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Research on Mobile Robot Path Planning Based on Improved Whale Optimization Algorithm

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Abstract

Aiming at the problems of slow convergence speed, insufficient precision, and easy trapping in local optima of the traditional Whale Optimization Algorithm (WOA) in mobile robot path planning, an Improved Whale Optimization Algorithm (IWOA) is proposed. The specific improvements include: using Logistic chaotic mapping to initialize the population, which enhances the randomness and diversity of initial solutions; designing a nonlinear convergence factor to avoid the algorithm entering the contraction encirclement phase prematurely and extend the global search time; introducing an adaptive spiral shape constant to dynamically adjust the search range for balancing exploration and exploitation capabilities; and integrating the bird navigation mechanism to optimize the individual update strategy through the companion position information, thereby improving the algorithm's stability and convergence speed. Path planning simulations were conducted in 30×30 and 50×50 grid maps. The results show that compared with WOA, GA, and PSO, the path length of IWOA is shortened by 3.21%, 2.00%, and 7.76% respectively in the 30×30 map, and by 4.88%, 10.50%, and 23.55% respectively in the 50×50 map. The research indicates that IWOA has significant advantages in both path planning precision and efficiency, verifying its feasibility and superiority.

Keywords: path planning; nonlinear convergence factor; whale optimization algorithm; bird navigation mechanism

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1. Introduction

Path planning, as one of the key technologies for mobile robots, plays a crucial role in their navigation [1, 2]. Currently, path planning algorithms are mainly classified into two categories: traditional path planning algorithms and intelligent path planning algorithms [3]. Traditional path planning algorithms include Dijkstra's algorithm [4], A* algorithm [5], artificial potential field method [6], etc., while intelligent path planning algorithms consist of particle swarm optimization (PSO) [7], genetic algorithm (GA) [8], whale optimization algorithm (WOA) [9], etc. Among them, the Whale Optimization Algorithm (WOA) is a novel intelligent optimization algorithm inspired by the hunting behavior of whale groups in nature. WOA has advantages such as a simple structure and few optimization parameters, but it also faces problems like slow convergence speed, easy trapping in local optimal solutions, and low optimization accuracy [10, 11], which need to be further addressed.

Existing studies have improved WOA through various strategies: Reference [12] proposed the Improved Modified Whale Optimization Algorithm (IMWOA) based on the trapezoidal traction strategy and variable strategy idea, enhancing the balance between global exploration and local exploitation of the algorithm and significantly improving convergence performance and accuracy. However, in some multi-modal function and high-dimensional non-convex function optimization problems, the algorithm tends to suffer from insufficient convergence accuracy. Reference [13] adopted an adaptive step Gaussian walk strategy for global search to balance the global and local exploitation capabilities of WOA. Reference [14] integrated the golden sine algorithm to enhance the global search ability of WOA and accelerate convergence speed, but this may lead to a decrease in the algorithm's search ability when dealing with unknown data. Reference [15] proposed a whale optimization algorithm based on improved predation and feedback mechanisms, improving the stability of the algorithm, but there is still room for further optimization in terms of global search ability, convergence speed, and population diversity.

To address the problems of WOA such as slow convergence speed, low accuracy, and easy trapping in local optima, this paper proposes a path planning method based on the Improved Whale Optimization Algorithm (IWOA). To solve the problem of low quality of the initial population of the algorithm, Logistic chaos mapping [16] is used to initialize the population, enhancing the randomness of the initial population. A nonlinear convergence factor is designed to improve convergence accuracy. An adaptive spiral parameter is introduced to optimize search performance. The bird navigation mechanism is integrated to enhance the stability and convergence speed of the algorithm. Through simulation experiments, IWOA is compared with the traditional WOA, particle swarm optimization algorithm, and genetic algorithm, verifying the superiority and stability of IWOA.

2. Whale Optimization Algorithm

The Whale Optimization Algorithm (WOA) simulates the unique hunting behavior of humpback whales. It updates the position of individual whales through three methods—random search, encircling prey, and spiral predation—to obtain the optimal solution.

