

Article

Not peer-reviewed version

Construction and Study of a Probabilistic Model for the Sliding Mode Along and Across the Slip Line

[Gurami Tsitsiashvili](#) *

Posted Date: 6 March 2026

doi: 10.20944/preprints202603.0356.v1

Keywords: slip line; regenerative process; Qi Fan metric; central limit theorem



Preprints.org is a free multidisciplinary platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This open access article is published under a [Creative Commons CC BY 4.0 license](#), which permit the free download, distribution, and reuse, provided that the author and preprint are cited in any reuse.

Disclaimer/Publisher's Note: The statements, opinions, and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.

Article

Construction and Study of a Probabilistic Model for the Sliding Mode Along and Across the Slip Line

Gurami Tsitsiashvili 

Institute for Applied Mathematics, Far Eastern Branch of Russian Academy of Sciences, Radio Str. 7,
690041 Vladivostok, Russia; guram@iam.dvo.ru; Tel.: +7-914-693-2749

Abstract

In this paper, we construct a probabilistic model of a sliding mode. This model is based on the moment a random walk with positive jumps crosses a certain critical level. It is assumed that the jump magnitude has a geometric distribution. If the initial state is negative and the critical level is zero, then after crossing this level, a random walk begins in the opposite direction until it crosses zero again. As a result, motion orthogonal to the slip line is defined as a regenerative process, in which the moments of regeneration are the moments of zero crossings from right to left. An estimate of the Qi Fan metric of the maximum deviation of this random walk over a certain time interval is constructed under the assumption that the time and magnitude of the jumps are reduced by a factor of m . This estimate is found to be of the order of $\ln m/m$ as $m \rightarrow \infty$ and characterizes the deviation of a random trajectory orthogonal to the slip line. In the model of motion along a slip line, its velocity is assumed to assume fixed values when the trajectory of motion orthogonal to the slip line is above or below zero. Using the central limit theorem for the integral of a regenerative process, an estimate of the non-uniformity of motion of a random trajectory along the slip line is constructed. It is found that the characteristic magnitude of this non-uniformity is of the order of $1/m$ as $m \rightarrow \infty$. This indicates that the accumulation of random errors during motion along the slip line is significantly faster than during motion orthogonal to the slip line.

Keywords: slip line; regenerative process; Qi Fan metric; central limit theorem

MSC: 60J28

1. Introduction

The study of motion along a slip line has long attracted the attention of automatic control specialists. The deterministic models used in the studies allow the derivation of averaged sliding equations, albeit ambiguously [1]. These models find application in studies of robotic manipulators and nanorobots [2–5], in the design of converters providing a constant energy load [6], in cascaded DC power supply systems [7], in calculating stable modes in energy and micromanipulation systems [8–10], in sliding mode control using a neural network [11], in the control of biochemical processes [12], in systems with a time delay [13], and in many other technical systems [14]. It should also be noted that sliding modes can arise when solving nonlinear equations of mathematical physics (see, for example, [15]).

However, the study of deterministic models of sliding mode, movement along the slip line, is complicated by the fact that in these models periodic sequences arise, which in some cases are transformed into pseudo-random ones [16–18] due to the nonlinearity of the control processes. Indeed, the sliding-mode control strategy transforms unstable trajectories into fairly stable periodic ones. This circumstance necessitates the development of models in which this periodicity is expressed most explicitly.

In this paper, we propose to use a probabilistic model of a regenerative random process with the simplest possible structure. This refers to a process in which the first component, which characterizes

the deviation of the sliding process orthogonally to the slip line, successively moves from the negative semi-axis to the positive one and vice versa, while the speed of movement along the slip line at each of these time intervals also successively changes. Thus, the stochastic model of motion along the slip line is in many ways similar to the deterministic periodic model.

The choice of the laws for the distribution of the first component residence times in the positive and negative semi-axes plays an important role. In this case, the regeneration period, which determines the periodicity of the sliding process, contains the time of the jump from the negative semi-axis to the positive one and the time of the reverse jump. The distribution of the jump magnitude from the negative to the positive region (and vice versa) is chosen so that it is independent of movement in the negative (or positive) region. A geometric distribution, which has the property of no aftereffect, is most convenient in this case.

