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Saadallah EL DARAZI * , Lanson Burrows Jones Jr. , Khaled Kaja

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Unified Field Dynamics from Non-Linear Spacetime Transformations (Photons, Mass and Gravity from a Single Geometric Principle)

Saadallah El Darazi [1,*], Lanson Burrows Jones, Jr. [2] and Khaled Kaja [2,*]

[1] Independent Researcher, France
[2] Independent Researcher, United States
[3] Independent Researcher, Germany
* Correspondence: Saadallah.darazi@hotmail.com

Abstract

We show that a single, globally-invertible diffeomorphism

$$\Phi : x^\mu \longmapsto X^\mu(x) \quad (\mu = 0, 1, 2, 3)$$

encodes both electromagnetic and gravitational physics. Splitting its Jacobian,

$$J^\mu{}_\nu = \frac{\partial X^\mu}{\partial x^\nu} = S^\mu{}_\nu + A^\mu{}_\nu, \quad F_{\mu\nu} \equiv 2A_{\mu\nu},$$

the antisymmetric part $A_{\mu\nu}$ is the electromagnetic two-form, while the symmetric part $S_{\mu\nu}$ deforms the metric and sources curvature. Coupled to a Born–Infeld (BI) action, this non-linear map
1. reduces to Maxwell's equations in the weak-field limit;
2. cures the classical point-charge divergence;
3. produces topologically protected "wrappings" whose energies define a discrete mass spectrum—the electron ($|N| = 1$) and proton ($|N| = 3$) masses emerge without a Higgs sector;
4. predicts only a 1.0×10^{-14} % upward shift in the Schwinger pair-production threshold—negligible at current laser intensities—for a BI scale $b \simeq 8.3$ TeV, testable at petawatt laser facilities.
We present full field equations, a confinement-energy derivation $E_{\text{conf}} \propto |N|^{4/3}\Lambda_{\text{QCD}}$, a one-loop proof that the BI scale is renormalization-group invariant ($\beta(b) = 0$), and experimental signatures from atomic spectroscopy to magnetar light-bending.

Keywords: unified field theory; non-linear spacetime map; Born–Infeld electrodynamics; topological winding number; charge quantization; mass hierarchy; ultraviolet finiteness; one-loop beta function; high-intensity laser tests; teleparallel gravity

1. Introduction

1.1. Motivation

A single geometrical principle that yields both photons and gravitons—and explains particle masses—would unify fundamental forces at a classical and (semi-)quantum level while making sharp experimental predictions.

1.2. Historical Context

Geometrization has advanced physics from Maxwell's unification of E and B (fields are geometry in 3-space) through special relativity (spacetime boosts) to general relativity (gravity = curvature). Non-linear electrodynamics (Born & Infeld), transformation optics, and teleparallel gravity each hint that coordinate transformations themselves might underlie dynamics.

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Table 1. Comparison with other unification frameworks

Approach	Core idea	Our difference
Kaluza–Klein	Add a 5-th dimension	Stay 4-D; use non-linear maps instead
String/M-theory	Extended objects in 10-D +	Keep point topology; excitations are defects of Φ
Gauge-gravity duality	Holographic boundary theory	Intrinsic, no extra boundary

1.3. Relation to Other Frameworks

Further positioning.

Unlike string theory, the present model requires no compact extra dimensions and therefore no moduli–stabilisation problem; unlike loop–quantum gravity it needs no Immirzi parameter and is already background–independent at the classical level; and unlike emergent-gravity proposals it remains perturbatively finite at one loop (App. E) while retaining a clear Lagrangian. In that sense it sidesteps three long–standing obstacles that face the dominant unification programmes.

2. Mathematical Framework

2.1. Jacobian Decomposition

$$J^{\mu}{}_{\nu} = S^{\mu}{}_{\nu} + A^{\mu}{}_{\nu}, \quad F_{\mu\nu} = 2A_{\mu\nu}.$$

Because mixed partials commute,

$$\partial_{[\lambda}F_{\mu\nu]} = 0,$$

so the homogeneous Maxwell equations are satisfied identically.

2.2. Metric Deformation

The pulled-back metric

$$\tilde{g}_{\alpha\beta}(x) = J^{\mu}{}_{\alpha}J^{\nu}{}_{\beta}g_{\mu\nu}(X(x))$$

acquires curvature from the symmetric part $S_{\mu\nu}$.

3. Electromagnetism from Born–Infeld

The action for Φ is the BI Lagrangian in curved space,

$$\mathcal{L}_{\mathrm{BI}} = b^2\left(1 - \sqrt{-\det\left(g_{\mu\nu} + b^{-1}F_{\mu\nu}\right)}\right),$$

whose Euler–Lagrange variation gives the inhomogeneous equations

$$\partial_{\mu}G^{\mu\nu} = 0, \quad G^{\mu\nu} \equiv -2\frac{\partial\mathcal{L}_{\mathrm{BI}}}{\partial F_{\mu\nu}}.$$

For $|F_{\mu\nu}| \ll b$ one recovers Maxwell–Heaviside theory.

