradation of agricultural soils (while maintaining production at an acceptable level). At the same time, the so-called controlled traffic technology has been developed, in which the wheels of agricultural aggregates run throughout the entire vegetation period on the same tracks, which are neither worked nor sown [11].

Improving the energy efficiency of soil work by optimizing the operating characteristics of the aggregates (including the aggregate that has a disc harrow as a component) is one of the most commonly used methods. In principle, the aim is for the disc harrow aggregate to work with minimal specific energy consumption, i.e., energy consumption related either to the soil surface worked in cross-section or, more rarely, to the working width.

Therefore, this paper presents theoretical notions relating to the disc harrow equipment/aggregate, analyzing the phenomena of the dynamics of the interaction of the working organs (discs) and the soil, only from the point of view of energy aspects.

2. Materials and Methods

In accordance with the purpose of the work, the soil cultivation through the disc-soil interaction of a disc harrow was analyzed to increase energy efficiency, given that the soil is the loose layer formed under natural conditions on the surface of the earth, which ensures plant life [12]. The role of the soil is essential for the normal functioning of all terrestrial and aquatic ecosystems, which, through various auto-morphic processes, permanently produces phytomass that forms the basis for the development of all organisms [12,13]. The soil performs functions essential for existence on Earth, following the accumulation and supply of nutrients and energy to all living organisms [12] (Fig. 1).

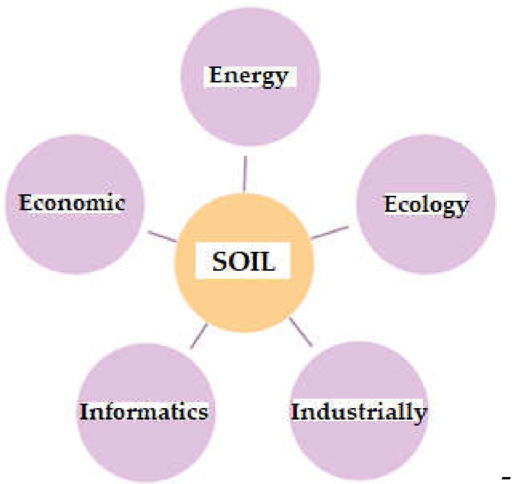

Figure 1. Soil functions

Disc harrows have main purpose is to prepare the seedbed for sowing or planting, usually performing superficial soil work consisting of loosening it, breaking and shredding the clouds resulting from plowing, leveling the land surface, breaking the crust, harrowing the seeds, and incorporating fertilizers, herbicides, etc. They consist of a frame on which several sections called

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batteries are mounted. A disc battery includes in its composition the battery frame and a number of discs mounted on a square-section shaft spaced apart by means of spacer bushes. The active elements are concave discs (spherical caps) with a sharp, continuous (smooth) or discontinuous (notched) periphery, made of steel (Figure 2) [14,15] and have a more energetic action on the soil.

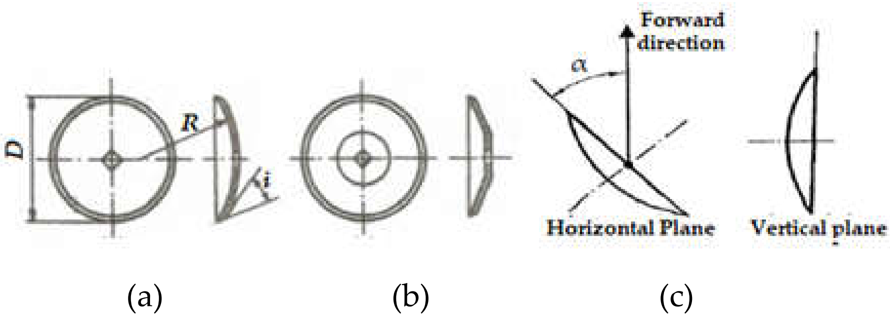

Figure 2. Types of discs and disc position during work: (a) spherical cap with full contour; (b) truncated cone [16]; (c) disc position during work; D – disc diameter; R - radius of the spherical cap; i - angle of incidence (sharpening); α - angle of attack.

If the discs are mounted in two rows, the discs on the rear row are placed on the center of the intervals between the discs on the front row and have the concavities facing in the opposite direction to those on the front row, for the purpose of leveling the ground.

Discs are grouped in batteries arranged perpendicular to the soil surface and at an angle, α, of 10 – 30^0 to the direction of advance, called the angle of attack. Due to contact with the soil, in the process of work, the discs rotate and penetrate the soil to a certain depth that depends on the mass that returns to a disc and on the vertical component of the resistance opposed by the soil, conditioned by the size of the angle of attack.

The shaft with discs is attached to the battery frame by bearings and supports. Scrapers are mounted on the battery frame to clean the active parts and unclog them.

For a more energetic action on plant debris and clods, in some combined disc harrows, the front battery consists of notched discs, and the rear battery consists of smooth discs (Figure 3).

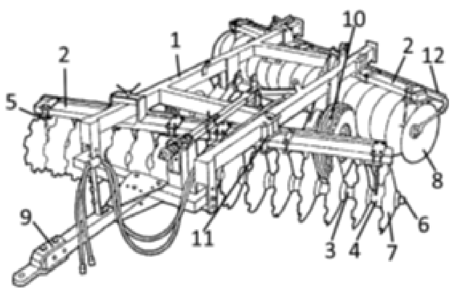

Figure 3. Semi-mounted disc harrow: 1 – harrow frame; 2 – battery frame; 3 – spacers; 4 – battery bearing; 5 – battery support; 6 – battery shaft; 7 - notched disc; 8 – smooth disc; 9 - traction triangle; 10 – transport wheels; 11 – battery angle adjustment device; 12 – scraper support.

