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Spinors, Quarks and Color Confinement

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Abstract

The notion of spinor is generalized. The article offers a deeper understanding of quarks and gluons by directly relating their spinor formulation to the Bloch vector associated with quantum computing. It also helps us better understand the color confinement problem of quantum chromodynamics.

Keywords: spinors; quarks; gluons; Bloch vector; color confinement; GHZ states

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1. Introduction

All atoms in the periodic table are composed of nuclei made of protons and neutrons, which in turn are composed of three colored quarks and held together by eight gluons, which mediate the strong force. As it turns out, protons and neutrons are two examples of baryons, particles composed of three quarks. Intrinsically associated with baryons is the concept of color charge.

The novelty of this article lies in its approach. By generalizing Hermann Weyl's notion of spinor [1,2], we show that the threefold color charge associated with quarks and the strong force can be represented by a 3-spinor, which in turn can be linked to the Bloch vector used in quantum computing. This approach presents new insights into the nature of color confinement in that the 3-spinor representation of colored quarks can be used to better understand GHZ states of three particles [3]. Also, every n-dimensional spinor can be associated with an n^2 isotropic vector (see equation (16)).

Historically, Oscar Greenberg in 1964 implicitly introduced "color charge" to avoid a violation of the Pauli exclusion principle in relation to the Δ^{++} particle [4,5], although the actual name 'color' was later coined by Gell-Mann. Also in 1965, Han and Nambu explicitly incorporated the color concept into an SU(3) model [6]. Later, in 1973, Gell-Mann and Fritzsch futher developed the SU(3) color representation for quarks, not only to preserve the exclusion principle but also to better explain strong force interactions. Specifically, although all baryons are colorless, the Pauli exclusion principle could be preserved by allowing particles like the Ω^- and Σ^{++} to be composed of three different colored quarks. In addition, "color" charge being analogous to "electric charge" in electromagnetism, offered an explanation as to why baryons are colorless.

As it happens, Pauli's exclusion principle rests upon the validity of the principle of microcausality. Usually, this is interpreted to mean that signals cannot communicate beyond the light cone and results in "bosons" and "fermions" being distinguished by their numerical spin values. Bosons and fermions are defined as particles with integral and half-integral spin respectively. When Pauli published his paper on the spin-statistics theorem [7], the notion of entanglement was at best confined to philosophical interpretations of quantum mechanics and in that regard, Einstein held that certain hidden parameters existed that would help avoid the EPR paradox [8] and "spooky action at a distance."

Pauli was aware that if the principle of microcausality were violated (although he did not use these words, [7] p. 721) then the numerical distinction between bosons and fermions also broke down. This is not to say that there is no difference between them, but that distinction rests not upon spin value but entanglement. With the experimental confirmation that Bell's inequality was violated [9,10], the notion of nonlocality became a physical reality, which in the case of spin-singlet states means that their spin values are equal and opposite in all directions at once but in an indeterministic way.

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There is a perfect isotropic spin correlation between the particles, charaterized by an inseparable mathematical relationship, although their distinct spin values cannot be factored into two distinct parts. One observation yields two pieces of information. No signal is transmitted, but it does mean that mathematically (and physically) speaking the spin operator observables of spin-singlet states do not necessarily commute beyond the light cone and that microcausality, in the mathematical sense, is violated [11] in this instance. It can be interpreted as equivalent to the violation of Bell's inequality for paired entangled particles irrespective of color.

In itself, this does not challenge the theory of color. Rather, it establishes it as a theory in its own right, independent of the usual form of the Pauli principle. The fact that colored quarks obey the Pauli exclusion principle follows directly from the $SU(3)$ theory of color entanglement (see equation (19)), and not from the principle of microcausality. Indeed, in this article, we will give a justification for color confinement based on the unique properties of $SU(3)$ and link color confined states to 3-particle GHZ states.

2. Review of $SL(2,C)$ Spinor Methodology

Before entering into a discussion of $SU(3)$ and quark color, we review the $SU(2)$ theory of spinors and photon mediation of the EM force. Following Hermann Weyl's approach to spinors (in contrast to the approach of Elie Cartan) [1,2,12] we define

> a two component spinor as a vector in the 2-dimensional (self-) representation of the group $SL(2;C)$,
> $$\phi \to \phi' = S(l)\phi \qquad \text{where} \qquad S(l) = e^{\omega_{ab}\sigma_{ab}} \in SL(2,C), \tag{1}$$
> and the $\omega's$ and $\sigma's$ are the parameters and generators of $SL(2,C)$ respectively.

It follows that [1]

> the bilinears
> $$v_a = \phi^\dagger \sigma_a \phi \qquad \sigma = \{1, \vec{\sigma}\}, \tag{2}$$
> where $\vec{\sigma}$ are the Pauli matrices and the dagger denotes hermitian conjugation, transform according to the 4-vector representation of $SL(2,C)$ and that this representation of $SL(2,C)$ is isomorphic to the connected part of the Lorentz group.

