

The influence of time in a theory of nature

Richard Oldani Illinois Institute of Technology

Abstract: We think of time as one of four dimensions appearing in the equations of motion of Newtonian and quantum physics, but participating independently of space. In these pages we explore the way the time variable affects how space is perceived. We show first that non-commutation may be interpreted as the difference between electron and photon cycling during radiation processes. For every electron cycle of a quantum oscillator there are two photon cycles. Secondly, the difference between relativistic and non-relativistic quantum mechanics is shown to be related to time evolution. Only relativistic models evolve continuously. The reasoning of Einstein is implicit to an understanding of both physical models. Previously obtained derivations of relativistic quantum mechanics are reaffirmed by following the ideas of Dirac.

Keywords: Non-commutation; quantum mechanics; time; quantum oscillator; Planck's law; matrix mechanics; relativity theory; Schrödinger equation

1. Introduction

By following theoretical reasoning from Einstein in his 1917 theory it becomes apparent that overemphasis of the importance of emissions has caused experimentalists to ignore the need to investigate absorption [1]. We include absorption formally in a theory of radiation to show that new perspectives are possible. For example, we can view radiation processes in either of two ways: as the result of a complete electron cycle, excitation and decay; or as the result of absorbed and emitted photons, and two cycles of an electromagnetic wave. An electron is excited when a photon is absorbed and a photon is emitted when an electron decays. Thus the cycling of a quantum oscillator occurs at two distinct rates. By comparing both possibilities simultaneously non-commutation is immediately recognizable as having a physical origin since one wave cycle corresponds to one-half of a quantum oscillator cycle and a difference in angular momentum of \hbar [2]. Both the particle and wave models are valid methods for interpreting quantum mechanics since both obey the conservation of energy. The electron oscillator approach leads to matrix mechanics and the wave cycle approach leads to wave mechanics.

Another example of the under appreciation of Einstein's intuitive logic is in his insistence on considerations of the conservation of momentum. As he repeatedly emphasized in his derivation of the A and B coefficients and elsewhere, the momentum of energy absorption is in the direction of the propagation of radiation and the momentum of energy emission is in a direction opposed to the direction of propagation of radiation, thereby indicating that time asymmetry is due to probability. He even states this explicitly [3]. "Einstein believes that irreversibility is exclusively due to reasons of probability." Ordinarily arguments based on momentum exchange provide sufficient reason to question a theory. They resulted in Pauli's proposal for the existence of an unknown particle to explain beta decay, and in Fermi's theory of the neutrino. Even though Einstein's theory of the A and B coefficients is believed accurate and it has proven fundamental to an understanding of lasers, the time symmetry of the Schrödinger equation has not been questioned despite being in direct contradiction to momentum conservation. In a recent communication it was also pointed out that the Schrödinger equation is an approximation for it yields twice the allowable action minimum.

2. Interpretations of time

2.1. Einstein's quantum theory

Einstein's methods are also instrumental in demonstrating a second way that time is important for evaluating theories of nature. In most text books Planck's law $B_\nu(\nu, T)$ is described as the relationship between the temperature T and the frequency ν of the emitted radiation. Explicit use of photons in a radiation equation is for the purpose of experimental derivation.

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}, \quad 1)$$

In contrast Einstein's derivation of Planck's law is based on statistical considerations [4]. Due to the conservation of energy a precise balance exists between the absorption of energy by classical Maxwell Boltzmann statistics and Planck's radiation law. In other words, in the case of black body radiation the energy absorbed classically from thermal energy must equal the energy emitted quantum mechanically as radiation. Energy conservation is maintained by time-averaging without consideration of detailed molecular interactions. Energy balance between absorption and emission at the molecular level is determined statistically by momentum exchange between radiation and atoms and is due to the structural characteristics of atoms, in particular their discrete energy levels as given by the A and B coefficients. By adhering to the requirements of the conservation laws he obtains a detailed balance between the statistically defined fluctuation of energy due to interference on one hand and the fluctuation of energy due to photon numbers on the other.

$$\langle \epsilon(\nu, T) \rangle = \left(h\nu\rho + \frac{c^3}{8\pi\nu^2} \rho^2 \right) \nu d\nu \quad 2)$$

"According to the current theory, the expression would be reduced to the second term (fluctuation due to interference). If the first term alone were present, the fluctuations of the radiation pressure could be completely explained by the assumption that the radiation consists of independently moving, not too extended complexes of energy $h\nu$. In this case, too, the formula says that in accordance with Planck's formula the effects of the two causes of fluctuation mentioned act like fluctuations (errors) arising from mutually independent causes."

