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Article

Vacuum Energy with Natural Bounds: A Spectrally Bounded Effective Explanation

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Abstract

We propose an observationally motivated effective framework for the gravitationally relevant vacuum sector, in which only a spectrally bounded subset of quantum fluctuations contributes to the vacuum energy density. The construction is defined by two physically motivated scales: a short-wavelength ultraviolet bound associated with confinement-scale physics, and a long-wavelength infrared scale arising from thermodynamic and entropic structuring in the late-time universe. Within this bounded spectral domain, the vacuum energy density is governed by a characteristic geometric scale defined by the ultraviolet and infrared bounds, leading to a robust inverse fourth-power scaling that is largely insensitive to the detailed form of the spectral kernel. This effective description provides a structured interpretation of the observed smallness of vacuum energy in terms of spectral selection rather than ultraviolet cancellation. The model yields testable predictions at low redshift, including percent-level deviations in the equation of state and in the growth of structure, offering a falsifiable alternative to a purely phenomenological cosmological constant.

Keywords: vacuum energy; cosmological constant problem; dark energy; quantum vacuum; spectral cutoff; effective field theory; emergent gravity; coarse-graining; cosmology; QCD confinement

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1. Introduction

The accelerated expansion of the universe remains one of the central open problems of modern cosmology [22–24]. Within the standard ΛCDM model, this behavior is attributed to a cosmological constant. While phenomenologically successful, this interpretation leaves unresolved the large discrepancy between the observed vacuum density and naive quantum zero-point estimates [19,25,28].

A possible resolution is that gravity does not couple equally to all formally allowed vacuum modes. Instead, only a bounded and physically realizable subset of vacuum fluctuations may contribute to the effective gravitational vacuum sector. In this paper, we develop this viewpoint as an effective framework referred to as the Quantum Entropic Vacuum (QEV) model.

The central premise is that the observable vacuum response is controlled by two physical scales: a short-distance ultraviolet boundary associated with hadronic or confinement physics, and a large-scale infrared boundary associated with cosmological coherence. The resulting vacuum density is then governed not by the unrestricted zero-point spectrum, but by a bounded spectral domain.

This work does not claim a complete first-principles derivation of vacuum gravity. Rather, it proposes a physically motivated effective description with testable late-time cosmological consequences.

The paper is organized as follows. Section 2 introduces the bounded spectral framework. Section 3 derives the effective vacuum-density scaling. Section 4 discusses observational implications. Section 5 summarizes interpretation, limitations, and outlook.

2. QEV Framework and Physical Scope

The Quantum Entropic Vacuum (QEV) framework describes the gravitationally relevant vacuum sector as an effective, spectrally bounded domain of quantum fluctuations, characterized by physically motivated ultraviolet and infrared scales.

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We formulate the gravitationally relevant vacuum energy as an effective contribution arising from a restricted subset of quantum fluctuations. Rather than summing over the full spectrum of vacuum modes, we consider a bounded spectral domain defined by physically motivated ultraviolet and infrared scales.

This approach is motivated by the observation that the standard identification of vacuum energy with a sum over all modes leads to ultraviolet-divergent expressions [4], while cosmological observations indicate a finite and small vacuum energy density [23,29,30]. This tension suggests that only a limited subset of vacuum fluctuations may contribute effectively to the gravitational sector.

2.1. *Effective Spectral Representation*

We represent the vacuum energy density as an effective spectral integral over wavelength space,

$$\rho_{\mathrm{vac}} = \int_0^\infty f(\lambda)\, \rho(\lambda)\, d\lambda, \tag{1}$$

where $\rho(\lambda)$ denotes the spectral energy density per wavelength and $f(\lambda)$ is a smooth suppression function that selects the gravitationally relevant portion of the spectrum.

In contrast to conventional approaches, the function $f(\lambda)$ is not interpreted as a fundamental modification of quantum field theory, but as an effective description of the subset of modes that contribute coherently to gravitational dynamics. Similar spectral cutoff ideas have been considered in other contexts [16], although the present framework emphasizes a physically motivated ultraviolet–infrared structure.

To implement the bounded domain, we adopt a smooth suppression kernel of the form

$$f(\lambda) = \exp\left[-\left(\frac{\lambda_{\min}}{\lambda}\right)^{\alpha}\right] \exp\left[-\left(\frac{\lambda}{\lambda_{\max}}\right)^{\beta}\right], \tag{2}$$

with $\alpha, \beta > 0$, ensuring rapid suppression outside the interval $\lambda \in [\lambda_{\min}, \lambda_{\max}]$ while preserving analytical tractability.

For the spectral density, we use the dimensional form

$$\rho(\lambda) = A_0(\hbar c)\, \lambda^{-5}, \tag{3}$$

which corresponds to the standard scaling of vacuum energy contributions per wavelength interval.

2.2. *Characteristic Scale and Scaling Behavior*

The bounded spectral integral is dominated by contributions near a characteristic scale determined by the interplay between the ultraviolet and infrared bounds. This scale is given by the geometric mean

$$L = \sqrt{\lambda_{\min}\lambda_{\max}}. \tag{4}$$

Using asymptotic methods (e.g. Laplace approximation), one finds that the vacuum energy density exhibits a robust scaling behavior of the form

$$\rho_{\mathrm{vac}} \propto L^{-4}. \tag{5}$$

This result is largely insensitive to the detailed form of the kernel exponents (α, β), indicating that it reflects a structural property of the bounded spectral domain rather than a fine-tuned feature of the model. Similar robustness properties are known in other contexts involving spectral integrals and asymptotic approximations [21].

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2.3. Normalization and Effective Amplitude

The normalization constant A_0 encodes the amplitude of the spectral density. In the present framework, we distinguish between a microscopic amplitude and an effective late-time amplitude,

$$A_{0,\text{eff}} = A_0 e^{-\Xi}, \tag{6}$$

where Ξ represents an integrated attenuation factor summarizing the cumulative suppression of vacuum contributions from earlier cosmological epochs.

The present-day vacuum energy density can then be expressed as

$$\rho_{\text{vac}}(t_0) = A_{0,\text{eff}}(\hbar c)\, C\, L^{-4}, \tag{7}$$

where C is a dimensionless constant determined by the form of the spectral kernel.

We emphasize that Ξ is introduced here as an effective parameter rather than a dynamical quantity. Its role is to encode the empirical difference between microscopic energy scales and the observed vacuum energy density, without specifying the detailed mechanism responsible for this suppression. The parameter Ξ should not be interpreted as a freely adjustable quantity. Its magnitude is constrained by the observed vacuum energy density and is expected to be of order $\mathcal{O}(10^2)$ when expressed in logarithmic units. In this sense, Ξ represents an integrated historical suppression factor linking microscopic energy scales to the present-day effective vacuum energy, rather than a tunable parameter introduced to match observations.

2.4. Physical Interpretation of the Attenuation Factor

The attenuation parameter Ξ introduced in the effective amplitude

$$A_{0,\text{eff}} = A_0 e^{-\Xi} \tag{8}$$

should not be interpreted as a freely adjustable parameter, but as a cumulative measure of physical suppression processes acting over cosmic history.

Rather than representing a single mechanism, Ξ summarizes the integrated effect of multiple, physically distinct contributions that limit the gravitationally coherent vacuum sector. Conceptually, it can be decomposed as

$$\Xi = \Xi_G + \Xi_T + \Xi_E, \tag{9}$$

where each term represents a class of suppression effects.

The term Ξ_G encodes gravitational and cosmological effects associated with the expansion of the universe. As spacetime evolves, causal horizons and dynamical dilution reduce the coherence of vacuum modes on large scales, effectively limiting their contribution to the gravitational sector.