In the definition of the stochastic sliding process, a small parameter $1/m$ is also introduced, which characterizes both the time step and the magnitude of the jump orthogonal to the slip line at this step. This construction of a stochastic sliding model allows us to determine the maximum deviation of the sliding trajectory in the orthogonal direction using the Qi-Fan probabilistic metric [19]. In turn, the motion trajectory along the slip line allows for research using the law of large numbers and the central limit theorem for regenerative processes [20].

The result of such an analysis is the dependences of the maximum deviation perpendicular to the slip line of the order of $\ln m/m$ and the unevenness of movement along the slip line of the order of $1/\sqrt{m}$. The presence of fairly clear differences in these values indicates a significant difference between the constructed model and a conventional periodic process, despite the fact that the stochastic model is in many ways similar to a deterministic periodic model.

The main result of this work is the construction of a probabilistic model of the sliding mode and a detailed study of its convergence to the averaged motion with increasing switching frequency. Probabilistic concepts such as the regenerative process and limit theorems for this process, as well as the Qi Fan probabilistic metric, are used for this purpose. Each detail of the constructed probabilistic model is selected in such a way as to maximally simplify its study and comparative analysis of movement across and along the slip line. This is because the primary goal of this work is not to construct the most general model possible, but to ensure that it can yield the most accurate and contrasting results. Specifically, to achieve this, the model's jump distribution is chosen to be geometric rather than exponential.

2. Construction of a Probabilistic Sliding Mode Model

Let $y_0 = -\zeta^-(0)$, be a negative random variable, where $\zeta^-(0)$ has a geometric distribution defined by the equality $P(\zeta^-(0) > i) = (1 - p_-)^i$, $i = 1, 2, \dots$, $0 < p_- < 1$. Consider a discrete random walk $y(0), y(1), \dots, y(\tau_1^-)$ with jumps $\zeta^+(1), \zeta^+(2), \dots, \zeta^+(\tau_1^-)$, having a geometric distribution of the form $P(\zeta^+ > i) = (1 - p_+)^i$, $i = 1, 2, \dots$, $0 < p_+ < 1$:

$$y(1) = y(0) + \zeta^+(1), y(2) = y(1) + \zeta^+(2), \dots, y(\tau_1^-) = y(\tau_1^- - 1) + \zeta^+(\tau_1^-).$$

This wandering continues until the moment $\tau_1^+ = \min(k : y(k) > 0)$ of the first jump through the point 0, $P(y(\tau_1^+) > t) = (1 - p_+)^t$.

After jumping over the point 0 on the left, the random walk $y(k)$ continues in the opposite direction until the moment $\tau_1^+ + \tau_1^- = \min(k > \tau_1^+ : y(\tau_1^+ + \tau_1^-) < 0)$ in accordance with the equalities

$$y(\tau_1^+ + 1) = y(\tau_1^+) - \zeta^-(1), \dots, y(\tau_1^+ + \tau_1^-) = y(\tau_1^+ + \tau_1^- - 1) - \zeta^-(\tau_1^-),$$

where the independent random variables $\xi^-(1), \xi^-(2), \dots$ coincide in distribution with $\xi^-(0)$. The described random walk is then repeated from the point $y(\tau_1^+ + \tau_1^-)$ etc. As a result, a regenerative process is constructed between states

$$y(0), y(\tau_1), y(\tau_1 + \tau_2), \dots, \tau_k = \tau_k^+ + \tau_k^-, k = 1, 2, \dots$$

Let us denote $T_0 = 0, T_1 = \tau_1, T_2 = T_1 + \tau_2, \dots$, due to the property of absence of aftereffect of the geometric distribution, all random variables $y(T_0), y(T_1), \dots$ are independent, less than zero and their absolute values have a geometric distribution with parameter p_- . Similarly, random variables $y(T_0 + \tau_1^+), y(T_1 + \tau_2^+), \dots$ are also independent and have a geometric distribution with parameter p_+ . Moreover, sequences of random variables $\tau_k^+, k = 1, 2, \dots$ and random variables $\tau_k^-, k = 1, 2, \dots$ are also independent.