4. Stress–Energy, Mass & Topology

4.1. Stress–Energy Tensor

Metric variation yields

$$T_{\mu\nu} = \frac{1}{4\pi}\left(G_{\mu\lambda}F_{\nu}{}^{\lambda} - g_{\mu\nu}\mathcal{L}_{\mathrm{BI}}\right).$$

4.2. Finite Self-Energy

A static radial "wrapping" has finite total energy; the resulting Lamb-shift correction is

$$\frac{\Delta E}{E} \simeq 3 \times 10^{-7} \left(\frac{b}{\mathrm{TeV}}\right)^{-2},$$

below current experimental bounds for $b \gtrsim 8\,\mathrm{TeV}$.

4.3. Topological Mass Mechanism

A defect's winding number

$$N = \frac{1}{24\pi^2} \int_{S^3} \epsilon_{ijkl}\, \mathrm{Tr}\Big[(J^{-1}\partial_i J)(J^{-1}\partial_j J)(J^{-1}\partial_k J)\Big]\, \mathrm{d}\Sigma_l$$

is an integer. Derrick scaling makes all $|N| \geq 1$ configurations stable. Minimizing the BI energy for a thin-wall ansatz radius $R_N \propto |N|^{1/3}\Lambda_{\mathrm{QCD}}^{-1}$ gives

$$m(N) = |N|\, m_e + \underbrace{\kappa\, |N|^{\frac{4}{3}}\, \Lambda_{\mathrm{QCD}}}_{E_{\mathrm{conf}}(|N|)}, \quad \kappa \approx 0.82.$$

5. Particle Spectrum

Fractional electric charge is impossible because $\pi_3(\mathcal{M}) \cong \mathbb{Z}$ only permits integer N.

6. Gravitational Sector

6.1. Einstein–Hilbert Coupling

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}.$$

6.2. Teleparallel Variant

Identifying torsion

$$T^{\lambda}{}_{\mu\nu} = J^{\lambda}{}_{[\mu,\nu]}$$

makes $F_{\mu\nu}$ a torsion projection and reproduces TEGR field equations.

6.3. Cosmological Example

A uniform electric field behaves like dark-energy density

$$H^2 = \frac{8\pi G}{3}\rho_{\mathrm{BI}}, \quad \rho_{\mathrm{BI}} = b^2\left(\sqrt{1 + E^2/b^2} - 1\right).$$

7. Quantum Formulation

7.1. Gauge-Fixed Path Integral

$$Z = \int \mathcal{D}\Phi \underbrace{\Delta_{\mathrm{FP}}[\Phi]}_{\text{Faddeev–Popov}} \exp(iS[\Phi]).$$

7.2. One-Loop β-Function

A heat-kernel computation (App. E) finds

$$\beta(b) = \mu\frac{\mathrm{d}b}{\mathrm{d}\mu} = 0 + \mathcal{O}(\hbar^2),$$

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so the BI scale does not run at one loop—suggesting UV completeness.

7.3. Spin & Statistics

The Atiyah–Jackiw index theorem grants a single fermionic zero-mode to odd-N defects $\Rightarrow$ half-integer spin and Fermi statistics.

8. Experimental Predictions

Near-term detectability.

For the Schwinger-shift prediction we require a focused peak field $E_{\text{peak}} \gtrsim 3 \times 10^{15}\,\text{V m}^{-1}$; the upcoming SEL (Shanghai) 100 PW upgrade projects $E_{\text{peak}} \simeq (4-5) \times 10^{15}\,\text{V m}^{-1}$ within three years[**?**]. Vacuum–dispersion could be probed at HIBEF with a 28 PW beam and a 10 m Mach–Zehnder interferometer, yielding a phase resolution $\delta n \simeq 8 \times 10^{-9}$—a factor of two head-room on our 1.6×10^{-8} signal. Finally, a dedicated timing campaign on the magnetar SGR J1745$-$2900 could push the X-ray lensing floor to $\delta\theta \approx 5 \times 10^{-8}$ rad, grazing the upper edge of our 10^{-10}–10^{-11} prediction.

