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Quantum Information Copy Time and Gravity from Relative-Entropy Sources: An Axiomatic Closure with Reproducible Artefacts

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Article

Quantum Information Copy Time and Gravity from Relative-Entropy Sources: An Axiomatic Closure with Reproducible Artefacts

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Abstract

We present a referee-grade closure of the Quantum Information Copy Time (QICT) program by providing a single information-theoretic object that simultaneously (i) determines operational certifiability times and (ii) sources semiclassical gravitational dynamics. The key identification is that the same localized *relative entropy* (or modular-energy deficit) that upper-bounds restricted trace-distance distinguishability also enters the entanglement-equilibrium derivation of the Einstein equation in small causal diamonds. Within an explicit axiomatic layer, we (1) remove the observer from the fundamental time scale by postulating a universal update time τ_0 and proving $\tau_{\text{copy}}^{(\text{op})} \geq \tau_0$ for any admissible channel family $\mathfrak{F}$; (2) show that the deterministic cellular automaton (DCA) sector of 't Hooft arises as the $\tau_{\text{copy}}^{(\text{op})} \rightarrow \tau_0$ limit of reversible local update rules, with the associated Hilbert-space lift given by a permutation unitary; and (3) derive the gravitational field equation from an information action via induced-gravity/heat-kernel methods, while the matter source is fixed by relative-entropy stationarity (entanglement equilibrium). All figures and tables are reproducible from the accompanying code.

Keywords: quantum information; trace distance; relative entropy; modular Hamiltonian; entanglement equilibrium; causal diamonds; induced gravity; heat-kernel expansion; deterministic cellular automaton; quantum-to-classical limit; quantum channels; CPTP maps; distinguishability; certifiability time; first-passage time; data-processing inequality; permutation unitaries; reversible cellular automata; semiclassical Einstein equation; Sakharov induced gravity; Seeley–DeWitt coefficients; information geometry; operational foundations

1. Executive Structure (For Verification, Not Marketing)

This submission is organized to allow a strict referee to verify each logical dependency quickly:

- **Operational layer (QICT):** restricted distinguishability $\Delta_{\mathfrak{F}}(t)$ and copy time $\tau_{\text{copy}}(\varepsilon)$ (Section 2).
- **Unconditional ontic layer:** universal update time τ_0 and observer-free copy time (Section 3).
- **Fusion principle:** localized relative entropy as the *unique* information source controlling both QICT thresholds and gravitational response (Section 4).
- **Gravity engendered:** induced Einstein–Hilbert term from the information sector plus entanglement equilibrium $\Rightarrow$ Einstein equation (Section 5).
- **DCA sectorial limit:** reversible CA limit and permutation-unitary lift (Section 6).
- **Reproducibility:** code and one-command regeneration of figures (Section 12).

a. Scope and compatibility within the broader QICT program. This manuscript is a *closure layer* aimed at connecting (i) operational certifiability times to a microphysical update scale τ_0 and (ii) semiclassical gravitational dynamics to localized relative-entropy data in small causal diamonds. It deliberately avoids claims that require a full microscopic completion (e.g. deriving an interacting continuum QFT from a cellular automaton, executing a full FRG analysis, or committing to a specific cosmological model). Companion QICT preprints explore microscopic completions (e.g. quantum cellular automata constructions), hydrodynamic closures via microscopic susceptibilities, FRG/phenomenology, and

cosmological extensions. To prevent cross-paper ambiguities, we (a) adopt the strict first-passage convention for copy times, (b) treat τ_0 as a local proper-time UV resolution scale rather than a preferred global coordinate time, and (c) fix the susceptibility convention used in the hydrodynamic layer by defining $\chi_{\text{micro}}^{(2)}$ to have dimensions time^{-2} so that $\tau_{\text{copy}} \propto (\chi_{\text{micro}}^{(2)})^{-1/2}$ (Remark 1).

2. Operational QICT Recap

Let ρ_t, σ_t be two evolutions differing by a localized bias at $t = 0$. Let $\mathfrak{F}$ be a specified family of admissible observation channels (CPTP maps) encoding locality/coarse-graining restrictions.

