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On Structural Identifiability of Decentralised Systems

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Abstract. The identifiability problem nonlinear decentralised systems (NDS) were not considered. Synthesis of mathematical models for NDS having a complex structure is an urgent problem. The presence of internal relationships and nonlinearities in NDS complicates the identifiability estimation. Therefore, models with uncertainty are used to design control systems for NDS. To describe the processes in NDS, it is necessary to apply adequate models based on verifying their identifiability. We propose an approach for assessing the local structural identifiability of nonlinear decentralised systems. Conditions of local parametric identifiability are obtained as functional constraints for parameters and nonlinearity.The effect of the constant excitation of experimental data on the S-synchronizability of DS is shown. The S-synchronizability condition fulfilment guarantees the structural identifiability estimate of DS with nonlinearity belonging to a certain sector. We introduce the S-congruence concept and propose a method for estimating the S-synchronizability of an input. Geometric structures and criteria are used to assess NDS identifiability. The simulation results are presented.

Keywords: identifiability; geometric structure; decentralised system; S-synchronizability; constant excitation; S-congruence; quadratic coupling; nonlinearity

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1. Introduction

Increasing the requirements for the DS mathematical description requires solving the structural identifiability (SI) problem. A lot of attention is paid to SI issues. This is relevant for biological systems that are described by models with many parameters [1,2]. Various problems of SI are discussed in [3–9]. The identifiability problem is parametric, and its solution is based on ensuring the completeness (rank) of the information matrix. Its solution is based on Taylor series expansion [10], series generation method, similarity transformation [11] and differential algebra. In identification theory, this requirement is equivalent to the condition of constant excitation [5].

The parametric paradigm [11] as the main research in the identification is being transformed into nonlinear systems. The issues of parametric identifiability (PI) of nonlinear systems are considered in many works (see, for example, [12-15 et al.]). Sensitivity methods are used [12], conditions for local parametric identifiability are got based on experimental data [13] analysis. A study of approaches used to assess the identifiability of biological models is given in [14]. A critical analysis of the data requirements used in parametric identifiability problems is considered in [15]. The study of global, local, structural and practical identifiability is considered in [1, 5, etc.].

There are several interpretations of the structural identifiability concept. Various authors understand the SI as the possibility of an unambiguous estimation of system parameters. SI [16–18] is interpreted as the possibility of an unambiguous theoretical assessment of model parameters based on given data. In [19], SI is understood as the a priori identifiability (AI) of a model with the set structure, since identifiability properties are based on the transformation of the model equation (such transformations are aimed at the possibility of estimating model parameters, rather than choosing the model structure). Most of SI approaches are a priori method. The analysis of existing approaches to AI is given in [19,20]. In this paper, SI is understood as an opportunity to evaluate the DS structure.

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The identifiability of decentralised systems (DS) has not been studied. In principle, DS is a multivariable system (MS), whose SI has been studied by various authors. The main difference between DS and MS is internal connections, which complicates the study of SI.

In this paper, we consider the SI problem of nonlinear decentralised systems. The concepts and definitions of parametric identifiability (PI) are given. We consider conditions of constant excitation and S-synchronizability, which guarantee the SI of the NDS. We study the PI of decentralised systems with nonlinearities satisfying the sector condition and obtain functional conditions that ensure local PI. Conditions for the relationship between subsystems that guarantee the identifiability of the nonlinear part of the DS are considered. Geometric structures that are the basis for SI analysis are presented. In the final part of the paper, we describe the approach used to estimate the structural identifiability of NDS. In the final part of the article, we consider the approach used to evaluate the NDS structural identifiability and present simulation results.

2. Problem Statement

Consider a system comprising m interconnected subsystems

$$S_i : \begin{cases} \dot{X}_i = A_i X_i + B_i u_i + \sum\limits_{j=1, j\neq i}^{m} \bar{A}_{ij} X_j + F_i\left(X_i\right), \\ Y_i = C_i X_i, \end{cases} \tag{1}$$

where $X_i \in \mathbb{R}^{n_i}$, $Y_i \in \mathbb{R}^{q_i}$ are vectors of the state and output of S_i-subsystem, $u_i \in \mathbb{R}$ is control, $i = \overline{1,m}$, $\sum\limits_{i=1}^{m} n_i = n$. Coefficients of the matrices $A_i \in \mathbb{R}^{n_i \times n_i}, B_i \in \mathbb{R}^{n_i}, \bar{A}_{ij} \in \mathbb{R}^{n_i \times n_j}$ are unknown, $C_i \in \mathbb{R}^{q_i \times n_i}$. The matrix $\bar{A}_{ij}$ reflects the mutual influence of the S_j subsystem. $F_i\left(X_i\right) \in \mathbb{R}^{n_i}$ consider the nonlinear state of the S_i-subsystem, and A_i is Hurwitz matrix (stable).

Assumption 1. $F_i\left(X_i\right)$ *belongs to the class*

$$\mathscr{N}_F\left(\pi_1, \pi_2\right) = \left\{F(X) \in \mathbb{R}^n : \pi_1 X \leq F(X) \leq \pi_2 X, F(0) = 0\right\} \tag{2}$$

and satisfies the quadratic condition

$$\left(\pi_2 X - F\left(X\right)\right)^T \left(F\left(X\right) - \pi_1 X\right) \geq 0, \tag{3}$$

Information set for subsystems S_i

$$\mathbb{I}_{o,i} = \left\{X_i(t),\ u_i(t),\ X_j(t),\ t \in \mathbb{J} = \left[t_0, t_k\right]\right\}.$$

Mathematical model for

$$\dot{\hat{X}}_i = K_i\left(\hat{X}_i - X_i\right) + \hat{A}_i X_i + \hat{B}_i u_i + \sum\limits_{j=1, j\neq i}^{m} \hat{\bar{A}}_{ij} X_j + \hat{F}_i\left(X_i\right), \tag{4}$$

where $K_i \in \mathscr{H}$ is Hurwitz matrix with known parameters (reference model); $\hat{A}_i$, $\hat{B}_i$, $\hat{\bar{A}}_{ij}$ are tuning matrices of corresponding dimensions, $\hat{F}_i$ is a priori given nonlinear vector function.

Problem: based on the analysis $\mathbb{I}_{o,i}$, determine the SI conditions for the S_i-subsystem.