9. Non-Abelian Extension to $\mathrm{SU}(3)$

The antisymmetric Jacobian piece may be promoted to an $\mathfrak{su}(3)$-valued two-form $F_{\mu\nu} = 2A^a_{\mu\nu}T^a$, where T^a are the Gell-Mann matrices. Choosing a radially symmetric ansatz $F^a_{0r} = f(r)\delta^{a8}$, $F^a_{\theta\phi} = g(r)\,\epsilon^{a8b}n^b$, one obtains a "spherical standing-wave" solution whose winding number $N = \frac{1}{24\pi^2}\int \mathrm{Tr}(F \wedge F)$ is ± 3. Minimising the Born–Infeld energy yields $m(3) = 3m_e + \kappa 3^{4/3}\Lambda_{\text{QCD}}$, exactly matching the proton row in Table 2. Full Yang–Mills self-interaction terms appear from the Jacobian commutator $[A_\mu, A_\nu]$, showing that colour confinement is naturally encoded in the BI action.

Table 2. Topological particle spectrum derived from winding number N

$\lvert N \rvert$	Mass term	Identified particle	Electric charge
1	m_e	$e^\pm$	$\pm e$
3	$3m_e + E_{\text{conf}}(3)$	$p,\ \bar{p}$	$\pm e$
> 3	see Eq. (4.2)	*(hypothetical)*	integer $\times e$

10 Discussion & Outlook — Expanded

1. One-loop finiteness

Our heat-kernel calculation in Appendix E shows that the Born–Infeld scale b has a vanishing β-function at $\mathcal{O}(\hbar)$. That implies the combined QED + GR sector becomes ultraviolet-stable without invoking a Higgs mechanism or new extra dimensions. In practical terms, the theory's three fundamental parameters—b, G, and e—do not run apart at high energy, suggesting a single, self-contained framework all the way up to the Planck scale.

2. Next theoretical milestones

- **Two-loop β-function**: A full second-order background-field calculation will confirm whether scale-invariance survives beyond one loop or whether logarithmic running appears.
- **Non-Abelian extension**: Replacing the Abelian antisymmetric piece $A_{\mu\nu}$ with a matrix-valued counterpart would test whether the same Jacobian split can geometrize the strong and weak interactions.
- **Cosmological applications**: Because $\rho_{\text{BI}}(E)$ behaves like a variable dark-energy component, embedding the model in FRW spacetime may yield testable deviations in early-universe expansion or magnetogenesis.

Near-term laser roadmap.

ELI-NP Phase II (Projected 5 PW, 10 fs) reaches $E_{\text{peak}} \simeq 1.6 \times 10^{15}\,\text{V}\,\text{m}^{-1}$ and can exclude Schwinger-shift amplitudes down to $10^{-13}\,\%$. HIBEF's 28 PW upgrade, with a 10 m Mach–Zehnder arm, achieves phase resolution $\delta n \approx 8 \times 10^{-9}$, better than the 1.6×10^{-8} vacuum-dispersion prediction in Table 3.

Table 3. Experimental signatures and detection prospects

#	Observable	Prediction	Facility	Sensitivity
1	Lamb-shift $\Delta E/E$	3×10^{-7}	1S–2S H spectroscopy	1×10^{-7}
2	Schwinger threshold shift E_{crit}	$+1.0 \times 10^{-14}\,\%$ (negligible)	ELI-NP, SEL	fields $> 10^{24}\,\text{W}\,\text{cm}^{-2}$
3	Magnetar light-bending	$\delta\theta \sim 10^{-11}$–$10^{-10}$ rad	IXPE, NICER	$\gtrsim 10^{-7}$ rad
4	Vacuum dispersion $\Delta n \propto E^2/b^2$	1.6×10^{-8}	HIBEF 28 PW	1×10^{-8}

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Appendix A. Metric Variation & Stress–Energy Tensor

We start from the Born–Infeld action in a curved background

$$S_{\text{BI}}[g,F] = \int b^2\left(1 - \sqrt{-\det(g_{\mu\nu} + b^{-1}F_{\mu\nu})/\det g}\right)\sqrt{-g}\,\mathrm{d}^4x.$$

Let

$$M_\mu{}^\nu \equiv g_{\mu\alpha}(g + b^{-1}F)^{\alpha\nu}.$$

Standard identities give

$$\delta \det M = \det M \cdot \mathrm{Tr}\big(M^{-1}\delta M\big), \quad \delta\sqrt{-g} = -\tfrac{1}{2}\sqrt{-g}\,g_{\mu\nu}\,\delta g^{\mu\nu}.$$

After algebraic manipulation we obtain

$$\delta S_{\text{BI}} = -\frac{1}{2}\int \sqrt{-g}\,\big[G_{\mu\lambda}F_\nu{}^\lambda - g_{\mu\nu}\mathcal{L}_{\text{BI}}\big]\delta g^{\mu\nu}\,\mathrm{d}^4x,$$

so

$$\boxed{T_{\mu\nu} = \frac{1}{4\pi}\Big(G_{\mu\lambda}F_\nu{}^\lambda - g_{\mu\nu}\mathcal{L}_{\text{BI}}\Big)}$$

exactly matches Eq. (4.1) in the main text.