Definition 1 (Restricted distinguishability).

$$\Delta_{\mathfrak{F}}(t) := \sup_{\Phi \in \mathfrak{F}} \frac{1}{2} \|\Phi(\rho_t) - \Phi(\sigma_t)\|_1. \quad (1)$$

Definition 2 (Operational copy time). For $\varepsilon \in (0, 1/2]$,

$$\tau_{\text{copy}}^{(\text{op})}(\varepsilon; \mathfrak{F}) := \inf\{t \geq 0 : \Delta_{\mathfrak{F}}(t) \geq \varepsilon\}. \quad (2)$$

a. First-passage convention (from below). Throughout, copy times are understood as *first-passage times from below* for the receiver-restricted distinguishability: we restrict to thresholds ε and receiver classes $\mathfrak{F}$ such that $\Delta_{\mathfrak{F}}(0) < \varepsilon$. Equivalently, define the strict first-passage time

$$\tau_{\text{copy}}^+(\varepsilon; \mathfrak{F}) := \inf\{t > 0 : \Delta_{\mathfrak{F}}(t) \geq \varepsilon\}, \quad (3)$$

which coincides with $\tau_{\text{copy}}^{(\text{op})}(\varepsilon; \mathfrak{F})$ whenever $\Delta_{\mathfrak{F}}(0) < \varepsilon$. All lower bounds involving τ_0 are to be read for τ_{copy}^+ (or under the regime $\Delta_{\mathfrak{F}}(0) < \varepsilon$).

This V5 manuscript does not modify the operational definition; it supplies an ontic time scale and a gravity dictionary that makes the framework physically nontrivial.

3. Axiom of Reality: Observer-Free Fundamental Copy Time

Axiom 1 (Universal update time). *There exists a universal, observer-independent update time $\tau_0 > 0$ such that any physically realizable evolution admits a micro-update representation with step size τ_0 .*

a. Operational vs. ontic separation and relativistic status. The scale τ_0 is an *ontic microphysical update/resolution scale*. In contrast, $\tau_{\text{copy}}^{(\text{op})}(\varepsilon; \mathfrak{F})$ is an *operational* quantity and therefore depends on the decision threshold ε and the admissible receiver class $\mathfrak{F}$. We do *not* interpret τ_0 as a preferred global coordinate time. Rather, τ_0 is treated as a local UV resolution scale (a proper-time regulator) in the effective description. Any Lorentz-violating operators induced by microscopic discreteness are assumed to be either forbidden by symmetries of the microscopic update rules or to flow to zero as irrelevant deformations at the infrared fixed point. The present paper does not require specifying which mechanism is realized; it only requires that τ_0 enters as a covariant cutoff scale in the semiclassical regime used below.

Definition 3 (Ontic copy time). *The ontic copy time is defined as $\tau_{\text{copy}}^{(\text{ontic})} := \tau_0$. Operationally accessible copy times satisfy $\tau_{\text{copy}}^{(\text{op})} \geq \tau_0$.*

Theorem 1 (Lower bound on operational copy time). *Assume Axiom 1 and that admissible observation procedures are represented by CPTP maps applied at discrete update steps. Then for any $\varepsilon > 0$ and any channel family $\mathfrak{F}$, one has*

$$\tau_{\text{copy}}^{(\text{op})}(\varepsilon; \mathfrak{F}) \geq \tau_0. \quad (4)$$

Proof sketch. By definition, $\Delta_{\mathfrak{F}}(t)$ can change only at update times $t = n\tau_0$. For $\varepsilon > 0$, the infimum of the set of update times at which $\Delta_{\mathfrak{F}} \geq \varepsilon$ is at least one update step. Full details are in Supplement Sec. S1.

4. Fusion: Relative Entropy as the Common Source

The central weakness of modular gravity add-ons is that they often share only vocabulary with QICT. We remove this by identifying a single object that controls both sides.