The term Ξ_T represents thermodynamic and entropic suppression. As the universe cools, finite correlation lengths emerge, and high-frequency or incoherent fluctuations become increasingly irrelevant for macroscopic gravitational dynamics [6,15].

The term Ξ_E reflects emergent or coarse-grained aspects of gravity. In approaches where gravitational dynamics arise from collective or entanglement-based degrees of freedom, only a restricted subset of microscopic fluctuations contributes effectively to the macroscopic vacuum energy [11,12].

Importantly, the detailed partitioning of Ξ into these contributions is not required for the present effective description. What matters is that Ξ represents a cumulative, history-dependent suppression factor arising from physically motivated processes.

In this sense, the smallness of the observed vacuum energy does not result from a fine-tuned cancellation of large contributions, but from the integrated effect of multiple suppression mechanisms acting across different physical regimes. The parameter Ξ therefore encodes a logarithmic measure of this cumulative attenuation, rather than a free parameter adjusted to match observations.

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This interpretation aligns with the broader perspective that the cosmological constant problem may be understood as a problem of effective spectral selection rather than ultraviolet divergence [16,18,25].

2.5. Interpretation of the Effective QEV Framework

The spectral formulation presented above should be understood as an effective representation of the gravitationally relevant vacuum sector. It does not modify the underlying structure of quantum field theory, but instead introduces a projection onto the subset of modes that contribute coherently at macroscopic scales.

In this sense, the function $f(\lambda)$ acts as a selection function, encoding the empirically motivated restriction of the vacuum spectrum. The bounded spectral domain is therefore not a fundamental property imposed at the level of microscopic theory, but an effective description inferred from observational consistency.

From this perspective, the vacuum energy density is not interpreted as the sum over all quantum modes, but as an emergent quantity associated with a restricted spectral domain. This viewpoint is consistent with approaches that relate vacuum energy to coarse-grained, entropic, or emergent gravitational degrees of freedom [11,12].

The framework developed here provides a minimal realization of this idea, linking the effective vacuum energy scale to physically motivated ultraviolet and infrared bounds while remaining compatible with standard cosmological observations.

3. Physical Origin of the Spectral Bounds and Attenuation

The effective framework introduced in the previous section is based on three central ingredients: an ultraviolet boundary λ_{UV}, an infrared boundary $\lambda_{\mathrm{IR}}(t)$, and an attenuation factor $\Xi(t)$. In the present work, these quantities are not introduced as arbitrary fitting parameters, but are associated with physically motivated processes connected to strong-interaction physics, cosmological coherence, and cumulative cosmic evolution.

The purpose of this section is not to claim a complete first-principles derivation, but to demonstrate that each ingredient can be linked to known physical regimes in a consistent and non-arbitrary way.

3.1. Ultraviolet Boundary from Hadronic Spectral Reorganization

3.1.1. Confinement as a Reorganization of the Physical Spectrum

In quantum chromodynamics (QCD), the ultraviolet sector of the formal field description is not identical to the spectrum of physically realized late-time states. At sufficiently high energies or short distances, quarks and gluons appear as the relevant microscopic degrees of freedom, while at lower energies the observable spectrum reorganizes into confined hadronic bound states.

This transition is qualitative rather than merely quantitative. The physically realized excitations of the late-time universe are predominantly baryons, nuclei, and composite hadronic matter rather than free colored partons. The confinement scale therefore marks a natural boundary between two physical regimes:

- a microscopic partonic regime described by quark and gluon fields,
- a macroscopic hadronic regime containing stable matter.

Within the present framework, this distinction is central because the gravitationally relevant vacuum sector is associated with the physically realized matter content of the late universe.

3.1.2. Hadronic Matter as the Anchor of the Late-Time Gravitational Sector

The observable matter content of the present universe is built from hadronic structures. Protons and neutrons form atomic nuclei, which support atoms, molecules, stars, planets, and all known baryonic macroscopic systems.

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Although electromagnetic radiation is mediated by photons, its ordinary astrophysical and laboratory sources depend on stable charged matter, atomic transitions, plasma processes, and nuclear structure. Likewise, the dominant localized gravitational sources in the visible universe are organized through hadronic rest mass.

For this reason, the present framework does not treat the gravitational vacuum sector as an abstract sum over all formal quantum modes, but as the vacuum sector coupled to the physically realized late-time matter structure.

3.1.3. Effective Ultraviolet Boundary

The claim of the present model is *not* that quantum fields cease to exist above the confinement scale. Rather, the claim is that the sector relevant to stable late-time matter and its gravitational organization does not extend unchanged into the ultraviolet partonic regime.

Modes associated with scales beyond hadronic organization may exist formally in the microscopic theory, but they need not contribute coherently to the emergent vacuum sector relevant for macroscopic gravity. In this sense, confinement acts as a spectral reorganization that limits which ultraviolet modes remain physically relevant after hadron formation.

We therefore interpret the effective ultraviolet boundary as a realizability boundary rather than a hard fundamental cutoff:

$$k_{\mathrm{UV}} \sim \Lambda_{\mathrm{QCD}}, \tag{10}$$

or equivalently

$$\lambda_{\mathrm{UV}} \sim \frac{1}{\Lambda_{\mathrm{QCD}}} \sim 1\,\mathrm{fm}, \tag{11}$$

up to factors of order unity.

3.2. Infrared Boundary from Cosmological and Thermodynamic Coherence

3.2.1. Physical Motivation

In conventional quantum field theory, vacuum modes with arbitrarily long wavelengths are formally present in the spectrum. However, the formal existence of such modes does not imply that they contribute coherently to the late-time gravitational vacuum sector.

The observable universe possesses a finite causal structure, undergoes continuous expansion, and is characterized by a finite thermodynamic temperature. These properties naturally restrict the range of long-wavelength modes that can remain physically coherent on cosmological scales.

Within the present framework, the infrared boundary is therefore interpreted as an effective coherence limit rather than a fundamental cutoff.

3.2.2. Cosmological Horizon Scale

A first natural infrared scale is provided by the causal horizon of the expanding universe,

$$\lambda_H(t) = \frac{c}{H(t)}, \tag{12}$$

where $H(t)$ is the Hubble parameter.

The associated infrared momentum scale is

$$k_{\mathrm{IR},H}(t) = \frac{H(t)}{c}. \tag{13}$$

3.2.3. Thermodynamic Coherence Scale

A second natural infrared scale arises from finite-temperature coherence,

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$$\lambda_T(t) = \frac{\hbar c}{k_B T_{\text{eff}}(t)}, \tag{14}$$

where $T_{\text{eff}}(t)$ is an effective cosmological temperature scale.
The corresponding momentum scale is

$$k_{\text{IR},T}(t) = \frac{k_B T_{\text{eff}}(t)}{\hbar c}. \tag{15}$$

3.2.4. Effective Infrared Boundary

The physically relevant infrared limit is taken as the most restrictive of the two coherence scales:

$$\lambda_{\text{IR}}(t) = \min\left[\frac{c}{H(t)}, \frac{\hbar c}{k_B T_{\text{eff}}(t)}\right]. \tag{16}$$

Equivalently,

$$k_{\text{IR}}(t) = \max\left[\frac{H(t)}{c}, \frac{k_B T_{\text{eff}}(t)}{\hbar c}\right]. \tag{17}$$

Only modes satisfying

$$k_{\text{IR}}(t) \lesssim k \lesssim k_{\text{UV}} \tag{18}$$

are assumed to contribute coherently to the gravitationally relevant vacuum sector.

3.2.5. Present-Epoch Limit

At the present epoch,

$$\frac{H_0}{c} \ll \frac{k_B T_{\text{CMB}}}{\hbar c}, \tag{19}$$

so the thermal scale dominates, implying

$$\lambda_{\text{IR}}(t_0) \approx \frac{\hbar c}{k_B T_{\text{CMB}}} \sim 10^{-3}\,\text{m}. \tag{20}$$

Thus, the present infrared boundary naturally lies near the millimeter regime.