The study of the interaction (dynamic effect) in the soil-working organ (disc) system is important, first of all, for the analysis of energy consumption in the work process, but the intended goal must also be taken into account, i.e., the desired final state of the soil, correlated with the initial state. In this context, the energy balance in the work process of the disc harrow must be studied, more precisely the energy balance rate, through the ratio between the amount of energy contained in the final product (primary and/or secondary) of the seedbed preparation process and the energy consumed in the process of making the finished product (see Figure 1).

It is known that, in the case of any economic activity, in addition to an increase in production, it is necessary to improve economic efficiency. In the special case of seedbed preparation, an important goal is to increase energy efficiency, which, in principle, can take place in two ways: reducing the amount of energy introduced into the production process and increasing the amount of energy contained in the final products.

Regarding the initial state of the soil, it is necessary to define its physical, dynamic, chemical, and biological characteristics. It is obvious that through soil work, only the physical and dynamic characteristics can be influenced in the short term. From the point of view of the requirements of the cultivated plants, these do not remain unchanged throughout the vegetation period. The germination bed must allow good contact of the seeds with the moist soil, but, on the other hand, it must be loose enough to allow rapid germination. Thus, the desired final state is usually characterized by a spatially variable structure, both vertically and horizontally, but below the working depth, a compacted soil is produced; an effect that is amplified if the works are carried out several times at the same working depth [17].

Knowing the initial and final state of the soil, the method/methods by which these changes can be achieved must be defined, to which it must be specified that it is not enough to achieve a certain state of the soil at a given time, but this state must be maintained over a longer term. In resolving these issues, they must be correlated with the requirements of the plants that will be cultivated, the preceding plant, the entire rotation of which the respective crops are part, the pedo-climatic conditions, the requirements for maintaining soil fertility in the long term, and slowing down the process of soil destruction.

In this sense, the attempts of several soil tillage machine manufacturers to design working tools that can efficiently loosen the soil but produce as few surface changes as possible [17–19].

The most recent ones refer to the development of automatic control and operation strategies, respectively, to the development of sensors necessary for the automation of soil tillage processes [3,20]. Figure 4 shows the block diagram of an automatic soil tillage control system.

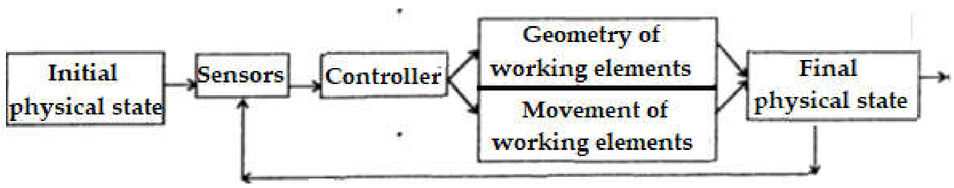

Figure 4. Block diagram of an automatic soil tillage control system.

As can be seen from the block diagram, at present this research is in continuous expansion; there are still no solutions to the question of what to do in the case of automatic real-time control, if the result of the work does not fall within the prescribed interval of the physical state of the soil.

When studying the energy consumption in the case of soil tillage with a disc harrow, the energy consumption of the entire aggregate, consisting of the energy source (tractor) and the trailed agricultural machinery, is often considered through the energy correlation between them. Thus, the energy consumption has several components related to the movement of the energy source on the field, these being the energy necessary to overcome rolling resistance, the energy dissipated due to the slippage of the drive wheels, the energy necessary to move, and possibly, to drive the agricultural machinery to till the soil.

3. Results and Discussions

3.1. Germination Bed Preparation

To ensure appropriate conditions for seed germination, it is necessary to loosen the soil to an appropriate depth that favors aeration, storage and retention of the largest possible amount of water,

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the accumulation of optimal amounts of nutrients necessary for plant growth, in short "hard bed and soft blanket" [21], in other words a "perfect" bed in which the seeds have a "blanket" without plant debris and weeds and loosened over the entire surface (Figure 5).

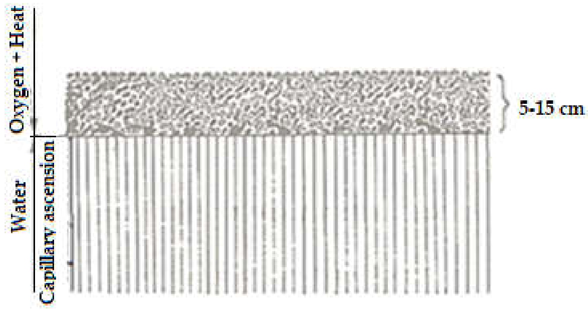

Figure 5. Germination bed structure.

A suitable and good germination bed ensures quality sowing on a weed-free field, seed germination and uniform plant emergence, achieving optimal density and ultimately large and high-quality harvests.

For this, it is necessary to determine the qualitative indices regarding the preparation of the germination bed for sowing and planting. Qualitative indices ensure maximum economic effects in the specific conditions of each plant exploitation and represent a complex of measurable or determinable quantities using measuring devices, instruments, or equipment. These indices characterize the degree to which the imposed agrotechnical requirements [21, 22, to ensure optimal germination [23,24] have been met and are presented with indicative values [25–27], in Table 1.

Table 1. Guideline values of qualitative indices for seedbed preparation in the field.