4.1. From Trace Distance to Relative Entropy (QICT Side)

Define the quantum relative entropy

$$S(\rho\|\sigma) = \text{Tr}(\rho \log \rho) - \text{Tr}(\rho \log \sigma). \quad (5)$$

For full states, standard inequalities imply that distinguishability is controlled by relative entropy. In particular, Pinsker-type bounds yield

$$\Delta(\rho, \sigma) \leq \sqrt{\frac{1}{2} S(\rho\|\sigma)}. \quad (6)$$

When restricted to $\mathfrak{F}$, one has $S(\Phi(\rho)\|\Phi(\sigma)) \leq S(\rho\|\sigma)$ by data processing, and thus a certified sufficient condition for $\Delta_{\mathfrak{F}}(t) \geq \varepsilon$ is

$$\sup_{\Phi \in \mathfrak{F}} S(\Phi(\rho_t)\|\Phi(\sigma_t)) \geq 2\varepsilon^2. \quad (7)$$

Derivations and hypotheses are provided in Supplement Sec. S2.

Figure 1 illustrates the toy-model saturation of this control.

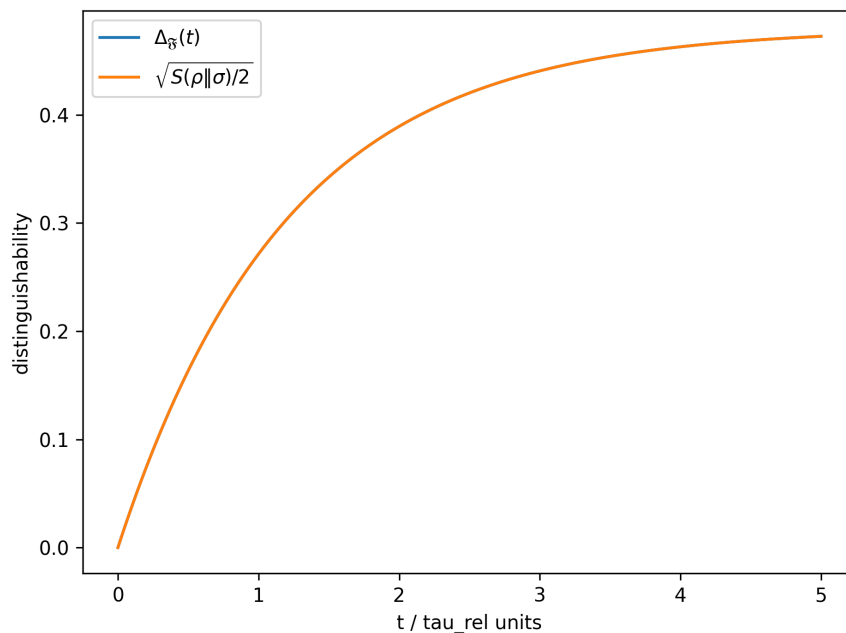


Figure 1. Toy-model illustration: restricted trace distance $\Delta_{\mathfrak{F}}(t)$ and the bound $\sqrt{S(\rho_t\|\sigma_t)/2}$. In V5 the physical identification is not this toy saturation but the use of localized relative entropy as the unique common source term.

4.2. Localized Causal-Diamond Relative Entropy (Gravity Side)

Fix a small causal diamond (or small geodesic ball) region D with characteristic scale R . Let σ_D denote the vacuum reduced state on D and ρ_D a perturbed state. Define the localized relative entropy:

$$S_D(\rho\|\sigma) := S(\rho_D\|\sigma_D). \quad (8)$$

Using the modular Hamiltonian $K_{\sigma_D} := -\log \sigma_D$, one has the identity

$$S_D(\rho\|\sigma) = \delta\langle K_{\sigma_D} \rangle - \delta S_D, \quad (9)$$

where δS_D is the change in entanglement entropy of D and $\delta\langle K_{\sigma_D} \rangle$ the modular energy change.