3.3. Attenuation Factor as Effective Cumulative Cosmological Suppression

3.3.1. Status of the Parameter

Within the present framework, the attenuation factor Ξ is not interpreted as a fundamentally derived microscopic constant. Instead, it is introduced as an effective quantity summarizing the cumulative influence of processes that reduce the coherent contribution of the vacuum sector during cosmic evolution.

3.3.2. Effective Amplitude

We write

$$A_{\text{eff}}(t) = A_0 e^{-\Xi(t)}, \tag{21}$$

where A_0 is the microscopic reference amplitude.
The corresponding vacuum energy density becomes

$$\rho_{\text{vac}}(t) = A_0 e^{-\Xi(t)} C\, L(t)^{-4}, \tag{22}$$

with

$$L(t) = \sqrt{\lambda_{\mathrm{UV}}\lambda_{\mathrm{IR}}(t)}. \tag{23}$$

3.3.3. Why an Exponential Form?

If the amplitude changes through many small suppression steps according to

$$\frac{dA}{A} = -\Gamma(t)\,dt, \tag{24}$$

then integration yields

$$A_{\mathrm{eff}}(t) = A_0 \exp\left[-\int_{t_i}^{t} \Gamma(t')dt'\right]. \tag{25}$$

This motivates

$$\Xi(t) = \int_{t_i}^{t} \Gamma(t')dt'. \tag{26}$$

3.3.4. Minimal Cosmological Approximation

As a first approximation one may assume

$$\Gamma(t) = \alpha H(t), \tag{27}$$

which gives

$$\Xi(t) = \alpha \ln\left(\frac{a(t)}{a_i}\right), \tag{28}$$

and therefore

$$e^{-\Xi(t)} = \left(\frac{a_i}{a(t)}\right)^{\alpha}. \tag{29}$$

3.3.5. Interpretation

The small present vacuum energy density is attributed not to fine-tuned cancellation, but to two sequential mechanisms:

1. spectral restriction through ultraviolet and infrared boundaries,
2. cumulative historical suppression encoded in $\Xi(t)$.

Together these effects yield a naturally reduced late-time effective vacuum contribution.

3.3.6. Scope and Limitation

The present treatment remains an effective parametrization rather than a complete first-principles derivation. A deeper theory would require deriving $\Gamma(t)$ from microscopic vacuum dynamics, nonequilibrium quantum field theory, horizon thermodynamics, or emergent gravity.

Nevertheless,

$$\Xi(t) = \int \Gamma(t)\,dt \tag{30}$$

provides a concrete and physically meaningful basis for the attenuation factor used in the present framework.

4. Results and Discussion

The QEV framework developed in this work should be understood as an effective description of the gravitationally relevant vacuum sector, capturing structural features of vacuum energy without

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relying on a detailed microscopic model. The baseline formulation adopted here is the propagation-only scenario, in which no explicit energy exchange, dissipation, or decay of vacuum energy is assumed at late times.

4.1. Propagation-Only Interpretation

In the baseline Quantum Entropic Vacuum (QEV) realization, thermal physics does not enter through explicit damping of the vacuum energy density. In particular, the thermal attenuation channel is set to zero,

$$\Xi_{\rm th} \equiv 0, \tag{31}$$

and no microphysical interaction between matter, radiation, and the vacuum is introduced.

Instead, temperature enters solely through the effective infrared scale,

$$\lambda_{\max}(t) \simeq \frac{hc}{k_B T_{\rm eff}(t)}, \tag{32}$$

which determines the largest vacuum modes contributing coherently to the gravitational sector. This formulation ensures that thermal effects are encoded only through the propagation of the spectral window, avoiding double counting.

Within this interpretation, the vacuum does not undergo a physical phase transition, nor does it become thermal. Terms such as "freeze-out" or "stabilization" are therefore to be understood in an effective sense: as the universe expands and cools, the evolution of $\lambda_{\max}(t)$ slows, leading to an approximately constant geometric scale

$$L(t) = \sqrt{\lambda_{\min}\lambda_{\max}(t)}. \tag{33}$$

As a result, the vacuum energy density

$$\rho_{\rm vac} \propto L^{-4} \tag{34}$$

approaches a constant value, and the equation-of-state parameter converges to

$$w \to -1, \tag{35}$$

consistent with cosmological observations [23,29,30].

As shown in Figure 1. the deviation from $w = -1$ is maximal at intermediate epochs and decreases toward low redshift, reflecting the gradual stabilization of the effective spectral scale. This deviation is illustrated in Figure 2.

4.2. Early-to-Late-Time Evolution

At early times, the bounded spectral structure is dominated by the ultraviolet scale $\lambda_{\min}$, which is associated with confinement physics and becomes fixed after the QCD transition [15,26]. As the universe cools, the infrared scale continues to evolve, reflecting the thermodynamic and cosmological conditions.

In the late-time regime relevant for observations ($z \lesssim 1$), the evolution of $\lambda_{\max}(t)$ becomes slow, and the spectral window effectively saturates. The resulting vacuum energy density remains nearly constant, with any residual evolution directly traceable to the mild time dependence of the infrared scale.

This behavior provides a natural explanation for the observed stability of the cosmological constant without invoking a fundamental constant term or fine-tuned cancellation.

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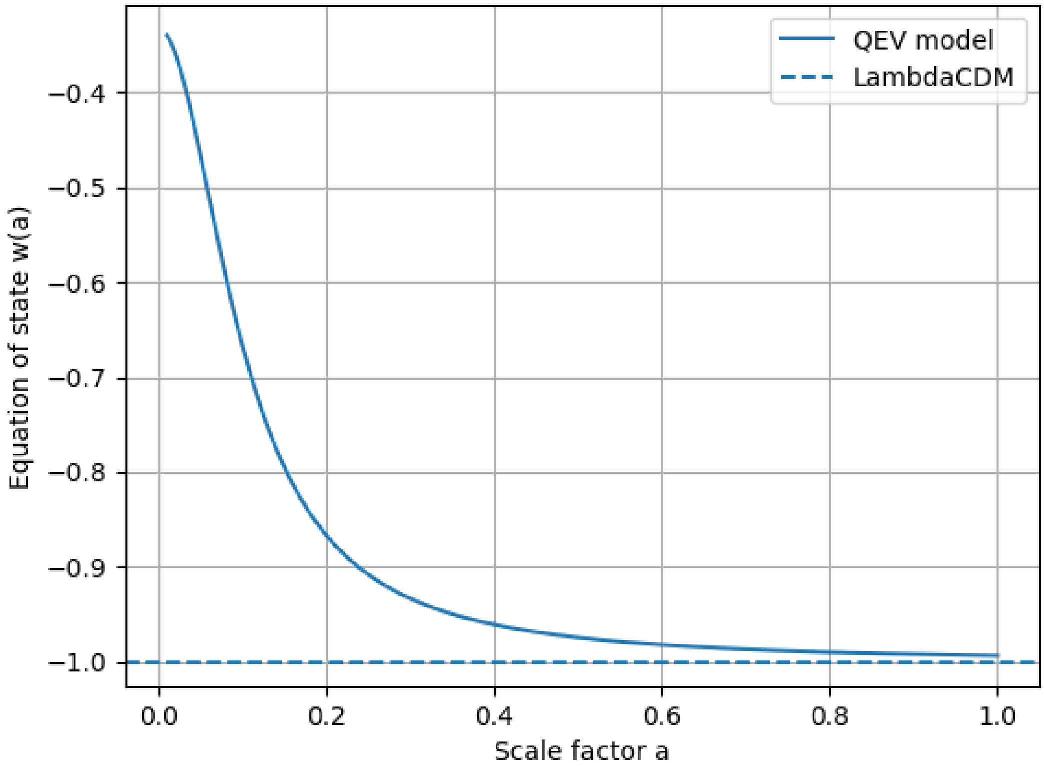

Figure 1. Evolution of the effective equation-of-state parameter $w(a)$ in the spectrally bounded vacuum QEV framework (solid line), compared to the constant ΛCDM value $w = -1$ (dashed line). The model shows a small deviation from $w = -1$ at intermediate epochs, followed by convergence to a cosmological-constant-like behavior at late times as the effective spectral scale stabilizes.

