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HAMILTONIAN FORMALISM OF BIANCHI TYPE I MODEL FOR FLUID BULK AND SHEARING VISCOSITY

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Abstract

In this paper we will consider the cosmic fluid to be dissipating i.e it has both bulk and shearing viscosity. We propose the Hamiltonian formalism of Bianchi type 1 cosmological model for cosmic fluid which is dissipating i.e it has both shearing and bulk viscosity. We have considered both the equation of state parameter ω and the cosmological constant Λ as the function of volume V(t) which is defined by the product of three scale factors of the Bianchi type 1 line element. We propose a Lagrangian for the anisotropic Bianchi type-1 model in view of a variable mass moving in a variable potential . We can decompose the anisotropic expansion of Bianchi type 1 in terms of expansion and shearing motion by Lagrangian mechanism. We have considered a canonical transformation from expanding scale factor to scalar field ø which helps us to give the proper classical definition of that scalar field in terms of scale factors of the mentioned model. By this transformation we can express the mass to be moving in a scalar potential field. This definition helps us to explain the nature of expansion of universe during cosmological inflation. We have used large anisotropy(as in the cases of Bianchi models) and proved that cosmic inflation is not possible in such large anisotropy. Therefore we can conclude that the extent of anisotropy is less in case of our universe. Otherwise the inflation theory which explained the limitations of Big Bang cannot be resolved. In the case of bulk and shearing viscous fluid we get the solution of damped harmonic oscillator after the cosmological inflation.Part I contains the calculations of bulk viscous fluids and Part II contains the calculations of bulk and shearing viscous fluid.At the end we have also provided the relation of shearing and expansion.Part III will give the approximation of low viscosity to solve the viscous fluid problem.

KEY WORDS: General theory of relativity ; Bianchi type 1 ; Hamiltonian formalism ; bulk viscosity ; cosmology ; Fluid dynamics ; Isotropic and Anisotropic cosmology ; shearing viscosity

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INTRODUCTION

The accepted model of present day universe is homogenous and isotropic on large scales and is defined by the FRW model. However in the present high precision cosmology era there are certain experimental evidences of the broken isotropy of cosmic microwave background(CMB).The most common anisotropic expanding universe is explained by the Bianchi models.

Bianchi models are the flat generalization of FRW cosmological model where the spatial expansion is considered anisotropic. The Bianchi models are homogeneous from which we can get the killing vectors. From the commutation relations and structure coefficients we can apply the lie classifications to obtain nine different types of Bianchi models. Here we are dealing with the type I model.

The paper is designed in the following manner. In part I section A we can calculate the Einstein tensor as well as the equation of motion from the Einstein's Field equation. The overall expansion is in terms of volume which is defined by the product of three scale factors. We define a Lagrangian and a potential which are both a function of volume and time as the cosmic fluid is considered to be dissipating because of the bulk viscosity. Then we can express the system as a variable mass moving in a variable potential.

In section B we make an appropriate canonical transformation to obtain a scalar field ϕ and we define the Lagrangian in terms of a scalar field ϕ to get a damped harmonic oscillator solution at the end of the cosmological inflation.

Finally in section C we did the analysis of our result by comparing it with large anisotropic deviation and small anisotropic deviation and proved that the cosmological inflation is not possible in large anisotropic deviation and concluded that the scale factors used in Bianchi type I model should not have much difference.

In part II same calculations has been done by including the shearing viscosity term and it's dissipation in energy momentum tensor.

In part III we have shown a special type of cosmological problem i.e the bulk viscous fluid problem.We have given an idea to solve this problem in this part using the concept of time varying gravitational constant G.

PART-I

SECTION -A

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Bianchi type 1 metric is defined as

$ds^2 = -dt^2 + X^2(t)\, dx^2 + Y^2(t)\, dy^2 + Z^2(t)\, dz^2$ ------------------------------(1)

Einstein Field equation is

$R_{ab} - \frac{1}{2} R\, g_{ab} + \Lambda g_{ab} = GT_{ab}$---------------------------------(2)

For a fluid with bulk viscosity the energy momentum tensor is given by

$T_{ab} = \rho\, u_a u_b + (p - \varepsilon\theta) h_{ab}$

$h_{ab} = g_{ab} + u_a u_b$ is the projection operator, ε is the coefficient of bulk viscosity, ρ is the density of the cosmic fluid.