No. crt.	Qualitative indices in seedbed preparation	Values
1	Average deviation from average working depth	$\delta_a \leq \pm 0{,}10 \cdot a_m$
2	Maximum deviation from average working depth	$\Delta a \leq \pm 0{,}20 \cdot a_m$
3	Standard deviation of working depth	$\delta_s \leq \pm 0{,}10 \cdot a_m$
4	Coefficient of variation of working depth	$c_a \leq \pm 0{,}10$
5	Degree of soil fragmentation	$D_{fs} > 75\%$
6	Soil penetration resistance	- light soil: $\sigma_p < 15$ kPa - medium soil: $\sigma_p = 15\text{-}50$ kPa
		- heavy soil: $\sigma_p = 50 - 150$ kPa - very heavy soil: $\sigma_p > 150$ kPa
7	Degree of soil loosening	$D_{ls} \approx 20 \ldots 30\%$
8	Degree of soil leveling	$D'_{ls} > 40\%$
9	Degree of weed destruction	$D_d > 95\%$

3.2. Theoretical Research on the Disc-Soil Interaction of the Disc Harrow

Soil cultivation through the interaction between discs and soil can be presented as a mechanical system that illustrates the complexity of the interdependence between factors influencing soil quality and energy consumption [1,28]. An example of systems thinking is presented in Figure 6. The input to the system is represented by the initial properties and characteristics of the soil, S_i, and the energy applied to the system, E, to obtain the final characteristics S_f at the output.

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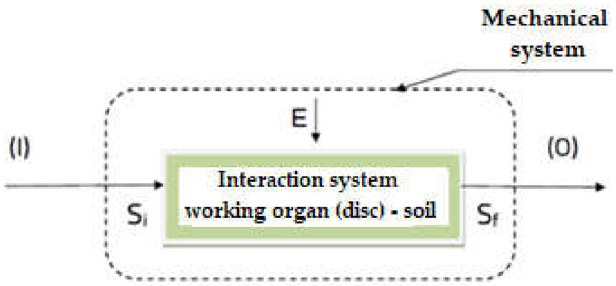

Figure 6. Representation of the soil processing system: (I) – Initial; (O) - Obtaining.

Thus, the input to the system is represented by the initial properties and characteristics of the soil (S_i) and the energy applied to the system (E) to obtain the final characteristics (S_f) at the output. This can be represented by a mathematical equation of the form:

$$S_f = f(S_i, E) \quad (1)$$

This equation represents the functional relationship between the disc and the soil. If the elements of the equation are expressed appropriately, this will lead to an explicit mathematical expression that defines the process of soil processing by the discs of the disc harrow. In order to quantify the parameters involved in the process [28–30] it is necessary to divide the system (relation (1)) into a series of subsystems (relations), Figure 7.

Figure 7. Decomposition of the interaction system between the working body (disc) and the soil.

Thus, the energy, *E* can be expressed by the relationship:

$$E = f(C_g, M_m, S_i), \quad (2)$$

where, C_g – the geometric characteristics of the disc, M_m - the mode of movement during the working process.

As a result, the relation (1) can be written as follows:

$$S_f = f(C_g, M_m, S_i). \quad (3)$$

Each element/parameter of the relationship/equation can be characterized as a subsystem that can be described by its own equations:

$$C_g = f_1(\text{geometric parameters of the working body (angles, dimensions, …)}); \quad (4)$$

$$M_m = f_2(\text{motion parameters (velocities, displacements, …)}); \quad (5)$$

$$S_i = f_3(\text{parameters regarding the physical-mechanical properties of the soil (tensions, densities, deformations, humidity, porosity, compactness, …)}). \quad (6)$$

Based on the previous relationships ((1) … (6)) and the scheme in Figure 8, a quantifiable mathematical model is obtained [31], and the interaction between the working body and the soil can be traced according to Figure 9.

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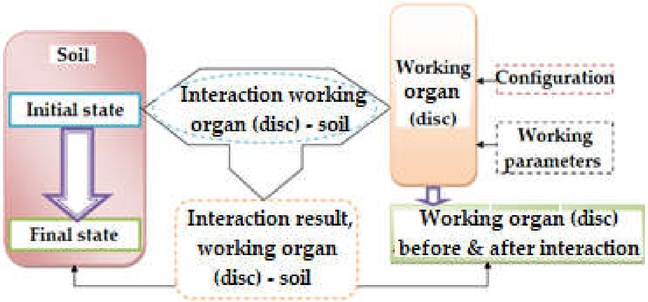

Figure 8. Scheme of interaction between the working body (disc) and the soil

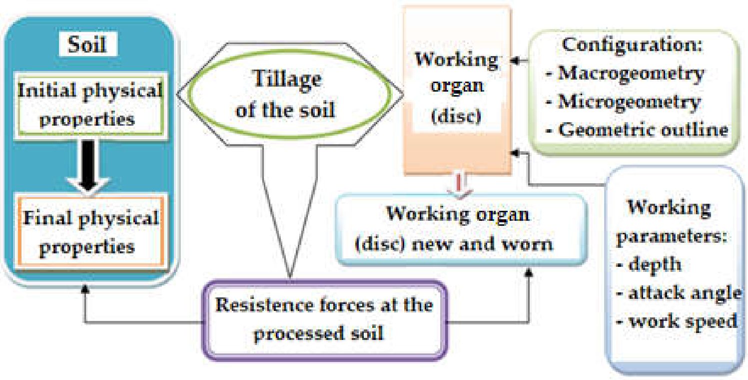

Figure 9. Result of the interaction between the working body (disc) and the soil

3.3. Mathematical Modeling of the Energy Consumption of Disc Harrows for Seedbed Preparation

In general, the technical equipment used for seedbed preparation can positively or negatively influence the physical and mechanical properties of the soil. The total mechanical energy, E_t, required for soil processing with the disc harrow can be determined with the relationship [25,27]:

$$E_t = E_w + E_l, \text{ [J]} \tag{7}$$

where, E_w - the energy required to perform mechanized soil work, in J; E_l - the energy consumed through losses in the disc shaft bearings, in J.