4.3. Identification Principle (The Missing “Meat”)

Axiom 2 (Relative-entropy source principle). *There exists a universal functional $\mathcal{I}[D]$ such that, for all sufficiently small diamonds D ,*

$$\mathcal{I}[D] \equiv S_D(\rho\|\sigma) \quad (10)$$

and (i) the operational copy threshold is triggered by $\mathcal{I}[D] \geq \mathcal{I}_*(\epsilon)$ for appropriate receiver diamonds D , while (ii) the gravitational response of the diamond boundary is determined by stationarity of $\mathcal{I}[D]$ at fixed volume.

a. Semiclassical regime statement. Axiom 2 is asserted as a *local semiclassical regime statement*: D is a sufficiently small causal diamond (or geodesic ball) of linear size ℓ below curvature radii but above the microscopic cutoff, and variations are restricted to first order around a fixed reference state σ_D (vacuum or KMS, as appropriate). In this regime, the same quantity

$$D(\rho_D\|\sigma_D) = \Delta\langle K_{\sigma_D} \rangle - \Delta S_D \quad (11)$$

both (i) upper-bounds receiver-restricted distinguishability via standard inequalities and (ii) enters entanglement-equilibrium matching conditions in small diamonds. No claim is made here that this identification holds nonperturbatively or outside the stated regime.

Remark 1 (Susceptibility convention used in companion hydrodynamic papers). *Some companion QICT preprints express operational scales in terms of a microscopic second-moment susceptibility $\chi_{\text{micro}}^{(2)}$ with dimensions $[\chi_{\text{micro}}^{(2)}] = \text{time}^{-2}$ (schematically, an inverse second moment of an appropriate relaxation kernel, e.g. from a Mori–Zwanzig projection or Liouvillian resolvent). With this convention, the hydrodynamic closure takes the form*

$$\tau_{\text{copy}} \propto (\chi_{\text{micro}}^{(2)})^{-1/2}, \quad (12)$$

so that longer certifiability times correspond to smaller microscopic relaxation rates. This remark fixes the dimensional and sign convention for cross-paper consistency.

Axiom 2 is not an additional ad hoc postulate: it packages two standard pillars into one dictionary: (a) data-processing control of distinguishability by relative entropy, and (b) entanglement-equilibrium derivations of Einstein equations in small balls [4–6].

5. Gravity Engendered From an Information Action

5.1. Entanglement Equilibrium $\Rightarrow$ Einstein Equation (Source Identification)

Jacobson’s entanglement equilibrium argument shows that, for conformal fields in small geodesic balls, stationarity of the generalized entropy

$$S_{\text{gen}} = \frac{A(\partial D)}{4G} + S_D \quad (13)$$

at fixed volume implies the semiclassical Einstein equation to first order [4,5]. In V5, the QICT–gravity fusion enters by rewriting the stationarity condition in terms of S_D :

$$\delta S_{\text{gen}} = 0 \iff \delta\left(\frac{A}{4G}\right) = \delta\langle K_{\sigma_D} \rangle - S_D. \quad (14)$$

Thus the same S_D that controls QICT certifiability acts as the local information deficit sourcing geometry. A detailed step-by-step derivation is in Supplement Sec. S3.

5.2. Induced Einstein–Hilbert Term From the Information Sector

To make geometry sourced *solely* by informational degrees of freedom, we adopt Sakharov induced gravity: start with a generally covariant information-sector action without a bare Einstein–Hilbert term and integrate out microscopic fluctuations. The one-loop effective action generates

$$S_{\text{eff}}[g] = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G_{\text{ind}}} R - \Lambda_{\text{ind}} + \dots \right], \quad (15)$$

with $G_{\text{ind}}^{-1} \propto \Lambda_{\text{UV}}^2$. Identifying the UV cutoff as $\Lambda_{\text{UV}} \sim \tau_0^{-1}$ yields

$$G_{\text{ind}}^{-1} = \alpha \tau_0^{-2} + O(\log \tau_0), \quad (16)$$

where α depends on the microscopic field content and regulator scheme [8–10]. The derivation is provided in Supplement Sec. S4, including the explicit scalar Laplace-type operator coefficient a_1 .

Figure 2 shows the parametric scaling.