3. Some Concepts and Definitions

Consider a dynamic system

$$\Sigma(P):\begin{cases}\dot{X}(t)=F\left(X(t),P\right)+G\left(X(t),P\right)U(t),\\ Y\left(t,P\right)=H\left(X(t),P\right),\end{cases}$$

where $X\in\mathbb{R}^n$ is state vector, $U\in\mathbb{R}^m$ is input vector (control), $Y\in\mathbb{R}^h$ is output, $X(t_0)=X_0(P)$, $X_0(P)$ is the initial condition, $P\in\mathbb{R}^q$ is vector of unknown parameters.

Definition 1 [21]. The parameter $P\in\mathbb{R}^q$ is structurally globally identifiable if for almost any $P^*\in\mathbb{R}^q$

$$\Sigma(P)=\Sigma(P^*)\Rightarrow P=P^*.$$

Definition 2 [21]. Let there be a neighbourhood $\mathscr{V}(P)$ such that

$$\left(P^*\in\mathscr{V}(P)\right)\&\left(\Sigma(P)=\Sigma(P^*)\Rightarrow P=P^*\right).$$

Then P is structurally locally identifiable for any $P^*\in\mathbb{R}^q$.

If the neighbourhood of $\mathscr{V}(P)$ does not exist, then the parameter P is called structurally locally unidentifiable.

Definition 3 [21]. The model $\mathscr{M}$ structure is globally identifiable at point K^* if

$$\mathscr{M}\left(K\right)=\mathscr{M}\left(K^*\right),\ \ K\in\mathbb{D}_{\mathscr{M}}\Rightarrow K=K^*,$$

where $\mathbb{D}_{\mathscr{M}}\subset\mathbb{R}^r$ is a connected open subset of parameters.

Fulfilment of definition 3 conditions guarantees the identifiability of the model $\mathscr{M}$ at the point.

4. Condition of Constant Excitation and S-Synchronizability

A priori identifiability is based on the analysis of rank conditions for the information matrix. In adaptive systems, conditions based on the constant excitation (CE) of set $\mathbb{I}_{o,i}$ elements are used to analyse properties of the identification system.

Definition 4. The vector $X_i(t)\in\mathbb{R}^{n_i}$ is continuously excited if

$$\mathscr{E}\mathscr{C}_{\underline{\alpha}_{X_i},\bar{\alpha}_{X_i}}:\underline{\alpha}_{X_i}I_{n_i}\leq X_iX_i^T(t)\leq\bar{\alpha}_{X_i}I_{n_i}\ \ \forall t\in\left[t_0,t_0+T\right],\tag{5}$$

where $I_{n_i}\in\mathbb{R}^{n_i\times n_i}$ is the identity ma, $\underline{\alpha}_{X_i},\bar{\alpha}_{X_i}$ are positive numbers, $T>0$.

We will write about the CE condition as $X_i(t)\in\mathscr{E}\mathscr{C}_{\underline{\alpha}_{X_i},\bar{\alpha}_{X_i}}$. If condition (5) for $X_i(t)$ is not satisfied, then we will write $X_i(t)\notin\mathscr{E}\mathscr{C}_{\underline{\alpha}_{X_i},\bar{\alpha}_{X_i}}$ or $X_i(t)\notin\mathscr{E}\mathscr{C}$

Since DS (1) is nonlinear, then we represent condition (5) as

$$\mathscr{E}\mathscr{C}^S_{\underline{\alpha}_{X_i},\bar{\alpha}_{X_i}}:\left(\underline{\alpha}_{X_i}I\leq X_iX_i^T(t)\leq\bar{\alpha}_{X_i}I\right)\&\left(\Omega_{X_i}(\omega)\subseteq\Omega^S_{X_i}(\omega)\right),$$

where $\Omega_{X_i}(\omega)$ is the set of frequencies for X_i; $\Omega^S_{X_i}(\omega)$ is the set of acceptable frequencies for X_i, guaranteeing S-synchronizability. Next, we will denote the CE property as $\mathscr{CE}_{\underline{\alpha}_{X_i},\bar{\alpha}_{X_i}}$, assuming that it guarantees $\mathscr{CE}^S_{\underline{\alpha}_{X_i},\bar{\alpha}_{X_i}}$..

The equation for the S_i-subsystem identification error $E_i = \hat{X}_i - X_i$

$$\dot{E}_i = K_i E_i + \Delta A_i X_i + \Delta B_i u_i + \sum_{j=1,\, j\neq i}^{M} \Delta \bar{A}_{ij} X_j + \Delta F_i\left(X_i\right), \tag{6}$$

where $\Delta A_i = \hat{A}_i - A_i, \Delta B_i = \hat{B}_i - B_i, \Delta\bar{A}_{ij} = \hat{\bar{A}}_{ij} - \bar{A}_{ij}, \Delta F_i = \hat{F}_i - F_i$ are parametric residuals.

Definition 5. The input $u_i(t)$ will be called S-synchronising and written $u_i \in \mathrm{S}$ if it reflects all the nonlinear properties of the S_i-subsystem.

5. PI Conditions

We consider the DS identifiability based on analysis $\mathbb{I}_{o,i}$. This is practical identifiability.

Assumption 2. Let 1) $A_i \in \mathscr{H}$; 2) $\operatorname{rank}\left[C_i \ C_i A_i \ C_i A_i^2 \ \ldots \ C_i A_i^{n_i-1}\right]^T = n_i$.

Definition 6 [22]. Let assumption 2 hold. Then the pair (A,C) is called fully recoverable, and the S_i-subsystem is fully recoverable.

Definition 7 [22]. Let the pair (A,C) be fully recoverable. Then subsystem S_i will be locally parametrically identifiable on the set $\mathbb{I}_{o,i}$, if estimates of the model (4) parameters belong to some limited region $\mathscr{G}_A \subset \mathbb{R}^q$ of the parametric space $\mathscr{P}_{S_i}$.

We will consider the results [23], which will be used.

Lemma 1. If the nonlinearity $F(X) \in \mathbb{R}^n$ belongs to the class $F(X) \in \mathscr{N}_F\left(\pi_1,\pi_2\right)$, then

$$\left\|F\left(X\right)\right\|^2 \leq \eta\bar{\alpha}_X, \tag{7}$$

where $X \in \mathbb{R}^n$, $\eta = \eta\left(\pi_1,\pi_2\right) > 0, \bar{\alpha}_X = \bar{\alpha}_X\left(X\right) > 0$.