The total mechanical energy of a system, E_t, is its capacity to perform useful mechanical work, L, and is the product of the power consumed, P, in W, and the operating time, t, in s, that is:

$$E_t = L = P \cdot t. \text{ [J]} \tag{8}$$

The power consumed, P, is the product of the forces acting on the machine while performing the work, F, and the speed, v, according to the relationship:

$$P = F \cdot v. \text{ [W]} \tag{9}$$

Schematically, the energy study of the disc harrow is presented in Figure 10.

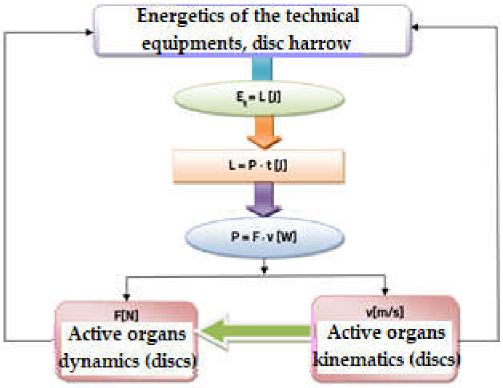

Figure 10. Energy study of the disc harrow for soil cultivation.

In turn, the energy required for soil processing with the disc harrow, E_c is composed of:

$$E_p = E_m + E_{c/l}, \text{[J]} \tag{10}$$

$$\text{so:} \quad E_t = E_m + E_{c/l} + E_{lb}, \text{[J]} \tag{11}$$

where, E_m – the energy required to move the harrow, in J; $E_{c/l}$ - the energy required to break up/loosen the soil, in J; E_{lb} - the energy consumed through losses in the disc shaft bearings, in J.

Based on the relation (11), the mathematical model of energy consumption in the working process of the disc harrow can be established. Thus, the energy required for movement, E_m is transformed into mechanical work not required for the harrow movement, L_m and is directly proportional to the weight of the harrow, W_h and to the coefficient of resistance to movement, c_r between the harrow and the soil, that is:

$$E_m = L_m = P_m \cdot t_m = f(W_h, c_r), \text{[J]} \tag{12}$$

where: P_m - the power required to move the harrow, in W, and t_m - the time, how long the movement is performed, in s.

Reducing energy consumption when moving the harrow can be achieved by reducing W_h even from the design phase and using appropriate materials, and reducing the c_r by using discs with the largest diameters and concavities.

The energy required to perform mechanical shredding/loosening operations, $E_{s/l}$ is transformed into mechanical work not required for shredding/loosening, $L_{s/l}$ by the harrow and is used to break boulders, crush and loosen the soil and is directly proportional to the working depth, a, the working width of the disc, b, the number of discs in the soil at a given time, z and the physical and mechanical properties of the soil, k_s, as follows:

$$E_{s/l} = L_{s/l} = P_{s/l} \cdot t_{s/l} = f(a, b, z, k_s), \quad \text{[J]} \tag{13}$$

where, $P_{s/l}$ - the power required for soil shredding/loosening, in W; $t_{s/l}$ - the time during which the shredding/loosening is performed, in s.

The specific shredding/loosening surface, $S_{s/l,}$ is the geometric parameter of the displaced amount of soil and represents the ratio between the soil shredding/loosening (cutting) surface, S_t, and its volume, V:

$$S_{sp} = \frac{S_t}{V}, \ [m^{-1}] \tag{14}$$

where, S_t – cutting surface of the soil (of the spherical curvilinear segment ABE in Figure 11), whose working width is b, in m^2; V – volume of displaced soil, in m^3.

The surface, S_t, is approximately determined by the relation:

$$S_t = S_{ABD} = S_{ABE} \cdot \sin\alpha, \ [\text{m}^2] \tag{15}$$

where, the notations correspond to Figure 11 and have the following meanings: α - is the angle of attack, in radians; s - the advance of the disc harrow (and therefore of each disc), in m, when moving at its average working speed, v, in m/s; a - the working depth (shredding/loosening), in m; D - the

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diameter of a disc (all discs of a harrow have the same diameter); φ - the angle of contact of the disc with the soil, in radians.

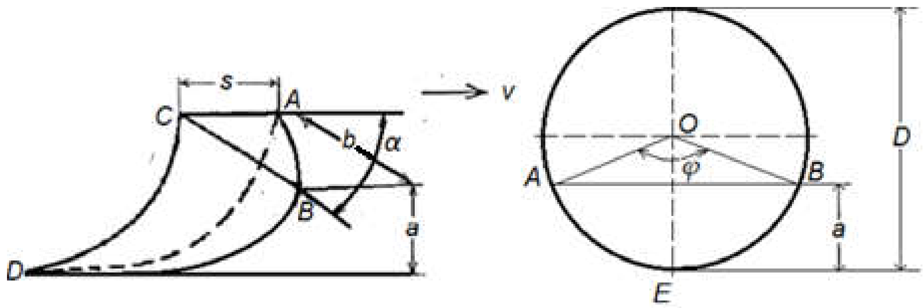

Figure 11. Dimensions of the displaced soil slice and the contact angle of the disc with the soil.