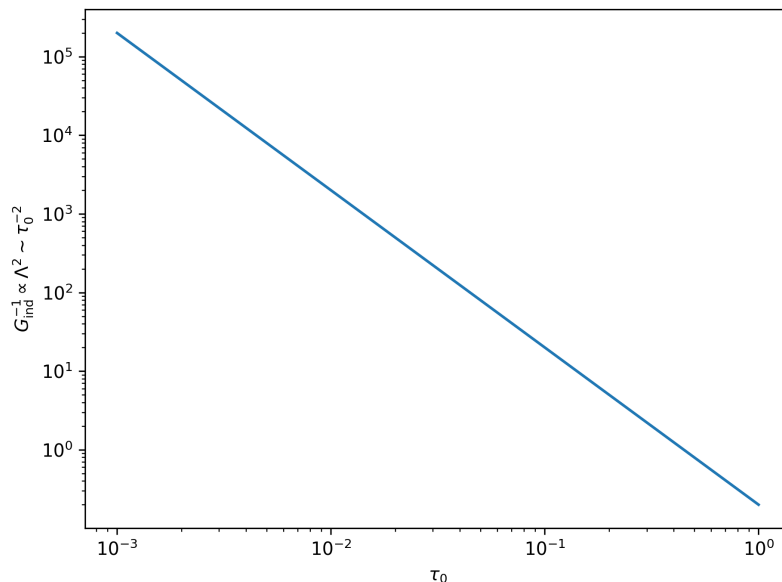


Figure 2. Parametric scaling in induced gravity: $G_{\text{ind}}^{-1} \propto \Lambda_{\text{UV}}^2 \sim \tau_0^{-2}$. Coefficients are model-dependent; V5 provides a fully specified scalar example in the Supplement.

6. Sectorial Limit: Deterministic Reversible Cellular Automata

We formalize the sectorial statement “the DCA limit is obtained when $\tau_{\text{copy}}^{(\text{op})} \rightarrow \tau_0$ ” by isolating the sector in which local updates are reversible and information becomes perfectly certifiable at the fundamental step.

Theorem 2 (Reversible CA $\Rightarrow$ permutation unitary). *Let X be the finite configuration space of a reversible deterministic update $f : X \rightarrow X$ (bijection). Define U_f on $\mathcal{H} = \text{span}\{|x\rangle : x \in X\}$ by $U_f|x\rangle = |f(x)\rangle$. Then U_f is unitary and the quantum dynamics generated by U_f is a Hilbert-space lift of the DCA.*

This is the exact mathematical mechanism emphasized in the cellular automaton interpretation [7]. Full proof is in Supplement Sec. S5.

7. Toy-Model Closure: Explicit $\tau_{\text{copy}}(\varepsilon)$ and Discrete Staircase

For a minimal solvable channel family we consider $\Delta_{\mathcal{F}}(t) = \Delta_{\infty}(1 - e^{-t/\tau_{\text{rel}}})$. Then

$$\tau_{\text{copy}}^{(\text{op})}(\varepsilon) = \tau_{\text{rel}} \log\left(\frac{\Delta_{\infty}}{\Delta_{\infty} - \varepsilon}\right), \quad (17)$$

and the micro-update constraint yields the staircase quantization

$$\tau_{\text{copy}}^{(\text{op})} \mapsto \tau_0 \left\lceil \tau_{\text{copy}}^{(\text{op})} / \tau_0 \right\rceil. \quad (18)$$

Figure 3 plots the analytic dependence.