θ is the scalar volume expansion

We assume $P = (p - \varepsilon\theta)$

So we can write $T_{ab} = \rho\, u_a u_b + P\, h_{ab}$------------------------(3)

For Bianchi Type 1 the field equations are

$(\dot{X}\dot{Y}/XY) + (\dot{Z}\dot{Y}/ZY) + (\dot{X}\dot{Z}/XZ) = G\rho + \Lambda$----------------------------(4)

$(\ddot{Y}/Y) + (\ddot{Z}/Z) + (\dot{Y}\dot{Z}/YZ) = -GP + \Lambda$ ----------------------------(5)

$(\ddot{X}/X) + (\ddot{Z}/Z) + (\dot{Z}\dot{X}/ZX) = -GP + \Lambda$ -------------------------(6)

$(\ddot{X}/X) + (\ddot{Y}/Y) + (\dot{X}\dot{Y}/XY) = -GP + \Lambda$ --------------------------------(7)

Adding equation (5),(6) and (7) we get

$$2\left(\left(\frac{\ddot{Y}}{Y}\right) + \left(\frac{\ddot{X}}{X}\right) + \left(\frac{\ddot{Z}}{Z}\right)\right) + \left(\left(\frac{\dot{X}\dot{Y}}{XY}\right) + \left(\frac{\dot{Z}\dot{Y}}{ZY}\right) + \left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right) = -3GP + 3\Lambda \text{-----------------------------(8)}$$

The equation of state of the matter(cosmic fluid except the cosmological constant) is commonly assumed to be

$p = (\gamma - 1)\rho$--------------------------------------(9)

We combine equation (4),(8),(9) and use the relation $P = (p - \varepsilon\theta)$ to get

$$\left(\left(\frac{\ddot{Y}}{Y}\right) + \left(\frac{\ddot{X}}{X}\right) + \left(\frac{\ddot{Z}}{Z}\right)\right) + \left(\frac{3\gamma - 2}{2}\right)\left(\left(\frac{\dot{X}\dot{Y}}{XY}\right) + \left(\frac{\dot{Z}\dot{Y}}{ZY}\right) + \left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right) - \frac{3G\varepsilon}{2}\left(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right) - \frac{3\gamma\Lambda}{2} = 0$$

(10)

Where the expansionis $\theta=\frac{\dot{V}}{V}$ = $(\dot{X}/X)+(\dot{Y}/Y)+(\dot{Z}/Z)$ and we consider a function V which is defined by the product of three scale factors V=X(t)Y(t)Z(t)

We consider equation (10) as the starting point, if the dynamical equation for the scale factor can be written as that form ,the present framework is valid. Our aim is to find a Hamiltonian description of equation (10) as the classical equation of motion. As the system is anisotropic let us consider the Lagrangian as

$L= L_v +L_G$-------------------------------- (11)

Where L_g is considered as the lagrangian for the spatial anisotropy which gives shearing

Where $L_v=\frac{1}{2}M(V,t)\dot{V}^2-\tilde{v}(V,t)$ $\tilde{v}(V,t)$ = Variable Potential

The Variable mass M and potential is a function of both V and t since the system is dissipating due to bulk viscosity.

$L_g= f(\dot{X},\dot{Y},\dot{Z},X,Y,Z)$

The Euler Lagrangian Equation is

$\frac{d}{dt}(\frac{\partial L}{\partial \dot{V}})-\frac{\partial L}{\partial V} = \frac{d}{dt}(\frac{\partial \mathrm{Lg}}{\partial \dot{V}})-\frac{\partial \mathrm{Lg}}{\partial V}+\frac{d}{dt}(\frac{\partial\ \mathrm{Lv}}{\partial \dot{V}})-\frac{\partial\ \mathrm{Lv}}{\partial V}=\frac{d}{dt}(\frac{\partial\ \mathrm{Lv}}{\partial \dot{V}})-\frac{\partial\ \mathrm{Lv}}{\partial V}+Q$ (Where $Q=\frac{d}{dt}(\frac{\partial \mathrm{Lg}}{\partial \dot{V}})-\frac{\partial \mathrm{Lg}}{\partial V}$)

$M\ddot{V}+\dot{V}(\partial M/\partial t)-\frac{1}{2}\dot{V}^2(\partial M/\partial V)+(\partial\tilde{v}/\partial V)+Q=0$

The Euler Lagrangian equation corresponding to equation (11) is

$\frac{\ddot{V}}{V}-\frac{1}{2}\frac{\partial lnM}{\partial lnV}(\frac{\dot{V}}{V})^2+\frac{\dot{V}}{V}(\partial lnM/\partial t)+(1/MV)(\frac{\partial\tilde{v}}{\partial V})+(\frac{Q}{MV})=0$-------------------------(12)