Lemma 2. If the conditions of Lemma 1 are satisfied, then

$$\Delta F^T \Delta F \leq 2\eta\bar{\alpha}_X + \delta_F,$$

where $\eta = 2\bar{\pi} + \pi^2$, $\bar{\pi} = \pi_1\pi_2$, $\pi = \pi_1 + \pi_2$, $\delta_F > 0$.

Consider the system (6) and the Lyapunov function (LF) $V_i\left(E_i\right) = 0.5E_i^T R_i E_i$, where $R_i = R_i^T > 0$ is the positive symmetric matrix. Let $\left\|\Delta A_i\right\| = \sqrt{\operatorname{tr}(\Delta A_i^T \Delta A_i)}$, where $\|\cdot\|$ is the matrix norm, tr is the trace of the matrix.

The following conditions for the local identifiability of subsystem S_i are valid in the parametric space. They are a modification of Theorem 1 [23].

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Theorem 1. Let *1)* $A_i \in \mathscr{H}$ *; 2)* $X_i(t) \in \mathscr{EC}_{\underline{\alpha}_{X_i},\bar{\alpha}_{X_i}}$ *,* $X_j(t) \in \mathscr{EC}_{\underline{\alpha}_{X_j},\bar{\alpha}_{X_j}}$ $u_i(t) \in \mathscr{EC}_{\underline{\alpha}_{u_i},\bar{\alpha}_{u_i}}$ *; 3) Lemma 1, 2 conditions of are fulfilled for* $F_i\left(X_i\right)$*; 4) assumption 2 holds. Then the system (1) is locally parametrically identifiable on the set* $\mathbb{I}_{o,i}$ *if*

$$2\left(\bar{\alpha}_{X_i}\left\|\Delta A_i\right\|^2+\bar{\alpha}_{u_i}\left\|\Delta B_i\right\|^2+\sum_{j=1,\, j\neq i}^{m}\bar{\alpha}_{X_j}\left\|\Delta \bar{A}_{ij}\right\|^2+2\eta\bar{\alpha}_{X_i}+\delta_{F_i}\right)\leq \bar{\lambda}_i V_i, \tag{8}$$

where $\bar{\lambda}_i=\lambda_i-k_i$, $\lambda_i>0$ *is the minimum eigenvalue of the matrix* Q_i, $k_i>0$, $K_iR_i+K_i^TR_i=-Q_i$, Q_i *is the positive symmetric matrix,* $\eta=2\bar{\pi}+\pi^2$, $\pi=\pi_1+\pi_2$, $\bar{\pi}=\pi_1\pi_2$, $\delta_{F_i}\geq 0$.

Corollary of Theorem 1. *Let the conditions of Theorem 1 be fulfilled. Then the nonlinearity* F_i *is locally structurally identifiable in the sector*

$$\mathscr{S}_{X_i}\left(\pi_1,\pi_2\right)=\left\{X_i\in\mathbb{R}^{n_i}:\pi_1X_i\leq F_i(X_i)\leq\pi_2X_i\right\},$$

if

$$\left\|\Delta F_i\right\|^2\leq 0.25\bar{\lambda}_iV_i+z_i, \tag{9}$$

where $z_i\geq 0$.

The proof of the corollary from Theorem 1 is given in Appendix A.

We see that the S_i-subsystem will be parametrically identifiable if the nonlinearity dominates on the effects of the S_j subsystem.

The corollary from Theorem 1 gives the conditions under which the local structural identifiability is possible for the nonlinear part of subsystem S_i in sector $\mathscr{S}_{X_i}\left(\pi_1,\pi_2\right)$.

Definition 8. If conditions (8), (9) are fulfilled, then subsystem S_i is locally structurally identifiable at the parametric level.

Remark 1. Theorem 1 provides conditions for the local SI of the system R1 in the form of estimates for the model (2) parameters belonging to the limited region of parametric space.

Consider the input-output information set

$$\mathbb{I}_{o,Y_i}=\left\{Y_i(t),\, u_i(t),\, Y_j(t),\, t\in\mathbb{J}=\left[t_0,t_k\right]\right\}.$$

For subsystem S_i, we get the representation [23]

$$\dot{Y}_i=A_{\#,i}Y_i+B_{\#,i}u_i+\sum_{j=1,\, j\neq i}^{m}\bar{A}_{\#,ij}Y_j+\tilde{C}_i^{\#}F_i\left(\tilde{C}_iY_i\right), \tag{10}$$

where $\tilde{C}_i=\left(C_i^TC_i\right)^{\#}C_i^T$, $A_{\#,i}=\tilde{C}_i^{\#}A_i\tilde{C}_i$, $B_{\#,i}=\tilde{C}_i^{\#}B_i$, $\bar{A}_{\#,ij}=\tilde{C}_i^{\#}\bar{A}_{ij}\tilde{C}_j$, # is the sign of the pseudo-conversion of the matrix.

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Model for the system (10)

$$\dot{\hat{Y}}_i = K_{\#,i}\left(\hat{Y}_i - Y_i\right) + \hat{A}_{\#,i}Y_i + \hat{B}_{\#,i}u_i + \sum_{j=1,\, j\neq i}^{m} \hat{\bar{A}}_{\#,ij}Y_j + \hat{F}_{\#,i}\left(\tilde{C}_i Y_i\right) \tag{11}$$

and the equation for the error

$$\dot{E}_{\#,i} = K_{\#,i}E_{\#,i} + \Delta A_{\#,i}Y_i + \Delta\hat{B}_{\#,i}u_i + \sum_{j=1,\, j\neq i}^{m} \Delta\hat{\bar{A}}_{\#,ij}Y_j + \Delta F_{\#,i}\left(\tilde{C}_i Y_i\right),$$

where $\hat{F}_{\#,i} = \tilde{C}_i^{\#}\hat{F}_i$, $\Delta A_{\#,i} = \hat{A}_{\#,i} - A_{\#,i}, \Delta B_{\#,i} = \hat{B}_{\#,i} - B_{\#,i}, \Delta\bar{A}_{\#,ij} = \hat{\bar{A}}_{\#,ij} - \bar{A}_{\#,ij}, \Delta F_{\#,i} = \hat{F}_{\#,i} - F_{\#,i}$, $\bar{\lambda}_{\#,i} > 0$. The $K_{\#,i} \in \mathscr{H}$ matrix is selected to ensure the stability of the identification process.

Definition 9. If the conditions of assumption 2 are fulfilled and the pair $\left(A_{\#,i}, \tilde{C}_i\right)$ is fully recoverable and the pair $\left(A_{\#,i}, \bar{A}_{\#,ij}\right)$ is stabilised, then subsystem (10) is called fully recoverable.