Let us now turn to the description of a continuous-time process. Let $h = 1/m$, define $Y_m(kh) = y(k)/m, k \geq 0$, where $Y_m(kh + \delta) = Y_m(kh), 0 \leq \delta < h$. This definition of the function $Y_m(kh)$ can be rewritten as $Y_m(t) = y(mt)/m, 0 \leq t \leq 2$. Our task is to estimate the maximum deviation from zero of the process $Y_m(t), 0 \leq t \leq 2$.

Let's denote

$$U_m = \max_{0 \leq k < m} \frac{|y_m(T_k)|}{m}, V_m = \max_{0 \leq k < m} \frac{y_m(T_k + \tau_{k+1}^+)}{m} \quad \lambda_- = -\ln q_-, \lambda_+ = -\ln q^+.$$

From the definition of the geometric distribution it follows that the inequalities $1 \leq \tau_k^+, 1 \leq \tau_k^-, k = 1, 2, \dots$ are almost certainly satisfied, and, therefore $2m \leq T_m, 1 \leq m$, hence

$$|Y_m(0)| \leq \max_{0 \leq t < 2} |Y_m(t)| \leq \max(U_m, V_m), \quad (1)$$

$$P(U_m > u) = 1 - (1 - \exp(-m\lambda_-u))^m, P(V_m > u) = 1 - (1 - \exp(-m\lambda_+u))^m, u \geq 0. \quad (2)$$

Let $f(Y_m(t)) = v_x^+, Y_m(t) \leq 0; f(Y_m(t)) = v_x^-, Y_m(t) > 0$, where $v_x^+, v_x^- > 0$ – are the velocities of the sliding mode along the x -coordinate along the slip line x . Then, on the half-intervals $[T_0, T_0 + \tau_1^+), [T_1, \tau_1^+), \dots$ the velocity of motion along the x is equal to v_x^+ . And on the half-intervals $[T_0 + \tau_1^+, T_2), [T_1 + \tau_2^+, T_3), \dots$ the velocity of motion along the x is equal to v_x^- . We now define the regenerative process $X_m(kh) = f(Y_m(kh)) = v(k), k \geq 0$, and $X_m(kh + \delta) = X_m(kh), 0 \leq \delta < h$. We now investigate the behavior of the motion velocity of the slip line in the thus constructed stochastic model using the integral $S(t) = \int_0^t f(Y_m(s))ds$.

3. Estimating the Deviation of the Stochastic Model Orthogonally to the Slip Line

Let $\kappa(A, B) = \inf\{u : P(|A - B| > u) \leq u\}$ be the Ki-Fan distance (see [19, p. 35]) between random variables A, B . We formulate our problem in the form of a definition of $\kappa\left(0, \sup_{0 \leq t \leq 2} |Y_m(t)|\right)$.

We say that sequences of numbers A_m (numbers B_m), $1 \leq m$ satisfy the relation $A_m \preceq B_m$, (the relation the relation) as $m \rightarrow \infty$, if

$$\limsup_{m \rightarrow \infty} \frac{A_m}{B_m} \leq 1 \left(\lim_{m \rightarrow \infty} \frac{A_m}{B_m} = 1 \right).$$

Theorem 1. For $0 < \lambda_-, \lambda_+$ the upper estimate is valid for

$$\kappa\left(0, \max_{0 \leq t < 2} |Y_m(t)|\right) \leq \kappa(0, \max(U_m, V_m)) \preceq \frac{2 \ln m}{\min(\lambda_-, \lambda_+)m}. \quad (3)$$

Proof. From inequality (1) and the absence of aftereffect for the geometric distribution, it follows that the random variables U_m, V_m are independent. Therefore, from formula (2) we obtain

$$P(\max(U_m, V_m) > u) = 1 - (1 - \exp(-m\lambda_-))^m (1 - \exp(-m\lambda_+))^m.$$