4.3. Falsifiability and Near-Term Tests

A key feature of the present QEV framework is that it leads to concrete, testable predictions at low redshift.

Equation of state.

The effective equation-of-state parameter is given by

$$w(a) = -1 + \frac{4}{3}\frac{d\ln L}{d\ln a}, \tag{36}$$

and may exhibit small deviations from $w = -1$ during the transition to the late-time regime. In the baseline scenario, these deviations are expected to be at the level

$$|w+1| \lesssim 10^{-2}, \tag{37}$$

within the reach of precision cosmological measurements.

Growth of structure.

The modified background evolution leads to percent-level deviations in the growth-rate observable $f\sigma_8(z)$ relative to ΛCDM. This effect is illustrated in Figure 2. Current redshift-space distortion data already probe this regime, and upcoming surveys are expected to improve sensitivity [29,30].

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BAO evolution.

The slow evolution of the infrared scale introduces a mild modification in the propagation of the baryon acoustic oscillation (BAO) scale. This may manifest as a small shift in the inferred BAO ruler at low redshift, potentially detectable by high-precision surveys such as Euclid and DESI [5,29].

Observational prospects.

Future experiments, including DESI, Euclid, LSST, and the Simons Observatory, are expected to reach the precision required to test these deviations [1–3,5,29,30].

As shown in Figure 2, the deviation from the ΛCDM prediction is most pronounced at low redshift ($z \lesssim 1$), where the slow evolution of the infrared scale leads to percent-level modifications in structure growth.

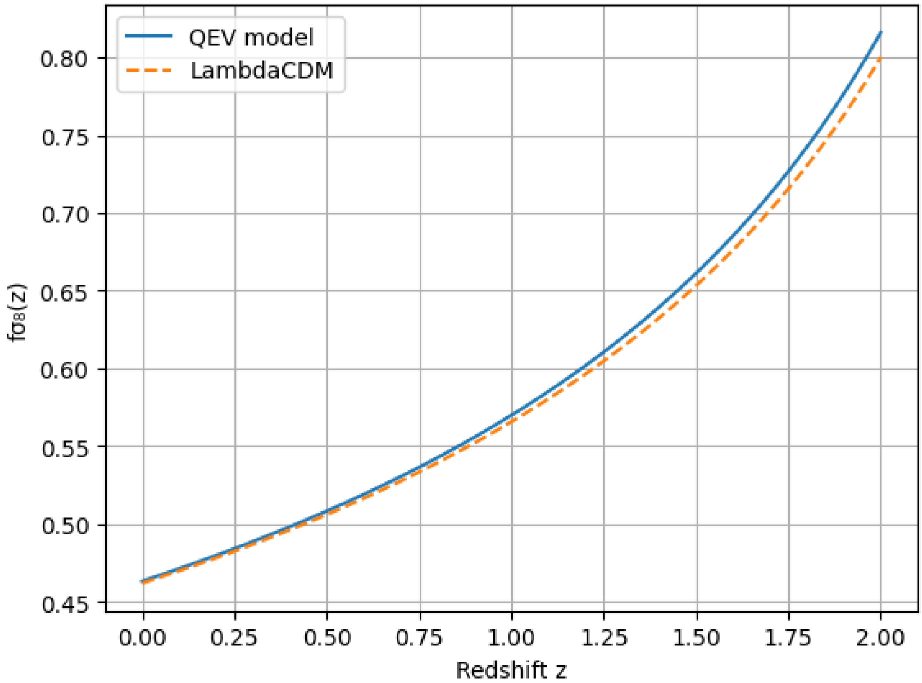

Figure 2. Comparison of the growth-rate observable $f\sigma_8(z)$ between the spectrally bounded vacuum model (solid line) and the standard ΛCDM prediction (dashed line). The QEV framework predicts percent-level deviations at low redshift, providing a potential observational signature accessible to current and upcoming surveys.

4.4. Interpretational Scope

The present QEV framework is restricted to the gravitationally relevant vacuum sector. Laboratory experiments probing electromagnetic vacuum fluctuations, such as Casimir or cavity measurements, access a distinct photonic sector and should not be interpreted as direct tests of the hadronically motivated spectral bounds considered here [9,10].

The time dependence of the vacuum energy density arises entirely from the evolution of the spectral window and should be understood as an effective or kinematic feature rather than a dynamical process. No physical energy exchange or dissipation is assumed.

A complete understanding of the infrared scale and the attenuation parameter Ξ remains an open question. These may ultimately require input from quantum gravity, nonequilibrium thermodynamics, or emergent spacetime QEV frameworks [11,12].

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4.5. Summary

The propagation-only realization of the spectrally bounded vacuum QEV framework provides a minimal, observationally consistent description of late-time vacuum dynamics. It reproduces the observed near-constancy of vacuum energy while predicting small, testable deviations from ΛCDM.

Within this interpretation, the cosmological constant is not a fundamental parameter, but an emergent quantity associated with a bounded spectral domain. This shifts the focus from fine-tuning to identifying the physical origin of spectral selection, providing a concrete direction for future theoretical and observational investigation.

5. Discussion, Consistency, and Broader Context

The Quantum Entropic Vacuum (QEV) framework developed in this work is intended as a late-time effective description of the gravitationally relevant vacuum sector. It does not claim to replace the microscopic structure of quantum field theory, nor does it provide a complete first-principles derivation of vacuum energy. Its purpose is more modest: to identify a minimal spectral structure that is physically motivated, observationally viable, and capable of reproducing the small positive vacuum energy density inferred from cosmology.

In this perspective, the cosmological constant problem is reformulated. Rather than asking why a formally divergent ultraviolet vacuum contribution almost exactly cancels to an extremely small residual value, the present framework asks which subset of formally available quantum fluctuations remains physically relevant for large-scale gravity. The emphasis is therefore shifted from ultraviolet cancellation to spectral participation.

5.1. Consistency with Quantum Field Theory

The QEV framework does not modify local particle physics, gauge interactions, or the renormalized formalism of standard quantum field theory. The full formal spectrum of vacuum fluctuations may still exist at the microscopic level.

What changes is the interpretation of gravitational relevance. The bounded spectral window introduced in Sec. 2 should not be understood as a hard cutoff imposed on fundamental theory, but as an effective weighting of those modes that contribute coherently to the macroscopic gravitational sector.

This distinction is important. In non-gravitational physics, only energy differences are usually measurable, whereas gravity is sensitive to the background energy content of spacetime. It is therefore plausible that the gravitational vacuum sector may differ from naive flat-space mode-counting arguments while remaining fully compatible with established quantum theory [4].

5.2. Physical Meaning of the Three Ingredients

The framework is organized around three linked ingredients.

Ultraviolet boundary.

The ultraviolet scale is associated with confinement-scale hadronic physics. At short distances, QCD is described in terms of quarks and gluons, whereas the late-time observable universe is built from confined hadronic states such as protons, neutrons, and nuclei.

In the present interpretation, this transition acts as a natural reorganization scale. Modes deep in the partonic regime are not assumed to disappear, but they need not contribute coherently to the emergent gravitational vacuum sector relevant for late-time cosmology.

Infrared boundary.