Definition 10. Let assumption 2 hold. Then 1) the system (10) is detectable if the subspace of non-recoverable states is contained in the subspace of stable states; 2) the pair $\left(A_i, C_i\right)$ is detectable.

Introduce the error $E_{\#,i} = \hat{Y}_i - Y_i$ and ФЛ $V_{\#,i}\left(E_{\#,i}\right) = 0.5E_{\#,i}^T R_i E_{\#,i}$.

Theorem 2. *Le: 1)* $A_{\#,i} \in \mathscr{H}$; *2)* $Y_i(t) \in \mathscr{EC}_{\underline{\alpha}_{Y_i}, \bar{\alpha}_{Y_i}}$, $Y_j(t) \in \mathscr{EC}_{\underline{\alpha}_{Y_j}, \bar{\alpha}_{Y_j}}$ $u_i(t) \in \mathscr{EC}_{\underline{\alpha}_{u_i}, \bar{\alpha}_{u_i}}$; *3) Lemma 1, 2 conditions of are fulfilled for* $F_i\left(X_i\right)$; *4) the system (10) is observable, recoverable, and stabilised. Then the system (10) is locally structurally parametrically identifiable on the set G2 if*

$$2\left(\bar{\alpha}_{Y_i}\left\|\Delta A_{\#,i}\right\|^2 + \bar{\alpha}_{u_i}\left\|\Delta B_{\#.i}\right\|^2 + \sum_{j=1,\, j\neq i}^{m} \bar{\alpha}_{Y_j}\left\|\Delta\bar{A}_{\#,ij}\right\|^2 + 2\eta\bar{\alpha}_{Y_i} + \delta_{F_{\#,i}}\right) \leq \bar{\lambda}_{\#,i}V_{\#,i}, \tag{12}$$

$$\left\|\Delta F_{\#,i}\right\|^2 \leq 0.25\bar{\lambda}_{\#,i}V_{\#,i} + z_{\#,i}, \tag{13}$$

where $\bar{\lambda}_{\#,i} > 0,\ z_{\#,i} > 0.$

Theorem 2 is proved similarly to Theorem 1.

So, conditions (12), (13) guarantee the local parametric identifiability of the system (1) in the output space.

Remark 2. In Theorems 1 and 2, the concept of the "structurally identifiable system" is understood in a parametric sense. This follows from estimates for nonlinearity obtained in Lemmas 1 and 2. The concept of structural refers to non-linearity, which must belong to class $\mathscr{N}_F$ and satisfy the functional constraint (13).

Consider the SI problem. Apply the approach based on the special class analysis of geometric frameworks [24].

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6. Geometric Frameworks

Consider the system S_i. Let $u_i \in \mathscr{EC}_{\underline{\alpha}_i,\bar{\alpha}_i}$ and $y_{i,k} \in Y_i$ is k th element of the output vector. Calculate the derivative $y_{i,k}$ and denote it as $\dot{y}_{i,k}$. Construct a phase portrait $S_{i,k}$ for the S_i-system in the space $\left(y_{i,k}, \dot{y}_{i,k}\right)$. Consider structures $S_{u_i,y_{i,k}}$ $\forall k = 1, q_i$ described by the functions $\gamma_{i,k}: u_i \to y_{i,k}$. Introduce the model for $\dot{y}_{i,k}$:

$$\dot{\hat{y}}_{i,k} = a_{i,k,0} + a_{i,k,1}u_i + a_{i,k,2}y_{i,j} \quad \forall j \geq 1 \tag{14}$$

on the set $\mathbb{I}_{o,y_{i,k}} = \left\{y_{i,k}(t),\, u_i(t),\, y_{j,l}(t),\, t \in \mathbb{J} = \left[t_0, t_k\right]\right\}$ and calculate the error $e_{i,k} = \dot{\hat{y}}_{i,k} - \dot{y}_{i,k}$, where $a_{i,k,0}, a_{i,k,1}, a_{i,k,2}$ are numbers determined using the least squares method.

Define the structures $S_{u_{i,k}y_{i,k}}$ and $S_{u_{i,k}\dot{y}_{i,k}}$ described by the mappings $\gamma_{y_{i,k}}: u_i \to y_{i,k}$ and $\gamma_{\dot{y}_{i,k}}: u_i \to \dot{y}_{i,k}$.

Remark 3. The model (14) structure can change. The number of independent variables on the right-hand side (14) depends on the properties of the subsystem S_i. The choice of the model (14) structure is based on the iterative immersion method [24].

For the system S_i, we construct the geometric framework (GF) $S_{e_iy_i}$ described by the function $\gamma_{e,y_{i,k}}: y_{i,k} \to e_{i,k}$. Then, the subsystem S_i can be represented as:

$$\mathscr{C}_i: \begin{cases} S_{y,i}: \begin{cases} \dot{\tilde{X}}_i = A_i\tilde{X}_i + \sum\limits_{j=1,\, j\neq i}^{m} \bar{A}_{ij}\tilde{X}_j + G_i, \\ \tilde{Y}_i = C_i\tilde{X}_i, \end{cases} \\ S_{\varphi,i,k}: e_{i,k} = f\left(y_{i,k}, y_j\right), \end{cases} \quad \forall (k,j) \geq 1, \tag{15}$$

where $\tilde{X}_i \in \mathbb{R}^{n_i}$ is a variable describing the general solution of the system (1); $G_i \in \mathbb{R}^{n_i}$ is a limited disturbance that appears as a result of the allocation of the variable $e_{i,k} \in \mathbb{R}2$.

7. SI of Subsystem S_i

Let $u_i \in \mathscr{EC}$ and $A_i \in \mathscr{H}$.

Definition 10 [24]. The input $u_i(t) \in \mathbb{U}_{\mathrm{S},i} \subseteq \mathbb{U}_i$ is an S-synchronising for system (1) if the domain $\mathscr{D}_y$ of the structure $S_{e_iy_i}$ has a maximum diameter D_{y_i} on the set $\left\{y_{i,k}(t),\, t \in \mathbb{J}\right\}$, where $\mathbb{U}_{\mathrm{S},i}$ is multiple synchronization inputs having the CE property; $\mathbb{U}_i$ is the set of acceptable inputs for the S_i-subsystem.

If the input is S-synchronizing, then we will write $u_i \in \mathrm{S}$. We understand the diameter D_{y_i} as the maximum distance between two points $S_{e_iy_i}$.