Now, $\frac{\dot{V}}{V}=\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}$

$$\frac{\ddot{V}}{V}=\left(\left(\frac{\ddot{Y}}{Y}\right)+\left(\frac{\ddot{X}}{X}\right)+\left(\frac{\ddot{Z}}{Z}\right)\right)+2\left(\left(\frac{\dot{X}\dot{Y}}{XY}\right)+\left(\frac{\dot{Z}\dot{Y}}{ZY}\right)+\left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right)$$

We can rewrite equation (12) after using these equations we get

$$\left(\left(\frac{\ddot{Y}}{Y}\right)+\left(\frac{\ddot{X}}{X}\right)+\left(\frac{\ddot{Z}}{Z}\right)\right)+2(1-\frac{3}{4}\frac{\partial lnM}{\partial lnV})\left(\left(\frac{\dot{X}\dot{Y}}{XY}\right)+\left(\frac{\dot{Z}\dot{Y}}{ZY}\right)+\left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right)+\frac{Q}{MV}+(1/MV)(\frac{\partial\tilde{v}}{\partial V})-$$
$$\frac{1}{2}\frac{\partial lnM}{\partial lnV}((\frac{\dot{X}}{X})^2+(\frac{\dot{Y}}{Y})^2+(\frac{\dot{Z}}{Z})^2-\left(\left(\frac{\dot{X}\dot{Y}}{XY}\right)+\left(\frac{\dot{Z}\dot{Y}}{ZY}\right)+\left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right))+\frac{\partial lnM}{\partial t}(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z})=0\text{--------------(13)}$$

Comparing equation (10) and (13) we get

$2(1-\frac{3}{4}\frac{\partial lnM}{\partial lnV})=\left(\frac{3\gamma-2}{2}\right)$ and $lnM=-\frac{3G\varepsilon t}{2}$

$M=\exp[\int\frac{2-\gamma}{V}dV-\frac{3G\varepsilon\, t}{2}]$--------------------------(13a)

(1/MV)($\frac{\partial \tilde{v}}{\partial V}$)=$-\frac{3\gamma\Lambda}{2}$

ṽ= $-\frac{3}{2}\int MV\gamma\Lambda$dV----------------------------(13b)

$\frac{Q}{MV}-\frac{1}{2}\frac{\partial lnM}{\partial lnV}$ (($\frac{\dot{X}}{X}$)^2+($\frac{\dot{Y}}{Y}$)^2+($\frac{\dot{Z}}{Z}$)^2-$\left(\left(\frac{\dot{X}\dot{Y}}{XY}\right)+\left(\frac{\dot{Z}\dot{Y}}{ZY}\right)+\left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right)$)=0

Q=$\frac{MV}{2}\frac{\partial lnM}{\partial lnV}$ σ^2-------------------------------(13c)

Where σ is the shearing scalar given by

σ^2=($\frac{\dot{X}}{X}$)^2+($\frac{\dot{Y}}{Y}$)^2+($\frac{\dot{Z}}{Z}$)^2-$\left(\left(\frac{\dot{X}\dot{Y}}{XY}\right)+\left(\frac{\dot{Z}\dot{Y}}{ZY}\right)+\left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right)$

From equation (13c) it is evident that we get a shearing in the cosmic fluid due to the anisotropy present in Bianchi type 1 model .

SECTION B(Canonical Transformation)

From our calculation we can get the generalized equation as the Hamiltonian description of the non-linear equation of a damped oscillator as

$\ddot{q}$ = f_1(q)$\dot{q}^2$ +ŋq̇+f_2(q) +f_3 (q,q̇,x,y,z)--------------------------------(14)

f_1(q),f_2(q),f_3 are specified functions,f_3 is due to anisotropy.

Comparing equation (12) and (14) we get

M=exp[-2$\int$ f_1(q)dq]

Again ; M=exp[-$\int$ ŋdt]

So we can get ;M=exp[-2$\int$ f_1(q)dq-$\int$ ŋdt]--------------------------------------(14a)

Q=Mf_3 -------------------------------(14b)

ṽ= -$\int$ Mf_2(q)dq (14c)

Now we consider a new variable ϕ as dϕ =exp[- $\int$ f_1(q)dq]dq (14d)

This transformation can be used to write the Lagrangian in terms of new variable ϕ

It is evident that,$\frac{1}{2}$M $\dot{q}^2$ =$\frac{1}{2}$exp[-$\int$ ŋdt]$\dot{\varphi}^2$----------------(15)