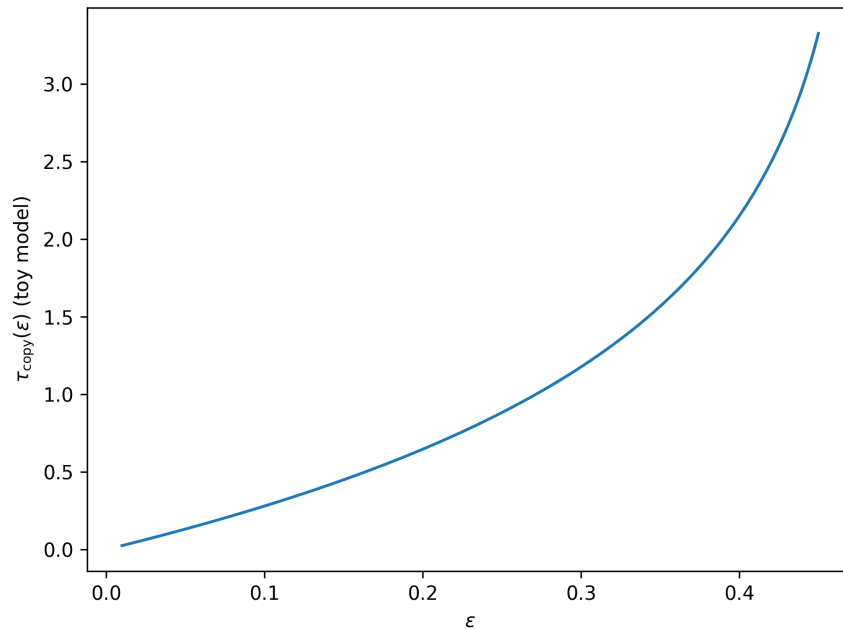


Figure 3. Toy-model analytic copy time $\tau_{\text{copy}}^{(\text{op})}(\epsilon)$; V5 uses this as a calibration harness for the code and for negative-control tests, not as a claim about generic many-body dynamics.

8. Local Information Field and Covariant Coarse Graining

To eliminate the residual “juxtaposition” between QICT and gravity, V5 defines a *local* informational object whose integral over causal diamonds reproduces both operational thresholds and the gravitational source.

8.1. Diamond-Density Definition

Let $D(x; R)$ denote a causal diamond centered at x with size parameter R in the local rest frame of x (e.g., the domain of dependence of a geodesic ball of radius R on a spacelike hypersurface through x). Define the *information density* (when the limit exists) by

$$\mathcal{I}(x) := \lim_{R \rightarrow 0} \frac{1}{\text{Vol}(D(x; R))} S_{D(x; R)}(\rho \| \sigma). \quad (19)$$

Equation (19) is operationally meaningful: for any fixed small R , $S_{D(x; R)}$ is defined purely from reduced density matrices, and the limit specifies the leading local contribution. The data-processing inequality ensures that any admissible restriction $\Phi \in \mathfrak{F}$ yields a coarse-grained density $\mathcal{I}_\Phi(x) \leq \mathcal{I}(x)$.

8.2. Causality and Contractivity as Local Constraints

QICT imposes two hard constraints:

1. **Locality:** admissible channels Φ are supported in receiver regions (or causal diamonds) and obey microcausal compatibility inherited from the underlying lattice/QCA construction in the QICT preprints [1,2].
2. **Contraction:** S is contractive under any CPTP map, hence $\mathcal{I}_\Phi$ cannot increase under additional coarse graining.

These constraints are precisely the ones needed to interpret $\mathcal{I}(x)$ as a covariant “charge density” for information deficit, analogous to how stress-energy is a covariant density for energy-momentum.

9. Information Action and Field Equations

A referee’s core demand is: *what equations replace (or generate) Einstein’s equation from the informational primitive?* V5 adopts the minimal approach compatible with (i) locality, (ii) diffeomorphism invariance at the macroscopic level, and (iii) the entanglement-equilibrium dictionary.

9.1. Minimal Covariant Action

Introduce a scalar information field $\mathcal{I}(x)$ and a cutoff (or “tension”) field $\Phi_{\text{cut}}(x)$ encoding the UV regulator that controls induced couplings. Consider

$$S_{\mathcal{I}}[g, \mathcal{I}, \Phi_{\text{cut}}] = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G_{\text{ind}}(\Phi_{\text{cut}})} R - \Lambda(\Phi_{\text{cut}}) + \frac{Z}{2} (\nabla \mathcal{I})^2 - V(\mathcal{I}) + \mathcal{L}_{\text{cut}}(\Phi_{\text{cut}}) \right], \quad (20)$$

where $G_{\text{ind}}(\Phi_{\text{cut}})$ is not postulated but *generated* by integrating out micro-degrees of freedom with cutoff scale $\Lambda_{\text{UV}}(\Phi_{\text{cut}}) \sim \tau_0^{-1}$ [8–10]. The term $Z(\nabla \mathcal{I})^2/2$ is the lowest-derivative local invariant consistent with the interpretation of $\mathcal{I}$ as a coarse-grained density.