Consider a system $S_{\varphi,i,k}$ and frameworks $S_{e_iy_i}$, $S_{u_{i,k}y_{i,k}}$ and $S_{u_{i,k}\dot{y}_{i,k}}$.

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Definition 11. Frameworks $\mathcal{S}_{e_i y_i}$ and $\mathcal{S}_{u_{i,k}\dot{y}_{i,k}}$ are S-congruent if their structural features are similar.

Definition 12. If $\mathcal{S}_{ey}$, and $\mathcal{S}_{u_{i,k}\dot{y}_{i,k}}$ are S-congruent, then the input $u_i(t)$ is S-synchronising and guarantees the S-synchronizability of the system $S_{\varphi,i,k}$.

Definition 12 provides a criterion for selecting the required input having the specified properties and the property $\mathcal{EC}_{\underline{\alpha}_{u_i},\bar{\alpha}_{u_i}}$ is valid for it.

As shown in [25], $u_i \in \mathrm{S}$ ($u_i \triangleq u_{h,i}$) guarantees the SI of NDS, where the index h is a sign of identifiability.

There may be some subset $u_{h,k}(t) \in \mathbb{U}_{\mathrm{S},k} \subseteq \mathbb{U}_k$ ($k \geq 1$) whose elements have the property of S-synchronizability. The structure $\mathcal{S}_{ey,k}\left(u_{h,k}\right)$ with a diameter $D_{y,k}$ corresponds to each $u_{h,k}(t)$. Since $u_{h,k}(t) \in \mathrm{S}$, the diameters $D_{y,k}$ will have the $\mathcal{d}_{h,\Sigma}$-optimality property [25].

Let the hypothetical framework $\mathcal{S}_{ey}$ (structure $\mathcal{S}_{ey}^{ref}$) of the system $S_{\varphi,i,k}$ have diameter $\mathcal{d}_{h,\Sigma}$. The reference framework $\mathcal{S}_{ey}^{ref}$ is the structure $\mathcal{S}_{ey}$, reflecting all the properties of the function $f(\cdot)$.

Designate the diameter $D_y\left(\mathcal{S}_{ey}^{ref}\right)$ as D_y^{ref}. D_y^{ref} exists for the $S_{\varphi,i,k}$-system having the S-synchronising input.

If $\mathcal{S}_{ey} \cong \mathcal{S}_{ey}^{ref}$, then $\left|D_y - D_y^{ref}\right| \leq \varepsilon_y$, where $\varepsilon_y \geq 0$, $\cong$ is a proximity sign. The elements of the subset $\mathbb{U}_{\mathrm{S}}$ have the property

$$\left|D_y\left(\mathcal{S}_{ey}\left(u(t)\big|_{u\in\mathbb{U}_{\mathrm{S}}}\right)\right) - D_y^{ref}\right| \leq \varepsilon_y . \tag{16}$$

Definition 13 [25]. The framework $\mathcal{S}_{ey,i}$ has the $\mathcal{d}_{h,\Sigma}$-optimality property on the set $\mathbb{U}_h$ if there exists $\varepsilon_{\Sigma_i} > 0$ such that $\left|\mathcal{d}_{h,\Sigma} - D_{y,i}\right| \leq \varepsilon_{\Sigma_i}$ $\forall i = \overline{1,\#\mathbb{U}_h}$, where $\#\mathbb{U}_h$ is cardinality of set $\mathbb{U}_h$.

Definition 14 [25]. If the subset of inputs $\left\{u_{h,k}(t)\right\} = \mathbb{U}_h \subset \mathbb{U}$ ($k \geq 1$) the elements of which are $u_{h,k}(t) \in \mathrm{S}$ and their corresponding frameworks $\mathcal{S}_{ey,k}\left(u_{h,k}\right)$ have the of $\mathcal{d}_{h,\Sigma}$-optimality property, then frameworks $\mathcal{S}_{ey,k}\left(u_{h,k}\right)$ are structurally indistinguishable on sets $\mathbb{U}_h$ and $\mathbb{J}_u\left(u(t) = u_{h,k}(t)\right)$.

It follows from definitions 13 and 14 that if there is the set $\mathbb{U}_h$, then the h_{δ_h}-identifiability (SI) estimate is obtained for any input $u(t) \subset \mathbb{U}_h$.

Definition 15 [25]. Frameworks $\mathcal{S}_{ey,i}\left(u_i \notin \mathbb{U}_{\mathrm{S}}\right)$ that do not have the $\mathcal{d}_{h,\Sigma}$-optimality property are locally structurally unidentifiable (SNI) on the set $\mathbb{U}_h$.

Remark 4. The **framework** $\mathcal{S}_{ey,i}\left(u_i \notin \mathbb{U}_{\mathrm{S}}\right)$, which is SNI on the set $\mathbb{U}_h$, defines the class $\mathcal{N}_\varphi^{N} \not\subset \mathcal{N}_F\left(\pi_1, \pi_2\right)$. Here, $\mathcal{S}_{ey,i}\left(u_i \notin \mathbb{U}_{\mathrm{S}}\right)$ belongs to the insignificant structures $\mathcal{NC}_{ey}$ for which the $\mathcal{d}_{h,\Sigma}$ -optimality condition is not fulfilled.

Definition 16. Frameworks $S_{ey,k}(u_{h,k})$ defining the class $\mathscr{F}_f$ for the function $f(\cdot)$ and having the $d_{h,\Sigma}$-optimality property are locally structurally identifiable on the set $\mathbb{U}_h$.

Notation:

(a) $S_{ey,i}^{\Sigma}$ is the framework $S_{ey,i}(u_{h,i})$, which has the $d_{h,\Sigma}$-optimality property;

(b) $S_{ey,i}^{LSI}$ is a locally structurally identifiable framework $S_{ey,i}(u_{h,i})$.

Let $S_{ey_i} = F_{S_{ey_i}}^{l} \cup F_{S_{ey_i}}^{r}$, where $F_{S_{ey_i}}^{l}, F_{S_{ey_i}}^{r}$ are the left and right fragments S_{ey}. The fragments are determined on areas $\mathscr{D}_y^{l}, \mathscr{D}_y^{r}$ with diameters $D_{y_i}^{l}, D_{y_i}^{r}$. Then the system $S_{\varphi,i,k}$ is locally structurally identifiable or h_{δ_h}-identifiable and $S_{ey,i}(u_{h,i}) \subset \{S_{ey,i}^{LSI}\}$ if

$$\left| D_{y_i}^{l} - D_{y_i}^{r} \right| \le \varepsilon_{y_i,(l,r)}, \tag{17}$$

where $\varepsilon_{y_i,(l,r)} \ge 0$.