The infrared scale reflects the largest wavelengths over which vacuum fluctuations remain cosmologically coherent. It is linked to horizon structure, finite temperature, and the gradual thermodynamic organization of the universe.

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At late times, the thermal branch dominates and naturally leads to an infrared scale in the millimeter regime, as discussed in Sec. 3. This late-time stabilization is central to the emergence of an approximately constant vacuum density.

Attenuation factor.

The attenuation factor $\Xi(t)$ summarizes cumulative suppression processes acting over cosmic history. These may include expansion, entropy growth, decoherence, phase transitions, or other coarse-graining effects.

In this sense, the observed hierarchy between microscopic scales and the present vacuum density is interpreted not as miraculous cancellation, but as the integrated result of many small suppressive processes over time.

5.3. Lorentz Symmetry and Effective Cosmological Frame

Because the spectral bounds evolve with cosmic time, one may ask whether the framework conflicts with Lorentz invariance.

At the microscopic level, no such violation is required. The underlying laws may remain Lorentz covariant. However, cosmology already defines an effective preferred frame through the Friedmann–Lemaître background and the cosmic microwave background rest frame.

Quantities such as the Hubble scale, cosmological temperature, and horizon size are naturally tied to this evolving background. The ultraviolet and infrared bounds should therefore be interpreted as emergent cosmological properties rather than as violations of fundamental symmetry.

5.4. Relation to Laboratory Vacuum Effects

It is useful to distinguish the gravitational vacuum sector considered here from laboratory manifestations of vacuum fluctuations such as Casimir measurements or cavity experiments.

Such experiments probe electromagnetic vacuum modes under controlled material boundary conditions. They access a photonic sector that is conceptually distinct from the hadronically motivated gravitational vacuum sector emphasized in the present framework.

Accordingly, the absence of a direct numerical relation between Casimir-scale energies and cosmological vacuum energy is not in conflict with the QEV model.

5.5. Observational Interpretation

A central strength of the framework is that it remains falsifiable.

Because the infrared scale evolves slowly with cosmic time, the effective vacuum density need not remain exactly constant. This generically allows small late-time deviations from the cosmological-constant limit $w = -1$.

As illustrated in Figure 1, the effective equation-of-state parameter can deviate mildly from $w = -1$ at intermediate epochs and gradually converge back toward the cosmological-constant value at late times. This behavior reflects the progressive stabilization of the spectral window.

Likewise, Figure 2 shows that the modified background evolution can induce percent-level changes in the growth observable $f\sigma_8(z)$ relative to ΛCDM. The largest effects occur at low redshift, where residual infrared evolution is most relevant.

Figure 3 presents an explicit low-redshift realization of the model developed in Appendix F. It illustrates that the QEV framework may remain observationally close to ΛCDM while still allowing measurable departures from a strictly constant vacuum term.

These signatures place the framework within reach of current and upcoming surveys such as DESI, Euclid, LSST, Roman, and related probes.

5.6. Strengths and Present Limitations

The principal strengths of the framework are:

- physically motivated ultraviolet and infrared scales,

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- a natural route toward small late-time vacuum energy without explicit fine-tuning,
- compatibility with standard local quantum field theory,
- direct phenomenological testability.

At the same time, several limitations remain:

- the framework is effective rather than fundamental,
- the detailed spectral kernel is representative rather than microscopically derived,
- the suppression rate entering $\Xi(t)$ remains phenomenological,
- the relation to quantum gravity or emergent spacetime is open.

These limitations should be viewed as directions for future development rather than as inconsistencies.

6. Final Conclusions and Outlook

We have presented a spectrally bounded effective description of vacuum energy in which the gravitationally relevant vacuum sector is associated not with the full formal quantum spectrum, but with a restricted subset of fluctuations selected by physically motivated bounds.

The framework is governed by three linked ingredients:

1. an ultraviolet boundary connected to confinement-scale hadronic physics,
2. an infrared boundary connected to cosmological and thermodynamic coherence,
3. a cumulative attenuation factor encoding the integrated suppressive history of the universe.

Together these ingredients generate a characteristic scale determined by the interplay of ultraviolet and infrared physics. This naturally leads to a strongly reduced late-time vacuum density with approximate convergence toward the observationally favored limit

$$w \to -1. \tag{38}$$

In this interpretation, the cosmological constant is not introduced as a bare fundamental parameter. Rather, it emerges as the quasi-static late-time limit of a bounded vacuum structure shaped by physical selection processes over cosmic history.

The retained figures summarize the phenomenology of the framework:

- Figure 1 shows convergence of the effective equation of state toward $w = -1$,
- Figure 2 shows small but testable deviations in cosmic structure growth,
- Figure 3 shows an explicit low-redshift realization consistent with current observational bounds.

The present model remains explicitly phenomenological and does not yet derive the spectral bounds or attenuation factor from first principles. Nevertheless, it provides a coherent bridge between microscopic physical scales and observable cosmological behavior.

Future progress should focus on:

- deriving the spectral kernel from microscopic theory,
- obtaining a dynamical model for $\Xi(t)$,
- confronting the framework with precision low-redshift data,
- exploring links with nonequilibrium thermodynamics,
- investigating possible embedding within emergent-gravity or quantum-gravity scenarios.

Whether or not the present realization proves complete, it demonstrates that the observed smallness of vacuum energy may be approached through bounded spectral participation and cumulative suppression, rather than through extreme ultraviolet cancellation alone.

A forthcoming overview article will place the present QEV results in the broader context of cosmology, gravitational response, and related applications.

Comparative schematic of vacuum-energy models.

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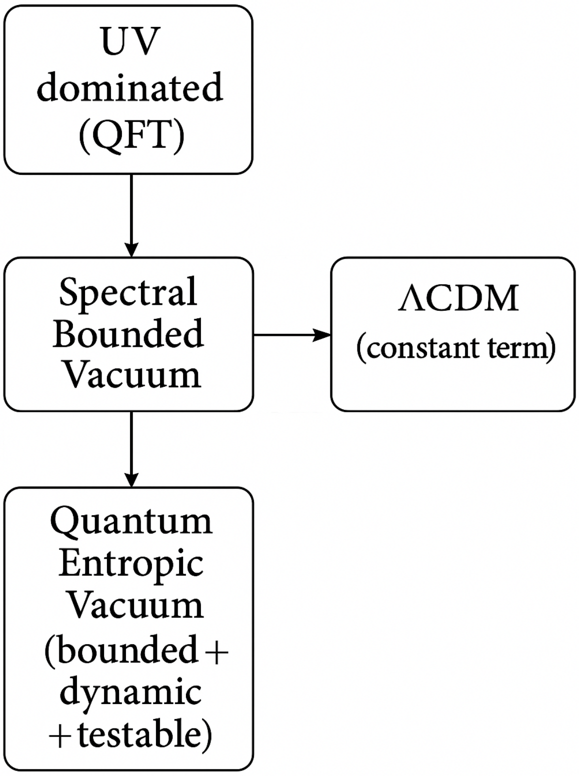
Table 1. Conceptual comparison of the QEV framework with representative approaches to vacuum energy.

Aspect	**Conventional ΛCDM / Λ term**	**Spectral Bounded Vacuum + QEV (this work)**
Physical basis	Phenomenological constant Λ without microphysical linkage.	Vacuum energy from a bounded quantum spectrum with natural UV/IR limits.
Naturalness problem	Large hierarchy $\sim 10^{120}$ between QFT and cosmological scales.	Bridged dynamically via integrated damping $\Xi \simeq 101$; no fine-tuning.
Time dependence	Strictly constant ρ_Λ.	Mild late-time evolution, $\lvert w+1\rvert \lesssim 10^{-2}$.
Laboratory connection	None (purely gravitational).	Indirectly testable through cosmological observables; photonic realizations probe a distinct sector and are not direct tests of the gravitational vacuum contribution.
Free parameters	Ω_Λ phenomenological.	$\{\lambda_{\min}, T_{\mathrm{IR}}, \Xi\}$ with physical interpretation.
Predictive falsifiability	Indirect only (cosmological fits).	Direct (mm-band null test + cosmology).



