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A Simpson–Type Decomposition of the Euler–Mascheroni Constant

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Abstract

An elementary and self-contained approach to the Euler–Mascheroni constant γ is presented, based solely on Simpson's quadrature rule and the convexity of the function $f(x) = 1/x$. By introducing Simpson-type weighted harmonic sums, local logarithmic increments are approximated by simple finite linear combinations of reciprocal integers. Sharp two-sided inequalities, derived from monotonicity and convexity, yield explicit control of the quadrature error and provide a purely numerical proof of the classical limit defining γ, without recourse to the Euler–Maclaurin summation formula. A key structural outcome of this framework is a decomposition $\gamma = (\log[2] + 1)/3 + \delta$, where the constant δ arises naturally as the limit of a rational sequence associated with Simpson-regularized harmonic sums. This formulation isolates a dominant oscillatory component and leads to a sequence with markedly faster convergence. This work highlights an unexpected connection between elementary numerical quadrature and one of the fundamental constants of analysis.

Keywords: Euler–Mascheroni constant; simpson-type decomposition; harmonic series; numerical quadrature

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1. Introduction

The Euler–Mascheroni constant is a classical object of analysis and number theory [1]. Let

$$H(N) = \sum_{k=1}^{N} \frac{1}{k} \tag{1}$$

denote the N-th harmonic number. Then the Euler–Mascheroni constant can be written succinctly as

$$\gamma = \lim_{N\to\infty} (H(N) - \log[N]) \approx 0.57721\ldots. \tag{2}$$

Figure 1 illustrates the difference between the integral of $1/x$ and its approximation by the rectangular rule. The existence of γ is usually established via asymptotic expansions derived from the Euler—Maclaurin summation formula or related analytic techniques [2]. While this limit is familiar, most standard proofs rely on the Euler–Maclaurin summation formula, and it is natural to ask whether γ can be understood using only elementary numerical ideas.

As noted by Havil [3], the Euler–Mascheroni constant γ arises naturally in the study of the harmonic series and its relation to the logarithmic function, and has attracted considerable attention due to its slow convergence in its classical definition. This slow convergence motivates the search for alternative approximations with provably faster convergence. In this paper, we study a class of rational approximations arising from Simpson-type quadrature formulas and analyze their convergence behavior.

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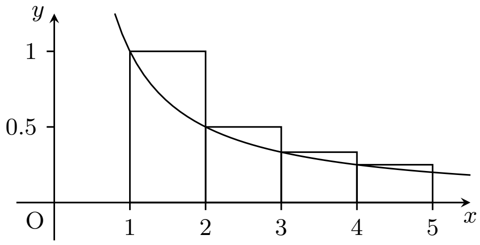

Figure 1. Approximation of the integral of $1/x$ by the left-endpoint rectangular rule. Each rectangle has unit width and height $1/k$, producing the harmonic sum $H(N)$. The difference between this sum and $\log[N]$ converges to the Euler–Mascheroni constant γ.

A classical acceleration of the harmonic sequence is due to DeTemple [4], who showed that a simple shift in the logarithmic term yields a convergence to the Euler–Mascheroni constant of order $O(n^{-2})$.

Classical acceleration techniques are typically based on the Euler–Maclaurin expansion, where the difference $\gamma_n - \gamma$ is expressed as an asymptotic series whose coefficients involve Bernoulli numbers. Accelerated sequences are then constructed by introducing polynomial corrections to cancel leading terms in this expansion. Such approaches have been systematically developed, for example by Mortici [5], and further refined by Chen and Mortici [6,7], who constructed sequences involving polynomial corrections in the logarithmic term. These methods achieve increasingly higher-order convergence at the cost of more complex expressions and coefficients. Further developments by Yang [8] provide a systematic framework for determining the optimal correction coefficients using Bell polynomials, thereby enabling arbitrarily high-order asymptotic acceleration.

