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Article

Towards Scalar-Field Actions in General Relativity from a Maximum-Entropy Displacement Ensemble

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Abstract

We present a maximum-entropy (MaxEnt) derivation of spacetime geometry starting from a quantum thermal ensemble of local displacement fluctuations. The sole constraint imposed is the expectation value of a quadratic line-element observable. Maximization of entropy yields a Gaussian displacement kernel whose second moments encode an emergent metric structure. Beginning in a locally inertial (flattened Minkowski) frame, we show how curved spacetime geometry and field-space metrics arise through pushforward of the same MaxEnt measure, performed entirely inside the defining integrals. We demonstrate the equivalence of this formulation with the quantum thermal (Matsubara) density-matrix description, without assuming a prior Hilbert-space structure. The resulting geometry is expectation-valued and information-theoretic in origin. This framework provides a unified statistical foundation for spacetime geometry consistent with information geometry, quantum statistical mechanics, and covariant field theory.

Keywords: maximum entropy; information geometry; quantum thermal ensembles; emergent space-time; statistical geometry; Matsubara formalism

1. Maximum-Entropy Displacement Ensembles: Scope and Strategy

The purpose of this work is to clarify how familiar scalar-field actions in curved spacetime may be understood as emergent structures arising from a maximum-entropy (MaxEnt) ensemble of local displacement fluctuations. The approach is deliberately conservative in its assumptions and proceeds entirely at the level of statistical observables before any operator or field-theoretic structure is invoked.

1.1. Displacements as Primitive Observables

We take as primitive objects local spacetime displacements

$$\Delta X^a = X^a - X'^a, \tag{1}$$

defined in a locally inertial (flattened) frame with Minkowski metric

$$\eta_{ab} = \text{diag}(-1, 1, 1, 1). \tag{2}$$

The fundamental observable of the theory is the quadratic line-element increment

$$\Delta s^2 \equiv \eta_{ab} \Delta X^a \Delta X^b, \tag{3}$$

which plays a role analogous to an “energy” or quadratic constraint in statistical mechanics.

Importantly, the variables ΔX^a are not assumed to be geometric operators or dynamical fields. They serve as integration variables in a statistical ensemble describing local fluctuations. Geometry, when it emerges, will be reconstructed from the statistical properties of this ensemble rather than postulated *a priori*.

1.2. Continuous Versus Discrete Formulations

Throughout the main text we work with continuous displacement variables $\Delta X^a \in \mathbb{R}^4$. This choice is natural for differential geometry and information-theoretic entropy maximization, where metrics, connections, and curvature are defined through local second moments and their **derivatives** [1,8].

A discrete formulation, in which ΔX^a takes values on a lattice or finite set (for example $\Delta X^a = \ell n^a$ with $n^a \in \mathbb{Z}^4$), leads to a discrete Gaussian ensemble. In the local limit $\Delta X^a \rightarrow 0$, both continuous and discrete formulations yield identical second-moment structures. Since all geometric quantities constructed below depend only on these second moments, the distinction between continuous and discrete ensembles is immaterial at the level relevant for the present analysis. Discrete formulations may therefore be viewed as regulators or primitive approximations without altering the emergent geometry.

1.3. Maximum-Entropy Construction in a Flattened Frame

We seek the least-biased probability density $\rho(\Delta X)$ subject to normalization and a fixed expectation value of the quadratic observable Δs^2 :

$$\int d^4(\Delta X) \rho(\Delta X) = 1, \quad (4)$$

$$\int d^4(\Delta X) \rho(\Delta X) \eta_{ab} \Delta X^a \Delta X^b = \sigma^2. \quad (5)$$

Maximization of the Shannon–Jaynes entropy

$$S[\rho] = - \int d^4(\Delta X) \rho(\Delta X) \ln \rho(\Delta X) \quad (6)$$

yields the Gaussian displacement ensemble

$$\rho(\Delta X|\beta) = \frac{1}{Z_X(\beta)} \exp[-\beta \eta_{ab} \Delta X^a \Delta X^b], \quad (7)$$

with partition function

$$Z_X(\beta) = \int d^4(\Delta X) \exp[-\beta \eta_{ab} \Delta X^a \Delta X^b]. \quad (8)$$

The parameter β acts as an inverse variance and admits interpretation as a thermal (Matsubara or Euclidean) parameter. Quantum aspects of the construction enter through this thermal weighting, not through any assumed operator character of the displacement variables.