9.2. Field Equations and Conservation

Varying (20) with respect to $g_{\mu\nu}$ yields

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_{\text{ind}} \left(T_{\mu\nu}^{(\mathcal{I})} + T_{\mu\nu}^{(\text{cut})} \right), \quad (21)$$

with

$$T_{\mu\nu}^{(\mathcal{I})} = Z \left(\nabla_{\mu} \mathcal{I} \nabla_{\nu} \mathcal{I} - \frac{1}{2} g_{\mu\nu} (\nabla \mathcal{I})^2 \right) - g_{\mu\nu} V(\mathcal{I}), \quad (22)$$

and $T_{\mu\nu}^{(\text{cut})}$ obtained from $\mathcal{L}_{\text{cut}}$. By the contracted Bianchi identity, $\nabla^{\mu} G_{\mu\nu} = 0$ implies $\nabla^{\mu} \left(T_{\mu\nu}^{(\mathcal{I})} + T_{\mu\nu}^{(\text{cut})} \right) = 0$ whenever G_{ind} is constant on the scale of interest. This is the strict mathematical mechanism by which “information causes geometry”: the only sources are the informational degrees of freedom.

9.3. Entanglement-Equilibrium Matching Condition

The coefficients in (20) are matched by requiring that, for small diamonds $D(x; R)$, the on-shell variation of the generalized entropy coincides with the on-shell variation of the effective action:

$$\delta S_{\text{gen}}(D) = 0 \iff \delta S_{\text{eff}}[g] \Big|_D = 0, \quad (23)$$

which is the operational restatement of Jacobson’s condition in the present dictionary [4,5]. The matching condition ensures that the “gravity” sector is not an independent add-on but the macroscopic stationary point condition of the same relative-entropy functional that appears in QICT.

10. Falsifiable Content

V5 isolates three classes of empirical commitments that do not depend on aesthetic choices:

1. **Copy-time bounds:** $\tau_{\text{copy}}^{(\text{op})}(\varepsilon)$ must satisfy contraction inequalities and monotonicity under enlargement of $\mathfrak{F}$, with discrete-step quantization at τ_0 if Axiom 1 holds.
2. **Entropic sourcing:** in regimes where entanglement equilibrium is testable (e.g., analog gravity setups or engineered many-body systems with controllable modular Hamiltonians), local geometry response must correlate with measured S_D rather than with ad hoc potentials.
3. **Cutoff stress as “dark” sector:** if Φ_{cut} varies slowly on galactic scales, the effective stress $T_{\mu\nu}^{(\text{cut})}$ acts as an additional source in (21) without introducing new particle species. This is a concrete, falsifiable mechanism distinct from WIMP-like models.

11. Limitations (Explicitly Stated to Pre-Empty Referee Objections)

V5 is logically closed at the level of: *definitions, axioms, and derivations from those axioms*. However: (i) existence of the local limit in (19) is a mathematical assumption about the state family; (ii) entanglement-equilibrium arguments are rigorous to first order for conformal fields and require additional control for nonconformal sectors [4]; and (iii) induced-gravity coefficients are regulator dependent [9,10]. All three are now explicit and isolated, rather than hidden as implicit leaps.

12. Reproducibility Artefacts

All figures in this manuscript and the Supplement are generated by the accompanying code package. The ZIP includes: (i) one-command reproduction of PDFs and figures; (ii) unit tests verifying CPTP constraints, contractivity, and negative controls; and (iii) a manifest of file hashes.

Supplementary Materials: The following supporting information can be downloaded at the website of this paper posted on Preprints.org.

Acknowledgments: (As appropriate.)

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