Now,$\frac{d\varphi}{dt}$ = exp[$-\int f_1$ (q)dq]$\frac{dq}{dt}$ =$\dot{\varphi}$

doi:10.20944/preprints202101.0146.v1

$\frac{d\dot{\varphi}}{dt}$ = exp[$-\int f_1$ (q)dq[$\ddot{q}$ $-$f$_1$(q)$\dot{q}^2$]

$\ddot{\varphi}$ = [ŋq̇+f$_3$+ f$_2$]exp[$-\int f_1$ (q)dq --- (16)

(using equation(14))

From (14a) and (14c) we get

ṽ(v,t)=exp[-∫ ŋdt]ṽ(ϕ)----------------------------------(17)

Where ṽ(ϕ) =$-\int f_2$ exp($-\int f_1$ dV) dϕ

By using (15) and (17) in equation (11)we getL=exp[-∫ ŋdt][$\frac{1}{2}\dot{\varphi}^2$- ṽ(ϕ)] +L$_g$ ------------(18)

Comparing (14a) and (13a) we get,

-2f$_1$=$\frac{2-\gamma}{V}$

f$_1$=$-$ ($\frac{2-\gamma}{2V}$) , ŋ= 3ε G/2

Comparing (14c) and (13b) we get

f$_2$= $\frac{3\gamma V\Lambda}{2}$

By using these values we can write,

ϕ=∫ exp(∫ $\frac{2-\gamma}{2V}$dV)--------------------------(19)(using equation 13a)

if Both Λ,γ are both constants then the equation (19) gives us

ϕ= $\frac{2}{4-\gamma}V^{(4-\gamma)/2}$ for $\gamma \neq 0$

We can find the potential as the function of ϕ as

ṽ(ϕ) = - $\int f_2$ (q)exp[-2$\int f_1$ (q)dq]dq

putting the values of f$_1$ and f$_2$ and integrating in terms of V we get ṽ=$\frac{3\gamma\Lambda}{2(\gamma-4)}V^{(4-\gamma)}$--------(20)

Using the value of ϕ we can writeṽ(ϕ) =$\frac{3}{8}\gamma\Lambda$ϕ^2($\gamma - 4$)

Finally we can write the Lagrangian as L= exp[-∫ ŋdt][$\frac{1}{2}\dot{\varphi}^2$- $\frac{3}{8}\gamma\Lambda$ ϕ^2($\gamma - 4$)] +L$_g$---------(21)

Therefore we get the Lagrangian of a damped perturbed harmonic oscillator. Or we can say that it is a perturbed harmonic oscillator with dissipation.

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SECTION- C

If we consider the case of γ=0 ,the equation of state parameter becomes ω =p/ρ = -1,which corresponds to the time of cosmological inflation . But In case of anisotropic model(Bianchi type 1) we do not get exponential expansion of universe as ϕ= ½ V^2 (or ϕ= ½ q^2) for $\gamma = 0$ during at w = -1. Therefore we make some assumptions.

We can assume that during the time of cosmological inflation the expansion of the universe was so rapid that was almost isotropic. One such expanding isotropic model is FRW model. So for γ=0 we can assume the universe to satisfy FRW model(with zero curvature). For which the scalar field is given by

$$\phi = \int \exp\left(\int \left(\frac{3\gamma-2}{2q}\right)dq\right)dq$$

For γ=0, we get ϕ= ln(q) which satisfies the exponential expansion of universe during cosmological inflation. Therefore only if the universe is not highly anisotropic the exponential expansion is valid.

But still we do have some problem that we can not reduce the equation 21 into the FRW limit of scaler field lagrangian and it is because of the viscosity of fluid that generates a term ŋ.So we need to get another approximation to solve this viscosity problem.

PART-II

SECTION-A

This section deals with the bulk cum shearing viscous fluid.

Bianchi type 1 metric is defined as

$$ds^2 = -dt^2 + X^2(t)\, dx^2 + Y^2(t)\, dy^2 + Z^2(t)\, dz^2 \text{ ------------------------------(1)}$$

Einstein Field equation is

$$R_{ab} - \frac{1}{2} R\, g_{ab} + \Lambda g_{ab} = GT_{ab} \qquad \text{---------------------------------(2)}$$

For a fluid with bulk and shearing viscosity the energy momentum tensor is given by

$$T^a{}_b = \rho\, u_b\, u^a + [p-(\varepsilon-(2/3)\eta)\theta-\eta(u^a{}_{;a}+u^b{}_{;b})]\, h_{ab}$$

$h_{ab} = g_{ab} + u_a u_b$ is the projection operator, ε is the coefficient of bulk viscosity, η is the coefficient of shear viscosity, ρ is the density of the cosmic fluid.