Related approaches have also been proposed by other authors, who construct explicit parameterized correction formulas to improve convergence and derive refined inequalities; see, for example, Crîngan u [9,10]. More recently, Mortici [**?**] considered modifications of the logarithmic term based on local averaging structures, leading to sequences with improved convergence rates and explicit double inequalities. Such approaches highlight the effectiveness of incorporating local information in the construction of accelerated sequences, while maintaining relatively simple functional forms. However, all these methods ultimately rely on modifying the logarithmic approximation by introducing correction terms, either through asymptotic expansions or local averaging.

In contrast, the present approach follows a fundamentally different route, focusing on the structural interpretation of the harmonic sum. Rather than modifying an asymptotic expansion, we reinterpret the harmonic sum as a quadrature problem for the function $1/x$. By applying a Simpson-type decomposition, the error is represented as a summable sequence of local quadrature errors. This structure leads to a convergence rate of order $O(N^{-4})$ without the use of Bernoulli numbers or polynomial corrections. Moreover, the resulting expressions involve only simple rational coefficients, reflecting the structural nature of the method.

2. Analytic Approximation of the Logarithmic Integral

2.1. Simpson–Type Approximation of Logarithmic Increments

Definition 1. For integers $n \geq 1$, define

$$f(n) := \int_{2n-1}^{2n+1} \frac{1}{x}\,dx = \log[2n+1] - \log[2n-1]. \tag{3}$$

We note that summing the increments defined in (3) forms a telescoping sum:

$$\sum_{n=1}^{\frac{N-1}{2}} f(n) = \log[N], \tag{4}$$

for all odd integers $N \geq 3$.

This integral is approximated using Simpson's rule with a step size of 1, as illustrated in Figure 2. Using the local Simpson weight

$$g(n) = \frac{1}{3}\left(\frac{1}{2n-1} + \frac{4}{2n} + \frac{1}{2n+1}\right), \tag{5}$$

we now define a global Simpson-regularized approximation of the harmonic sum. Notably, $g(n)$ is a finite linear combination of reciprocal integers and depends solely on the values of $1/x$ at the integer points.

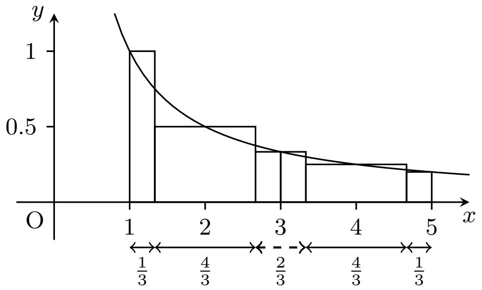

Figure 2. Simpson–type numerical integration of $1/x$, producing the regularized sum $h(N)$. The horizontal width of each rectangle represents the Simpson weight: endpoint contributions carry weight $1/3$, interior subintervals have width $4/3$, and interior boundary nodes contribute a combined weight $2/3$ arising from adjacent subintervals, indicated by a dashed line. The alternating excess and deficit areas explain the higher accuracy compared with the rectangular approximation in Figure 1.

2.2. Convexity and Comparison Inequalities

The local quadrature error is defined as follows:

$$d(n) = g(n) - f(n). \tag{6}$$

The function $1/x$ is strictly decreasing and convex on $(0, \infty)$. Let $p_n(x)$ denote a quadratic polynomial interpolating $1/x$ at the three points.

$$x = 2n-1, 2n, 2n+1. \tag{7}$$

By construction,

$$g(n) = \int_{2n-1}^{2n+1} p_n(x)dx. \tag{8}$$

As $1/x$ is convex, the interpolation polynomial $p_n(x)$ lies above $1/x$ on $[2n-1, 2n+1]$. Thus, $p_n(x) - f(x) > 0$ except at the midpoint, and integration yields $d(n) > 0$. Consequently,

$$d(n) > 0 \qquad \text{for } n \geq 1. \tag{9}$$

Moreover, convexity implies a comparison across adjacent intervals; in particular,

