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Article

Completing Quantum Mechanics Within the Framework of Local Realism

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Abstract: In the Einstein-Bohr debate, Einstein considered quantum mechanics incomplete and disagreed with Born's probabilistic interpretation of wave-functions, which collapse abruptly once measurements are performed on the corresponding microscopic objects. Inspired by Einstein, Bell and his followers intended to complete quantum mechanics within the framework of local realism. Regrettably, the *deterministic correlation* between *distant components* of a *separable* system in Einstein's local-realist description of the world is mistaken for the so-called non-locality or "nonlocal-interaction" in the world described by Bell's theorem, which leads to the questionable interpretation of the experimental results obtained by testing Bell inequalities. This article introduces a new principle, *the general principle of measurements*, which is proved as a mathematical theorem and allows quantum mechanics to be completed within the framework of local realism while keeping the formal axiomatic definition of a *general* Hilbert space essentially unchanged. Using disjunction ("or") as the logical relation between orthonormal vectors spanning a given Hilbert space, the completed quantum theory precludes inexplicable collapses of wave-functions and is intuitively comprehensible, thus alleviating much difficulty in understanding quantum mechanics. Among various world views, Einstein's local-realist world view is correct.

Keywords: einstein-bohr debate; local realism; einstein's separability principle; bell inequalities; bell's theorem; quantum correlation; quantum measurements; non-locality (nonlocal-interaction)

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1. Introduction

Consider two *spatially separated, individual* microscopic objects. A *correlation* between them always exists. If the condition demanded by Einstein's *separability principle* is satisfied [1], namely, the objects possess their autonomous, real states independent of each other, then they constitute a *separable* system, and the possibility of describing the objects in any nonlocal way will be precluded: the so-called non-locality or "nonlocal-interaction" ([2], pp. 1886-1887) does not exist. For ease of exposition, call such objects *distant components* of the separable system, or simply call them distant components, when no confusion will arise if the corresponding system is not mentioned. In Einstein's local-realist description of the world, values of all variables describing a distant component exist objectively according to the assumption of realism, and according to the assumption of locality, measurements performed on either distant component cannot immediately affect the other distant component as shown in the Einstein-Bohr debate [3,4].

The essence of the Einstein-Bohr debate is the legitimacy of quantum superpositions. Einstein disagreed with Born's probabilistic interpretation of wave-functions, which are expressed as quantum superpositions. Born's probabilistic interpretation cannot account for indeterminism in quantum physics. Once a measurement is performed on a quantum-mechanically described object, the corresponding wave-function collapses abruptly. Because the quantum-mechanical description "cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance" ([5], p.158), Einstein called it "the fundamental dice-game" ([5], p.149).

Inspired by Einstein, Bell and his followers intended to complete quantum mechanics within the framework of local realism [6]. Bell derived the first of Bell inequalities to be tested by experiments [7–10] and proved Bell's theorem to interpret the experimental results [7]. To his surprise, his theorem shows Einstein to be wrong in the Einstein-Bohr debate [2,7], which is the opposite of what Bell intended [6,11]. Why does Bell's approach lead him to prove the opposite of what he intended? Can quantum mechanics be completed in a way different from Bell's approach? Concerning different understandings of Bell's theorem [1,12–18], including those disproving or questioning Bell's theorem [17,18], the questions are still open. This study aims to answer the above questions by introducing a new principle, *the general principle of measurements*, which is proved as a mathematical theorem. The main findings are as follows.

Bell's approach is problematic; the *deterministic correlation* between *distant components* of a separable system in Einstein's local-realist description of the world is mistaken for the so-called non-locality or "nonlocal-interaction" in the world described by Bell's theorem [2]. Quantum mechanics can be completed within the framework of local realism in a way consistent with the formal axiomatic definition of a *general* Hilbert space. Using disjunction ("or") as the logical relation between superposed orthonormal vectors, the completed quantum theory precludes inexplicable collapses of wave-functions and is intuitively comprehensible, thus alleviating much difficulty in understanding quantum mechanics. Among various world views, Einstein's local-realist world view is correct. Relinquishing Einstein's separability principle as suggested in [1] is unnecessary.

2. Local Realism and EPR Argument

According to [1], Einstein himself did not write the EPR paper (i.e., [3]). He neither mentioned the "elements of physical reality" in his standard argument for the incompleteness of quantum mechanics nor considered the well-known EPR argument (as given in [3]) satisfactory; Podolsky wrote the EPR paper. Thus the EPR argument cannot fully reflect Einstein's own views and deeper philosophical assumptions. Nevertheless, because the EPR argument is widely known, it is still necessary to analyze the role played by the assumptions of locality and realism (local realism) underlying the EPR argument in the context of its logical structure.

