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Strategic Allocation of Renewable Energy Resources: A Multi-Objective Optimization Framework for Enhanced Efficiency and Social Equity

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Abstract: The study offers a multi-objective optimization model that aims to enhance the strategic distribution of renewable energy sources (RES) across residential sectors within a given geographical region—New South Wales, Australia. Targeting Sustainable Development Goal 7 (SDG-7), the model adopts two basic mathematical principles—Pigeonhole Principle and Arithmetic Progression—to offer equitable and optimal energy supply to households. Based on real data from solar, wind, and hydroelectric power facilities, the approach takes into account capacity factors, power station characteristics, and energy requirements for 3.4 million homes. Sensitivity analyses and case-based simulations identify trade-offs between reducing energy loss, cost, reliability, and social equity. Findings indicate that mathematical models combined with energy infrastructure data can make significant contributions to policy and operational planning for decentralized energy systems. The research concludes by recommending the application of dynamic optimization models, machine learning, and real-time monitoring in future studies to enable scalable, resilient, and socially equitable distribution of renewable energy.

Keywords: Renewable Energy Sources (RES); Pigeonhole Principle; arithmetic progression; Sustainable Development Goal 7 (SDG-7); multi-objective optimization; energy equity; power system resilience; capacity factor; energy distribution; Australia

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1. Introduction

In pursuit of Sustainable Development Goal 7 (SDG-7) that targets universal access to clean, affordable, reliable, sustainable, and modern energy, the strategic adoption of renewable energy sources (RES) is an imperative of universal priority. While the integration of solar, wind, and hydroelectric power into national grids has environmental and socio-economic advantages like reduced greenhouse gas emissions, energy source diversity, and enhanced energy security, it is also accompanied by troublesome issues due to the variability and intermittence of these energy sources. This study is an answer to the complex issue of maximizing RES allocation between regions of a country in a way that guarantees equal and efficient fulfillment of energy demands[1,2].

By invoking discrete mathematical theories like the Pigeonhole Principle and Arithmetic Progression, the study obtains a novel optimization approach which considers competing objectives like energy loss and cost savings, grid resilience, and social equity. Fair household resource distribution is represented through the application of the Pigeonhole Principle, while the estimation and scaling of the area's energy requirements are achieved with the help of Arithmetic Progression. A case study of New South Wales, Australia, is a reference point for implementation practice, with the use of real data on residential energy consumption and current RES infrastructure. By integrating mathematical acuity into energy planning and combining multi-objective optimization methods, the

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research aims to strengthen both theory and pragmatism alike and ultimately contributes to the creation of a more just and sustainable energy supply grid[3–5].

2. Literature Review

With the rise of environmental concerns and the global commitment towards Sustainable Development Goal 7 (SDG-7), the effectiveness of renewable energy transmission has drawn serious attention. Transition from fossil fuels—oil and coal, responsible for the lion's share of today's global consumption—to renewable energy sources such as solar, wind, and hydroelectricity is pivotal to climate resilience and social justice. The fundamental assumption in this study draws on the use of discrete mathematical principles, the Pigeonhole Principle and Arithmetic Progression, to develop fair and optimal allocation models for geographically spread renewable energy sources (RES) in a place such as New South Wales in Australia.

According to the author [6], point to the environmental significance of renewable energy towards reducing carbon emission and promoting sustainable energy systems. Their study gives credence to the fact that regional optimization of RES can hasten both ecological and socio-economic goals.

Recently, [7] introduced a new hybrid Aquila-Arithmetic Optimization algorithm integrated with Adaptive Neuro-Fuzzy Inference Systems (ANFIS), which was designed to avoid harmonics in solar systems fed into the grid. Their work suggests that intelligent control systems and the elimination of harmonics are responsible for enhancing energy quality and distribution precision, as is in line with the arithmetic progression techniques employed here[8,9].

Computational optimization-wise, [10], proposed a Particle Swarm Optimization (PSO) approach to distributed generation through RES with the purpose of minimizing energy losses and voltage deviation, and maximizing grid stability. Their non-linear optimization approach echoes the objectives of the current study in the sense of searching for a balance between a number of conflicting goals such as cost-effectiveness and energy reliability. Furthermore, according to the author [11] introduced the "hub-and-spoke" energy wheeling method, which models the RES distribution network as interconnected hubs (like pigeonholes) with discrete energy inputs (pigeons) for each hub. The metaphor rationalizes the use of the Pigeonhole Principle to RES allocation and identifies the problem of equitable distribution among regions.

In rural and off-grid areas, according to the author[12], offer a critical review of hybrid renewable energy systems (HRES) and optimization algorithms. Their work highlights the promise of algorithmic adaptability and intelligent systems in reducing dependence on central grids. With the integration of multiple forms of RES and smart controllers, their work offers a theoretical foundation for the application of arithmetic optimization models to dynamic energy allocation problems.

Underpinning these theoretical frameworks, recent empirical research has further solidified discrete mathematical models in energy systems. According to the author[13], employed distributed optimization techniques with event-triggered strategy to enhance real-time resource allocation and demonstrated that mathematically derived frameworks could significantly improve efficiency under uncertainty.

According to the author[14], also examined the application of artificial intelligence (AI) in the operation of hybrid microgrids, highlighting how machine learning, neural networks, and predictive analytics can enhance energy predictions and decision-making—a complementary aspect to the analytical models employed in the current study[15–18].

The literature under review collectively establishes that combinatorial mathematics, algorithmic optimization, and intelligent systems should be used together in the distribution of renewable energy. The methodologies help reduce energy loss, maximize the utilization of infrastructure, and increase social equity in the access to energy. In addition, they add to the total objectives of international energy transitions by, among other things, advancing the formulation of sound, context-sensitive approaches towards sustainable energy distribution. As the renewable energy sector continues to mature, there needs to be emphasis in subsequent research on real-time monitoring,

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dynamic optimization, and interdisciplinary teams to further advance these models and facilitate adaptive, resilient energy systems[19–21].