The volume, V is determined, approximately, as follows:

$$V \approx \frac{s\cdot b_a\cdot a}{3}, [m^3], \qquad (16)$$

$$\text{and} \quad s = v/n, \text{ [m]} \qquad (17)$$

where, n – rotation speed of the drive and fixing shaft of the discs in the working process, in rpm.

With these notations, the surface of the spherical curvilinear segment, S_{ABE} becomes:

$$S_{ABE} = \frac{\frac{D}{2}\cdot\frac{\varphi D}{2}}{2} - \frac{2D}{2}\sin\frac{\varphi}{2}\cos\frac{\varphi}{2} = \frac{1}{8}D^2(\varphi - \sin\varphi). \qquad (18)$$

$$\text{Therefore:} \quad S_t = \frac{1}{8}D^2(\varphi - \sin\varphi)\cdot\sin\alpha, \qquad (19)$$

where, φ - the angle of contact of the disc with the ground (see Figure 12), results from the equality:

$$\cos\frac{\varphi}{2} = \frac{\frac{D}{2}-a}{\frac{D}{2}} \Rightarrow \varphi = 2\,arccos\left(1-\frac{2a}{D}\right). \qquad (20)$$

The working width of the disc, b will be:

$$b = AB\cdot sin\ \varphi, \qquad (21)$$

where AB – the length of the chord of the circle with the diameter, D, at the distance, a, and the length of the chord, AB, results from the expression:

$$(AB)^2 = (D/2)^2 - [(D/2) - a]^2, \text{ astfel } AB = 2\sqrt{a(D-a)}, \qquad (22)$$

$$\text{respectively:} \quad b = 2\sqrt{a(D-a)}\cdot sin\ \varphi. \quad \text{[m]} \qquad (23)$$

$$\text{Then:} \quad V = \frac{\frac{v}{n}\cdot 2\sqrt{a(D-a)}\cdot\sin\varphi\cdot a}{3} = \frac{2a\cdot v\cdot sin\varphi\cdot\sqrt{a(D-a)}}{3n}. \text{ [m}^3\text{]} \qquad (24)$$

By substituting relations (19) and (24) into (14), the specific surface area relation, S_{sp} of shredding/thinning, becomes:

$$S_{sp} = \frac{\frac{1}{8}D^2(\varphi-\sin\varphi)\cdot sin\ \alpha}{\frac{2a\cdot v\cdot sin\varphi\cdot\sqrt{a(D-a)}}{3n}} = \frac{3nD^2(\varphi-\sin\varphi)\cdot\sin\alpha}{16a\cdot v\cdot sin\varphi\cdot\sqrt{a(D-a)}} \text{ [m}^2\text{]} \qquad (25)$$

Under specific working conditions: D = 0.51 m, b = 0.12 m, v = 4 m/s, n = 2.55 rot/s, α = 15 degree, at different working depths, a, respectively at different working speeds, v and the same working depth, a = 0.15 m, the following values were obtained, presented in Table 2 and graphically in Figure 12.

Table 2. Calculated values of the specific cutting surface, Ssp, in concrete working conditions.

Nr. crt.	Working depth, *a* (m)	Contact angle disc - soil, φ (degree)	Specific surface, S_{sp} (m^{-1})	Working speed, v (m/s)	Specific surface, S_{sp} (m^{-1})
1.	0.05	72.98	1.18	1	8.88
2.	0.07	86.98	1.36	2	4.44
3.	0.09	99.36	1.57	3	2.96
4.	0.11	110.69	1.85	4	2.41
5.	0.13	121.29	2.22		
6.	0.15	131.37	2.41		

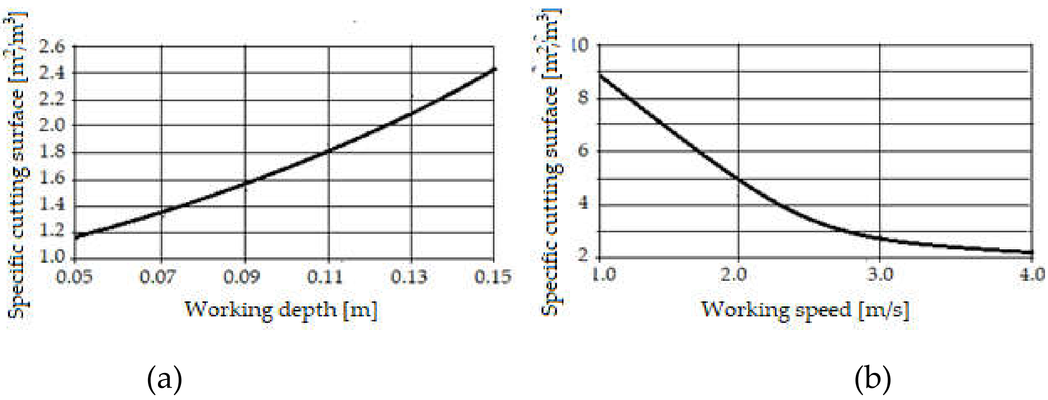

Figure 12. Variation of specific surface area when working the soil with a disc harrow: (a) - with working depth, and (b) - with working speed.

From the graphical representations, it can be seen that S_{sp} increases with the working depth, a and decreases with the increase in the average working speed, v, so that the degree of shredding/loosening is optimal.

Regarding the working depth, its value is determined primarily according to agrotechnical criteria, which results in an optimal working depth range. The actual value at which the work will be carried out can also be determined according to other requirements, such as energy ones.