Remark 4. We have considered the case of non-linearities belonging to the class $\mathscr{N}_F(\pi_1, \pi_2)$. $\mathscr{N}_F(\pi_1, \pi_2)$ has a set of non-linearities, which are both symmetric and non-symmetric. If $f_{i,k} \in F_i(X_i)$ is non-symmetric and $f_{i,k} \notin (\pi_1 X_i, \pi_2 X_i)$, then we use the approach [26] to estimate SI. Here, the SI assessment is reduced to ensuring ε_a-identifiability.

Remark 5. Structural identification depends on the connections between subsystems. These connections must be stable and satisfy the corollary of Theorem 1. This condition guarantees the identifiability of the nonlinear part of the S_i-subsystem at the parametric level. This approach allows you to localise the nonlinearity to evaluate the SI.

Definition 16. NDS is locally structurally identifiable if conditions of Theorems 1, 2 are fulfilled, and the nonlinear part is SI.

8. Simulation Results

1. Consider the system

$$\begin{aligned} &S_1 : \begin{cases} \begin{bmatrix} \dot{x}_{11} \\ \dot{x}_{12} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_{21} & -a_{22} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{a}_1 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ b_1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ c_1 \end{bmatrix} f_1(x_{11}), \\ y_1 = x_{11}, \end{cases} \\ &S_2 : \begin{cases} \dot{x}_2 = -a_2 x_2 + \bar{a}_2 x_{11} + b_2 u_2 + c_2 f_2(x_2), \\ y_2 = x_2, \end{cases} \end{aligned} \tag{18}$$

where $X_1 = [x_{11} \; x_{12}]^T$ and y_1 are the state vector and output of the subsystem S_1; u_1, u_2 are input (control)); $f_1(x_{11}) = \mathrm{sat}(x_{11})$ is saturation function; $f_2(x_2)=\mathrm{sign}(x_2)$ is sign function; y_2 is

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subsystem S_2 output. System parameters (18): $a_{21}=2$, $a_{22}=3$, $\bar{a}_1=1.5$, $b_1=1$, $c_1=1$, $a_2=1.25$, $\bar{a}_2=0.2$, $b_2=1$, $c_2=0.25$. Inputs $u_i(t)$ are sinusoidal.

For the S_1-subsystem, the phase portrait is shown in Figure 1. The processes are nonlinear. There is a relationship $y_2=y_2(y_1)$ between y_1 and y_2 (the determination coefficient is 75%), which affects the properties of the subsystem S_1 (see Figure 1).

The results of the evaluation of the S-congruence for subsystems are presented in Figures 2 and 3, which show phase portraits and frameworks.

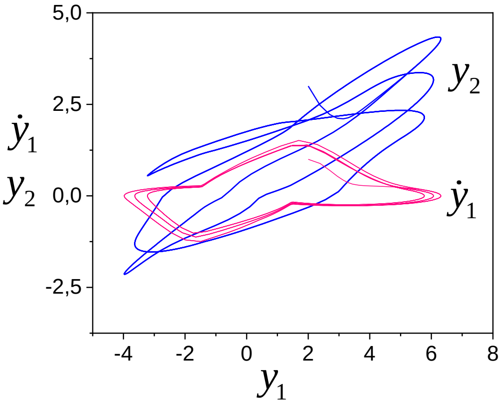

Figure 1. Phase portrait of the subsystem S_1.

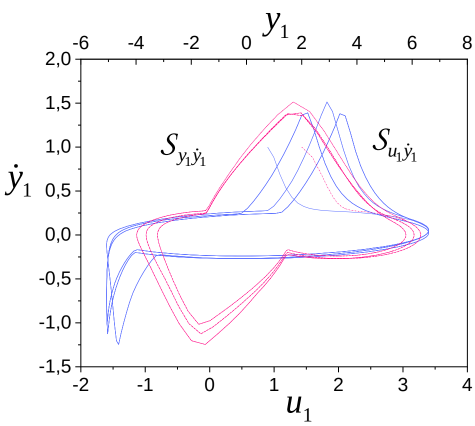

Figure 2. S-congruence estimation for the subsystem S_1.

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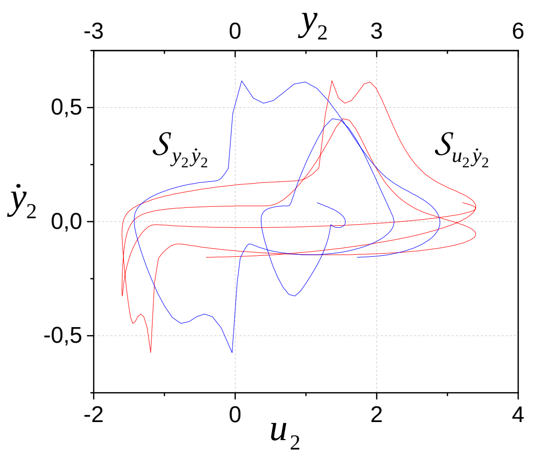

Figure 3. S-congruence estimation for the subsystem S_2.

We see the frameworks are S-congruent, and the inputs (controls) u_i are S-synchronising.

Remark 5. In the space (u_1, y_1), the subsystem S_1 has the representation

$$\dot{y}_1 = -\alpha_1 y_1 + \alpha_2 p_{y_1} + \beta_{12} p_{x_2} + b_1 p_{u_1} + c_1 p_{f_1}, \tag{19}$$

where α_1, α_2, β_{12}, b_1, c_1 are coefficients; $\mu > 0$,

$$\begin{aligned} \dot{p}_{y_1} &= -\mu p_{y_1} + y_1, \ \dot{p}_{x_2} = -\mu p_{x_2} + x_2, \\ \dot{p}_{u_1} &= -\mu p_{u_1} + u_1, \ \dot{p}_{f_1} = -\mu p_{f_1} + f_1. \end{aligned} \tag{20}$$

The application of filters (20) changes the original frequency spectrum of the system (18). Here, the S-congruence estimation procedure also works, but the set $\Omega_{u_i}(\omega)$ of u_i inputs must change. This is confirmed by the simulation results.

The structure $S_{e_1 y_1}$ is shown in Figure 4. It is obtained as follows. Since there is the relationship $y_2 = y_2(y_1)$, the iterative immersion method is used to obtain $S_{e_1 y_1}$.