The locally inertial (flattened) formulation is essential. It ensures that the MaxEnt ensemble is defined independently of curvature and that local flatness is built into the statistical description from the outset, in accordance with the equivalence principle.

1.4. Equivalence with the Quantum Thermal Density Matrix

The MaxEnt displacement ensemble admits an equivalent representation in terms of a quantum thermal density matrix. Specifically, expectation values of observables $\mathcal{O}$ computed as ensemble averages,

$$\langle \mathcal{O} \rangle = \int d^4(\Delta X) \mathcal{O}(\Delta X) \rho(\Delta X|\beta), \quad (9)$$

may equally be written as thermal traces

$$\langle \mathcal{O} \rangle = \text{Tr}(\hat{\rho} \hat{\mathcal{O}}), \quad \hat{\rho} = Z^{-1} e^{-\beta \hat{H}}, \quad (10)$$

where $\hat{H}$ is the quadratic generator associated with the line-element observable. No additional physical assumptions are introduced by passing between these representations; they constitute equivalent descriptions of the same quantum thermal ensemble [1,2]. Recall that [2] Caticha explicitly establishes

that: quantum theory can be formulated as entropic / inferential dynamics, thermal / Matsubara structure can arise without postulating operators first, and that PDFs and density matrices are representationally equivalent, not new physics.

In this work, the MaxEnt formulation is adopted as primitive. The density-matrix language is introduced only as a representational convenience and to connect with standard quantum statistical mechanics.

1.5. Motivation for Pushforward to Coordinates and Fields

Once the statistical structure of the displacement ensemble is established, it becomes natural to ask how familiar geometric and field-theoretic objects arise from it. Curved spacetime geometry will be introduced through pushforward of the MaxEnt measure under local frame (tetrad) transformations, performed entirely inside the defining integrals. Similarly, scalar-field actions will emerge by pushing the same ensemble forward into field space via differentiable coordinate–field maps.

By organizing the construction in this way, geometry and field dynamics appear as consequences of a single underlying statistical structure, rather than as independent postulates.

2. From Flat-Space Fluctuations to an Emergent Spacetime Metric

This section clarifies the logical sequence by which a spacetime metric emerges from the maximum-entropy displacement ensemble. We emphasize that no geometric structure beyond local flatness is assumed initially. Objects defined in the flattened frame are therefore interpreted as statistical response tensors rather than spacetime metrics. Only after a change of variables to general coordinates does a general-relativistic metric acquire its standard interpretation.

2.1. Statistical Covariance in a Flattened Frame

We begin with the maximum-entropy displacement ensemble defined in a locally inertial (flattened) frame,

$$\rho(\Delta X|\beta) = \frac{1}{Z_X(\beta)} \exp\left[-\beta \eta_{ab} \Delta X^a \Delta X^b\right], \quad (11)$$

where η_{ab} is the Minkowski metric. The second moments

$$C^{ab}(\beta) \equiv \langle \Delta X^a \Delta X^b \rangle = \int d^4(\Delta X) \Delta X^a \Delta X^b \rho(\Delta X|\beta) \quad (12)$$

define the covariance structure of the ensemble.

At this stage, C^{ab} is not interpreted as a spacetime metric. Rather, it is a statistical object encoding directional fluctuation strengths in a locally inertial frame. Because the ensemble is Gaussian, these second moments fully characterize the fluctuation structure.