2.1. Random Search

When some individual whales are far from the current optimal solution, they move with a random position as a reference. This stage can explore solutions beyond the current optimal one, avoiding trapping in local optima. The mathematical model of this process can be expressed as:

$$D_{rand} = \left| C \times X_{rand}(t) - X(t) \right| \tag{1}$$

$$X(t+1) = X_{rand}(t) - A \times D_{rand} \tag{2}$$

Among them, D_{rand} denotes the distance between the position of the current individual whale and the position of a random individual whale; $X_{rand}(t)$ represents the position of the random individual whale; $X(t)$ is the position of the current individual whale; t denotes the current iteration number; and A and C re coefficient vectors. The calculation formulas of A and C are as follows:

$$A = 2a \times r_1 - a \tag{3}$$

$$C = 2 \times r_2 \tag{4}$$

Among them, r_1, r_2 is a random vector within the range [0, 1]; a is a linear convergence factor that decreases from 2 to 0. The expression of a is as follows:

$$a = 2 - \frac{2t}{T} \tag{5}$$

Among them, T denotes the maximum number of iterations.

2.2. Encircling Prey

When the position of an individual whale is close to the current optimal solution, it moves toward the position of this current optimal solution. The mathematical model at this stage can be expressed as:

$$D_{best} = \left| C \times X_{best}(t) - X(t) \right| \tag{6}$$

$$X(t+1) = X_{best}(t) - A \times D_{best} \tag{7}$$

Among them, $X_{best}(t)$ epresents the position of the current optimal individual, and D_{best} denotes the distance between the current individual whale and the optimal individual.

2.3. Spiral Predation

When humpback whales hunt, they release bubbles to form a spiral bubble net. This strategy is translated into a process where search agents perform a spiral search around the optimal solution [17]. The mathematical model for this stage is:

$$D_{best} = \left| X_{best}(t) - X(t) \right| \tag{8}$$

$$X(t+1) = D_{best} \times e^{bl} \times \cos(2\pi l) \times X_{best}(t) \tag{9}$$

Among them, b is a constant for the spiral shape, usually set to 1; l is a random number within the range [-1, 1].

After individual whales approach their prey through both prey encircling and spiral predation, one of the two methods is chosen for predation based on probability. The specific formula is as follows:

$$X(t+1) = \begin{cases} X_{best}(t) - A \times D_{best}, p \le 0.5 \\ D_{best} \times e^{bl} \times \cos(2\pi l) + X_{best}(t), p > 0.5 \end{cases} \tag{10}$$

Among them, p is the predation probability, and it is a random number within [0, 1].

For the random search and prey encircling stages, the magnitude of vector A determines which behavior mode to adopt. When $|A| > 1$ the algorithm focuses on global exploration and adopts the random search mode; when $|A| \le 1$ the algorithm focuses on local exploitation and adopts the prey encircling mode. Details are as follows:

$$X(t+1) = \begin{cases} X_{rand}(t) - A \times D_{rand}, |A| > 1 \\ X_{best}(t) - A \times D_{best}, |A| \le 1 \end{cases} \tag{11}$$

3. Improved Whale Optimization Algorithm (IWOA)

To address the problems of WOA, such as insufficient initial population quality, local optimality caused by the linear decline of the convergence factor, and limited search diversity affected by the fixed spiral parameter, this paper proposes improvements from four aspects: adopting Logistic chaos mapping for population initialization; designing a nonlinear convergence factor; introducing an adaptive spiral shape constant; and integrating the bird navigation mechanism to enhance stability.

3.1. Logistic Chaotic Mapping

The initial positions of population individuals are initialized using Logistic chaotic mapping, which helps the algorithm escape local optima, search for the global optimal solution more effectively, and improve the performance of the optimization algorithm. The mathematical model is as follows:

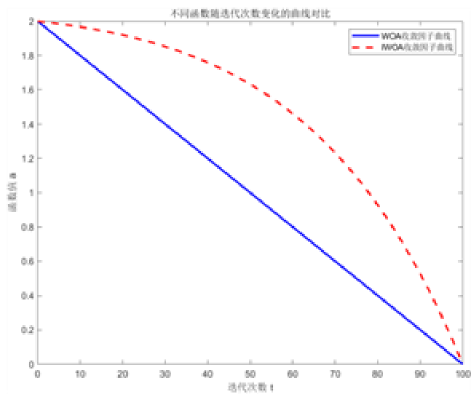
$$x_{n+1} = \mu x_n \left(1 - x_n\right) \tag{12}$$

Among them, μ is the control parameter. When μ =4, the Logistic chaotic mapping is in a fully chaotic state, exhibiting the strongest randomness and unpredictability. Therefore, μ =4 is adopted in this paper.