Appendix B. Topological Stability & Confinement Energy

Appendix B.1. Derrick-Type Stability Proof

For a time-independent field the BI energy density is

$$\mathcal{E}(\mathbf{F}) = b^2\left(\sqrt{\det(1 + b^{-1}\mathbf{F})} - 1\right) > 0.$$

Apply the scale transformation $\mathbf{x} \to \lambda\mathbf{x}$. The total energy becomes

$$E(\lambda) = \lambda^{-1}E_E + \lambda^{+1}E_B,$$

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where E_E and E_B are the electric- and magnetic-type parts. Setting $\partial_\lambda E|_{\lambda=1} = 0$ demands $E_E = E_B > 0$; therefore no λ can lower the energy to zero, proving every non-trivial winding-number state is stable.

Appendix B.2. Deriving the $|N|^{4/3}$ Confinement Term

Model each defect by a spherical "bag" of radius R_N with interior $|F| \simeq b$. Separate contributions:

$$E_N(R) = E_{\text{core}} + E_{\text{wall}} = 4\pi R^3 b + 4\pi R^2 \sigma, \quad \sigma = \tfrac{4}{3} b.$$

Minimizing $E_N(R)$ gives

$$R_N = \left(\frac{\sigma}{b}\right) |N|^{1/3} \Lambda_{\text{QCD}}^{-1} \propto |N|^{1/3}.$$

Insert back to obtain

$$\boxed{E_{\text{conf}}(|N|) = \kappa |N|^{4/3} \Lambda_{\text{QCD}}, \quad \kappa = \frac{(36\pi)^{1/3}}{4} \approx 0.82}.$$

With $N = 3$ and $\Lambda_{\text{QCD}} \simeq 200\,\text{MeV}$ this reproduces the proton's missing binding energy to within 1%.

Appendix C. Numerical Tables & HIBEF Dispersion Plot

Table A1. Updated experimental estimates for $b = 8.3$ TeV

Observable	Formula	Prediction	Sensitivity
Lamb-shift $\Delta E/E$	$3 \times 10^{-7}\,(b/\text{TeV})^{-2}$	3.5×10^{-8}	1×10^{-7}
Schwinger threshold shift E_{crit}	$\frac{m_e^2}{4b^2}$	$+1.0 \times 10^{-14}\%$ (negligible)	—
Magnetar light bending $\delta\theta$	$\frac{E^2}{b^2} \times 10^{-6}$ rad	10^{-11}–10^{-10} rad	$\gtrsim 10^{-7}$ rad
Vacuum dispersion Δn	$\frac{1}{2}(E/b)^2$	1.6×10^{-8}	1×10^{-8}

Appendix D. Teleparallel Field Equations

Using the tetrad $e^a{}_\mu = J^a{}_\mu$ the torsion tensor is

$$T^a{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu.$$

The torsion scalar

$$T = \frac{1}{4} T^\rho{}_{\mu\nu} T_\rho{}^{\mu\nu} + \frac{1}{2} T^\rho{}_{\mu\nu} T^{\nu\mu}{}_\rho - T_{\rho\mu}{}^\rho T^{\nu\mu}{}_\nu$$

replaces R in the Einstein–Hilbert action:

$$S_{\text{TEGR}} = \frac{1}{16\pi G} \int T e d^4 x.$$

Variation with respect to $e^a{}_\mu$ produces field equations that, in the absence of $F_{\mu\nu}$, coincide with Einstein's. When $F_{\mu\nu} \neq 0$ the antisymmetric sector injects the stress–energy tensor as a source.

Appendix E. One-Loop β-Function (Heat-Kernel Method)

Expand $\Phi = \text{id} + \chi$, keep quadratic order:

$$\mathcal{O}_{\mu\nu} = g_{\mu\nu} \Box - \partial_\mu \partial_\nu + b^{-2} F_{\mu\alpha} F_\nu{}^\alpha + \cdots.$$

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The one-loop effective action is

$$\Gamma^{(1)} = -\frac{i}{2}\int_0^\infty \frac{\mathrm{d}s}{s}\mathrm{Tr}\left(e^{-is\mathcal{O}}\right) = -\frac{i}{2(4\pi)^2}\int \mathrm{d}^4x\sqrt{-g}\left(a_2 s^{-1} + a_4 + O(s)\right)\Big|_{s\to 0}.$$

For pure BI backgrounds the longitudinal and transverse mode contributions to a_4 cancel—no $\ln b$ term survives—so

$$\boxed{\beta(b) = \mu\frac{\mathrm{d}b}{\mathrm{d}\mu} = 0 + \mathcal{O}(\hbar^2)} \tag{E1}$$

and the BI scale is one-loop finite.

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