To probe how this structure responds to changes in the overall fluctuation scale, we consider the derivative with respect to the inverse-variance parameter β . A direct calculation yields

$$\partial_\beta C^{ab} = -\langle \Delta X^a \Delta X^b Q \rangle + \langle \Delta X^a \Delta X^b \rangle \langle Q \rangle, \quad Q \equiv \eta_{cd} \Delta X^c \Delta X^d. \quad (13)$$

This expression represents the covariance between directional fluctuations and the full quadratic invariant that defines the ensemble.

For a Gaussian distribution, Wick factorization applies. The disconnected contribution proportional to $\langle \Delta X^a \Delta X^b \rangle \langle Q \rangle$ cancels exactly, leaving a purely tensorial response constructed from products of second moments. We therefore interpret $\partial_\beta C^{ab}$ as a *statistical response tensor* characterizing the susceptibility of local fluctuations to changes in scale. No spacetime curvature or nontrivial geometry is implied at this level.

2.2. Pushforward from Flat to General Coordinates

To introduce spacetime geometry, we now perform a change of variables from the locally inertial coordinates X^a to general coordinates x^μ . This is implemented via a tetrad field $e^a_\mu(x)$,

$$\Delta X^a = e^a_\mu(x) \Delta x^\mu. \quad (14)$$

The quadratic invariant transforms as

$$\eta_{ab} \Delta X^a \Delta X^b = g_{\mu\nu}(x) \Delta x^\mu \Delta x^\nu, \quad g_{\mu\nu}(x) \equiv e^a_\mu e^b_\nu \eta_{ab}. \quad (15)$$

Crucially, this transformation is performed *inside the defining integrals* of the maximum-entropy ensemble. The resulting displacement distribution in general coordinates is

$$\rho(\Delta x|\beta) = \frac{1}{Z_x(\beta)} \exp[-\beta g_{\mu\nu}(x) \Delta x^\mu \Delta x^\nu]. \quad (16)$$

At this point, the ensemble explicitly depends on a nontrivial tensor $g_{\mu\nu}(x)$, which now plays the role of a spacetime metric in the general-relativistic sense. Local flatness is preserved by construction, while curvature enters through the spacetime dependence of the tetrad.

2.3. Covariance Structure in Curved Coordinates

The second moments in general coordinates are

$$\langle \Delta x^\mu \Delta x^\nu \rangle_x = \frac{1}{Z_x(\beta)} \int d\Omega_x \Delta x^\mu \Delta x^\nu \exp[-\beta g_{\alpha\beta}(x) \Delta x^\alpha \Delta x^\beta]. \quad (17)$$

These moments now encode information about the local spacetime geometry through $g_{\mu\nu}(x)$.

To extract the metric from the ensemble, we again examine the response with respect to β . Defining

$$\mathcal{G}^{\mu\nu}(x) \equiv -\frac{n}{Z_x(\beta)} \frac{\partial}{\partial \beta} \int d\Omega_x \Delta x^\mu \Delta x^\nu \exp[-\beta g_{\alpha\beta}(x) \Delta x^\alpha \Delta x^\beta], \quad (18)$$

with n a normalization constant fixed by dimensional considerations, we obtain an expectation-valued tensor constructed entirely from the maximum-entropy ensemble.

This object has the same transformation properties and index structure as a spacetime metric. Moreover, for a Gaussian ensemble, the β -response isolates precisely the connected covariance between directional displacements and the quadratic line element, ensuring that $\mathcal{G}^{\mu\nu}(x)$ is a well-defined rank-two tensor.

Measure transformation and invariance.

The change of variables from locally inertial coordinates X^a to general coordinates x^μ induces a corresponding transformation of the integration measure. Writing the displacement relation $\Delta X^a = e^a_\mu(x) \Delta x^\mu$, the flat measure transforms as

$$d\Omega_X \equiv d^4(\Delta X) = |\det e^a_\mu(x)| d^4(\Delta x) \equiv \sqrt{|g(x)|} d\Omega_x, \quad (19)$$

where $g(x) = \det g_{\mu\nu}(x)$. This Jacobian factor is precisely the one required for invariance of the Gaussian weight under coordinate transformations.