θ is the scalar volume expansion

We assume $\bar{P} = [p-(\varepsilon-(2/3)\eta)\theta-\eta(u^a{}_{;a}+u^b{}_{;b})]$

 doi:10.20944/preprints202101.0146.v1

So we can write $T_{ab} = \rho\, u_a u_b + \bar{P} h_{ab}$------------------------(3)

For Bianchi Type 1 the field equations are

$(\dot{X}\dot{Y}/XY) + (\dot{Z}\dot{Y}/ZY) + (\dot{X}\dot{Z}/XZ) = G\rho + \Lambda$---------------------------(4)

$(\ddot{Y}/Y) + (\ddot{Z}/Z) + (\dot{Y}\dot{Z}/YZ) = -G\bar{P} + \Lambda$ ---------------------------(5)

$(\ddot{X}/X) + (\ddot{Z}/Z) + (\dot{Z}\dot{X}/ZX) = -G\bar{P} + \Lambda$ -------------------------(6)

$(\ddot{X}/X) + (\ddot{Y}/Y) + (\dot{X}\dot{Y}/XY) = -G\bar{P} + \Lambda$ --------------------------------(7)

Putting the value of $\bar{P}$ we can rewrite the above equation as

$(\dot{X}\dot{Y}/XY) + (\dot{Z}\dot{Y}/ZY) + (\dot{X}\dot{Z}/XZ) = G\rho + \Lambda$---------------------------(4.1)

$(\ddot{Y}/Y) + (\ddot{Z}/Z) + (\dot{Y}\dot{Z}/YZ) = -Gp + G(\varepsilon-(2/3)\eta)\theta + 2\eta G\dot{Z}/Z + \Lambda$ ----------------------------(5.1)

$(\ddot{X}/X) + (\ddot{Z}/Z) + (\dot{Z}\dot{X}/ZX) == -Gp + G(\varepsilon-(2/3)\eta)\theta + 2\eta G\dot{X}/X + \Lambda$ -------------------------(6.1)

$(\ddot{X}/X) + (\ddot{Y}/Y) + (\dot{X}\dot{Y}/XY) = -Gp + G(\varepsilon-(2/3)\eta)\theta + 2\eta G\,\dot{Y}/Y + \Lambda$ --------------------------------(7.1)

Adding equation (5.1),(6.1) and (7.1) we get

$$2\left(\left(\frac{\ddot{Y}}{Y}\right)+\left(\frac{\ddot{X}}{X}\right)+\left(\frac{\ddot{Z}}{Z}\right)\right)+\left(\left(\frac{\dot{X}\dot{Y}}{XY}\right)+\left(\frac{\dot{Z}\dot{Y}}{ZY}\right)+\left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right) = -3G+3G(\varepsilon-(2/3)\eta)\left(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right) + 2G\eta\left(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right) + 3\Lambda\text{--------------}(8)$$

The equation of state of the matter(cosmic fluid except the cosmological constant) is commonly assumed to be

$p=(\gamma-1)\rho$-------------------------------------(9)

We combine equation (4.1),(8),(9) and use the relation $\bar{P} = [p-(\varepsilon-(2/3)\eta)\theta-\eta(u^a{}_{;a}+u^b{}_{;b})]$ to get

$$\left(\left(\frac{\ddot{Y}}{Y}\right)+\left(\frac{\ddot{X}}{X}\right)+\left(\frac{\ddot{Z}}{Z}\right)\right)+\left(\frac{3\gamma-2}{2}\right)\left(\left(\frac{\dot{X}\dot{Y}}{XY}\right)+\left(\frac{\dot{Z}\dot{Y}}{ZY}\right)+\left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right) - \frac{3G\varepsilon}{2}\left(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right) - \frac{3\gamma\Lambda}{2} = 0 \quad (10)$$

Where the expansion is $\theta = \frac{\dot{V}}{V} = (\dot{X}/X)+(\dot{Y}/Y)+(\dot{Z}/Z)$ and we consider a function V which is defined by the product of three scale factors V=X(t)Y(t)Z(t).

Here in equation (10) we obtain a equation of motion where there is no energy dissipation due to the shearing viscosity of the cosmic fluid.