3.2. Nonlinear Convergence Factor

In the algorithm's search process, the vector coefficient A determines whether to search for solutions beyond the optimal one, preventing trapping in local optima. The magnitude of the convergence factor a dictates the size of A [18]. A larger a leads to a larger A, while a smaller a results in a smaller A. When $|A| > 1$, the algorithm tends to conduct global exploration, guiding whale individuals away from the currently known optimal position to search a broader solution space. This helps discover new and potentially better regions, avoiding premature convergence to local optima. When $|A| \le 1$, the algorithm enters exploitation mode [19], favoring local refinement. It enables whale individuals to perform fine-grained searches around the currently found suboptimal solutions, continuously optimizing existing solutions and exploring better solutions corresponding to higher fitness values in this region, thereby improving convergence accuracy. Therefore, to ensure the algorithm can better escape local optima, the decrease of A can be appropriately slowed to extend the global search duration. Based on this idea, introducing a nonlinear convergence factor is a feasible approach. The comparison of the convergence factors before and after improvement is shown in **Figure 1**. The mathematical formula for the improved convergence factor is as follows:

$$a = 2 - \frac{2t}{T \times e^{0.02 \times (T-t)}}, 0 \le t \le T \tag{13}$$



(a)

Figure 1. Comparison of the curves of convergence factor a before and after improvement.

3.3. Adaptive Spiral Shape Constant

In the original WOA, the spiral shape parameter b is usually set as a constant, leading to a lack of diversity in the search and predation postures of whale individuals [20]. The value of the spiral shape constant b determines the size of the spiral during the bubble-net predation phase. A larger b results in a larger spiral and a wider search range; a smaller b leads to a smaller spiral and a narrower

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search range. Therefore, this paper adopts an adaptive spiral shape constant, where the value of b decreases with the increase in the number of iterations, and its mathematical expression is shown in Equation (14). In the early stage of iterations, a larger b value is assigned to expand the search range. As the number of iterations increases, the value of b gradually decreases, favoring local precise search and improving the algorithm's accuracy.

$$b = b_{\max} - (b_{\max} - b_{\min})\frac{t}{T} \tag{14}$$

Among them, $b_{\max}$ is set to 2 and $b_{\min}$ to 0.1. When b=2, the spiral is relatively loose, facilitating global exploration. As the number of iterations increases, b decreases to 0.1, and the spiral gradually tightens, enhancing local exploitation capability. This aligns with the transition requirement from global exploration to local optimization.

3.4. Bird Navigation Mechanism

Birds judge their moving direction based on the moving directions of surrounding companions during migration, and this behavior can be integrated into the whale optimization algorithm. After introducing the bird navigation method, whale individuals determine their moving direction according to the positions of surrounding companions, which can avoid blindly searching in areas far from the optimal solution and improve the quality and stability of the algorithm. Secondly, influenced by companions, whale individuals trapped in local optima have the opportunity to continue searching for other solutions, enhancing the algorithm's accuracy. Finally, whale individuals will gradually approach companions that are already in relatively optimal regions, accelerating the algorithm's convergence speed. The specific method is as follows: Let the position of a randomly selected whale be z_r, randomly select n individuals from all individuals as companions of this whale, with their positions denoted as z_j, $\mathrm{j} = (1, 2, \ldots, n)$. Calculate the average position of all companions.

$$Z_j = \frac{\sum_{j=1}^{n} z_j}{n} \tag{15}$$

The weight coefficient w is used to adjust the influence degree of the average position of companions on the whale individual and update the whale's position. The specific approach is as follows:

$$z_r = (1 - w) z_r + w \times Z_j \tag{16}$$

Among them, w is a constant within the range of [0, 1].