There are very noticeable differences in the equations 1) and 2), both of which describe Planck's radiation law, for the simple reason that each one is designed for a particular purpose. Equation 1) is appropriate for experimental verification by including variables such as temperature that are measured at a particular time as the frequency of the radiation. It is used to give experimental evidence why a black body does not emit an unbounded amount of radiation as predicted by the ultraviolet catastrophe. On the other hand, Einstein's equation is based on a desire to understand how the elementary processes of black body radiation evolve in real time as a balance between the continuous and discrete properties of electromagnetic radiation. It is derived statistically as a time evolution. His equation is compatible with relativity theory because it is derived continuously in time. The equation 1) is incompatible with relativity theory for the simple reason that it is derived at a particular point in time. Temperature, for example, cannot be transformed relativistically [5]. By not specifying physical parameters at a specific point in time as in 1) Einstein's derivation of Planck's law shows an aversion to non-relativistic theory at a very fundamental level.

2.2 Matrix mechanics

Despite its incomprehensibility and highly complex mathematics, matrix mechanics is also related to the topic of black body radiation. To see how we shall inspect its formulation of the energy matrix.

$$\sum_k (p_{nk} q_{km} - q_{nk} p_{km}) = \begin{cases} i\hbar & \text{for } n=m \\ 0 & \text{for } n \neq m \end{cases} \quad 3)$$

The diagonal elements $n=m$ represent the observable properties of energy, the transition probabilities and frequencies, which are emissions formulated in coordinates of the atom. Non-diagonal matrix elements $n \neq m$ refer to the resonances of radiation with an atom's valence electrons which result in an exchange of momentum. Elements of the array E_{mn} above the diagonal have a plus sign because they represent energy absorption and elements of the array that are below the diagonal E_{nm} have a minus sign representing energy emission. Because the energy of an absorption offsets the energy of an emission except for a difference in phase a value of zero is assigned to these matrix elements when averaged over time. However, exchanges of momentum that occur instantaneously show that energy absorption leads to a positively directed momentum E/c and energy emission causes recoil momentum E/c directed in the opposite direction of propagation. They do not cancel and lead to a net increase in the kinetic energy of a molecule. Because momentum exchanges cannot be observed individually Heisenberg did not take them into account believing that quantum mechanics should be "founded exclusively upon relationships between quantities which in principle are observable" [6].

Although he eliminated exchanges of momentum, from consideration due to their unobservability Heisenberg soon realized that something was missing, lamenting to Pauli in a letter [7], 'But the worst thing is that I am quite unable to clarify the transition [of matrix mechanics] to the classical theory.' He could have easily resolved the confusion by consulting Einstein's 1917 paper. The off-diagonal matrix elements, which are assigned a value of zero in the energy matrix, give classically defined continuous contributions of momentum and kinetic energy to molecules, contributions which allow black bodies to radiate. He chose not to include classical variables because they could not be used to make experimental predictions and matrix mechanics is non-relativistic for that very reason. It rules out variables continuous in time which are relativistically correct..

2.3. Path integral and Schrödinger equation formulations

Feynman's path integral formulation, or sum over histories, is also limited to a non-relativistic interpretation due to its interpretation of time. The probability amplitudes of events are summed at a particular point in time and the result is squared to give the total amplitude. Similarly solutions to the Schrödinger equation are formulated as probabilities to be resolved experimentally at a particular point in time by a measurement. The reason quantum mechanics is restricted to non-relativistic formulations is in every case due to limitations placed on the time coordinate and the implicit requirement that experimental observations of natural phenomena always occur at specific points in time. Einstein avoided the limitations imposed by "non-relativistic" theory by using equations compatible with relativity theory and deriving quantum phenomena continuously in time. Because they are statistically motivated and do not make predictions that can be verified experimentally they have not received the attention they deserve.