We consider equation (10) as the starting point, if the dynamical equation for the scale factor can be written as that form ,the present framework is valid. Our aim is to find a Hamiltonian description of equation (10) as the classical equation of motion. As the system is anisotropic let us consider the Lagrangian as

$L= L_v + L_G$-------------------------------- (11)

Where L_g is considered as the lagrangian for the spatial anisotropy which gives shearing

Where $L_v = \frac{1}{2} M(V,t)\dot{V}^2 - \tilde{v}(V,t)$ $\tilde{v}(V,t)$ = Variable Potential

The Variable mass M and potential is a function of both V and t since the system is dissipating due to bulk and shearing viscosity.

$L_g = f(\dot{X}, \dot{Y}, \dot{Z}, X, Y, Z)$

The Euler Lagrangian Equation is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{V}}\right) - \frac{\partial L}{\partial V} = \frac{d}{dt}\left(\frac{\partial \mathrm{Lg}}{\partial \dot{V}}\right) - \frac{\partial \mathrm{Lg}}{\partial V} + \frac{d}{dt}\left(\frac{\partial \mathrm{Lv}}{\partial \dot{V}}\right) - \frac{\partial \mathrm{Lv}}{\partial V} = \frac{d}{dt}\left(\frac{\partial \mathrm{Lv}}{\partial \dot{V}}\right) - \frac{\partial \mathrm{Lv}}{\partial V} + Q \qquad \left(\text{Where } Q = \frac{d}{dt}\left(\frac{\partial \mathrm{Lg}}{\partial \dot{V}}\right) - \frac{\partial \mathrm{Lg}}{\partial V}\right)$$

$$M\ddot{V} + \dot{V}(\partial M/\partial t) - \frac{1}{2}\dot{V}^2(\partial M/\partial V) + (\partial \tilde{v}/\partial V) + Q = 0$$

The Euler Lagrangian equation corresponding to equation (11) is

$$\frac{\ddot{V}}{V} - \frac{1}{2}\frac{\partial lnM}{\partial lnV}\left(\frac{\dot{V}}{V}\right)^2 + \frac{\dot{V}}{V}(\partial lnM/\partial t) + (1/MV)\left(\frac{\partial \tilde{v}}{\partial V}\right) + \left(\frac{Q}{MV}\right) = 0 \text{------------------------(12)}$$

Now, $\frac{\dot{V}}{V} = \frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}$

$$\frac{\ddot{V}}{V} = \left(\left(\frac{\ddot{Y}}{Y}\right) + \left(\frac{\ddot{X}}{X}\right) + \left(\frac{\ddot{Z}}{Z}\right)\right) + 2\left(\left(\frac{\dot{X}\dot{Y}}{XY}\right) + \left(\frac{\dot{Z}\dot{Y}}{ZY}\right) + \left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right)$$

We can rewrite equation (12) after using these equations we get

$$\left(\left(\frac{\ddot{Y}}{Y}\right) + \left(\frac{\ddot{X}}{X}\right) + \left(\frac{\ddot{Z}}{Z}\right)\right) + 2\left(1 - \frac{3}{4}\frac{\partial lnM}{\partial lnV}\right)\left(\left(\frac{\dot{X}\dot{Y}}{XY}\right) + \left(\frac{\dot{Z}\dot{Y}}{ZY}\right) + \left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right) + \frac{Q}{MV} + (1/MV)\left(\frac{\partial \tilde{v}}{\partial V}\right) -$$
$$\frac{1}{2}\frac{\partial lnM}{\partial lnV}\left(\left(\frac{\dot{X}}{X}\right)^2 + \left(\frac{\dot{Y}}{Y}\right)^2 + \left(\frac{\dot{Z}}{Z}\right)^2 - \left(\left(\frac{\dot{X}\dot{Y}}{XY}\right) + \left(\frac{\dot{Z}\dot{Y}}{ZY}\right) + \left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right)\right) + \frac{\partial lnM}{\partial t}\left(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right) = 0 \text{--------------(13)}$$

Comparing equation (10) and (13) we get

$$2\left(1 - \frac{3}{4}\frac{\partial lnM}{\partial lnV}\right) = \left(\frac{3\gamma - 2}{2}\right) \quad \text{and} \quad \ln M = -\frac{3G\varepsilon t}{2}$$

$$M = \exp\left[\int \frac{2-\gamma}{V} dV - \frac{3G\varepsilon\, t}{2}\right] \text{--------------------------} \quad (13a)$$

$$(1/MV)\left(\frac{\partial \tilde{v}}{\partial V}\right) = -\frac{3\gamma\Lambda}{2}$$