First, we set (without loss of generality $\lambda_- < \lambda_+$) and choose $u^* = \frac{2 \ln m}{\min(\lambda_-, \lambda_+)m}$. Then, as $m \rightarrow \infty$ we obtain the relation

$$P(\max(U_m, V_m) > u^*) = m^{-1} + m^{-2\lambda_+/\lambda_-+1} - m^{-2\lambda_+/\lambda_-} \sim m^{-1} \preceq \frac{2 \ln m}{\min(\lambda_-, \lambda_+)m}.$$

A similar relation is obtained for $\lambda_+ < \lambda_-$. This implies formula (3).

We now consider the case when $\lambda_- = \lambda_+ = \lambda$ and obtain

$$P(\max(U_m, V_m) > u^*) = m^{-1} + m^{-2\lambda/\lambda+1} - m^{-2\lambda/\lambda} \sim 2m^{-1} \preceq \frac{2 \ln m}{\lambda m},$$

which also implies formula (3). Moreover, there exists M^* such that for any $m > M^*$ the inequality

$$\limsup_{m \rightarrow \infty} \kappa \left(0, \max_{0 \leq t < 2} |Y_m(t)| \right) \left[\frac{2 \ln m}{\lambda m} \right]^{-1} \leq 1. \quad (4)$$

holds. Theorem 1 is completely proven. $\square$

Theorem 2. For any ε , $0 < \varepsilon < 1/2$ with $0 < \lambda_- = \lambda_+ = \lambda$ the lower estimate is valid

$$\frac{(1-2\varepsilon) \ln m}{\lambda m} \preceq \kappa(0, |Y_m(0)|) \preceq \kappa(0, \max_{0 \leq t < 2} |Y_m(t)|). \quad (5)$$

Proof. The right inequality in formula (7) follows from the left inequality in formula (1). We now calculate $\kappa(0, |Y_m(0)|)$, for which we define for $u_* = \frac{2 \ln m^{1/2-\varepsilon}}{\lambda m}$ the probability

$$P(|Y_m(0)| > u_*) = \exp \left(-\lambda m \cdot \frac{2 \ln m^{1/2-\varepsilon}}{\lambda m} \right) = \frac{1}{m^{1-2\varepsilon}} \succeq \frac{2 \ln m^{1/2-\varepsilon}}{\lambda m} = \frac{(1-2\varepsilon) \ln m}{\lambda m}.$$

This implies the left inequality in formula (7). Theorem 4 is completely proved. $\square$

Theorem 3. If $\lambda_- = \lambda_+ = \lambda = 2$, then as $m \rightarrow \infty$ the following relation holds

$$\kappa \left(0, \max_{0 \leq t < 2} |Y_m(t)| \right) \sim \frac{\ln m}{m}. \quad (6)$$

Proof. We choose $u_{**} = \frac{\ln m^{1-1/(\ln m)^\gamma}}{\lambda m}$ for $0 < \gamma < 1$, then due to this condition on γ

$$\begin{aligned} P(|Y_m(0)| > u_{**}) &= \exp(-(1-1/(\ln m)^\gamma) \ln m) = \frac{m^{1/(\ln m)^\gamma}}{m} = \\ &= \frac{\exp((\ln m)^{1-\gamma})}{m} \succeq \frac{\exp(\ln \ln m)}{m} = \frac{\ln m}{m}. \end{aligned}$$

Moreover, there exists an M_* , such that for any $m > M_*$ the following inequality holds:

$$\frac{\ln m}{m} \leq P(|Y_m(0)| > u_{**}) \leq P \left(\max_{0 \leq t < 2} |Y_m(t)| > u_{**} \right) \leq \kappa \left(0, \max_{0 \leq t < 2} |Y_m(t)| \right).$$

From this follows the relation

$$1 \leq \liminf_{m \rightarrow \infty} \kappa \left(0, \max_{0 \leq t < 2} |Y_m(t)| \right) \left[\frac{\ln m}{m} \right]^{-1}.$$