3.5. Flow of the Improved Whale Optimization Algorithm

The flow chart of the Improved Whale Optimization Algorithm (IWOA) is shown in Figure 2.The algorithm implementation steps are as follows:

Step 1: Set algorithm parameters, initialize the population using Logistic chaotic mapping, and generate random individuals.

Step 2: Calculate the individual fitness, and record the individual and global optimal positions.

Step 3: Calculate the convergence factor a, vector coefficients A and C, and generate a random number p.

Step 4: Calculate the fitness of the whale individual at the current position using the formula, and compare it with the fitness of the whale individual at the previous moment. If the fitness of the

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individual at the current position is better than that at the original position, update the optimal individual position to the global optimal position.

Step 5: Randomly select several individuals from the entire whale population as companions of a single whale. Calculate the average position of the surrounding companions, and combine it with its own position to obtain the new individual position.

Step 6: Determine whether the algorithm has reached the maximum number of iterations. If not, return to Step 3; if the maximum number of iterations is reached, output the optimal solution.

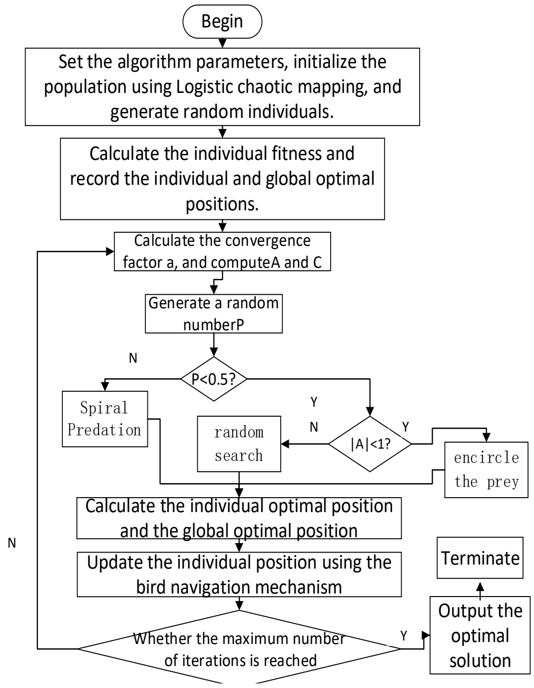


Figure 2. Flow Chart of the Improved Whale Optimization Algorithm.

4. Performance Test of the Improved Whale Optimization Algorithm

Five test functions from the CEC2005 test function set are selected to evaluate the performance of the Improved Whale Optimization Algorithm (IWOA), which is compared and analyzed with the traditional Whale Optimization Algorithm (WOA), Particle Swarm Optimization (PSO), and Genetic Algorithm (GA). The mathematical expressions and optimal values of the test functions are shown in Table 1. Among them, F1 is the Sphere function, F2 is Schwefel's Problem 2.21, F3 is the Rosenbrock function, F4 is the Rastrigin function, and F5 is the Griewank function.