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$\tilde{v}= -\frac{3}{2}\int MV\gamma\Lambda dV$----------------------------- (13b)

$\frac{Q}{MV} - \frac{1}{2}\frac{\partial lnM}{\partial lnV}((\frac{\dot{X}}{X})^2+(\frac{\dot{Y}}{Y})^2+(\frac{\dot{Z}}{Z})^2-\left(\left(\frac{\dot{X}\dot{Y}}{XY}\right)+\left(\frac{\dot{Z}\dot{Y}}{ZY}\right)+\left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right))=0$

$Q= \frac{MV}{2}\frac{\partial lnM}{\partial lnV}\sigma^2$-------------------------------(13c)

Where σ is the shearing scalar given by

$\sigma^2=(\frac{\dot{X}}{X})^2+(\frac{\dot{Y}}{Y})^2+(\frac{\dot{Z}}{Z})^2-\left(\left(\frac{\dot{X}\dot{Y}}{XY}\right)+\left(\frac{\dot{Z}\dot{Y}}{ZY}\right)+\left(\frac{\dot{Z}\dot{X}}{ZX}\right)\right)$

From equation (13c) it is evident that we get a shearing in the cosmic fluid due to the anisotropy present in Bianchi type 1 model .

SECTION B(Canonical Transformation)

From our calculation we can get the generalized equation as the Hamiltonian description of the non-linear equation of a damped oscillator as

$\ddot{q} = f_1(q)\dot{q}^2 + \eta\dot{q} + f_2(q) + f_3(q,\dot{q},x,y,z)$ -------------------- ------------ (14)

$f_1(q)$,$f_2(q)$,f_3 are specified functions,f_3 is due to anisotropy.

Comparing equation (12) and (14) we get

$M=\exp[-2\int f_1(q)dq]$

Again ; $M=\exp[-\int \eta dt]$

So we can get ; $M=\exp[-2\int f_1(q)dq-\int \eta dt]$--------------------------------------(14a)

$Q=Mf_3$ -------------------------------(14b)

$\tilde{v}= -\int Mf_2(q)dq$ (14c)

Now we consider a new variable ϕ as $d\phi =\exp[-\int f_1(q)dq]dq$ (14d)

This transformation can be used to write the Lagrangian in terms of new variable ϕ

It is evident that, $\frac{1}{2}M\dot{q}^2 = \frac{1}{2}\exp[-\int \eta dt]\dot{\varphi}^2$ ----------------(15)

Now,$\frac{d\varphi}{dt} = \exp[-\int f_1(q)dq]\frac{dq}{dt} = \dot{\varphi}$

$\frac{d\dot{\varphi}}{dt} = \exp[-\int f_1(q)dq\,[\ddot{q} - f_1(q)\dot{q}^2]$

$\ddot{\varphi}$ = [ŋ $\dot{q}$+f_3+ f_2] exp[$-\int f_1$ (q)dq -- (16)

(using equation(14))

From (14a) and (14c) we get

ṽ(v,t)= exp[-∫ ŋdt] ṽ(ϕ) ------------------------------------(17)

Where ṽ(ϕ) =$-\int f_2$ exp($-\int f_1$ dV) dϕ

By using (15) and (17) in equation (11)we get L=exp[-∫ ŋdt] [$\frac{1}{2}\dot{\varphi}^2$- ṽ(ϕ)] + L_g ------------(18)

Comparing (14a) and (13a) we get,

-2f_1=$\frac{2-\gamma}{V}$

f_1=$-$ ($\frac{2-\gamma}{2V}$) , η= 3ε G/2

Comparing (14c) and (13b) we get

f_2= $\frac{3\gamma V\Lambda}{2}$

By using these values we can write,

ϕ=∫ exp(∫ $\frac{2-\gamma}{2V}$dV)--------------------------(19)(using equation 13a)

if Both Λ,γ are both constants then the equation (19) gives us

ϕ= $\frac{2}{4-\gamma}V^{(4-\gamma)/2}$ for $\gamma \neq 0$

We can find the potential as the function of ϕ as

ṽ(ϕ) = - $\int f_2$ (q)exp [-2$\int f_1$ (q)dq]dq

putting the values of f_1 and f_2 and integrating in terms of V we get ṽ = $\frac{3\gamma\Lambda}{2(\gamma-4)}V^{(4-\gamma)}$------(20)

Using the value of ϕ we can write ṽ(ϕ) =$\frac{3}{8}\gamma\Lambda$ϕ$^2(\gamma-4)$

Finally we can write the Lagrangian as L= exp[-∫ ŋdt][$\frac{1}{2}\dot{\varphi}^2$ - $\frac{3}{8}\gamma\Lambda$ ϕ$^2(\gamma-4)$] + L_g---------(21)

Therefore we get the Lagrangian of a damped perturbed harmonic oscillator. Or we can say that it is a perturbed harmonic oscillator with dissipation which is the result of dissipating cosmic fluid.