Combining this inequality with inequality (4) for $\lambda = 2$, we obtain formula (8). $\square$

Theorem 4. For any ε , $0 < \varepsilon < 1/2$ with $0 < \lambda_- = \lambda_+ = \lambda$ the lower estimate is valid

$$\frac{(1-2\varepsilon) \ln m}{\lambda m} \preceq \kappa(0, |Y_m(0)|) \preceq \kappa(0, \max_{0 \leq t < 2} |Y_m(t)|). \quad (7)$$

Proof. The right inequality in formula (7) follows from the left inequality in formula (1). We now calculate $\kappa(0, |Y_m(0)|)$, for which we define for $u_* = \frac{2 \ln m^{1/2-\varepsilon}}{\lambda m}$ the probability

$$P(|Y_m(0)| > u_*) = \exp \left(-\lambda m \cdot \frac{2 \ln m^{1/2-\varepsilon}}{\lambda m} \right) = \frac{1}{m^{1-2\varepsilon}} \succeq \frac{2 \ln m^{1/2-\varepsilon}}{\lambda m} = \frac{(1-2\varepsilon) \ln m}{\lambda m}.$$

This implies the left inequality in formula (7). Theorem 4 is completely proved. $\square$

Theorem 5. If $\lambda_- = \lambda_+ = \lambda = 2$, then as $m \rightarrow \infty$ the following relation holds

$$\kappa \left(0, \max_{0 \leq t < 2} |Y_m(t)| \right) \sim \frac{\ln m}{m}. \quad (8)$$

Proof. We choose $u_{**} = \frac{\ln m^{1-1/(\ln m)^\gamma}}{\lambda m}$ for $0 < \gamma < 1$, then due to this condition on γ

$$\begin{aligned} P(|Y_m(0)| > u_{**}) &= \exp(-(1-1/(\ln m)^\gamma) \ln m) = \frac{m^{1/(\ln m)^\gamma}}{m} = \\ &= \frac{\exp((\ln m)^{1-\gamma})}{m} \succeq \frac{\exp(\ln \ln m)}{m} = \frac{\ln m}{m}. \end{aligned}$$

Moreover, there exists an M_* , such that for any $m > M_*$ the following inequality holds:

$$\frac{\ln m}{m} \leq P(|Y_m(0)| > u_{**}) \leq P \left(\max_{0 \leq t < 2} |Y_m(t)| > u_{**} \right) \leq \kappa \left(0, \max_{0 \leq t < 2} |Y_m(t)| \right).$$

From this follows the relation

$$1 \leq \liminf_{m \rightarrow \infty} \kappa \left(0, \max_{0 \leq t < 2} |Y_m(t)| \right) \left[\frac{\ln m}{m} \right]^{-1}.$$

Combining this inequality with inequality (4) for $\lambda = 2$, we obtain formula (8). $\square$

4. Study of the Motion Unevenness Along the Slip Line

Our goal is to study integral $S(t)$ using the law of large numbers and the central limit theorem for regenerative processes.

Obviously, the process $Y_m(t)$ is regenerative with regeneration times $T_0/m, T_1/m, \dots$. To use the law of large numbers and the central limit theorem for regenerative processes, we must introduce random variables

$$\tau = \tau_- + \tau_+, \quad \xi = \int_0^\tau f(Y_m(s)) ds = \tau^- v_x^+ + \tau_+ v_x^-, \quad (9)$$

where τ_- , τ_+ coincide in distribution with the random variables τ_1^+ , τ_1^- , respectively.

Theorem 6. When the conditions $0 < q_-, q_+ < 1$ are satisfied, the following characteristics exist: $a = M\tau$, $a_{\xi} = M\xi$, $r = a_{\xi}/a$, $d^2 = D(\xi - r\tau)$.