To exclude the effect of y_2 on y_1, we synthesized a model

$$\hat{y}_2 = 0.5626 + 0.4419\, y_1. \tag{21}$$

The variable $\hat{y}_2$ is used to construct the model (14), which has the form for the system (18)

$$\dot{\hat{y}}_1 = 0.07209 + 0,3602 u_1 + 0.1506 y_1 + 0.002 \hat{y}_2. \tag{22}$$

Apply (22) and get the structure $S_{e_1 y_1}$ described by the function $\gamma_{e_1, y_i} : y_{i,k} \to e_1$, where $e_1 = \dot{y}_1 - \dot{\hat{y}}_1$

The average diameters of fragments are equal $\bar{D}^l_{y_1} = 1.59$, $\bar{D}^r_{y_1} = 1.146$. If we put $\varepsilon_{y_1,(l,r)} = 0.2$ in (17), then we get that the system is h_{δ_h}-identifiable. It follows from Figure 4 that the nonlinearity of $f_1(y_1)$ is a function of saturation.

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So, the subsystem of the S_1-subsystem (18) is locally structurally identifiable.

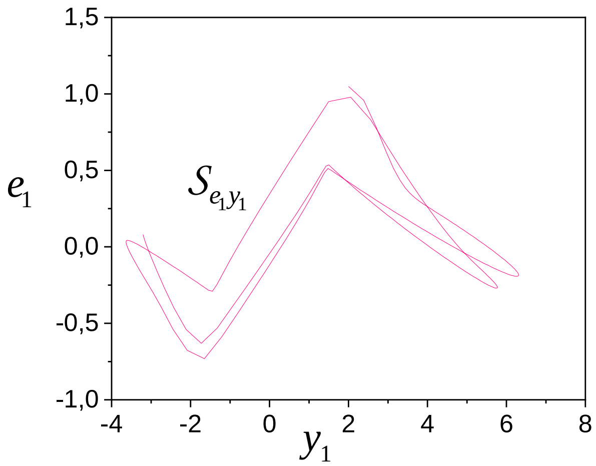

Figure 4. Structure $S_{e_1 y_1}$ for the subsystem S_1.

Consider the subsystem S_2. Data analysis has shown that there is an impact of $\dot{y}_1$ on $\dot{y}_2$. Therefore, we apply the iterative immersion method and use

$$\dot{\hat{y}}_2 = -0.0044 + 0.446\dot{y}_1$$

as the model (14), and calculate the error $e_2 = \dot{y}_2 - \dot{\hat{y}}_2$. Next, the geometric framework $S_{e_2 y_2}$ (Figure 5) described by the function $\gamma_{e_2,y_2} : y_2 \to e_2$ is definable. The results are shown in Figure 5. We see (a) subsystem S_2 is SI and contains non-linearity belonging to the class of sign functions; (b) structures $S_{e_2 y_2}$ and $S_{\dot{y}_1 y_1}$ are S-congruent.

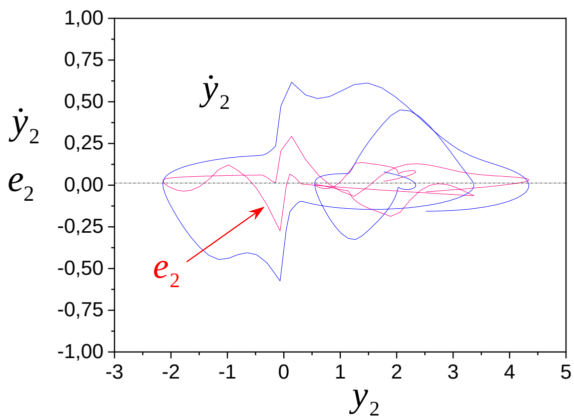

Figure 5. Structural identifiability evaluation of subsystem S_2.

Remark 6. If the S-synchronizability conditions are not fulfilment, then DS may be unidentifiable.

2. Consider the system

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$$\bar{S}_1:\begin{cases}\begin{bmatrix}\dot{x}_{11}\\ \dot{x}_{12}\end{bmatrix}=\begin{bmatrix}0 & 1\\ -a_{21} & -a_{22}\end{bmatrix}\begin{bmatrix}x_{11}\\ x_{12}\end{bmatrix}+\begin{bmatrix}0\\ \bar{a}_1\end{bmatrix}x_2+\begin{bmatrix}0\\ b_1\end{bmatrix}u_1+\begin{bmatrix}0\\ c_1\end{bmatrix}f_1(x_{11}),\\ y_1=x_{11},\end{cases}$$
$$\bar{S}_2:\begin{cases}\begin{bmatrix}\dot{x}_{21}\\ \dot{x}_{22}\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\tilde{a}_{21} & -\tilde{a}_{22}\end{bmatrix}\begin{bmatrix}x_{21}\\ x_{22}\end{bmatrix}+\begin{bmatrix}0\\ \tilde{a}_1\end{bmatrix}x_2+\begin{bmatrix}0\\ \tilde{b}_1\end{bmatrix}u_2+\begin{bmatrix}0\\ \tilde{c}_1\end{bmatrix}f_2(x_{21}),\\ y_2=x_{21},\end{cases} \tag{23}$$

where $X_1=[x_{11}\ x_{12}]^T$, $X_2=[x_{21}\ x_{22}]^T$ are state vectors of the subsystem $\bar{S}_1, \bar{S}_2$; is y_1, y_2 are; u_1, u_2 are inputs (control); $f_1(x_{11})=\mathrm{sat}(x_{11})$; $f_2(x_2)=|x_2|$. System parameters (23): $a_{21}=2$, $a_{22}=3$, $\bar{a}_1=0.5$, $b_1=2.9$, $c_1=2$, $\tilde{a}_{21}=2.5$, $\tilde{a}_{22}=.5$, $\tilde{a}_1=3.5$, $\tilde{b}_2=1$, $\tilde{c}_2=0.5$. Inputs $u_i(t)$ are sinusoidal.