The purpose of the EPR argument [3] is to question the completeness of quantum mechanics based on local realism and some less important assumption. As stated by EPR [3]: "In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system." In their argument [3], EPR then continued to reveal a contradiction in the conceptual foundations of current quantum theory: if quantum mechanics is *assumed* to be a complete theory, then this *assumption*, together with the criterion of reality, leads to a contradiction. But what is the contradiction they revealed?

To answer the above question, let us consider two well-known statements in the EPR argument [3]: "either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) two physical quantities described by two non-commuting operators cannot have simultaneous reality." One of the two statements must be wrong. According EPR [3]: "Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false."

In other words, the negation of (1) implies the negation of (2). By contra-position, the statement "the negation of (1) implies the negation of (2)" is equivalent to the statement "(2) implies (1)", i.e., "two physical quantities described by two non-commuting operators cannot have simultaneous reality" implies "the description of reality given by the wave function in quantum mechanics is not complete." If the two non-commuting operators are the position operator and the momentum operator, then (2) is a consequence of Heisenberg's uncertainty relation. The negation of (2), i.e., the statement "(2) is false", actually implies the statement "Heisenberg's uncertainty relation is false." Thus the negation of

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(2) further implies the falsity of Heisenberg's uncertainty relation. The negation of (1) is the statement "quantum mechanics is complete."

As shown above, the contradiction in the conceptual foundations of current quantum theory is this: the completeness of quantum mechanics implies the falsity of Heisenberg's uncertainty relation, or equivalently, Heisenberg's uncertainty relation, *if* it holds, implies the incompleteness of current quantum theory. Because the negation of (2), which implies the falsity of Heisenberg's uncertainty relation, is the only other alternative *if* Heisenberg's uncertainty relation holds, EPR "are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete." However, is it possible for EPR to endorse Heisenberg's uncertainty relation? As can be seen shortly, the answer to this question must be no.

But what forces EPR to reach their conclusion? In other words, what exactly in the EPR argument imply the incompleteness of current quantum theory? Can the incompleteness of quantum mechanics be implied by locality or realism? A positive answer to the above question would lead EPR to reject local realism. The purpose of the EPR argument is to question the completeness of current quantum theory based on locality and realism. It is the belief of EPR that a complete quantum theory within the framework of local realism is possible [3]. Considering locality or realism responsible for the incompleteness of quantum mechanics contradicts EPR's belief. Clearly, rejecting either of these assumptions would appear entirely unacceptable to EPR.

According to current quantum theory, *before* any measurement is performed on a particle, its state is described by a wave-function, which is a quantum superposition, such that the commutation relation for the position operator and the momentum operator holds for this wave-function; *after* a measurement is performed on the particle, the wave-function collapses immediately. The EPR argument rests on the existence of a *correlation* between two spatially separated particles ([19] p.225). According to Einstein's separability principle in his own incompleteness argument [1], the particles are *distant components* of a separable system. As distant components of a separable system, the particles always possess their autonomous, real states independent of each other. The collapses of wave-functions triggered by measurements are exactly due to what Einstein called "spooky actions at a distance". Therefore, it is impossible for EPR to endorse Heisenberg's uncertainty relation.

Bohr defended quantum mechanics. He raised issues concerning practical measurements related to the uncertainty relation but did not mention the correlation between the spatially separated particles [4]. Thus the contradiction in the conceptual foundations of current quantum theory cannot be explained away.

3. Bell's Approach Revisited

Consider the following opinions: 1) regarding hidden-variables theories, what Bell's theorem tell us is that for such theories to *reproduce* the *statistical* predictions given by current quantum theory, they must be either nonlocal or super-deterministic; the essence of Bell's investigation was whether the perfect correlations predicted by current quantum theory could be explained locally by introducing "hidden variables", and 2) questioning Bell's theorem amounts to suggesting that quantum mechanics is incorrect.

Such opinions reflect a fact: there are different understandings of Bell's theorem as shown in the literature [1,12–18]. But whatever Bell's theorem tell us may have nothing to do with Einstein's local-realist description of the world, and questioning Bell's theorem should not be considered as saying "quantum mechanics is incorrect." Bell's theorem is questionable, because Bell mistook the *correlation* between *distant components* of a separable system in Einstein's local-realist description of the world for the so-called non-locality or "nonlocal-interaction" in the world described by his theorem [2]. To address the above issues, let us begin with revisiting Bell's approach.