Of much greater importance is the correlation of the distance between the discs with the working depth so that both the specific traction resistance is minimized and the volume of loosened soil is as large as possible. The working speed is also particularly important. Modern tractors, especially those of medium and high power, have transmissions designed so that they work efficiently in the speed range from 8 to 12 km/h, in this range, the slip values are the lowest.

Thus, Berntsen and Berre [32] show that the effect of speed on the soil shredding process defines the shredding efficiency as the ratio between the energy required for shredding and the total energy consumed. Figure 13 shows the variation of the shredding efficiency as a function of the speed of the discs in one pass.

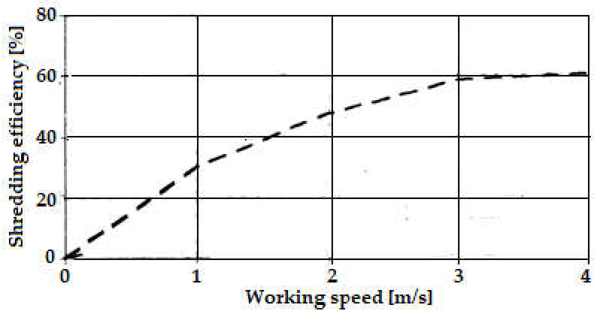

Figure 13. Shredding efficiency depending on working speed for the case of the disc harrow

3.4. Reducing Energy Consumption Related to Soil Cultivation with a Disc Harrow

A first solution to reduce the energy consumption of the disc harrow in the soil cultivation aggregate is related to the optimization of the operating characteristics, with reference to the optimal correlation of the tractive power of the energy source with the tractive resistance, to the optimal choice of the working speed, and sometimes - if not determined unambiguously for technological reasons - even the working depth. A second solution in the case of certain soil cultivation machines, where the working depth is determined by the weight of the machine (disc harrows, rolling knife harrows, etc.), is to reduce the weight of the machines. For this purpose, the so-called machine mass utilization index, I_{um}, is used, which defines the ratio between the useful mass, m_u of the soil tillage machine (depending on the type of machine) necessary to maintain the working tools in the soil at the maximum working depth and the total mass, m_t real [32,33], that is:

$$I_{um} = \frac{m_u}{m_t}. \tag{26}$$

In the case of disc harrows $I_{um} = 0.7$. The ideal value of I_{um} is equal to unity. A value of $I_{um} > 1$ means that the mass of the machine is not large enough for the machine to work stably at the maximum working depth,

A very important direction for reducing energy consumption is the optimization of the geometry of the working elements that are in contact with the soil (here the discs), because their geometry determines not only the energy consumption, but also the effect that the work has on the soil. For this reason, the optimization of the shape and size of the working elements (discs) should not be done exclusively according to energy criteria, but the effect of the changes on the soil must also be studied. From this point of view, two functional relationships are defined, particularly important, namely [33,34]:

$$F_o = f(G_o, M_o, S_i), \tag{27}$$

respectively: $S_f = g(G_o, M_o, S_i)$, (28)

where: F_o - function describing the forces acting on the working element (stellated disc); G_o - function describing the geometry of the working element; M_o - function describing the movement of the working element; S_i - function describing the initial state of the soil (see Figure 7); $\mathbf{S_f}$ - function describing the final state of the soil (see Figure 7), and f and g are two functional relations, which define how the process proceeds.

These are abstract, very complex mathematical functions, which in the simplest case can be considered as real functions of several variables each, and f and g can be different, or not.

Knowing the mathematical form of relations (27) and (28), would put designers in a particularly favorable situation: they could design both the shape and the movement of the working elements (discs) using exact deterministic methods. But even in the absence of these functional relationships, they provide guidelines for an empirical design based on correct mathematical relationships (even if they are abstract). Deriving relations (27) and (28), we can write:

$$dF_o = \frac{\partial f}{\partial G_o} dG_o + \frac{\partial f}{\partial M_o} dM_o + \frac{\partial f}{\partial S_i} dS_i \tag{29}$$

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and $$dS_f = \frac{\partial g}{\partial G_o} dG_o + \frac{\partial g}{\partial M_o} dM_o + \frac{\partial g}{\partial S_i} dS_i \tag{30}$$

From relations (29) and (30) the fundamental principle of qualitative designs of the working element results. If, for example, we modify the geometry of a working element (disc) without modifying its movement (type of movement, speed, working depth, spatial arrangement with respect to the velocity vector) and the tests are made in soil of the same type and having the same initial state, then all the changes in terms of the forces that appear in the process will be due to the geometric changes. This is also demonstrated mathematically, since:

$$dF_o = \left(\frac{\partial f}{\partial G_o}\right) dG_o \tag{31}$$

$$dS_f = \left(\frac{\partial g}{\partial G_o}\right) dG_o, \tag{32}$$

because $dMo = dSi = 0$.

In general, the working method used in practice is based on this approach, through which a local and partial optimization can be made.

At the same time, for discs that come into contact with the soil, the problem of reducing the friction between their surface and the soil is raised, as another possibility of reducing energy consumption. For this, there is research to reduce friction by modifying the roughness of the surface, covering the disc surfaces with materials with reduced friction in contact with the soil, and using various lubrication methods. There are several solutions/measures to reduce energy consumption, but more important are those from the point of view of preserving the fertility of agricultural soils, i.e. the development and use of working organs and machines that can combat soil compaction, and in this way, indirectly, also lead to an increase in the energy efficiency of the preparation of the seedbed for plant cultivation.