Proof. Indeed, let

$$\tau^-(s) = \inf(k : y(k) \geq 0 / y(0) = -s < 0),$$

$$\tau^+(s) = \inf(k : y(k) > 0 / y(0) = s \geq 0),$$

then the inequalities $(\tau^-(s))^2 \leq s^2$, $(\tau^+(s))^2 \leq s^2$, almost surely hold, hence $M(\tau^-(s))^2 \leq s^2$, $M(\tau^+(s))^2 \leq s^2$. And since

$$M(\tau^-)^2 = \int_0^\infty M(\tau^-(s))^2 P_+(ds), \quad M(\tau^+)^2 = \int_0^\infty M(\tau^+(s))^2 P_-(ds),$$

the inequalities

$$M(\tau^-)^2 \leq \sum_{k=1}^\infty k^2 p_+ q_+^{k-1} < \infty, \quad M(\tau^+)^2 \leq \sum_{k=1}^\infty k^2 p_- q_-^{k-1} < \infty,$$

leading to the inequalities $M\tau_-^2 < \infty$, $M\tau_+^2 < \infty$. These inequalities and equalities (9) prove the statement of Theorem 6. $\square$

Remark 1. More accurate (although not fundamentally different) estimates of the regeneration moment distribution are given in [20, Chapter 12].

Using [20, Chapter 13, Theorems 13.8.1, 13.8.4] and Theorem 6, as $m \rightarrow \infty$ we arrive at the convergence in probability of

$$\frac{S(m)}{m} \rightarrow \frac{a_{\xi}}{a}, \quad S(m) = \int_0^m f(Y_m(s))ds = m \int_0^1 f(Y_m(ms'))ds'$$

and at the convergence of the distribution of the random variable $\frac{S(m) - rm}{d\sqrt{m/a}} = \frac{\int_0^1 f(Y_m(ms'))ds' - r}{d\sqrt{1/ma}}$ to the standard normal distribution.

Corollary 1. It follows that the mean square distribution of the random variable $\int_0^1 f(Y_m(ms'))ds' - r$, characterizing the unevenness of motion along the slip line, is proportional $1/\sqrt{m}$. In turn, the Ki - Fan distance $\kappa\left(0, \sup_{0 \leq t \leq 2} |Y_m(t)|\right)$ of the transverse deviation of the trajectory from the slip line is proportional to $\frac{\ln m}{\lambda m}$ and, therefore, is significantly smaller. This means that random fluctuations in motion along the slip line accumulate significantly faster than fluctuations in motion orthogonal to the slip line.

5. Discussion

A comparison of the stochastic sliding model presented in this paper with known deterministic models shows that a deterministic sliding model is either periodic or aperiodic, pseudo-random. In the former case, there is no difference in the accumulation of deviations in this model along and across the slip line. In the latter case, problems arise associated with the analytical study of such an aperiodic model.

The above definition of regeneration periods is not entirely suitable when using limit theorems (the law of large numbers and the central limit theorem) for regenerative processes when considering a stochastic model of motion along a slip line. This model, strictly speaking, requires the regeneration period to begin upon reaching a designated point [20, Chapter 12] rather than the negative semi axis. Instead, the beginning of the regeneration moment can be defined as the moment when the

random walks right-to-left transition ends at point -1 . However, the transition to this definition of the regeneration moment is not significantly different from the one given above.

6. Conclusion

This paper constructs a probabilistic sliding mode model based on random variables with a geometric distribution and the property of no aftereffect. A large parameter m is introduced into this model, determining the frequency of model switching and allowing the model trajectory to approximate the slip line. Estimates of the maximum deviation from zero of the random walk trajectory are constructed, based on the Qi Fan metric. This allows, under certain conditions, to construct asymptotically accurate estimates in the Qi Fan metric. For motion along the slip line, estimates of the mean and variance of the integrals of regenerative processes are constructed using the well-known law of large numbers and the central limit theorem. A comparison of the results of the deviation analysis of the probabilistic model orthogonally to the slip line and along the slip line showed that random fluctuations accumulate significantly faster in the latter case.

Funding: This research received no external funding. The research was carried out within the state assignment for IAM FEB RAS (N 075-00460-26-00).

Data Availability Statement: Data supporting reported results were obtained by G.T.

Conflicts of Interest: The author declares no conflicts of interest.