When deriving Bell inequalities [7–10] by resorting to a hidden-variables theory [20–22], Bell and his followers merely tried to reinterpret quantum mechanics while keeping the theory in its current form intact [11]. Thus Bell's approach presumes the legitimacy of the current quantum theory.

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Purportedly obtained from the assumptions underlying the EPR argument, Bell inequalities cannot capture the essence of the Einstein-Bohr debate, namely, the legitimacy of quantum superpositions. Einstein never endorsed any hidden-variables theory ([19], p.254). Nevertheless, Bell regarded Einstein as a proponent of hidden variables [7] and maintained his views (see Ref. /23/ in [23,24]). Although neither Bell nor Einstein ever mentioned the "elements of physical reality" [1], some physicists believe that there exists a linkage between hidden variables and the "elements of physical reality" [2,10]. Such a linkage is nonexistent.

Bell inequalities are not results about quantum mechanics. But the world described by Bell's theorem is the world described by quantum mechanics, which differs substantially from Einstein's local-realist world view. In Einstein's local-realist description of the world, the *correlation* between *distant components* of a separable system is due to some interaction occurred *before* the components spatially separated; *after* the separation, there is no longer any interaction between the *distant components*. But the *correlation* still exists, even though the components have spatially separated [3]. Regrettably, the *correlation* between *distant components* of a separable system in Einstein's local-realist description of the world is mistaken for the so-called non-locality or "nonlocal-interaction" in the world described by Bell's theorem [2]; it is Bell's theorem that leads to the questionable interpretation of the experimental results obtained by testing Bell inequalities. The questionable interpretation has led to serious consequences as shown below.

One of Bell inequalities, the CHSH inequality [8], has been intensively tested by actual experiments with *single* pairs of *correlated* photons using technologies of modern optics [9,10]. According to Bell's theorem, the Einstein-Bohr debate seems to have been resolved "in the way which Einstein would have liked least" [22]. Nowadays people believe that Einstein's local-realist world view conflicts with the experimental results of testing the CHSH inequality [2,10]. However, derived based on the nonexistent linkage between hidden variables and the "elements of physical reality", the CHSH inequality merely represents an unsuccessful attempt to reinterpret quantum mechanics by reproducing its statistical predictions while presuming the legitimacy of quantum superpositions. Thus the CHSH inequality cannot capture the essence of the Einstein-Bohr debate, and the assumptions underlying the EPR argument as well as Einstein's separability principle are all irrelevant to Bell's theorem.

But why is the *correlation* between *distant components* of a separable system in Einstein's local-realist description of the world mistaken for the so-called non-locality or "nonlocal-interaction" in the world descried by Bell's theorem? Before answering this question, it is necessary to introduce a new principle, *the general principle of measurements*, which can be proved as a mathematical theorem.

4. General Principle of Measurements

Physical quantities are all measured in the real world based on mathematical models of space and time. Thus the corresponding models must be identified first before the new principle is introduced and proved mathematically. The mathematical model of space is the three-dimensional Euclidean space $\mathbb{R}^3$ endowed with the metric given by the usual distance function between two points in space; a point $z \in \mathbb{R}^3$ represents a precise space coordinate. The mathematical model of time is the set of nonnegative real numbers R_0 equipped with the metric given by the usual distance function between two nonnegative real numbers; an element $t \in R_0$ is a precise time coordinate.

To prove the general principle of measurements as a mathematical theorem, let us recall a few definitions in "metric space" and "point-set topology". A metric space is denoted by (X, d), where X is a set, and d is a metric on X. Let $r > 0$ be a real number. For $x \in X$, the open ball with center x and radius r is

$$B(x; r) = \{y \in X : d(x, y) < r\}. \tag{1}$$

Any open subset of X is a union of open balls. All open subsets of X constitute a metric topology $\mathcal{T}_X$ for X. The metric topology $\mathcal{T}_X$ and X form a metric topological space. Consider $x \in S$ where $S \in \mathcal{T}_X$. If there exists $r > 0$ such that

$$B(x; r) \cap S = \{x\}, \tag{2}$$

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then x is an isolated point of S. Denote by $\mathcal{T}_{\mathbb{R}^3}$ and $\mathcal{T}_{R_0}$ the metric topologies for $\mathbb{R}^3$ and R_0 associated with the metrics given by the corresponding distance functions. Apparently, measuring a point z in space perfectly precisely requires z to be an isolated point of $\mathbb{R}^3$. Similarly, unless time t is an isolated point of R_0, it is impossible to measure t perfectly precisely.