Because the quadratic form in the exponential transforms as $\eta_{ab} \Delta X^a \Delta X^b = g_{\mu\nu}(x) \Delta x^\mu \Delta x^\nu$, the combined object $d\Omega_X \exp[-\beta \eta_{ab} \Delta X^a \Delta X^b]$ is equal to $d\Omega_x \sqrt{|g(x)|} \exp[-\beta g_{\mu\nu}(x) \Delta x^\mu \Delta x^\nu]$. Thus the pushforward of the maximum-entropy ensemble preserves its normalization and covariance structure. No additional assumptions regarding the measure are required: the volume element is fixed uniquely by the tetrad transformation, exactly as in general-relativistic integration theory.

2.4. Identification with the Spacetime Metric

The tensor $\mathcal{G}^{\mu\nu}(x)$ constructed above coincides with the inverse metric $g^{\mu\nu}(x)$ that appears in general relativity. This identification follows from the fact that the quadratic form $g_{\mu\nu}(x)\Delta x^\mu\Delta x^\nu$ is the generator of the ensemble and that the β -response probes precisely the susceptibility of the second moments to variations of this generator.

Thus, the spacetime metric is recovered as an expectation-valued object derived from the covariance structure of a maximum-entropy displacement ensemble. No independent postulate of a metric field is required. Instead, the metric emerges as the unique tensor consistent with the statistical structure of local spacetime fluctuations.

Identification with the covariance-derived metric.

Equation (18) is not an independent postulate for the spacetime metric, but follows directly from the covariance structure of the maximum-entropy displacement ensemble. After the pushforward to general coordinates, the quadratic form $g_{\mu\nu}(x)\Delta x^\mu\Delta x^\nu$ plays the role of the generator of the ensemble. Taking a derivative with respect to the inverse-variance parameter β therefore probes the response of the directional second moments $\langle\Delta x^\mu\Delta x^\nu\rangle_x$ to variations of this generator. Note that $\langle\ldots\rangle_x$ denotes the un-normalized expectation.

For a Gaussian maximum-entropy distribution, this β -response isolates the connected covariance between $\Delta x^\mu\Delta x^\nu$ and the full quadratic line element. As shown explicitly in Section 2.1, the scalar contribution cancels, leaving a purely rank-two tensor constructed from contractions of second moments. Equation (18) therefore reproduces exactly the metric structure associated with the covariance of local spacetime displacements, expressed in a form that is manifestly covariant under general coordinate transformations.

In this sense, the spacetime metric emerges as the unique tensor encoding the local covariance of displacement fluctuations. The construction is fully equivalent to the standard metric structure used in general relativity, but is obtained here as an expectation-valued object derived from a maximum-entropy ensemble rather than introduced as a fundamental field.

2.5. Summary

In summary, the construction proceeds in four logically distinct steps: (i) definition of a maximum-entropy ensemble in a locally inertial frame; (ii) identification of statistical response tensors in flat space; (iii) pushforward of the ensemble to general coordinates via tetrads; and (iv) extraction of the spacetime metric as a β -response of the covariance structure. Only in the final step does the response tensor acquire the interpretation of a general-relativistic metric.

3. Pushforward to Field Space and Emergence of Scalar-Field Actions

In Section 2 the spacetime metric was shown to arise as an expectation-valued object determined by the covariance structure of a maximum-entropy displacement ensemble. In this section we extend the same construction to scalar fields defined on spacetime. Our goal is to show, step by step, how the standard scalar-field kinetic action in curved spacetime emerges from the pushforward of the displacement ensemble into field space.

No new statistical assumptions are introduced. All results follow from the covariance properties of spacetime displacements already established.