$$f(n) > g(n+1) \qquad \text{for } n \geq 2. \tag{10}$$

It follows that

$$0 < d(n) < g(n) - g(n+1) \qquad \text{for } n \geq 2. \tag{11}$$

2.3. *Telescoping Bounds and Error Convergence*

Summing the aforementioned inequalities for $n = 2, \dots, N$ yields

$$0 < \sum_{n=2}^{N} d(n) < \sum_{n=2}^{N} \{g(n) - g(n+1)\}. \tag{12}$$

Both bounds telescope:

$$0 < \sum_{n=2}^{N} d(n) < g(2) - g(N+1) \tag{13}$$

As $g(n) \to 0$ as $n \to \infty$ and $d(n) > 0$ for $n \geq 2$, the partial sums

$$\sum_{n=2}^{N} d(n) \tag{14}$$

form a monotone increasing sequence. Moreover, since

$$\sum_{n=2}^{N} d(n) < g(2) = \frac{23}{15} \approx 1.5333\dots, \tag{15}$$

the sequence of partial sums is bounded above and therefore converges.

Definition 2 (The constant δ)**.** The constant δ is naturally defined by

$$\delta := \lim_{N\to\infty} \sum_{n=2}^{N} d(n) = \lim_{N\to\infty} \sum_{n=2}^{N} \{g(n) - f(n)\}. \tag{16}$$

3. Arithmetic Consequences of Simpson-Regularized Sums

We now assemble the local Simpson approximations introduced above to obtain a global regularized analogue of the harmonic sum.

3.1. *Recovery of the Euler–Mascheroni Constant*

Definition 3 (Simpson-regularized harmonic sum)**.** Let N be a positive integer.

(i) N odd. If N is odd, we partition the interval $[1, N]$ into subintervals of length two,

$$[1,3], [3,5], \dots, [N-2, N], \tag{17}$$

and apply Simpson's rule to each subinterval to approximate the integral

$$\int_1^N \frac{1}{x} dx. \tag{18}$$

The resulting approximation is denoted by $h(N)$ and is given by

$$h(N) := \sum_{k=1}^{\frac{N-1}{2}} g(k). \tag{19}$$

(ii) N even. If N is even, we first define $h(N-1)$ by case (i), and then set

$$h(N) := h(N-1) + \frac{1}{6}\left(\frac{1}{N-1} + \frac{4}{N-0.5} + \frac{1}{N}\right). \tag{20}$$

For odd N, this corresponds to applying Simpson's rule on each subinterval of length two, while for even N, the final interval $[N-1, N]$ is additionally approximated by a Simpson rule with step size one.

Remark 4. The definition above is fully consistent with Simpson's rule. For odd N, the interval $[1, N]$ is partitioned into subintervals of length two, and Simpson's rule is applied to each part. For even N, the final interval $[N-1, N]$ is additionally approximated by a Simpson rule with step size one.

Lemma 5 (Representation by integer reciprocals)**.** *The Simpson-regularized harmonic sum $h(N)$ can be written in the form*

$$h(N) = \frac{1}{3} + \frac{1}{3}\sum_{n=2}^{N-1}\frac{c_n}{n} + q_N, \tag{21}$$

where the coefficients c_n are given by

$$c_n = \begin{cases} 4, & \text{if } n \text{ is even,} \\ 2, & \text{if } n \text{ is odd,} \end{cases} \tag{22}$$

and the boundary term q_N is defined by

$$q_N = \begin{cases} \dfrac{1}{3N} + \dfrac{1}{3(N-1)} + \dfrac{8}{3(2N-1)}, & \text{if } N \text{ is even,} \\ \dfrac{1}{3N}, & \text{if } N \text{ is odd.} \end{cases} \tag{23}$$

Lemma 6. *The constant δ defined in Definition 2 can be written as*

$$\delta = \lim_{N\to\infty}\{h(N) - \log[N]\} \tag{24}$$

Proof. By Definition 2, we have

$$\delta = \sum_{n=2}^{\infty} d(n), \qquad d(n) = g(n) - f(n). \tag{25}$$

From equations (19) and (4),

$$h(N) - \log[N] = \sum_{n=1}^{\frac{N-1}{2}}\{g(n) - f(n)\} = \sum_{n=1}^{\frac{N-1}{2}} d(n). \tag{26}$$