Theorem 1. *(The General Principle of Measurements): Precise space and time coordinates are practically unattainable by measurements, or equivalently, neither $\mathbb{R}^3$ nor R_0 have isolated points.*

Proof. Consider first an arbitrarily given $z \in S$, where $S \in \mathcal{T}_{\mathbb{R}^3}$ is arbitrary. Evidently, there is no $r > 0$ such that

$$B(z;r) \cap S = \{z\}. \tag{3}$$

Thus $\mathbb{R}^3$ has no isolated point. Now consider $t \in S$, where $S \in \mathcal{T}_{R_0}$ is arbitrary. An open "ball" now is an open interval

$$B(t;r) = (t-r, t+r). \tag{4}$$

There are two cases: $t = 0$, and $t > 0$. If $t = 0$, then $B(0;r) \notin \mathcal{T}_{R_0}$ for any $r > 0$, and there is no $S \in \mathcal{T}_{R_0}$ such that

$$S \cap B(0;r) = \{0\}. \tag{5}$$

Thus 0 is not an isolated point of R_0. If $t > 0$, there is no $r > 0$ such that

$$S \cap B(t;r) = \{t\}. \tag{6}$$

Thus t is not an isolated point of R_0 either. Consequently, R_0 has no isolated point. □

When physical events are considered in special relativity, the theorem proved above is also valid. Mathematically, the space-time of events in special relativity is a four-dimensional differential manifold called the Minkowski manifold, which is endowed with a topology, such that any subset of this manifold is either open or not in the usual sense. Thus the definition of an isolated point also applies to the Minkowski space-time manifold. Clearly, the Minkowski manifold has no isolated point.

Of course, real numbers and other mathematical objects constructed based on real numbers are all precisely *defined*. But the general principle of measurements is not about precise definitions of such mathematical objects; its significance is this: specified by precise space and time coordinates, "the same experimental conditions" are not physically meaningful. Any random phenomenon in physics can only be observed in *different* repetitions of a given experiment under the experimental conditions that can only be approximately the same. According to the general principle of measurements, if "the same experimental conditions" are specified by precise space and time coordinates, then such experimental conditions are physically meaningless, because precisely defined coordinates are unattainable by practical measurements. Proved as a mathematical theorem, the general principle of measurements does not involve issues concerning practical measurements raised by Bohr [4] or the accuracy of results obtained by measuring the values of space or time variables in various wave-functions.

Before the advent of quantum mechanics, physicists held a commonsense: "the same experimental conditions" always produce the same results. In other words, results produced by "the same experimental conditions" are deterministic. This commonsense is approximately true, because the experimental conditions in classical physics can be considered approximately the same, and random phenomena observed in classical physics are mainly due to lack of knowledge needed to describe physical situations that typically involve a large number of single microscopic objects; their behaviors are usually assumed to be independent. Randomness in such situations is explainable using statistical mechanics. The general principle of measurements is ignorable. Quantum mechanics changed this commonsense. Nowadays physicists hold a new commonsense: "the same experimental conditions" do not produce the same results in quantum physics, or equivalently, the results produced by "the same experimental conditions" in quantum physics are indeterministic [25]. However, this new com-

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monsense is misleading and largely responsible for erroneously interpreted experimental results in quantum physics.

Observed in *different* repetitions of an experiment in quantum physics, mutually exclusive properties actually correspond to *different* measurement outcomes associated with *different* objects of the same kind. But such properties are attached to an *imaginary* object. As a consequence of violating the general principle of measurements by taking precise space and time coordinates for granted to specify "the same experimental conditions", the *imaginary* object does not exist in the real world. Thus the origin of indeterminism in quantum physics is concealed by "the same experimental conditions". In various experiments involving quantum superpositions, indeterminism actually stems from violating the general principle of measurements by taking precise space and time coordinates for granted. Caused by lack of knowledge concerning precise space and time coordinates used to specify "the same experimental conditions", indeterminism in quantum physics is not explainable using statistical mechanics.