3. Non-relativistic quantum theory

Quantum mechanics is the most successful theory of nature ever discovered, and due to its highly accurate experimental predictions it will probably never be replaced by a more accurate theory. It is the lynch-pin of the standard model and the crown jewel of theoretical physics, a product of countless hours of studies by thousands of physicists and subjected to analysis by thousands of others in attempts to fine-tune or disprove its logic. It has stood the test of time and we are now nearing the one hundredth anniversary of the wave and matrix mechanical formulations where it all began. Based primarily on the spectroscopic measurements of a hydrogen atom's emission spectrum it has not changed significantly since other than in attempts to expand its influence to other areas, such as gravitation and cosmology.

In the old quantum theory electrons were believed to orbit the nucleus so it was an open question whether gravitational fields could affect their motion. However, with the arrival of matrix mechanics Heisenberg showed that there are no orbits; that is, no trajectories in atomic space, and therefore it seemed pointless to pursue that possibil-

ity further. Due to these specific conditions, which were determined at its infancy and have not changed, quantum mechanics is only valid in inertial frames; in frames within which there are no detectable gravitational potentials. If quantum mechanics is to be joined with general relativity theory it is believed that space-time itself must adapt by creating new physics at the Planck level in a theory of quantum gravity.

Recent changes in the initial conditions of quantum mechanics require re-thinking its foundations. Researchers designed an atomic clock using a single crystal of 100,000 strontium atoms together with ultraviolet light to differentiate between the gravitational potential of the crystal's upper and lower surfaces, a distance of one millimeter [8]. Collapse of the wave function does not occur when time measurements are performed. The ticks of the clock are referred to as "non-demolition measurements" because the uncertainty of the ticks does not increase from their measured value as the system evolves [9]. In fact the clock mechanism keeps time so accurately that its behavior is equivalent to that of an ideal quantum oscillator, the same theoretical model used a century ago to derive the founding principles of quantum theory in the formulation of non-relativistic theory. At the time the approximations allowed theoretical work on the quantum oscillator to proceed in ways that seemed perfectly reasonable. There were no orbits so there could be no direct influence by gravitational fields upon electron motion. However, experiment has now out-stripped theory, the ticks of a clock are directly influenced by gravitational fields, and the mass of the electron can no longer be considered negligible. We require a theory of quantum mechanics that accommodates recent clock experiments by not insisting on radical modifications of space-time geometry in order to allow the equivalence principle to be applied at the atomic level. Most importantly we seek a theory that is valid in non-inertial frames, frames which include gravitational fields.

3. The Lagrangian in quantum mechanics

We regularly compensate for error when clocks operate in non-inertial frames, as in the case of the atomic clocks orbiting the earth in GPS systems. The reason quantum mechanics cannot describe the behavior of atomic clocks in satellites is because it was derived non-relativistically. Much effort was expended in attempts to prove that quantum mechanics is incomplete by citing the EPR thought experiment, but quantum mechanics was formulated incompletely from the beginning. Dirac mentioned that a relativistic theory is possible, but did not pursue it [10]. "The theory is non-relativistic only on account of the time being counted throughout as a c-number [classically], instead of being treated symmetrically with the space coordinates." He took up the topic in more detail six years later by indicating a preference for Lagrangian mechanics [11]. "The Lagrangian method can easily be expressed relativistically, on account of the action function being a relativistic invariant; while the Hamiltonian method is essentially non-relativistic in form, since it marks out a particular time variable as the canonical conjugate of the Hamiltonian function. For these reasons it would seem desirable to take up the question of what corresponds in the quantum theory to the Lagrangian method of the classical theory." In this passage Dirac notes what we have previously emphasized in section 2.0, that Hamiltonian models of quantum mechanics are formulated at specific points in time and for that reason the Lagrangian method is preferred. He did not pursue the question further.

Feynman closely followed the "quantum analogue" suggested by Dirac to derive the path integral formulation with Lagrangian methods, but it remains a non-relativistic theory. In his thesis he states [12], "All of the analysis will apply to non-relativistic systems. The generalization to the relativistic case is not at present known." The reason that it is not known, we will maintain here, is that predictions cannot be made and measurements cannot be performed in the continuous time of a relativistic theory.