Note that v_a is also hermitian.

More explicitly, let $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$. Therefore

$$v_o = \tfrac{1}{2}(\bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2) \qquad v_1 = \frac{1}{2}(\bar{\psi}_1\psi_2 + \bar{\psi}_2\psi_1) \tag{3}$$
$$v_2 = \tfrac{i}{2}(-\bar{\psi}_1\psi_2 + \bar{\psi}_2\psi_1) \qquad v_3 = \frac{1}{2}(\bar{\psi}_1\psi_1 - \bar{\psi}_2\psi_2). \tag{4}$$

Direct calculation gives

$$v_0^2 - v_1^2 - v_2^2 - v_3^2 = 0 \tag{5}$$

and as Weyl noted, "there was no bilinear Lorentz *scalar* and concluded the 2-component spinors must be massless" ([1], p. 109). We also note that these bilinears, form a null four vector equivalent to a second rank spinor:

$$V = \phi\phi^\dagger \tag{6}$$
$$= \begin{pmatrix} v_0 + v_3 & v_1 - iv_2 \\ v_1 + iv_2 & v_o - v_3 \end{pmatrix} \tag{7}$$

such that

$$V\phi = v_o\phi. \tag{8}$$

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This means that the spinor ϕ is always an eigenvector of the 2nd rank spinor V, and $det(V) = v_0^2 - v_1^2 - v_2^2 - v_3^2 = 0$. It might be worth noting that for Elie Cartan's notion of spinor [12], equations (4) and (5) can be taken as the definition of a spinor ϕ provided $v_0 = 0$ and $v_1^2 + v_2^2 + v_3^2 = 0$. This also means that v_3 is a complex number and V is no longer hermitian, although his off-diagonal terms are all hermitian.

On a final note, we observe that the generators of $SL(2,C)$ obey the rule $\{\sigma_i, \sigma_j\} = 0$ for all i, j and consequently are all orthogonal. Moreover, if we replace the spinor ϕ with the spinor $\phi' = e^{ik\alpha(x)}\phi$ then equations (2) and (7) remain unchanged. In other words, each v_i and the second rank spinor V are gauge invariant under the action of $e^{ik\alpha(x)}$. This also means that $\alpha(x)$ is independent of any tetrad (vierbien) basis at a point x of a Lorentz manifold. Specifically, if we let $k = e$ be an electric charge (as did Hermann Weyl) then ϕ' can be used to generate the EM field and Maxwell's equation ([1], p. 118). It also follows on inspection of equations (3), (4), and (5) which define the photon mediation of the EM field, that all massless particles have to have zero charge in that the gauge term cancel for V. This also explains why in nature we never observe charged photons. When we switch to $SU(3)$ theory we will find that this property is no longer valid (see next section). We also note, that the vector V is defined on the Bloch sphere which is so important to quantum computing (see next section).

In general, a Lorentz four vector $\vec{x} = (x_0, x_1, x_2, x_3)$ can be identified with the hermitian matrix defined by

$$X = \begin{pmatrix} x_0 + x_3 & x_1 - ix_2 \\ x_1 + ix_2 & x_o + x_3 \end{pmatrix}, \tag{9}$$

with V being a representation of the Lorentz null vector $\vec{v} = (v_0, v_1, v_2, v_3)$. Finally to conclude this section, we note that if we normalize $v_o = 1$ in V, the vector $\frac{1}{2}V$ is a density matrix associated with the Bloch vector (v_1, v_2, v_3). In fact the Bloch density matrix is defined by

$$\rho = \frac{1}{2}(I + \vec{v} \cdot \vec{\sigma}) \tag{10}$$

and is associated with a pure state [13,14].

3. Results: $SL(3,C)$ Group and Spinors

Elie Cartan defines a "spinor" ξ as a (null) eigenvector of the equation $X\xi = 0$, where X is a $2^n \times 2^n$ matrix orthogonal to itself and a representation of a vector $x = (x_o, x_1, \dots, x_n)$ such that $X^2 = x_0^2 + \bar{x}_1 x_1 + \cdots + \bar{x}_n x_n$. The off diagonal terms in Cartan's representation are always hermitian. For example, the vector V in equation (7) serves as a representation of the null vector (v_1, v_2, v_3) provided $v_0 = 0$.