The simulation results confirm the S-synchronizability of the inputs. We apply the D2 model

$$\dot{\tilde{y}}_1=1.12+1{,}03u_1-0.7y_1+0.17y_2 \tag{21}$$

to construct the structure $\tilde{S}_{\tilde{e}_1y_1}$ and calculate the residual $\tilde{e}_1=\dot{y}_1-\dot{\tilde{y}}_1$. The structure $\tilde{S}_{\tilde{e}_iy_1}$, described by the function $\gamma_{\tilde{e}_1,y_1}:y_1\to\tilde{e}_1$, and the phase portrait $\tilde{S}_{y_iy_1'}$ are shown in Figure 6. We see frameworks $\tilde{S}_{\tilde{e}_iy_1}$ and $\tilde{S}_{y_iy_1'}$ are S-congruent, and subsystem $\bar{S}_1$ is structurally identifiable. The framework $\tilde{S}_{\tilde{e}_iy_1}$ represents the nonlinearity $f_1(y_1)$.

Consider the subsystem $\bar{S}_2$. The phase portrait $\tilde{S}_{y_2y_2'}$ and the structure $\tilde{S}_{\tilde{e}_2y_2}$ are shown in Figure 7. We see that subsystem $\bar{S}_2$ is structurally identifiable, and the framework $\tilde{S}_{\tilde{e}_2y_2}$ coincides with the function $f_2(y_2)$. The analysis showed that $\dot{y}_2$ is influenced by the variable $\dot{y}_1$. Therefore, apply the model:

$$\dot{\tilde{y}}_2=0.0144+1.7343\dot{y}_1,$$

which is the basis for calculating the residual $\tilde{e}_2$ and constructing the $\tilde{S}_{\tilde{e}_2y_2}$-structure. The oval (Figure 7) allocates the area that contains the $\tilde{S}_{\tilde{e}_2y_2}$-fragment corresponding to $f_2(y_2)$.

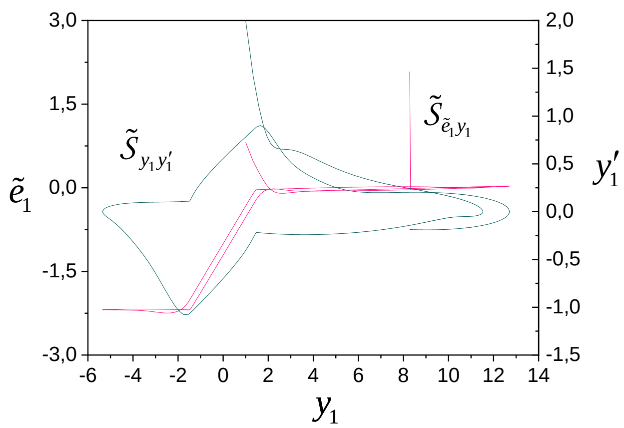

Figure 6. SI evaluation of subsystem $\bar{S}_1$.

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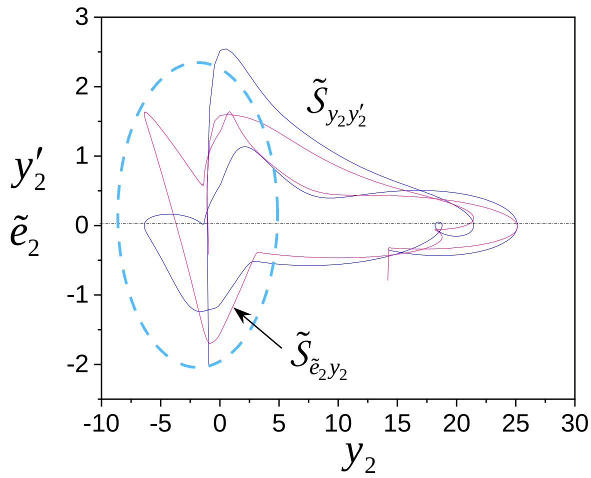

Figure 7. SI evaluation of subsystem $\bar{S}_2$.

So, simulation results confirm the effectiveness of the proposed approach to SI. The of the model structure (14) choice is the main problem of this method. The model identification depends on the DS type, experimental data properties, and the intuition of the researcher. Using the iterative immersion method solves this problem. The effectiveness of this method is showed by examples. The examples show it is difficult to propose a general approach to SI estimate. This is because of the peculiarities of the nonlinear system. Therefore, the DS detailed analysis and the results of data processing are the basis for deciding about SI.

9. Conclusions

An approach is proposed for assessing the local structural identifiability of a nonlinear decentralised system. The relationship is shown between the excitation constancy of the experimental data and the S-synchronizability, which guarantees the possibility of SI estimating for the NDS. We have obtained the local SI condition at the parametric level for subsystems, satisfying the quadratic condition for nonlinearity. The S-congruence concept is introduced, and the method is proposed, ensuring S-synchronizability of the input. Geometric structures and criteria are considered for estimating the structural identifiability of NDS.

Appendix

Corollary proof from Theorem 1. According to Theorem 1, NDS is locally parametrically identifiable if

$$2\left(\bar{\alpha}_{X_i}\left\|\Delta A_i\right\|^2+\bar{\alpha}_{u_i}\left\|\Delta B_i\right\|^2+\sum_{j=1,\,j\neq i}^{m}\bar{\alpha}_{X_j}\left\|\Delta \bar{A}_{ij}\right\|^2+2\eta\bar{\alpha}_{X_i}+\delta_{F_i}\right)\leq\bar{\lambda}_i V_i, \qquad (\Pi 1)$$

where $V_i\left(E_i\right)=0.5E_i^T R_i E_i$. From Lemmas 1 and 2, we get:

$$2\eta\bar{\alpha}_{X_i}+\delta_{F_i}=2\left(\eta\bar{\alpha}_{X_i}+\delta_{F_i}\right)-\delta_{F_i}=2\left\|\Delta F_i\right\|^2-\delta_{F_i}, \qquad (\Pi 2)$$

Therefore

$$\left\|\Delta F_i\right\|^2 \le 0.25\bar{\lambda}_i V_i + 0.5\left(\delta_{F_i} - \chi_i\theta_i - \zeta_{\alpha,j}\zeta_j\right), \tag{П3}$$

where

$$\theta_i = \min_i\left(\left\|\Delta A_i\right\|^2 + \left\|\Delta B_i\right\|^2\right),\ \ \zeta_j = \min_j \sum_{j=1,\, j\ne i}^{m} \left\|\Delta\bar{A}_{ij}\right\|^2,\ \ \zeta_{\alpha,j} = \min_j \sum_{j=1,\, j\ne i}^{m} \bar{\alpha}_{X_j}\ . \tag{П4}$$

From the stability condition, we obtain $\zeta_{\alpha,j}\zeta_j \le \delta_{F_i} - \chi_i\theta_i$, therefore $\left\|\Delta F_i\right\|^2 \le 0.25\bar{\lambda}_i V_i + z_i$. ■

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