4. Conclusions

By reducing energy consumption during soil cultivation (preparation of the seedbed) with the help of a disc harrow:

- a soil structure favorable to plant cultivation is obtained by: rapid infiltration and retention of water in the soil, necessary for seed germination; providing the seeds with a sufficient amount of air following the gas exchanges that take place in the soil; minimizing resistance to root system penetration.

- mechanical removal of weeds is ensured and allows the incorporation of plant residues after harvesting, mineral fertilizers, amendments, etc., into the soil.

- it combats diseases and pests as a result of the introduction/placement of plant residues at depth, where conditions are not favorable for them.

- positively influences the activity of aerobic microorganisms in the soil and the soil water regime (accumulation and retention of water in the soil, in case of excess moisture, it improves internal drainage, respectively positively influences soil porosity and the economic efficiency of soil works (fuels, lubricants, wear of the disc harrow equipment, etc.).

Following the analysis of the theoretical and partially experimental results, from the point of view of optimizing the studied parameters, it was highlighted that:

- the choice of working depth must be in correlation with the requirements of the plant (the agrotechnical interval corresponding to the plant), to avoid a significant increase in soil resistance to work (especially the traction component);

- the choice of the working speed of the soil crushing/loosening aggregate must be as high as possible, if the state of the land allows at the time of the work; the extra energy consumed due to the increase in soil resistance is reflected in the efficiency shredding); the operating mode should be optimal, from the point of view of specific fuel consumption.

The structure of the energy balance was highlighted, theoretically analyzing the energy aspects related to the technology of soil cultivation, for the preservation of its fertility, considering the appropriate parameters of the working organs (discs of the harrow).

A description of the aspects on which the theoretical model is based was made, which describes the interaction between the shredding/loosening organs (discs) and the soil, for increasing the energy efficiency in the soil working process.

The performance criterion of the working organ (disc) - soil system is given by the energy consumption, in the soil processing process, for preparing the germinal bed in the field.

The developed theoretical model highlighted the correlation between energy efficiency and depth, respectively working speed, validated for concrete conditions, in the sense that energy consumption increases with increasing working depth and decreases with increasing working speed, in the soil working process.

Also, the theoretical model describes well enough the effect of working depth and speed in the soil processing process and ensures the appropriate economic efficiency and clearly defines the crumbling/loosening area of disc harrows.

The mathematical modeling carried out to reduce the energy consumption of the working bodies (discs) of disc harrows was done with the aim of positively influencing the physical-mechanical properties of the soil and establishing the optimal parameters for this purpose.

Reducing the energy consumption for the technical equipment of disc harrows can also be achieved by reducing their weight, using lighter materials but appropriate in terms of resistance to mechanical stress, and by reducing the rolling resistance coefficient. The use of working bodies beyond the permissible wear limits leads to high energy consumption and failure to achieve the qualitative indices of the work.

The influence of the studied factors (depth and speed of work) is to be studied experimentally in the field, in order to observe the level of correlation between theory and practice. In addition, it is necessary to continue theoretical and experimental research, to determine the energy consumption and for other aggregates for soil crushing/loosening, and their optimization. Also, to carry out theoretical and experimental studies, using the finite element method of the interaction between the working bodies and the soil to find the best shapes of the technical equipment.

Author Contributions: For research articles with several authors, a short paragraph specifying their individual contributions must be provided. The following statements should be used "Conceptualization, F.I.; methodology, F.I. and D.A.B.; software, F.I and D.A.B.; validation, F.I and D.A.B.; formal analysis, D.A.B..; investigation, F.I and D.A.B.; resources, F.I; data curation, F.I. and D.A.B.; writing—original draft preparation, F.I.; writing—review and editing, F.I. and D.A.B.; visualization, F.I. and D.A.B.; supervision, D.A.B.; project administration, F.I.; funding acquisition, F.I. and D.A.B. All authors have read and agreed to the published version of the manuscript." Please turn to the CRediT taxonomy for the term explanation. Authorship must be limited to those who have contributed substantially to the work reported.

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References

1. Gheres M.I., Mathematical model for studying the influence of tillage tool geometry on energy consumption, *INMATEH – Agricultural Engineering* **2014**, *42*(1), 5-12.
2. Maharjan G.R., Prescher A-K., Nendel C., Frank Ewert F., Mboh C.M., Gaiser T., Seidel S., Approaches to model the impact of tillage implements on soil physical and nutrient properties in different agro-ecosystem models, *Soil and Tillage Research* 2018, 180, 210-221, https://doi.org/10.1016/j.still.2018.03.009.