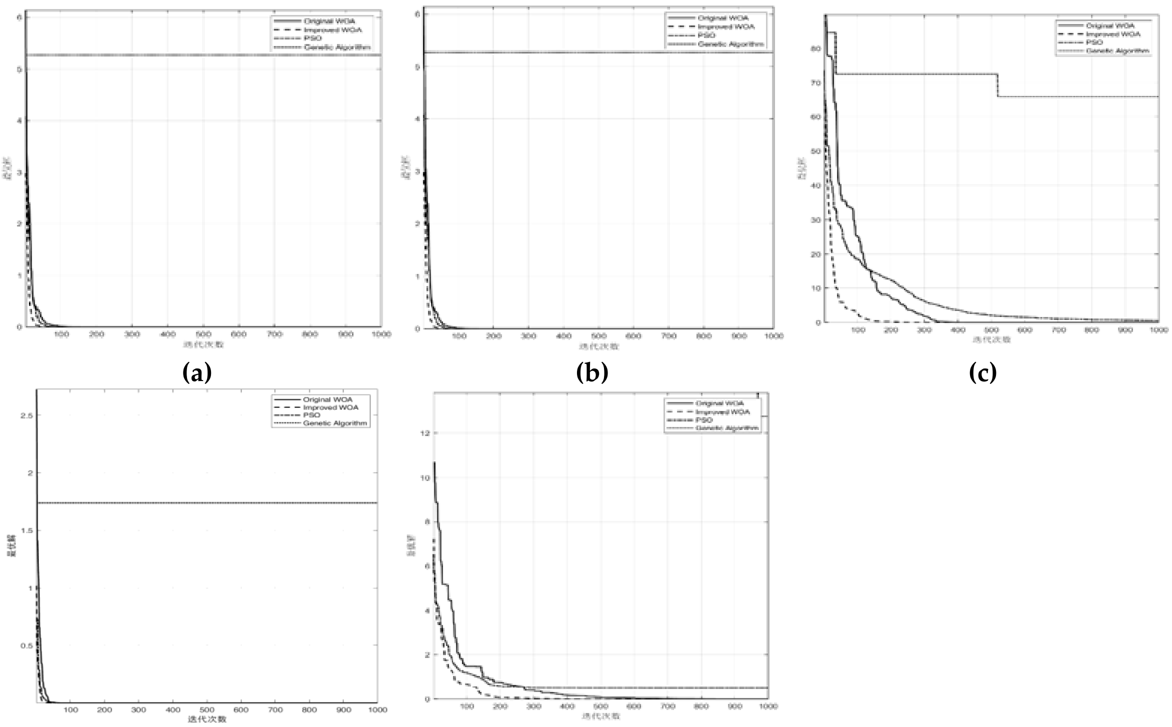
Table 1. Mathematical Expressions, Function Intervals and Optimal Values of Test Functions.

Function	Interval	Optimal Value
$F_1=\sum_{i=1}^{n}x_i^2$	$[-100,100]$	0
$F_2=\max_{i=1}^{n}\lvert x_i\rvert$	$[-100,100]$	0
$F_3=\sum_{i=1}^{n-1}\left[100\left(x_{i+1}-x_i^2\right)^2+\left(1-x_i\right)^2\right]$	$[-30,30]$	0
$F_4=10n+\sum_{i=1}^{n}\left[x_i^2-10\cos\left(2\pi x_i\right)\right]$	$[-5.12,5.12]$	0
$F_5=\sum_{i=1}^{n}\left(\sum_{j=1}^{i}x_i\right)^2$	$[-100,100]$	0

In this test, the algorithm population size is set to 30, the dimension is 30, the number of test runs is 10, and the maximum number of iterations is 1000. The test results are shown in Table 2, and the fitness vs. iteration number curves are presented in Figure 3. It can be seen from Table 2 that compared with other algorithms, the IWOA achieves better optimal values for each test function. For the unimodal functions F1-F2, both the IWOA and WOA can find the optimal values, but the IWOA exhibits a faster convergence speed in the early stage. For the multimodal functions F3-F5, the IWOA's optimal values on F4-F5 are significantly superior to those of other algorithms, and its optimal value on F3 is also the smallest. As observed from Figure 3, the IWOA can find smaller fitness values at the initial stage of iteration compared with other algorithms. With the increase in the number of iterations, the IWOA can converge to the optimal solution more quickly. Moreover, for different functions, the IWOA shows a faster fitness decrease in the early iteration stage and more stable convergence in the later stage, which verifies the improvement effect of the bird navigation mechanism on the algorithm's convergence speed and stability.

Table 2. Test Results of CEC2005 Under 30 Dimensions and 1000 Iterations.

Function	indicator	WOA	GA	PSO	IWOA
F1	optimal value	0	52713.6	0	0
	average	0	61304.12	0	0
F2	optimal value	1.56e-06	65.93	0.5	0
	average	2.16e-06	71.94	0.86	0
F3	optimal value	52.53	2.09×10^{8}	4819.52	28.18
	average	89.45	2.87×10^{8}	5534.73	46.58
F4	optimal value	24.82	301.07	90.54	0
	average	55.71	391.41	99.23	38.26
F5	optimal value	8.88	128912.8	5000.02	0.002
	average	15.87	134076.5	8086.5	0.086



(a) (b) (c)

(d) (e)

Figure 3. Curves of Fitness vs. Number of Iterations.