3.1. Field Variations Induced by Spacetime Displacements

Let $\phi(x)$ be a real scalar field defined on spacetime. For an infinitesimal spacetime displacement Δx^μ , the induced variation of the field is determined solely by differentiability:

$$\Delta\phi = \phi(x + \Delta x) - \phi(x) = \partial_\mu\phi(x)\Delta x^\mu + \mathcal{O}((\Delta x)^2). \quad (20)$$

At this stage no dynamics, equations of motion, or action principles are assumed.

The spacetime displacement Δx^μ is a random variable distributed according to the maximum-entropy ensemble constructed in Section 2. Consequently, the induced field variation $\Delta\phi$ is also a random variable, whose statistical properties are inherited from those of Δx^μ .

Taking the ensemble average of the squared field variation at a fixed spacetime point x , we obtain

$$\langle(\Delta\phi)^2\rangle_x = \langle\partial_\mu\phi\partial_\nu\phi\Delta x^\mu\Delta x^\nu\rangle_x. \quad (21)$$

3.2. Field-Space Covariance from Spacetime Covariance

The spacetime displacement ensemble is characterized by its covariance tensor

$$\langle\Delta x^\mu\Delta x^\nu\rangle_x \equiv C^{\mu\nu}(x), \quad (22)$$

which was shown in Section 2 to encode the local spacetime geometry.

Using Eq. (22) in Eq. (21) gives

$$\langle(\Delta\phi)^2\rangle_x = C^{\mu\nu}(x)\partial_\mu\phi\partial_\nu\phi. \quad (23)$$

In Section 2 it was established that the covariance tensor is proportional to the inverse spacetime metric,

$$C^{\mu\nu}(x) = \alpha g^{\mu\nu}(x), \quad (24)$$

where α is a constant determined by the normalization of the maximum-entropy ensemble. Substituting Eq. (24) into Eq. (23) yields

$$\langle(\Delta\phi)^2\rangle_x = \alpha g^{\mu\nu}(x)\partial_\mu\phi\partial_\nu\phi. \quad (25)$$

Equation (25) shows that the contraction of field gradients with the inverse spacetime metric arises directly from the covariance structure of spacetime displacements. No kinetic term has been postulated.

3.3. Construction of the Minimally Coupled Scalar Action

To obtain an action functional, we integrate the local variance Eq. (25) over spacetime using the invariant volume element associated with the metric $g_{\mu\nu}(x)$:

$$S_{\text{MC}} = \frac{1}{2\alpha} \int d^4x \sqrt{|g(x)|} \langle(\Delta\phi)^2\rangle_x. \quad (26)$$

Substituting Eq. (25) gives

$$S_{\text{MC}} = \frac{1}{2} \int d^4x \sqrt{|g(x)|} g^{\mu\nu}(x) \partial_\mu\phi\partial_\nu\phi. \quad (27)$$

Equation (27) is precisely the standard kinetic action for a minimally coupled scalar field in curved spacetime. In the present framework, however, it arises as the quadratic functional associated with the variance of field fluctuations induced by spacetime displacements.

3.4. Extension to Multiple Scalar Fields

For a collection of scalar fields $\phi^A(x)$, the induced variations are

$$\Delta\phi^A = \partial_\mu\phi^A\Delta x^\mu. \quad (28)$$

Their covariance is therefore

$$\langle\Delta\phi^A\Delta\phi^B\rangle_x = g^{\mu\nu}(x)\partial_\mu\phi^A\partial_\nu\phi^B. \quad (29)$$

This defines a field-space metric

$$\mathcal{G}_{AB}(x) = g^{\mu\nu}(x) \partial_\mu \phi^A \partial_\nu \phi^B, \quad (30)$$

leading to the action

$$S = \frac{1}{2} \int d^4x \sqrt{|g|} \mathcal{G}_{AB}(x) \partial_\mu \phi^A \partial^\mu \phi^B. \quad (31)$$

3.5. Curvature Content of the Minimally Coupled Action

The action (27) is minimally coupled in the standard sense: partial derivatives have been replaced by covariant derivatives, and no explicit curvature-dependent terms have been introduced. Nevertheless, spacetime curvature is already present implicitly.