3. Tripathi R.P. , Sharma P., Singh S., Tilth index: an approach to optimize tillage in rice–wheat system, *Soil and Tillage Research* **2005**, *80,* 125-137, https://doi.org/10.1016/j.still.2004.03.004.
4. Singh K.K., Colvin T.S.,.Erbach D.C, Mughal A.Q., Tilth index: An approach to quantifying soil tilth. Trans.ofthe ASAE **1992**, *35*(6), 1777-1785, DOI: 10.13031/2013.28797.
5. Obour P.B, Ugarte C.M., A meta-analysis of the impact of traffic-induced compaction on soil physical properties and grain yield, *Soil and Tillage Research* 2021, *211*, 105019, https://doi.org/10.1016/j.still.2021.105019.
6. *** ASAE Standard: ASAE S313.2, *Soil Cone Penetrometer*, ASAE Standards 41th edition **1994**. St. Joseph, MI.48.
7. Colvin, T.S., Erbach D.C., Buchele W.F., Cruse R.M., Tillage index based on created soil conditions. *Trans. of the ASAE* **1984**, *27*(2), 370-371, http://works.bepress.com/wesley_buchele/33/.
8. Singh M., Singh S., Deb S., Ritchie G., Root distribution, soil water depletion, and water productivity of sweet corn under deficit irrigation and biochar application, *Agricultural Water Management* **2023**, *279*, 108192, https://doi.org/10.1016/j.agwat.2023.108192.
9. Schafer, R.L., Johnson C.E., Elkins C B., Hendrick J.G., Prescription tillage: the concept and examples, *J. Agric. Eng. Res.* **1985,** *32*(2), 123-129, https://doi.org/10.1016/0021-8634(85)90072-1.
10. Hargreaves P.R, Peets S., Chamen W. C. T., White D. R., Misiewicz P. A., Godwin R. J., Improving grass silage production with controlled trafc farming (CTF): agronomics, system design and economics, *Precision Agriculture* **2019**, *20*, 260–277, https://doi.org/10.1007/s11119-018-09633-7.
11. Mihalache M., *Pedology - genesis, properties and taxonomy of soils* (in Romanian), Ceres Publishing House, **2006,** Bucharest, Romania.
12. Ahuja, L.R., Ma L., Timlin D.J., Trans-Disciplinary Soil Physics Research Critical to Synthesis and Modeling of Agricultural Systems, *Soil Sci. Soc. Am. J.* **2006**, *70*, 311–326. DOI:10.2136/sssaj2005.0207.
13. Căproiu Şt., *Agricultural machines for tilling the soil, sowing and crop maintenance* (in Romanian), Didactic and Pedagogical Publishing House, **1982,** Bucharest, Romania.
14. Hahn C.,Lindkvist E., Magnusson D., Johansson M., The role of agriculture in a sustainable energy system – The farmers' perspective, *Renewable and Sustainable Energy Reviews* 2025, *213*, 115437, https://doi.org/10.1016/j.rser.2025.115437.
15. Birkâs M., About the importance of loosening the soil (in Hungarian), *Agroforum* **1994**, *5*(8), 18-20.
16. Birkâs M., About the use of chisel cultivators (in Hungarian), *Mezogazdasăgi Technika* **1994**, *35*(8), 2-5.
17. Reeder R.C., Wood R.K., Finck C.L., Five subsoiler design and their effects on soil properties and crop yields, *Trans. of the ASAE* **1993**, *36*(6), 1525-1531.
18. Tupper G.R., Low-till parabolic subsoiler, *ASAE* **1994**, Papers No. 94-1008, St.Joseph, MI. ASAE.
19. Smith B.E., Schafer R.L., Johnson C.E., Using coulters to quantify the soil physical condition, *ASAE* **1994**, Papers No. 94-1047. St.Joseph, MI. ASAE.
20. ***http://www.agricultura365.ro/ quality-requirements-for-preparing-the-germination-bed-for-spring-crops /, acces. 30.11.2025.
21. ***https://www.finantariagricole.ro/ advice/preparing-the-germination-bed /, acces. 28.10.2025.
22. ***https://www.icpa.ro/ documents/codes/How_to_work_the_soil.pdf, acces. 08.12.2023.
23. ***https://www.lumeasatului.ro/articole-revista/2753-cum-trebuie-pregatit-patulgerminativ.html, acces. 30.11.2025.
24. Naghiu Al., Agricultural machines and installations, Risoprint Publishing House, **2004**, *I,* Cluj-Napoca, Romania.
25. Pásztor, J., *Research on the energy optimization of seed bed preparation works in greenhouses* (in Romanian), PhD Thesis **2009**, Transilvania University Braşov, Brasov, Romania.
26. Vlad, C., *Research on the energy optimization of the process of preparing the germinal bed for the establishment of vegetable crops*, PhD Thesis **2013,** Transilvania University Braşov, Brasov, Romania.
27. Ghereş, M. I., Mathematical model for studying the influence of tillage tool geometry onenergy consumption, *INMATEH Agricultural Engineering* **2014**,. *42* (1), 5-12, https://www.researchgate.net/publication/292258522.

 doi:10.20944/preprints202603.0625.v1

28. Farcaş, N., Popescu, O., Dobre, P., Simion C., Working organs of seed bed preparation machines (in Romanaian), "*Agriculture Mechanization" Magazine,AGRIS* **2001**, No. 4, 14-16.
29. Marin L.O., Dynamic three-dimensional simulation model of soil interaction estrecha narrow tillage tool using the finite element method. MOJ Research Review **2020**, *3*(1), 1–5. DOI: 10.15406/mojcrr.2020.03.00060.
30. Berntsen R., Berre B., Soil fragmentation and the efficiency of tillage implements, *Soil and Tillage Research* 2002, *64*(1-2), 137-147, https://doi.org/10.1016/S0167-1987(01)00251-3.
31. Pâiinkâs I., The useful mass and the coefficient of use of the mass of agricultural machines (în Hungarian). International scientific communications session "Scientific and technical research in support of the development and restructuring of agriculture" **1995**, . Mirton Publishing Timisoara 227-232, , Timisoara, Romania.
32. Marinello F., Yezekyan T., Armentano G., Sartori L., Modelling of Agricultural Machinery Trends for Power, Mass, Working Width and Price. In: Coppola, A., Di Renzo G., Altieri G., D'Antonio P. (eds) Innovative Biosystems Engineering for Sustainable Agriculture, Forestry and Food Production **2020**, MID-TERM AIIA **2019**. Lecture Notes in Civil Engineering, *67* Springer Cham. https://doi.org/10.1007/978-3-030-39299-4_54.