Aside from the formal reasons mentioned by Dirac there are also physical reasons for seeking a relativistically correct theory. Physical reasons, meaning the way natu-

ral phenomena evolve as opposed to the way we observe them. We observe phenomena at specific times by performing measurements, but they naturally occur continuously as evolutions in time. We can incorporate time evolution in relativistic theories by requiring them to be expressed as the time integral of a Lagrangian. To achieve a relativistically correct quantum theory we take advantage of Hamilton's principle function, $S = \int L dt$, where $L=T-V$. It provides for a more economical expression of the laws of motion by specifying fixed boundary conditions for transitions between energy states and using the calculus of variations to determine electron paths. The absorption of energy is conceived of as a continuous excitation of the electron and the assumption of orbital angular momentum due to a classical superposition of electromagnetic fields.

$$S[r(t)] = \int_{R_1}^{R_2} \int_{t_1}^{t_2} L dt = \hbar \quad (4)$$

In a separate analysis we have shown that in order for energy and momentum to be conserved absorption and emission must occur as separate processes [13]. Absorption is described above in relativistically correct form by a particle model. For emission we follow Dirac's suggestion to pursue a field model, "We may treat the problem of a vibrating medium in the classical theory by Lagrangian methods which form a natural generalization of those for particles. We choose as our coordinates suitable field quantities or potentials." We continue by following the methods of quantum field theory where the primary elements of reality are not assigned to particles as in (1), but to the underlying fields. Introducing a Lagrangian density of the fields and their first derivatives $\mathcal{L}(\phi_i, \phi_{i,\mu})$ allows for a complete accounting of the energy interactions between the excited state $R_2 = (x_2, y_2, z_2)$ at time t_2 and the ground state $R_1 = (x_1, y_1, z_1)$ at time t_1 , where ϕ_i is the current density and $\phi_{i,\mu}$ is the electromagnetic field strength. The action integral for a quantum oscillator with an outer electron that occupies either of two allowable energy states may now be formulated in a way that is consistent with special relativity theory. Applying Hamilton's principle we require the integral of the Lagrangian density over the region of space-time between the excited and ground states to be a minimum for all small variations of the coordinates within the region, where the action minimum for an arbitrary quantum system is defined in angular measure to be the reduced Planck's constant \hbar . The quantization of energy and its emission as a photon occurs due to a four-dimensional localization of field energy and is described by the action integral of a Lagrangian density \mathcal{L} .

$$S[\phi_i(t)] = \int_{R_2}^{R_1} \int_{t_2}^{t_1} \mathcal{L}(\phi_i, \phi_{i,\mu}) d^3x dt = \hbar$$

Lagrangian mechanics is a preferred model of atomic structure because it describes radiation processes as a time evolution rather than by introducing time with a propagator in step-wise fashion as is customary with Hamiltonian methods.

4. The "geonium atom"

The next step, which we wish to pursue here, is to compare the theoretically derived Lagrangian methods with non-relativistic methods in descriptions of natural phenomena, especially with respect to their time evolution. As a first example we take up a fascinating series of experiments that could not have possibly been conceived of by the founders of Hamiltonian models. They make use of intersecting electric and magnetic fields to manipulate the motion of an electron as it transitions between energy levels in a "Penning trap" at rates many magnitudes slower than the electrons in an atom [14]. Single electrons are trapped in a "magnetic bottle" by surrounding them with a homogeneous axial magnetic field and an inhomogeneous quadrupole electric field while non-demolition measurements are performed. The trapped electron constitutes an arti-

cial atom or “quantum cyclotron”, the simplest quantum mechanical system possible; also simpler than the hydrogen atom originally used to derive quantum theory. The measurements are so sensitive that the influence of the earth’s gravitational field is taken into account.

Applying a relatively large constant magnetic field to the trap causes the electron to execute two different types of motion simultaneously; circular orbits perpendicular to the field and axial drifts parallel to it. The experiments are used to precisely observe the absorption and emission of energy by a quantum oscillator. “There is a small alternating magnetic field in the particle’s rest frame, which is perpendicular to the large constant magnetic field. This alternating magnetic field has a frequency component and so a spin-flipping resonance occurs when the drive frequency equals the anomaly frequency.” In other words, the electron is stimulated by a small classically defined magnetic field with varying frequency. The drive frequency gradually shifts upwards with increasing energy until it causes a discrete “spin-flip” to occur.