Instead in this article, we generalize Hermann Weyl's 2-dimensional approach to spinors and define an n component spinor as an n-dimensional representation of the group $SL(n,C)$,

$$\xi \to \xi' = S(l)\xi \qquad \text{where} \qquad S(l) = e^{\omega_{ab}\lambda_{ab}} \in SL(n,C). \tag{11}$$

The $\omega's$ and $\lambda's$ are the parameters and generators of $SL(n,C)$ respectively. In the 3-dimensional case, the λ's are equivalent to the Gell-Mann matrices for quantum chromodynamics and are given by [14]:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0, & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0, & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0, & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1, & 0 \end{pmatrix}$$

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$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i, & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0, & -2 \end{pmatrix},$$

where the $\omega' s$ and $\lambda' s$ are the parameters and generators of $SL(3, C)$ respectively. It follows that the bilinears

$$w_a = \xi^\dagger \lambda_a \xi \qquad \lambda = \{1, \vec{\lambda}\} \tag{12}$$

where $\vec{\lambda}$ are the Gell-Mann matrices, and the dagger denotes hermitian conjugation, have the characteristics of "gluons" that mediate the color interactions of the strong force. Note that each w_a is hermitian. Specifically,

$$\begin{array}{lll} w_o = \frac{1}{2}(\bar{\xi}_1\xi_1 + \bar{\xi}_2\xi_2 + \bar{\xi}_3\xi_3) & w_1 = \frac{1}{2}(\bar{\xi}_1\xi_2 + \bar{\xi}_2\xi_1) & w_2 = \frac{i}{2}(-\bar{\xi}_1\xi_2 + \bar{\xi}_2\xi_1)) \\ w_3 = \frac{1}{2}(\bar{\xi}_1\xi_1 - \bar{\xi}_2\xi_2) & w_4 = \frac{1}{2}(\bar{\xi}_1\xi_3 + \bar{\xi}_3\xi_1) & w_5 = \frac{i}{2}(-\bar{\xi}_1\xi_3 + \bar{\xi}_3\xi_1) \\ w_8 = \frac{1}{2\sqrt{3}}(\bar{\xi}_1\xi_1 + \bar{\xi}_2\xi_2 - 2\bar{\xi}_3\xi_3) & w_6 = \frac{1}{2}(\bar{\xi}_2\xi_3 + \bar{\xi}_3\xi_2) & w_7 = \frac{i}{2}(-\bar{\xi}_2\xi_3 + \bar{\xi}_3\xi_2). \end{array} \tag{13}$$

Direct calculation gives

$$3(w_1^2 + w_2^2 + w_3^2 + w_4^2 + w_5^2 + w_6^2 + w_7^2 + w_8^2) = 4w_0^2, \tag{14}$$

which we refer to as the "quark confinement formula". This is the $SU(3)$ analogue of equation (5). It is also worth noting that if $\xi_3 = 0$ then $w_o = \sqrt{3}w_8$ and equation (14) reduces to

$$w_0^2 - w_1^2 - w_2^2 - w_3^2 = 0, \tag{15}$$

which is Lorentz invariant. Formula (14) can (with a little bit of work) be generalized to the n-dimensional case:

$$w_1^2 + w_2^2 + \cdots + w_{n^2-1}^2 = \frac{2(n-1)}{n} w_0^2. \tag{16}$$

Also, analogous to the two-dimensional case, we can define a second rank n-dimensional spinor

$$W = \xi\xi^\dagger \qquad \text{such that} \qquad W\xi = w_0\xi. \tag{17}$$

Note that $w_o = \bar{\xi}_1\xi_1 + \bar{\xi}_2\xi_2 + \cdots + \bar{\xi}_n\xi_n$ is an eigenvalue of the second rank spinor. When $n = 3$, it is often referred to as the color singlet state.

We also note that if $w_0 = 1$ then W corresponds to a density function such that

$$\rho = \frac{1}{n} I + \vec{w} \cdot \vec{\lambda}, \tag{18}$$

where $\vec{w} = (w_1, w_2, \ldots, w_{n^2-1})$ is a Bloch vector associated with a pure state [13,14]. In fact, it corresponds to a pure entangled state.

Finally, in addition to the invariant given by equation (14), there is another invariant under the action of the group $SL(n, C)$ that corresponds to a Fermi-Dirac statistic. Specifically, in the 3-dimensional case, the wedge (outer) product of the three color spinors, ξ, ν, ζ gives

$$\begin{aligned} v &\equiv \xi \wedge \nu \wedge \zeta \\ &= \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} \wedge \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \wedge \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{pmatrix}. \end{aligned}$$

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This reduces with a little algebra to

$$v = \begin{vmatrix} \xi_1 & \nu_1 & \zeta_1 \\ \xi_2 & \nu_2 & \zeta_3 \\ \xi_3 & \nu_2 & \zeta_3 \end{vmatrix} \mathbf{e}_1 \wedge \mathbf{e}_2 \wedge \mathbf{e}_3 \tag{19}$$

and corresponds to a Fermi-Dirac statistic. Note $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are unit vectors. It also means that while in this baryonic state no two quarks can have the same color.