Taking the limit as $N \to \infty$, we obtain

$$\lim_{N\to\infty}\{h(N) - \log N\} = \sum_{n=1}^{\infty} d(n) = \delta, \tag{27}$$

where the series converges as shown by equation (15). This completes the proof. $\square$

Theorem 7 (Limit formula for $H(N) - h(N)$)**.**

$$\lim_{N\to\infty}\{H(N) - h(N)\} = \frac{\log[2]+1}{3} = 0.56438\ldots. \tag{28}$$

Proof. Let us compare the coefficients of $1/n$ in the representations of $h(N)$ and $H(N)$.

For even n, the coefficient in $h(N)$ is $4/3$, hence

$$\frac{1}{n} - \frac{4}{3n} = -\frac{1}{3n}. \tag{29}$$

For interior odd $n \geq 3$, the coefficient in $h(N)$ is $2/3$, and thus

$$\frac{1}{n} - \frac{2}{3n} = \frac{1}{3n}. \tag{30}$$

Therefore, up to boundary terms, we obtain

$$H(N) - h(N) = \frac{1}{3}\left(\sum_{\substack{3 \le k \le N-2 \\ k \text{ odd}}} \frac{1}{k} - \sum_{\substack{2 \le k \le N-1 \\ k \text{ even}}} \frac{1}{k} \right) + \frac{2}{3} + O\left(\frac{1}{N}\right). \tag{31}$$

The alternating harmonic series

$$\sum_{k=1}^{\infty} (-1)^{k-1} \frac{1}{k} \tag{32}$$

is known to converge to $\log[2]$. Since the boundary contributions vanish as $N \to \infty$, we conclude that

$$\lim_{n \to \infty} \{H(n) - h(n)\} = \frac{\log[2] + 1}{3}. \tag{33}$$

□

Proposition 8 (Recovery of the Euler–Mascheroni constant)**.** *From equations (13) and (31), we obtain*

$$H(N) - \log[N] = \frac{\log[2] + 1}{3} + \delta + O\left(\frac{1}{N}\right). \tag{34}$$

In particular,

$$\gamma = \lim_{N \to \infty} \{H(N) - \log[N]\} = \frac{\log[2] + 1}{3} + \delta. \tag{35}$$

Equivalently, this yields the decomposition

$$\gamma = \frac{\log[2] + 1}{3} + \delta, \tag{36}$$

which expresses the Euler–Mascheroni constant, as defined in equation (2), in the form of a sum of two constants.

4. Convergence of the Simpson-Regularized Harmonic Sum

4.1. Convergence of $h(N)$

Theorem 9 (Convergence of $h(N)$)**.** *There exists a real constant δ and a function $r(N) \ge 0$ such that*

$$\big| h(N) - \log N - \delta \big| \le r(N) \tag{37}$$

for all integers $N \ge 3$, where $r(N)$ is defined as follows.

(i) If N is odd and $N \ge 3$, then

$$r(N) := \frac{1}{15\,(N-1)^4}. \tag{38}$$

(ii) If N is even and $N \ge 4$, then

$$r(N) := \frac{1}{15\,(N-2)^4} + \frac{1}{120\,(N-1)^5}. \tag{39}$$

We define the error term

$$\varepsilon(N) := h(N) - \log N - \delta. \tag{40}$$

Then $|\varepsilon(N)| \le r(N)$ for all $N \ge 3$. In particular, $h(N) - \log N \to \delta$ as $N \to \infty$.

Proof. We first treat the case where N is odd.

Let $f(x) = 1/x$. On each subinterval $[2j-1, 2j+1]$, application of Simpson's rule with step size 1 yields a local error of the form

$$-\frac{1}{90} f^{(4)}(\xi_j), \qquad \xi_j \in (2j-1, 2j+1). \tag{41}$$

See, for example, [2].