The figure illustrates the conceptual hierarchy among four representative approaches: the UV-dominated quantum field theoretic (QFT) vacuum, the *Spectral Bounded Vacuum* (SBV) introducing natural UV/IR limits, the *Quantum Entropic Vacuum* (QEV) as a bounded, dynamic and testable extension, and the conventional ΛCDM model treating ρ_Λ as a constant term. Arrows indicate theoretical

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progression and conceptual refinement, with QEV providing a consistent effective description that remains falsifiable in both cosmological and laboratory contexts.

7. Positioning of the QEV Research Framework

The present study should be viewed as the foundational layer of a broader QEV research program. Its primary purpose has been to formulate the central physical premise that gravity may couple not to the full formal vacuum sector, but only to a bounded and physically relevant subset of vacuum fluctuations. Within this perspective, the observed gravitational vacuum density emerges as an effective quantity shaped by natural ultraviolet and infrared response scales.

This foundational viewpoint motivates several companion developments that explore the consequences of the same principle at different levels of description.

First, a covariant extension investigates how bounded vacuum response may be formulated through relativistically consistent spectral operators, providing a field-theoretic route to dark-energy phenomenology.

Second, an effective cosmology study examines the homogeneous late-time limit of the framework, where near-ΛCDM expansion with small testable deviations can arise as the lowest-order observational sector.

Third, exploratory applications consider whether the same bounded-vacuum logic may also be relevant for galactic dynamics, cosmic expansion diagnostics, and regular black-hole interior models.

The QEV framework should therefore not be interpreted as a completed fundamental theory, nor as a collection of disconnected hypotheses. Rather, it is an evolving effective research program progressing from conceptual foundation, to covariant formulation, to observable cosmology, and finally to broader phenomenological applications.

Future work will determine which elements of this program remain viable under increasingly stringent consistency requirements, numerical tests, and observational constraints.

Appendix A. Spectral Scaling and Robustness

The bounded spectral formulation introduced in Sec. 2 leads to a vacuum energy density of the form

$$\rho_{\text{vac}} = \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} f(\lambda)\, \rho(\lambda)\, d\lambda, \tag{A1}$$

with $\rho(\lambda) \propto \lambda^{-5}$ and a smooth suppression kernel $f(\lambda)$.

The dominant contribution to this integral arises from the interplay between the ultraviolet and infrared bounds. Using asymptotic methods, the integral is maximized near the geometric mean

$$L = \sqrt{\lambda_{\text{min}}\lambda_{\text{max}}}, \tag{A2}$$

which defines the characteristic scale of the vacuum energy density.

This leads to the scaling behavior

$$\rho_{\text{vac}} \propto L^{-4}, \tag{A3}$$

as discussed in Sec. 2. This result is largely insensitive to the detailed form of the kernel, reflecting a structural property of the bounded spectral domain.

To demonstrate this robustness, one may consider an alternative logarithmic weighting,

$$f_{\log}(\lambda) \propto \frac{1}{\lambda}, \tag{A4}$$

which yields the same dominant scaling in the regime $\lambda_{\text{max}} \gg \lambda_{\text{min}}$. The kernel therefore acts primarily as a smooth implementation of the spectral bounds, rather than determining the scaling itself.

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Appendix B. Effective Attenuation and Amplitude Reduction

The effective amplitude introduced in Sec. 2 is given by

$$A_{0,\mathrm{eff}} = A_0 e^{-\Xi}, \tag{A5}$$

where Ξ encodes the cumulative attenuation of the gravitationally relevant vacuum contribution over cosmic history (Sec. 4).

The present-day vacuum energy density can then be written as

$$\rho_{\mathrm{vac}}(t_0) = A_{0,\mathrm{eff}}(\hbar c)\, C\, L^{-4}. \tag{A6}$$

The parameter Ξ should be interpreted as an effective quantity summarizing the hierarchy between microscopic energy scales and the observed vacuum energy density. It does not imply a specific dynamical process, but provides a compact representation of the integrated suppression discussed in Sec. 4.

Decomposing

$$\Xi = \Xi_g + \Xi_{\mathrm{th}} + \Xi_{\mathrm{had}}, \tag{A7}$$

allows for a physically motivated classification of contributions associated with gravitational expansion, thermodynamic evolution, and hadronic physics, respectively.

Appendix C. Effective Fluid Formulation

The time evolution of the vacuum energy density can be expressed in terms of an effective fluid description, consistent with the Friedmann equations.

Starting from

$$\rho_{\mathrm{vac}} \propto L^{-4}, \tag{A8}$$

and using $L(a) = \sqrt{\lambda_{\mathrm{min}}\lambda_{\mathrm{max}}(a)}$, one obtains

$$\frac{d\ln\rho_{\mathrm{vac}}}{d\ln a} = -4\frac{d\ln L}{d\ln a}. \tag{A9}$$

Identifying the effective equation-of-state parameter through

$$\frac{d\ln\rho}{d\ln a} = -3(1+w), \tag{A10}$$

yields

$$w(a) = -1 + \frac{4}{3}\frac{d\ln L}{d\ln a}, \tag{A11}$$

as used in Sec. 5.

This formulation ensures consistency with the standard cosmological continuity equation and demonstrates that the QEV framework can be embedded in the usual Friedmann–Robertson–Walker description without modification.

Appendix D. Thermal Implementation and Consistency

As discussed in Sec. 5, thermal effects can be incorporated in different ways. To avoid ambiguity, we define a consistent baseline implementation.

In the propagation-only realization, thermal effects enter exclusively through the infrared scale,

$$\lambda_{\mathrm{max}}(a) = \frac{hc}{k_B T_{\mathrm{eff}}(a)}, \tag{A12}$$

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while explicit thermal damping is set to zero,

$$\Xi_{\rm th}(a) \equiv 0. \tag{A13}$$

Alternatively, one may consider a source-based implementation in which $\lambda_{\max}$ is fixed and thermal effects enter through attenuation,

$$\rho_{\rm vac}(a) \propto e^{-\Xi_{\rm th}(a)}. \tag{A14}$$

To maintain physical consistency, these implementations must not be combined:

$$\frac{d\ln\lambda_{\max}}{d\ln a} \neq 0 \;\Rightarrow\; \Xi_{\rm th} = 0, \qquad \Xi_{\rm th} \neq 0 \;\Rightarrow\; \lambda_{\max} = \text{const}. \tag{A15}$$

In the late-time regime, the evolution of $\lambda_{\max}$ slows, leading to stabilization of the scale $L(a)$ and convergence toward

$$w \to -1, \tag{A16}$$

in agreement with observations (Sec. 5).

Appendix E. Time-Integrated Suppression and Entropic Interpretation

In the Quantum Entropic Vacuum (QEV) framework, the attenuation parameter Ξ is interpreted as a cumulative measure of suppression processes acting over cosmic history.

Appendix E.1. Time-Integrated Suppression

We model Ξ as the integral of an effective suppression rate $\Gamma_{\rm eff}(t)$,

$$\Xi = \int_{t_i}^{t_0} \Gamma_{\rm eff}(t)\, dt, \tag{A17}$$

where t_i denotes an early cosmological epoch (e.g. after relevant phase transitions) and t_0 the present time.

The effective rate summarizes the combined influence of cosmological expansion, thermodynamic evolution, and emergent coarse-graining effects,

$$\Gamma_{\rm eff}(t) = \Gamma_G(t) + \Gamma_T(t) + \Gamma_E(t), \tag{A18}$$

without requiring a detailed microscopic decomposition within the present effective description.