4. A Note on the Color Confinement Problem

We have already noted that charge e (or indeed rest mass m_o) can be introduced via a scalar factor attached to an abelian phase group. However when it comes to color, no such scalar exists. Colors are represented by vectors not scalars. In effect, physicists identify $\xi^t = (r, b, g)$ and respectively refer to them as the color charge: red, blue and green. The complex conjugates $\xi^\dagger = (\bar{r}, \bar{b}, \bar{g})$ are referred to as anti-red, anti-blue and anti-green color charge respectively. Indeed, as can be noted from equation (13), gluons represented by w_0, w_3, w_8 are colorless, but $w_1, w_2, w_4, w_5, w_6, w_7$ all carry some form of color charge. They can be used to explain quark interactions when one baryon transforms into another (via a decay process such as the neutron to a proton) or equivalently, as mediators of the strong force.

One might ask if w_o, w_3, w_8 can be observed? In reality, all mesons are a combination of $\bar{\xi}_i\xi_i$ (for some i) and are colorless. However, since they have mass, they are not considered to be gluons. One could ask, how colorless gluons would be recognized if they existed? Because they are massless, one could possibly identify them with photons, which are also colorless, spin 1 particles.

A second point to take into consideration is that the second order spinor W that generates the gluons (equations (13), (17)) is rank 1 and therefore has only one non-zero observable eigenvalue $w_o = \frac{1}{\sqrt{3}}(\bar{\xi}_1\xi_1 + \bar{\xi}_2\xi_2 + \bar{\xi}_3\xi_3)$. It is called the color singlet state. It is colorless and consequently cannot be observed. The analogue would be trying to observe the charge of a particle with zero electric charge. Note that w_0 is invariant under permutations.

The next point to note is the arbitrary identification of color charges with $r = \xi_1$, $b = \xi_2$ and $g = \xi_3$. The names red, blue and green are purely analogous expressions and have no relation to the primary colors. One could arbitrarily assign $r = \xi_2$ or $r = \xi_3$ without changing the mathematical structure. Indeed, there is an isomorphism between the different color representations.

Moreover, if individual quarks could be separated into three distinct colors, it would mean there are (hidden) local parameters in Einstein's sense with a built in Lorentz invariance (see equation (15)). More specifically, if the three-color particles could be separated into three beams they would be perfect candidates for the Greenberger, Horne, Zeilinger (GHZ) gedanken experiment associated with a three-particle interferometer [3]. One might expect that the three different colored quarks (if they existed as distinct colors) would emerge through apertures a, b, c with distinct colors (say) r, g, b and through apertures a', b', c' as g, b, r or b, r, g, depending on the size of the phase shift. We have agreed that color is a vector state and not a scalar and therefore, such phase shifts are always possible (in principle). Therefore, the three colored-particle state beyond the apertures will be the superposition

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|r\rangle_1|g\rangle_2|b\rangle_3 + |g\rangle_1|b\rangle_2|r\rangle_3), \tag{20}$$

where $|r\rangle_1$ denotes a "red" particle in beam 1 etc. Note that equation (20) is identical with equation (19) of the GHZ paper ([3], p.1135)), provided we replace (r, g, b) with (a, b, c). Repeating the same argument as GHZ, we can conclude "all the requirements are now met... [to exhibit] a contradiction in EPR's premises." In other words, distinct color parameters do not exist. They can coexist as a Fermi-Dirac (indistinguishable) state obeying equation (19). However, to be observed individually they would have to be scalars and not vectors or spinors.

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5. Conclusions

This article has generalized Hermann Weyl's notion of spinor to n-dimensions. As a consequence, it suggests a unified framework for the theory of quantum chromodynamics and quantum computing. The strong force mediated by gluons is found to obey an invariant equation (16) associated with the Bloch sphere. The article also offers an explanation of "color confinement" in that if individual quarks could be isolated with distinct colors, they would exhibit a mathematical contradiction associated with a GHZ state. To avoid such a contradiction, "color" should be considered not as a (hidden) parameter but as a pure entangled GHZ state that is inseparable and exists on the Bloch sphere.

Conflicts of Interest: The author declares no conflict of interest.

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Short Biography of Authors

Paul O'Hara holds a Ph.D. in mathematics from UCLA (1991) and an MA(research) in systematic theology from Catholic Theological Union (2009) in Chicago. He has published in theoretical physics and quantum mechanics, especially on the relationship between quantum entanglement and spin statistics and has presented on these topics at over 30 international conferences. He has coauthored two books, *Unified Field Theory and Occam's Razor* (WSP, 2022) and *Can AI Ever Be Human?* (CUA, 2026). Currently, he is an (emeritus) professor of scientific methodology at the Sophia University Institute, Italy. Previously, he was full professor and chair of the Department of Mathematics at Northeastern Illinois University in Chicago. He is an active member of the International Association for Relativistic Dynamics (IARD).