Since

$$f^{(4)}(x) = \frac{24}{x^5}, \tag{42}$$

and $f^{(4)}(x)$ is positive and strictly decreasing for $x > 0$, we obtain

$$|f^{(4)}(\xi_j)| \le \frac{24}{(2j-1)^5}. \tag{43}$$

Hence the absolute error on each subinterval is bounded by

$$\frac{4}{15}(2j-1)^{-5}. \tag{44}$$

Summing the exact logarithmic increments defined in (3) yields a telescoping sum:

$$\sum_{n=1}^{N} f(n) = \log[N]. \tag{45}$$

Therefore, for odd $N = 2K+1$,

$$h(N) - \log N = \sum_{j=1}^{K} E_j, \tag{46}$$

where

$$E_j := (\text{Simpson on } [2j-1, 2j+1]) - \int_{2j-1}^{2j+1} \frac{1}{x}\, dx. \tag{47}$$

Since $|E_j| \le \frac{4}{15}(2j-1)^{-5}$ and $\sum_{j\ge1}(2j-1)^{-5} < \infty$, the series $\sum_{j\ge1} E_j$ converges absolutely. We define

$$\delta := \lim_{N\to\infty} \big(h(N) - \log[N]\big), \tag{48}$$

which exists by the absolute convergence above. Moreover,

$$\sum_{j\ge(N+1)/2} (2j-1)^{-5} \le \int_{N-1}^{\infty} x^{-5}\, dx = \frac{1}{4\,(N-1)^4}. \tag{49}$$

Multiplying by the factor $4/15$ yields

$$\big|h(N) - \log N - \delta\big| \le \frac{1}{15\,(N-1)^4}, \tag{50}$$

which proves (i).

We next consider even N. By definition,

$$h(N) = h(N-1) + \frac{1}{3}\left(\frac{1}{N-1} + \frac{4}{N-\frac{1}{2}} + \frac{1}{N}\right). \tag{51}$$

The first term $h(N-1)$ is estimated by part (i) (since $N-1$ is odd). For the remaining interval $[N-1,N]$, Simpson's rule with step size 1 gives an error of the form

$$-\frac{1}{2880}f^{(4)}(\eta_N), \qquad \eta_N \in (N-1,N). \tag{52}$$

Since $f^{(4)}(x) = 24/x^5$ and $f^{(4)}(x)$ is strictly decreasing for $x > 0$, we have

$$\left|-\frac{1}{2880}f^{(4)}(\eta_N)\right| \le \frac{24}{2880}\cdot\frac{1}{(N-1)^5} = \frac{1}{120\,(N-1)^5}. \tag{53}$$

Combining these two estimates proves (ii) with

$$r(N) := \frac{1}{15\,(N-2)^4} + \frac{1}{120\,(N-1)^5}. \tag{54}$$

□

Definition 10 (A rational sequence associated with δ)**.** Let us define a sequence $a(N)$ by

$$a(N) := 2h(N) - h(N^2). \tag{55}$$

Since each $h(N)$ is rational, each $a(N)$ is also rational.

Lemma 11. *The constant δ satisfies*

$$\delta = \lim_{N\to\infty} a(N). \tag{56}$$

Proof. By Theorem 9, there exists a constant δ and a function $r(N) \ge 0$ such that

$$\big|\, h(N) - \log[N] - \delta \,\big| \le r(N) \tag{57}$$

for all integers $N \ge 3$.

Hence we may write

$$h(N) = \log[N] + \delta + \varepsilon(N), \tag{58}$$

where

$$|\varepsilon(N)| \le r(N). \tag{59}$$

Substituting into (55), we obtain

$$\begin{aligned} a(N) &= 2h(N) - h(N^2) && (60)\\ &= 2(\log[N] + \delta + \varepsilon(N)) - (\log[N^2] + \delta + \varepsilon(N^2)). && (61) \end{aligned}$$

Since $\log[N^2] = 2\log[N]$, this simplifies to

$$a(N) = \delta + 2\varepsilon(N) - \varepsilon(N^2). \tag{62}$$

Therefore,

$$|a(N) - \delta| \le 2|\varepsilon(N)| + |\varepsilon(N^2)| \le 2r(N) + r(N^2). \tag{63}$$