Varying Eq. (27) with respect to ϕ yields

$$\nabla^\mu \nabla_\mu \phi \equiv \square_g \phi = \frac{1}{\sqrt{|g|}} \partial_\mu \left(\sqrt{|g|} g^{\mu\nu} \partial_\nu \phi \right) = 0. \quad (32)$$

The Laplace–Beltrami operator $\square_g$ depends explicitly on the Christoffel symbols constructed from the metric and its derivatives. Thus curvature enters the dynamics through the geometry encoded in $g_{\mu\nu}$.

To make this dependence explicit, consider fluctuations $\phi \rightarrow \phi + \delta\phi$. The quadratic fluctuation operator is

$$\delta^2 S_{\text{MC}} = \frac{1}{2} \int d^4x \sqrt{|g|} \delta\phi (-\square_g) \delta\phi. \quad (33)$$

The operator $-\square_g$ is of Laplace type on a curved manifold. Its spectral properties are governed by curvature invariants.

Using the heat-kernel expansion, one finds

$$\text{Tr} e^{-s\square_g} = \frac{1}{(4\pi s)^2} \int d^4x \sqrt{|g|} \left(1 + \frac{s}{6} R + \mathcal{O}(s^2) \right), \quad (34)$$

which induces curvature-dependent contributions of the form

$$S_{\text{eff}} = \frac{1}{2} \int d^4x \sqrt{|g|} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{6} R \phi^2 + \dots \right]. \quad (35)$$

Equation (35) shows that curvature couplings of the form $\xi R \phi^2$ arise naturally from the minimally coupled action once the covariance-derived geometry is taken seriously. They are induced by fluctuations and do not represent additional classical postulates.

3.6. Relation to Non-Minimal Coupling and Conformal Frames

A scalar theory with explicit non-minimal coupling,

$$S = \frac{1}{2} \int d^4x \sqrt{|g|} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \xi R \phi^2 \right), \quad (36)$$

may be recast as a minimally coupled theory in a conformally related frame [9], up to potential terms. Thus minimal and non-minimal couplings correspond to different parametrizations of the same underlying geometric structure.

Within the present framework, both forms originate from the same maximum-entropy displacement ensemble. The distinction reflects a choice of representation rather than a difference in fundamental statistical principle. Throughout, potential terms often included in traditional action(s) are spectators and may be included independently without affecting the covariance-derived kinetic and curvature structure.

3.7. Interpretation

The pushforward of the maximum-entropy displacement ensemble from spacetime to field space yields the standard scalar-field action in curved spacetime without postulating field dynamics. Both geometry and field kinetics arise from the same covariance structure, providing an information-theoretic interpretation of scalar-field theory compatible with general relativity.

4. Quantum–Thermal Interpretation and Density–Matrix Formulation

The results of Sections 2 and 3 were obtained using a maximum-entropy probability distribution over spacetime displacements and their induced field variations. In this section we reinterpret the same construction in the language of quantum statistical mechanics. This serves two purposes. First, it clarifies the precise sense in which the framework is quantum rather than purely classical. Second, it establishes the equivalence between the maximum-entropy ensemble and a thermal density-matrix formulation familiar from finite-temperature quantum theory.

No new physical postulates are introduced. The section provides an alternative representation of the same statistical structure already developed.

4.1. Maximum–Entropy Ensemble and Density Matrix

The displacement ensemble constructed in Section 2 is defined by the maximum-entropy distribution

$$P(\Delta x) = \frac{1}{Z} \exp[-\beta Q(\Delta x)], \quad (37)$$

where $Q(\Delta x)$ is a quadratic form in the spacetime displacements and β is a Lagrange multiplier enforcing the covariance constraint.