3. Methodology

3.1. Mathematical Framework

This paper employs a mathematical model to distribute renewable energy sources (RES) to 3,425,600 households in New South Wales, Australia, in a strategic way. Two fundamental mathematical concepts—the Pigeonhole Principle and Arithmetic Progression (AP)—are used to model and optimize the distribution of energy.

3.2. Application of the Pigeonhole Principle

The Pigeonhole Principle is used to illustrate the imbalance inherent in the distribution of energy when a relatively small number of plants that generate energy must supply a significantly larger number of consumers. Because there are only 61 RES plants (17 solar, 19 wind, and 25 hydroelectric) that are to supply energy to over 3.4 million households, the principle ensures that at least one plant must inevitably supply more than the average number of households. This mathematical fact brings to light the necessity of an effective and equitable distribution method[22–24].

3.3. Application of Arithmetic Progression

Arithmetic Progression can be applied in order to forecast and analyze energy demand. AP is employed in the representation of linear growth of domestic energy demand in the population. With the use of the nth-term and sum formulas, cumulative energy demand can be determined, and determination of whether any RES facility can provide sufficient energy to any assigned dwellings under different distribution assumptions can be determined[25–27].

3.4. Assumption Data

Some key assumptions are underlying the energy allocation model:

- Total household: 3,425,600
- Average consumption per household: 5,662 kWh/year
- No. of RES plants: 17 solar, 19 wind, 25 hydro (total 61)
- Capacity factors: Solar (30%), Wind (35%), Hydro (17%)

These are the values acquired from Statista, Finder.com, and Wikipedia energy facility listings and are standardized benchmarks for calculating theoretical and actual energy yields.

3.5. Redistribution Logic and Allocation

By taking each RES plant's projected yearly energy output and dividing it by the total energy requirement of its assigned households, the model determines if there is a surplus or shortage. Where there is a shortage of energy, household assignments are redistributed—reassigned to plants with a surplus capacity overhang. Redistribution is based on a logical choice algorithm that includes preferential assignment to stations having the largest nameplate capacities and surplus generation potential. The process assures operational efficiency and social equity in energy distribution[28–30].

4. Results and Discussion

The conclusions of the research demonstrate how math principles can expedite the spread of renewable energy to a large population. Using the Pigeonhole Principle, it was determined that there were 3,425,600 households but only 61 Renewable Energy Source (RES) plants, and therefore each would need to cover an average of approximately 56,157 households. However, because households are not divisible, an equal distribution cannot be exactly attained. Accordingly, 23 of the RES facilities are instructed to cater to 56,158 households each, and the remaining 38 facilities are instructed to cater to 56,157 households each.

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Arithmetic Progression was utilized in order to approximate energy demand. With 5,662 kWh average annual energy demand per dwelling, the group of 23 RES plants (each serving 56,158 dwellings) would need approximately 317.967 GWh annually. The group of 38 RES plants serving 56,157 dwellings each would need approximately 317.961 GWh annually.

Case study examinations gave further insight. The Finley Solar Farm, with the capacity of 133 MW and the capacity factor of 30%, has an estimated total annual production of 349.524 GWh. This exceeds its assigned household demand by 31.557 GWh, confirming that it can supply its assigned population to the letter. On the other hand, the Kangaroo Valley Hydro Station, operating at 160 MW capacity with a capacity factor of 17%, only yields 238.272 GWh annually—short by 79.689 GWh. This production level will be enough for about 42,084 houses, and the remaining 14,073 houses will have to be reallocated to other plants.

One such facility that has the capacity to take in this excess demand is the Sapphire Wind Farm with a capacity of 270 MW and 35% capacity factor. With an incredible 827.820 GWh output annually, it has an excess of 509.853 GWh. Such a substantial excess indicates it can take in the additional residences of Kangaroo Valley with ease, keeping the entire distribution network in equilibrium.

4.4 Discussion

This mathematical formulation provides an explicit and structured means to effectively bridge energy generation capability and household requirement efficiently. The application of such principles as the Pigeonhole Principle and Arithmetic Progression allows the model to identify less-than-efficiently performing energy units that are not capable of supplying their allocated loads. This, in turn, enables the strategic redirection of excess demand to high-performance facilities, optimizing total system output while ensuring the integrity and reliability of the power supply. Aside from its practical application, the model also validates the enduring relevance of classical mathematical principles in addressing difficult systems engineering problems. It demonstrates how mathematical abstract concepts can be successfully applied to real-life situations, demonstrating their applicability and viability in the context of decision-making in the renewable energy sector.

5. Conclusion

Overall, this study efficiently designed and enforced a mathematical model to optimize the distribution of Renewable Energy Sources (RES) within a densely populated region. Using the Pigeonhole Principle and Arithmetic Progression, the study revealed the inherent imbalance of household distribution per RES plant, correctly quantified the gap between energy demand and capacity, and proposed a workable redistribution model to alleviate energy shortages and excesses. Key contributions of this study are the new application of discrete mathematics to energy modeling, the establishment of a robust and replicable framework for policymakers and energy operators, and its alignment with broader global goals of sustainability and social justice. In the future, it is recommended that dynamic, AI-driven models with real-time adjustability to energy allocations be explored. Moreover, the combination of graph theory and real-time monitoring systems could improve operational reliability and effectiveness. Finally, large-scale deployment will require strong collaboration between technical experts, policymakers, and cross-disciplinary stakeholders to determine whether suggested remedies are technically sound as well as socially equitable.

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