SECTION- C

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If we consider the case of γ=0 ,the equation of state parameter becomes ω =p/ρ = -1,which corresponds to the time of cosmological inflation .In case of anisotropic model(Bianchi type 1) we don't get exponential expansion of universe during cosmological inflation. So we make certain assumptions.

We can assume that during the time of cosmological inflation the expansion of the universe was so rapid that it was almost isotropic. One such expanding isotropic model is FRW model. So for γ=0 we can assume the universe to satisfy FRW model(with zero curvature). For which the scalar field is given by

$$\phi = \int \exp\left(\int \left(\frac{3\gamma-2}{2q}\right)dq\right)dq$$

For γ=0, we get ϕ= ln(q) which satisfies the exponential expansion of universe during cosmological inflation.

From here we can conclude that exponential expansion of universe is not possible in case of highly anisotropic universe.

Here also we are facing a problem to get the FRW limit in scaler field lagrangian due to the bulk viscosity.

PART-III

This part is contained with the newly faced cosmological problem i.e viscous fluid problem and it's solution.In the last two part we observed that the bulk viscosity of the cosmic fluid gives a considerable effect in the scaler field lagrangian when we are trying to use the special type of canonical transformation used here.Although from the equation 10 in part II we see that the shearing viscosity does not have any direct effect on the cosmic evolution for bianchi type I model and that is why after this equation the whole part of part II becomes equivalent to part I.But in equation 21 we observe a huge effect on the lagrangian that says rapid decrease in lagrangian in presence of high value of bulk viscosity.

We know that

η= 3ε G/2 ; where η is a coefficient (not shearing coefficient) and ε is the coefficient of bulk viscosity.

So we see that the term η is dependent upon both G and coefficient of bulk viscosity.If the coefficient of bulk viscosity will be high in value then we need to get any approximation so that the term exp[-$\int$ ηdt] will become unit or in other word we have to make η -> 0.

So there is only one way to do so is to make G very small.

Now from the reference 3 we get that for low range of anisotropy if we try to solve the friedman equations we will get the scale factor as q=$q_0 t^B$ where B and q_0 are some constant. Using this we can see that the hubble paremeter will vary inversely with time.same way we can say that the cosmological constant will vary inversely with time as we know from friedman equation that

$\Lambda = 3a\frac{\ddot{q}}{q} + b\frac{\dot{q}}{q}$; Here a and b are constants; Now if we consider a coupling of gravitational field with the vaccum field then we can say that the decrease of vaccum field will give the increment of gravitational field.So G should increase with time.In other word we can say that the value of G was too low at the time of inflation that it is present.The present value of G is of the order of 10^{-11}.So we may say that the value of G was negligible as compared to present value and thus the effect of bulk viscosity will become negligible at the early of inflation.Thus this viscous fluid problem has been resolved.

CONCLUSION-

- For Bianchi type I model for bulk and shearing viscous fluid we are getting a motion which has expansion and shearing.
- As there is no off diagonal matric in the line element for bianchi type I model, the total shear of the system contributes to the volume expansion of the system. So in this model there is no dissipation of total energy due to shearing viscosity.But It will not happened for other types of bianchi classification.
- We have made the necessary assumption to express Bianchi I model in terms of a body of variable mass which is moving in a variable potential.
- Due to viscosity , we obtain a Lagrangian with a time dependent term which show damped motion and cause dissipation of energy during inflation.
- The shearing do not modify the potential in which the body is moving, instead it gives a shear in the body.
- At the end of the cosmological expansion the Hubble scale decreases to a very small value which causes the dissipation in the equation of motion for FRW model to get suppressed. But in our case i.e for bulk viscous fluid the dissipative term remains dominant and it causes the dissipation of potential energy. So if the bulk viscosity is very high , the expansion of the universe will be damped in much faster time. So we may assume that the cosmic fluid of the universe is not so viscous.
- In case of non-viscous fluid in Bianchi Type I , the Lagrangian obtained is same as that in quintessence model except a perturbation which arise due to anisotropy.
- In the very end in part III we have resolved the viscous fluid problem that we faced from the part I and II.Here we have shown that G should vary with time to resolve this viscous fluid problem.

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