In quantum statistical mechanics, a formally identical structure arises from a thermal density matrix

$$\rho = \frac{1}{Z} \exp(-\beta \hat{Q}), \quad (38)$$

where $\hat{Q}$ is an operator whose expectation value is constrained. The partition function $Z = \text{Tr } e^{-\beta \hat{Q}}$ ensures normalization.

Expectation values computed using the classical ensemble,

$$\langle A \rangle = \int d\Omega_{\Delta x} P(\Delta x) A(\Delta x), \quad (39)$$

are therefore equivalent to quantum expectation values

$$\langle A \rangle = \text{Tr}(\rho \hat{A}), \quad (40)$$

provided the observables are quadratic or bilinear in the displacement variables. This equivalence is standard in maximum-entropy and entropic-dynamics approaches to quantum theory [2].

Within the present framework, the spacetime metric $g_{\mu\nu}(x)$ arises as such an expectation value. It may therefore be interpreted equivalently as the expectation of an operator-valued quadratic displacement observable.

4.2. Matsubara Representation and Thermal Geometry

The density-matrix formulation naturally admits an imaginary-time (Matsubara) representation. Introducing an imaginary time coordinate τ with periodicity β , the thermal trace may be written as a Euclidean path integral over one period in τ .

In this representation, spacetime displacements are extended to (τ, x^μ) , and the quadratic operator $\hat{Q}$ plays the role of a Euclidean action governing fluctuations. The covariance structure derived in Section 2 corresponds to the two-point function of these fluctuations in imaginary time.

The Matsubara formulation emphasizes that the displacement ensemble is intrinsically quantum-thermal rather than purely classical. The appearance of a metric tensor from displacement covariance is thus analogous to the emergence of effective geometries in finite-temperature quantum field theory and statistical field theory.

Importantly, no specific microscopic Hamiltonian is assumed. The thermal structure follows entirely from the entropy maximization subject to covariance constraints.

4.3. Operator-Valued Geometry and Expectation-Valued Metrics

It is useful to clarify the sense in which the spacetime metric is “quantum” in the present framework. The metric $g_{\mu\nu}(x)$ is not introduced as a fundamental operator to be quantized in the canonical sense. Instead, it is defined as the expectation value of an operator-valued quadratic displacement observable,

$$g^{\mu\nu}(x) \propto \langle \Delta x^\mu \Delta x^\nu \rangle_x. \quad (41)$$

This places the construction conceptually between classical general relativity and full quantum gravity. The geometry is neither fixed nor independently quantized; it emerges as a statistical object encoding the response of spacetime to quantum-thermal fluctuations. In this respect the framework is distinct from semiclassical gravity, where a classical metric is sourced by the expectation value of a stress-energy tensor.

Here, both geometry and field dynamics originate from the same underlying maximum-entropy ensemble.

4.4. Relation to Induced Actions and Curvature Terms

The density-matrix interpretation also clarifies the origin of curvature terms discussed in Section 3. Once scalar fields are treated as fluctuating degrees of freedom governed by the Laplace-Beltrami operator associated with the expectation-valued metric, curvature invariants arise naturally in the spectral expansion of the corresponding operators.

From this perspective, terms such as $\xi R\phi^2$ are understood as thermodynamic or fluctuation-induced contributions to the effective action. They do not signal the introduction of new interactions, but rather encode the response of quantum fields to the covariance-derived geometry.

This interpretation is consistent with standard results in quantum field theory in curved spacetime [10] and with the conformal improvement of scalar actions [9].

4.5. Scope and Limitations

The present framework establishes an information-theoretic and quantum-statistical origin for spacetime geometry and scalar-field actions. It does not attempt to provide a ultraviolet completion of quantum gravity, nor does it specify a microscopic theory of spacetime.

The results are kinematical and structural in nature. They show how familiar geometric and field-theoretic structures arise from a maximum-entropy displacement ensemble, but they do not address the dynamics of metric fluctuations beyond the level encoded in the covariance structure.

Within these limits, the framework provides a transformations based and conservative interpretation of geometry and fields compatible with both general relativity and quantum statistical mechanics.

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