Appendix E.2. Entropic Interpretation

A physically motivated interpretation arises by relating suppression to entropy production. As the universe evolves, increasing entropy is associated with decoherence and loss of microscopic phase information. This suggests

$$\Gamma_{\rm eff}(t) \propto \frac{dS}{dt}, \tag{A19}$$

where $S(t)$ is the entropy of the cosmological state.

Under this assumption, the attenuation parameter becomes

$$\Xi \propto \int_{t_i}^{t_0} \frac{dS}{dt}\, dt = \Delta S, \tag{A20}$$

i.e. proportional to the total entropy increase over cosmic history.

This relation should be understood as a physically motivated hypothesis within the effective framework.

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Appendix E.3. Interpretational Role

Within this formulation, Ξ provides a compact representation of cumulative decoherence and coarse-graining effects. The large hierarchy between microscopic vacuum energy scales and the observed cosmological value can then be interpreted as the result of many small suppression processes integrated over time.

This perspective supports the interpretation developed in Sec. 2 and Sec. 4, in which the cosmological constant emerges from spectral selection rather than ultraviolet divergence.

Appendix F. Explicit Low-Redshift Realization of the Equation of State

To illustrate the observational implications of the QEV framework in a concrete setting, we present an explicit low-redshift realization of the effective equation-of-state parameter $w(a)$. This construction complements the general discussion in Sec. 4 by specifying a phenomenological form for the evolution of the infrared scale and linking it to the redshift range probed by current cosmological observations.

Appendix F.1. Parametrization of the Infrared Scale

We model the infrared scale as

$$\lambda_{\max}(a) = \lambda_{\max,0}\, a^{\nu(a)}, \tag{A21}$$

where $\lambda_{\max,0}$ is the present-day value and $\nu(a)$ is an effective index describing the residual evolution of the spectral window.

To capture the late-time stabilization discussed in Sec. 4, we choose

$$\nu(a) = \nu_0(1-a)^m, \tag{A22}$$

with $\nu_0 > 0$ and $m \geq 1$. This form ensures that the evolution is maximal at intermediate redshift and vanishes smoothly as $a \to 1$, consistent with the thermodynamic stabilization of large-scale coherence discussed in Sec. 4 and Appendix D [6,15].

Appendix F.2. Derivation of $w(a)$

Using the QEV definition of the characteristic scale,

$$L(a) = \sqrt{\lambda_{\min}\lambda_{\max}(a)}, \tag{A23}$$

and the effective fluid relation derived in Appendix C,

$$w(a) = -1 + \frac{4}{3}\frac{d\ln L}{d\ln a}, \tag{A24}$$

we obtain an explicit expression for the equation of state.

Taking the logarithm,

$$\ln L(a) = \frac{1}{2}\ln\lambda_{\min} + \frac{1}{2}\ln\lambda_{\max,0} + \frac{1}{2}\nu(a)\ln a, \tag{A25}$$

and differentiating,

$$\frac{d\ln L}{d\ln a} = \frac{1}{2}\left[\nu(a) + \ln a\,\frac{d\nu}{d\ln a}\right]. \tag{A26}$$

For the chosen parametrization,

$$\frac{d\nu}{d\ln a} = a\frac{d\nu}{da} = -m\nu_0\, a(1-a)^{m-1}, \tag{A27}$$

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which yields

$$w(a) = -1 + \frac{2}{3}\Big[\nu_0(1-a)^m - m\nu_0\, a(1-a)^{m-1}\ln a\Big]. \tag{A28}$$

Appendix F.3. Late-Time Behavior

The model reproduces the cosmological-constant limit analytically:

$$a \to 1 \quad \Rightarrow \quad w(a) \to -1. \tag{A29}$$

This behavior reflects the stabilization of the infrared scale and is consistent with the observed near-constancy of the vacuum energy at late times [23,29,30].

Appendix F.4. Illustrative Parameter Choice

As a representative example, we consider

$$\nu_0 = 0.015, \qquad m = 2. \tag{A30}$$

This choice yields deviations from $w = -1$ at the level of

$$|w+1| \sim 10^{-3} - 10^{-2} \tag{A31}$$

over the redshift range

$$0 \le z \le 1, \tag{A32}$$

which corresponds to the regime where current and upcoming surveys are most sensitive to dark energy dynamics [1–3,5,23,29].

For illustration,

$$w(1) = -1, \tag{A33}$$
$$w(0.75) \approx -0.996, \tag{A34}$$
$$w(0.50) \approx -0.989. \tag{A35}$$

These values remain fully consistent with current observational constraints while preserving a small but potentially detectable deviation from ΛCDM.

Appendix F.5. Interpretation and Scope

This construction demonstrates that the QEV framework naturally leads to a controlled low-redshift deformation of $w = -1$ once the evolution of the infrared scale is specified. The parameters ν_0 and m may be interpreted as effective descriptors of the rate at which the spectral window stabilizes.

Importantly, the purpose of this appendix is not to define a unique functional form for $\lambda_{\max}(a)$, but to show that the QEV framework yields explicit, calculable predictions for the equation of state in the observationally relevant regime.

In this sense, the model connects the underlying spectral structure of the vacuum to directly measurable cosmological quantities, providing a concrete realization of the general framework discussed in Sec. 4.

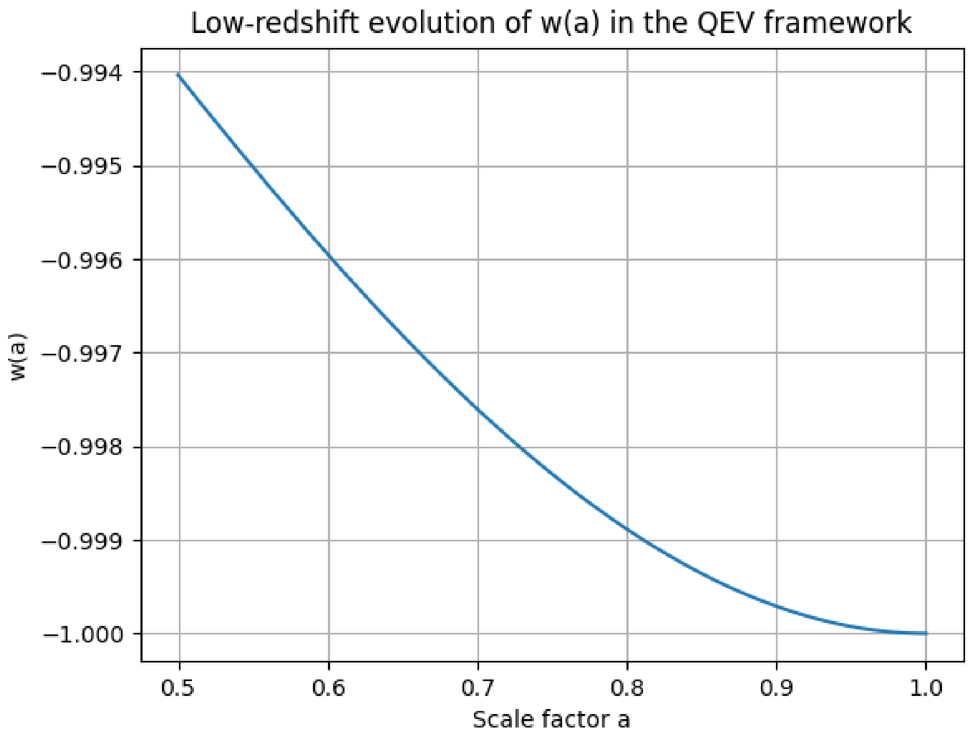

Figure A1. Low-redshift evolution of the effective equation-of-state parameter $w(a)$ in the QEV framework, for the illustrative parameter choice $\nu_0 = 0.015$ and $m = 2$ (Appendix F). The model shows a small but smooth deviation from the cosmological-constant value $w = -1$ at intermediate redshift, followed by convergence to $w \to -1$ as $a \to 1$. This behavior reflects the gradual stabilization of the infrared scale $\lambda_{\max}(a)$ and the associated spectral window. The magnitude of the deviation, $|w+1| \sim 10^{-3} - 10^{-2}$ over $0 \le z \le 1$, lies within the sensitivity range of current and upcoming cosmological surveys such as DESI, Euclid, LSST, and the Simons Observatory [1–3,5,29,30].