According to the Schrödinger wave equation and the standard model energy is quantized before it is absorbed. Experiments performed with the simplest possible quantum system clearly indicate otherwise. Energy is absorbed continuously due to classical magnetic resonance and emitted discretely in the form of spin-flips. The possibility that the Schrödinger equation possesses time reversal symmetry suggested by mathematical arguments is denied by these physical arguments. For a second time we have evidence that experimental techniques have advanced beyond theoretical principles derived non-relativistically a century ago when relatively simple spectroscopic measurements were all that was available and a reassessment is necessary.

5. Conclusion

Extensive effort is being expended in efforts to describe gravity according to the principles of quantum mechanics. It is an attempt to rewrite the history of quantum mechanics in order to right a wrong, because it should never have been left out to begin with. Gravitational fields were excluded from descriptions of atomic structure due to the belief that electron paths within atoms do not exist, non-relativistic models were derived based on that assumption, and now new equations are sought to add in what was left out a century ago. It is a pointless exercise in circular reasoning to combine fields after purposely eliminating them. If the premises are wrong no amount of evidence will assist in arriving at a correct conclusion. Attempts to combine the effects of electromagnetism and gravity are nevertheless being pursued and their legitimacy should be vigorously questioned.

The reason unification theories fail is due to the simple fact that the fields are already unified in the form of electrons and other particles. The electromagnetic and gravitational fields coexist harmoniously within these particles bound together in close proximity without interacting. If they do not affect each other during the superpositions that are particles we should not expect to detect a relationship in the far less intense setting of empty space. Everything we know about the gravitational and electromagnetic fields indicates that they have structural characteristics that are independent of the properties we experience. To attempt to unify fields by only looking at the properties we experience ignores their common origin. Instead we must take the opposite viewpoint and ask, “Why do fields that have the same physical origin interact according to completely distinct laws?”

References

- [1] A. Einstein, *Phys Z* **18**, 121 1917, p. 63, Doc 38 <https://einsteinpapers.press.princeton.edu/vol6-trans/>
- [2] Oldani, R. “Application of Einstein’s methods in a quantum theory of radiation” (Intechopen, London, 2021)
- [3] Ritz, W. and Einstein, A. “On the present status of the radiation problem” *Phys Z* **10** (1909) 323-4
- [4] Einstein, A. “On the present status of the radiation problem” *Phys Z* **10** (1909) 185-93.
- [5] Landsberg, P. and Matsas, G. “The impossibility of a universal relativistic temperature transformation” *Physica A* **340** (1-3) (2004), 92-94. doi: <https://doi.org/10.1016/j.physa.2004.03.081>

-
- [6] W. Heisenberg *Z Phys* **33** (1) (1925), in B.L. van der Waerden (ed.), *Sources of Quantum Mechanics* (Amsterdam, 1967).
- [7] Pauli, W. *Wissenschaftlicher Briefwechsel mit Bohr, Einstein, Heisenberg*. Vol 1 Hermann, A., von Meyenn, K., & Weisskopf, V. Eds. (NY: Springer, 1979), p. 251.
- [8] T. Bothwell *et al.* "Resolving the gravitational red shift within a millimeter atomic sample" *Nature* **602**, Issue 7897, 420 (2022) arXiv:2109.12238
- [9] Braginsky, V.B., Vorontsov, Y.I., Thorne, K.S. "Quantum nondemolition measurements" *Science* **209** 4456 (1980). doi: 10.1126/science.209.4456.547
- [10] P.A.M. Dirac, *Proc Roy Soc A* **114**, 243 (1927).
- [11] Dirac, P.A.M., "The Lagrangian in quantum mechanics" *Phys Zeit Sow* **3**, 1933, p.1.
- [12] Brown, L.M., *Feynman's thesis: A new approach to quantum theory* (World Scientific Publishing Co., 2005), p. 4.
- [13] Oldani, R. "The conservation laws in quantum mechanics" *J of Biomed Res and Env Sci* **4**(4): 654-659 (2023) doi: 10.37871/jbres1722
- [14] Brown, L.S.; Gabrielse, G. (1986). "Geonium theory: physics of a single electron or ion in a Penning trap" *Reviews of Modern Physics*. **58** (1): 233–311. doi:10.1103/RevModPhys.58.233.