5. Application of the Improved Whale Optimization Algorithm in Path Planning

5.1. Establishment of the Objective Function

In the path planning process, each node can be regarded as a whale individual. Through continuous update and screening of whale individuals, an optimal path is finally obtained. To determine whether this path is the optimal one, an evaluation function needs to be defined [21].

In point-to-point obstacle avoidance path planning, path length is an important evaluation index [22]. During the path planning process, the Euclidean distance is used to calculate the sum of distances between every two nodes, which is specifically expressed as:

$$L=\sum_{i=1}^{n-1}\sqrt{\left(x_{i+1}-x_i\right)^2+\left(y_{i+1}-y_i\right)^2} \tag{17}$$

Among them, $\left(x_i, y_i\right)$ denotes the coordinates of the i-th node, and n is the total number of nodes in the path. To ensure the robot does not collide during path planning, a collision penalty term C needs to be defined [23]. When the current node does not collide with obstacles, C should be 0. When the node collides with obstacles, C should be set to a large value to eliminate this path and ensure safety, and it is set to 500 in this paper. The specific expression is:

$$C=\begin{cases}0 & \textit{no collision} \\ 500 & \textit{collide with obstacles}\end{cases} \tag{18}$$

To ensure the smoothness of the path [24], the entire path should have fewer inflection points. The calculation of the number of inflection points is as follows:

Let the node sequence of a path be $\mathrm{k}=\left[\mathrm{k}_1, k_2, k_3 \ldots k_i\right]$), where $\mathrm{k}_i=\left(x_i, \mathrm{y}_i\right)$ represents the coordinates of the i-th node. Select every three consecutive nodes k_{i-1}, k_i, k_{i+1} in the current path, and let the direction vectors of two adjacent node pairs be $\overrightarrow{v_i}$ and $\overrightarrow{v_{i+1}}$. The included angleθ between the two direction vectors is calculated as follows:

$$\overrightarrow{v_i}=\left(x_i-x_{i-1}, y_i-y_{i-1}\right) \tag{19}$$

$$\overrightarrow{v_{i+1}}=\left(x_{i+1}-x_i, y_{i+1}-y_i\right) \tag{20}$$

$$\theta=\arccos\left(\frac{\overrightarrow{v_i}\cdot\overrightarrow{v_{i+1}}}{\left|\overrightarrow{v_i}\right|\cdot\left|\overrightarrow{v_{i+1}}\right|}\right) \tag{21}$$

Let an angle threshold be ε. When $|\theta|>\varepsilon$, the number of inflection points is increased by 1. Traverse the entire node sequence to obtain the final number of inflection pointsM. In this paper, ε is set to 5°

By integrating the above three indicators, an evaluation function is obtained as follows:

$$f=\alpha\cdot L+\beta\cdot C+\gamma\cdot M \tag{22}$$

Among them, α is set to 0.8, β is set to 0.1, and γ is set to 0.1.

5.2. Simulation of Path Planning Experiments

The experiments were conducted on a computer with the Windows 11 (64-bit) operating system, using MATLAB R2024a software. The computer is equipped with a 12th Gen Intel(R) Core(TM) i5-12450H processor and 16GB of memory. The grid method [25] was adopted to model the robot's working environment, with map sizes of 30×30 and 50×50.

A start point and an end point were set on the map. The path length, number of inflection points, and other metrics of the traditional Whale Optimization Algorithm (WOA), Particle Swarm Optimization (PSO), Genetic Algorithm (GA), and Improved Whale Optimization Algorithm (IWOA)

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when reaching the end point on different maps were observed and compared to analyze the feasibility of the IWOA in path planning.