References

1. Utkin, V.I. Variable Structure Systems—present and Future. *Automation and Remote Control* **1983**, *44* (9), 1105–1120.
2. Stepanenko, Y.; Su, C. Variable structure control of robot manipulators with non-linear sliding manifolds. *International Journal of Control* **1993**, *58* (2), 285–300.
3. Xu, R.; Xu, Q. A Survey of Recent Developments in Magnetic Microrobots for Micro-/Nano-Manipulation. *Micromachines* **2024**, *15*, 468.
4. Cao, H.X.; Nguyen, V.D.; Park, J.O.; et al. Acoustic Actuators for the Manipulation of Micro/Nanorobots: State-of-the-Art and Future Outlooks. *Micromachines* **2024**, *15*, 186.
5. Zhao, Y.; Tong, D.; Chen, Y.; et al. Microgripper Robot with End Electropermanent Magnet Collaborative Actuation. *Micromachines* **2024**, *15*, 798.
6. Martinez-Trevino, B.A.; Jammes, R.; Aroudi, A. et al. Sliding-mode control of a boost converter supplying a constant power load. *IFAC-Papers OnLine* **2017**, *50* (1), 7807–7812.
7. Zadeh, M.K.; Gavagsaz-Ghoachani, R.; Martin, J.P. et al. Discrete-time tool for stability analysis of DC power electronics-based cascaded systems. *IEEE Trans. Power Electron.* **2017**, *32* (1), 652–667.
8. Cheng, D.; Ma, J. Calculation of Stability Region. *Proceedings of the 2nd IEEE Conference on Decision and Control* **2003**, Maui, Hawaii, USA.
9. Adam, G.; Chidambaram, S.; Reddy, S.S.; et al. Towards a Comprehensive and Robust Micromanipulation System with Force-Sensing and VR Capabilities. *Micromachines* **2021**, *12*, 784.
10. He, J.; Liu, Y.; Yang, C.; et al. Design and Evaluation of an Adjustable Compliant Constant-Force Microgripper. *Micromachines* **2024**, *15*, 52.
11. Artetxe, E.; Barambones, O.; Calvo, I.; et al. Combined Control for a Piezoelectric Actuator Using a Feed-Forward Neural Network and Feedback Integral Fast Terminal Sliding Mode Control. *Micromachines* **2024**, *15*, 757.
12. Efe, M.O. Discrete time fuzzy sliding mode control of a biochemical process. *Proceedings of the 9-th WSEAS international conference on Automatic control, modelling and simulation* May 2007, 12–17.
13. Richard, J.-P.; Gouaisbaut, F.; Perruquetti, W.; et al. Sliding mode control in the presence of delay. *Kybernetika* **2001**, *37* (3), 277–294.
14. Shtessel, Yu.; Edwards, Ch.; Fridman, L.; et al. Sliding Mode Control and Observation. *Birkhauser Home Textbook Control Engineering* Germany 2014.
15. Korondi, P. et al. On the Mathematical Background of Sliding Mode-Based Friction Compensation of a Micro-Telemanipulation System. *Mathematics* **2024**, *12* (20), 3182.

16. Zhao, F.; May, J.A.; Loh, S.Ñ. Controller Synthesis and Verification for Nonlinear Systems: A computational approach using phase-space geometric models. *IEEE Control Systems Magazine* **1998**.
17. Utkin, V.; Poznyak, A.; Orlov, Yu.; Polyakov, A. Conventional and high order sliding mode control. *Journal of the Franklin Institute* **2020** *357* (15), 10244–10261.
18. Sira-Ramirez, H.; Gomez-Leon, B.C.; Aguilar-Orduna, M.A. A Geometric Algebra Approach to Invariance Control in Sliding Regimes for Switched Systems. *Advances in Applied Clifford Algebras* **2023** *33* (35).
19. Zolotarev, V.M. Modern Theory of Summation of Random Variables. *Modern probability and statistics* **1997** *1*.
20. Borovkov A.A. *Probability theory: textbook* Springer. 2012.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.