Appendix G. Why the Adopted Functional Forms Are Natural and Minimally Model-Dependent

The effective framework developed in this work employs three central ingredients:

1. a smooth bounded spectral kernel,
2. a characteristic intermediate scale given by the geometric mean of ultraviolet and infrared bounds,
3. an exponential attenuation factor.

A natural concern is whether these structures are arbitrary modeling choices. The purpose of this appendix is to show that they should instead be regarded as minimal effective forms expected in a broad class of bounded-vacuum descriptions [4,19,20].

Appendix G.1. Smooth Spectral Kernel

The framework requires suppression of contributions far below the infrared scale and far above the ultraviolet scale. Since the exact microscopic response profile is not yet known, a smooth kernel is the least model-dependent implementation.

We therefore adopt a representative form,

$$W(\lambda) = \exp\left[-\left(\frac{\lambda_{\rm UV}}{\lambda}\right)^{\alpha}\right] \exp\left[-\left(\frac{\lambda}{\lambda_{\rm IR}}\right)^{\beta}\right], \qquad \text{(A36)}$$

with $\alpha, \beta > 0$.

This choice satisfies three generic requirements:

- ultraviolet suppression for $\lambda \ll \lambda_{\mathrm{UV}}$,
- infrared suppression for $\lambda \gg \lambda_{\mathrm{IR}}$,
- smooth interpolation inside the allowed window.

Many alternative kernels (Gaussian, Lorentzian, sigmoid, compact windows) satisfy the same qualitative conditions. The main predictions of the framework depend primarily on the existence of a bounded spectral sector, not on the exact analytic profile [16,20].

Appendix G.2. Why the Geometric Mean Appears Naturally

The model contains two hierarchical scales,

$$\lambda_{\mathrm{UV}} \ll \lambda_{\mathrm{IR}}. \tag{A37}$$

When a system is governed by two extreme scales spanning many orders of magnitude, the natural midpoint is logarithmic rather than linear. In logarithmic variables,

$$\ln L = \frac{1}{2}(\ln \lambda_{\mathrm{UV}} + \ln \lambda_{\mathrm{IR}}), \tag{A38}$$

which yields

$$L = \sqrt{\lambda_{\mathrm{UV}}\lambda_{\mathrm{IR}}}. \tag{A39}$$

This scale is:

- symmetric under exchange of UV and IR labels,
- dimensionally consistent,
- stable under large hierarchy.

Geometric midpoint scales commonly appear in multiscale matching problems, asymptotic analysis, and saddle-point dominated integrals [21].

Appendix G.3. Why Exponential Attenuation Is Generic

The effective amplitude is written as

$$A_{\mathrm{eff}}(t) = A_0 e^{-\Xi(t)}. \tag{A40}$$

This is not an arbitrary ansatz. Exponential attenuation follows generically whenever suppression acts locally and multiplicatively.

If the amplitude changes through small fractional steps,

$$\frac{dA}{A} = -\Gamma(t)\, dt, \tag{A41}$$

then direct integration gives

$$A_{\mathrm{eff}}(t) = A_0 \exp\left[-\int_{t_i}^{t} \Gamma(t')dt'\right]. \tag{A42}$$

Thus,

$$\Xi(t) = \int_{t_i}^{t} \Gamma(t')dt' \tag{A43}$$

represents the cumulative suppressive history.

Such exponential forms are ubiquitous in physics: absorption, damping, radioactive decay, decoherence, thermal suppression, and survival probabilities [6,15].

Appendix G.4. Summary

The adopted ingredients are not claimed to be unique. Rather, they are chosen because they are the simplest effective forms compatible with broad physical principles:

- smooth kernels implement bounded spectral participation,
- geometric means arise naturally in hierarchical two-scale systems,
- exponentials arise from cumulative local suppression.

The predictive content of the model therefore lies mainly in the bounded spectral structure itself, rather than in the exact representative functional choices.

Appendix H. Physical Origin of the Spectral Bounds

The QEV framework assumes that the gravitationally relevant vacuum sector is limited by physically motivated ultraviolet and infrared scales. This appendix summarizes why such bounds are plausible within known physics.

Appendix H.1. Ultraviolet Scale from Confinement Physics

The ultraviolet scale λ_{UV} represents the shortest wavelengths contributing coherently to the late-time gravitational vacuum sector.

In standard quantum field theory, formal zero-point sums extend to arbitrarily short wavelengths. However, in quantum chromodynamics the physical content of the spectrum changes strongly with scale. At short distances, quarks and gluons are the relevant degrees of freedom, whereas at larger distances the realized low-energy spectrum is dominated by confined hadronic states [15,26,27].

This transition suggests a natural reorganization boundary near the confinement scale,

$$k_{\mathrm{UV}} \sim \Lambda_{\mathrm{QCD}}, \qquad \lambda_{\mathrm{UV}} \sim \Lambda_{\mathrm{QCD}}^{-1} \sim 1\ \mathrm{fm}, \tag{A44}$$

up to factors of order unity.

The claim is not that shorter-scale fields cease to exist, but that they need not contribute coherently to the emergent macroscopic vacuum sector relevant for gravity.

Appendix H.2. Infrared Scale from Cosmological Coherence

The infrared scale λ_{IR} represents the largest wavelengths that remain gravitationally coherent.

The late-time universe possesses:

- finite causal structure,
- cosmic expansion,
- finite effective temperature,
- evolving large-scale organization.

These properties naturally limit arbitrarily long-wavelength coherence [4,6,11].

Two representative scales are:

$$\lambda_H(t) = \frac{c}{H(t)}, \tag{A45}$$

and

$$\lambda_T(t) = \frac{\hbar c}{k_B T_{\mathrm{eff}}(t)}. \tag{A46}$$

The effective infrared scale may therefore be modeled as the most restrictive coherence scale,

$$\lambda_{\mathrm{IR}}(t) = \min\left[\frac{c}{H(t)}, \frac{\hbar c}{k_B T_{\mathrm{eff}}(t)}\right]. \tag{A47}$$

At the present epoch, the thermal branch naturally lies near the millimeter regime.

Appendix H.3. Combined Effect and Emergent Scale

The simultaneous presence of ultraviolet and infrared bounds defines a restricted spectral domain,

$$\lambda_{\mathrm{UV}} \leq \lambda \leq \lambda_{\mathrm{IR}}. \tag{A48}$$

A characteristic intermediate scale then emerges naturally:

$$L = \sqrt{\lambda_{\mathrm{UV}}\lambda_{\mathrm{IR}}}. \tag{A49}$$

This scale governs the dominant contribution to the bounded spectral integral and leads to the robust scaling relation

$$\rho_{\mathrm{vac}} \propto L^{-4}. \tag{A50}$$

The observed smallness of vacuum energy is therefore interpreted as a consequence of bounded spectral participation rather than unrestricted ultraviolet counting [18,25,28].

Appendix H.4. Scope

These arguments do not constitute a unique microscopic derivation. Rather, they show that known physical regimes provide plausible support for effective ultraviolet and infrared bounds.

The cosmological constant problem is thereby reframed as a problem of spectral selection: determining which subset of vacuum fluctuations contributes coherently to macroscopic gravity.

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