In this simulation, the algorithm population size was set to 200, the maximum number of iterations was 200, and the weighting coefficient w in the bird navigation mechanism was set to 0.3. Figure 4 shows the path planning results, and Table 3 records the path length, number of iterations, and number of inflection points of different algorithms on different maps.

Table 3. Simulation Results of Point-to-Point Path Planning.

Map Size	Map Size	WOA	PSO	GA	IWOA
30×30	Path Length	43.97	43.43	46.14	42.56
	Number of Iterations	8	13	28	15
	Number of Inflection Points	7	6	9	3
	Number of Inflection Points	35.876	35.344	37.812	34.348
50×50	Path Length	79.43	84.42	98.84	75.56
	Number of Iterations	70	75	72	65
	Number of Inflection Points	19	22	29	18
	Number of Inflection Points	65.444	69.736	81.972	62.248

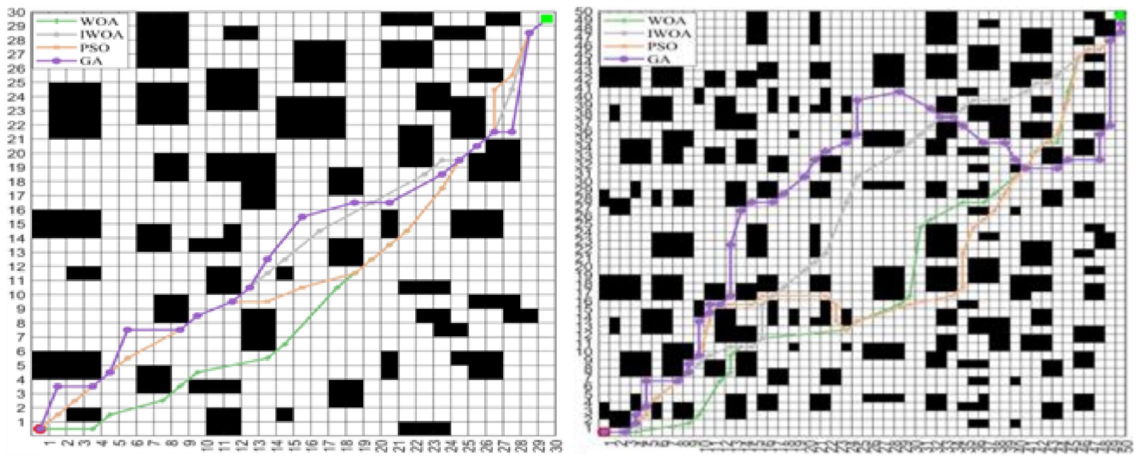

Figure 4. Comparison of Path Planning Results of Four Algorithms on 30×30 and 50×50 Maps.

It can be seen from the above table that in the 30×30 map, compared with the WOA, GA, and PSO algorithms, the IWOA shortens the path length by 3.21%, 2.00%, and 7.76% respectively. Although the number of iterations of the IWOA is slightly higher than that of the WOA and PSO algorithms, it has obvious advantages in the number of inflection points over the other algorithms. In the 50×50 map, the IWOA reduces the path length by 4.88%, 10.50%, and 23.55% respectively compared with the WOA, GA, and PSO algorithms, and also has significant advantages in both the number of iterations and the number of inflection points. Therefore, the feasibility of the IWOA in path planning is verified.

6. Conclusions

(1) The Logistic chaotic map is used to enhance the randomness of generating the initial population. A nonlinear convergence factor is adopted to balance the global and local search capabilities and avoid falling into local optima. An adaptive spiral parameter is utilized to improve the algorithm accuracy, and the bird navigation mechanism is integrated to enhance the algorithm stability.

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(2) Five test functions are employed to verify the performance of the IWOA, which is compared with the WOA and PSO algorithms. The final results indicate that the IWOA achieves excellent performance across different test functions.

(3) Path planning simulations are conducted on 30×30 and 50×50 grid maps. The results show that compared with the WOA, GA, and PSO algorithms, the IWOA shortens the path length by 3.21%, 2.00%, and 7.76% (on the 30×30 map) as well as 4.88%, 10.50%, and 23.55% (on the 50×50 map), verifying the feasibility of the IWOA in path planning.

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