(i) Suppose that N is odd. Then N^2 is also odd, and thus

$$r(N) = \frac{1}{15\,(N-1)^4}, \qquad r(N^2) = \frac{1}{15\,(N^2-1)^4}. \tag{64}$$

Hence

$$|a(N) - \delta| \le \frac{2}{15\,(N-1)^4} + \frac{1}{15\,(N^2-1)^4} \le \frac{1}{5\,(N-1)^4}. \tag{65}$$

(ii) Suppose that N is even. Then N^2 is odd, and

$$r(N) = \frac{1}{15\,(N-2)^4} + \frac{1}{120\,(N-1)^5}, \qquad r(N^2) = \frac{1}{15\,(N^2-1)^4}. \tag{66}$$

Thus

$$|a(N) - \delta| \leq \frac{2}{15\,(N-2)^4} + \frac{1}{60\,(N-1)^5} + \frac{1}{15\,(N^2-1)^4}. \tag{67}$$

In both cases, $|a(N) - \delta| \to 0$ as $N \to \infty$, which proves the statement. $\square$

The slow convergence traditionally observed in numerical approximations to the Euler–Mascheroni constant γ [3] is closely related to the presence of a leading $O(1/n)$ contribution, which in the present setting can be identified explicitly with the alternating harmonic series introduced in equation (32).

In the present framework, this contribution is isolated and evaluated separately, and subtracting it naturally leads to the constant δ, whose numerical values exhibit markedly faster convergence (see Table 1).

Table 1. Numerical convergence of $a(N)$ toward δ.

N	$a(N)$	$\delta - a(N)$	$r(N)$
3	0.012169312	6.64×10^{-4}	4.17×10^{-3}
4	0.012197696	6.36×10^{-4}	4.20×10^{-3}
5	0.012735433	9.78×10^{-5}	2.60×10^{-4}
6	0.012738721	9.46×10^{-5}	2.63×10^{-4}
7	0.012806754	2.65×10^{-5}	5.14×10^{-5}
8	0.012807457	2.58×10^{-5}	5.19×10^{-5}
9	0.012823395	9.88×10^{-6}	1.63×10^{-5}
10	0.012823611	9.66×10^{-6}	1.64×10^{-5}
11	0.012828805	4.47×10^{-6}	6.67×10^{-6}
12	0.012828888	4.38×10^{-6}	6.72×10^{-6}
13	0.012830969	2.30×10^{-6}	3.22×10^{-6}
14	0.012831007	2.26×10^{-6}	3.24×10^{-6}
15	0.012831968	1.30×10^{-6}	1.74×10^{-6}
16	0.012831987	1.28×10^{-6}	1.75×10^{-6}
17	0.012832480	7.92×10^{-7}	1.02×10^{-6}
18	0.012832490	7.82×10^{-7}	1.02×10^{-6}
19	0.012832763	5.08×10^{-7}	6.35×10^{-7}
20	0.012832769	5.02×10^{-7}	6.38×10^{-7}

5. Conclusion

This study demonstrated that the convergence of the harmonic series minus the logarithm can be established using solely Simpson's rule and the convexity of the function $1/x$. This provides a simple, purely numerical, and analytic alternative to classical proofs based on the Euler–Maclaurin formula.

In addition, we introduced the constant δ, defined as the limit of a rational sequence arising from Simpson–type harmonic approximations. Combined with the decomposition formula established in equation (36), this yields a representation of γ in which the dominant alternating component is isolated and treated separately.

From a numerical perspective, this formulation explains the accelerated convergence of the associated rational sequence, which results from the systematic cancellation of leading error terms.

More broadly, the approach illustrates how elementary quadrature methods can shed new light on classical constants and suggests that similar ideas may be fruitfully applied to higher-order Newton–Cotes formulas [11].

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Data Availability Statement: The implementation of the Simpson-regularized harmonic sum $h(N)$ and the numerical evaluation of the constant δ are publicly available at https://github.com/kazumisawa/delta_from_Simpson_harmonic.

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Conflicts of Interest: The author declares no conflicts of interest.

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