Consider, again, the optical experiment designed to test the CHSH inequality [10]. Expressed as a quantum superposition, an "entangled state" describes the single pairs of correlated photons and is used to calculate the probabilities of obtaining the corresponding outcomes by measuring the polarizations of the correlated photons in the pairs, which implies the legitimacy of Born's probabilistic interpretation of wave-functions. Thus the failure of the CHSH inequality is inevitable when tested by actual experiments against quantum mechanics. The "entangled state" depends on "the same experimental conditions" specified by precise space coordinates, which are the corresponding points on a unit sphere $U \subset \mathbb{R}^3$. This sphere U should not be confused with the "Bloch sphere", which is not contained in $\mathbb{R}^3$.

The points on U correspond to a) the polarizations as well as propagating directions of *different* photons detected in *different* repetitions of the experiment, and b) the orientations of the polarizers for measuring the polarizations of the photons. Taking the precise space coordinates for granted [10], the "entangled state" violates the general principle of measurements and is illegitimate. The indeterminism observed in the measurement outcomes is exactly due to lack of knowledge about the precise space coordinates used to specify "the *same* experimental conditions" for measuring the polarizations of *different* photons detected in *different* repetitions. Using statistical mechanics cannot explain this indeterminism.

Taking precise time coordinates for granted to specify "the same experimental conditions" can also conceal the origin of indeterminism in quantum physics and has caused much difficulty in understanding the quantum measurement problem concerning not only microscopic objects but also "macroscopic" objects [13,14]. Experiments with such "macroscopic" objects typically involve a "time ensemble". Measurements are performed on a "macroscopic" object in *different* repetitions of the corresponding experiment. In each repetition, the "macroscopic" object is measured at some *fixed* times used to specify "the same experimental conditions". The indeterminism due to lack of knowledge about precise time coordinates in the above situation can be analyzed similarly based on the general principle of measurements.

5. Hilbert Space in Quantum Mechanics

In 1927, John von Neumann provided the first formal axiomatic definition of a *general* Hilbert space. Based on the axiomatically defined *general* Hilbert space, von Neumann also provided an axiomatic formulation of quantum mechanics as its formal foundation. The name of "Hilbert space" is in honor of David Hilbert. At the beginning of the last century, Hilbert studied the classical prototype of what is known today as a Hilbert space in his work on the theory of integral equations. In functional analysis, mathematicians now denote this space by ℓ^2. As shown below, the general principle of measurements allows quantum mechanics to be completed within the framework of local realism in a way consistent with the formal axiomatic definition of the *general* Hilbert space.

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Concepts used by von Neumann to define axiomatically the *general* Hilbert space are all highly abstract notions and have no practical meanings. Specified by an inner product, orthogonality is a purely mathematical concept. Assigning practical meanings to orthogonality is unnecessary. Moreover, the logical relation between orthogonal vectors is not needed in the formal axiomatic definition of the *general* Hilbert space. Elements of ℓ^2, the prototypical Hilbert space, are infinite sequences of complex numbers. The logical relation between orthogonal vectors spanning ℓ^2 is neither conjunction ("and") nor disjunction ("or"); it is not necessary to assign any practical meaning to the logical relation. Only for a given application, practically meaningful concepts are necessary to define a specific Hilbert space used to describe practically meaningful objects.

However, if conjunction ("and") is the logical relation between orthogonal vectors spanning a Hilbert space, the orthogonal vectors must not correspond to mutually exclusive properties simultaneously belonging to the same object; such an imaginary object is a consequence of violating the general principle of measurements by taking precise space and time coordinates for granted to specify "the same experimental conditions". As shown in the last section, the "the same experimental conditions" are not physically meaningful, and the imaginary object does not exist in the real world. With the inner product defined for Euclidean vectors, $\mathbb{R}^3$ is a Hilbert space. Orthogonal vectors spanning $\mathbb{R}^3$ are orthogonal only in the sense of Euclidean geometry but do not represent mutually exclusive properties simultaneously belonging to any geometric object. Thus conjunction ("and") can serve as the logical relation between the orthogonal vectors spanning $\mathbb{R}^3$. The components of such vectors can be measured simultaneously. The measurements will not cause anything in $\mathbb{R}^3$ to collapse.

The *general* Hilbert space axiomatically defined by von Neumann differs from any Hilbert space in quantum mechanics. The difference between the former and the latter is that the concept of orthogonality in the latter has a specific meaning: conjunction ("and") is the logical relation between orthonormal vectors, which purportedly represent mutually exclusive properties simultaneously belonging to the same physical object *before* measurements. This specific meaning assigned to orthogonality makes the axiomatic formulation of quantum mechanics questionable. But von Neumann's formal axiomatic definition of the *general* Hilbert space is still valid and allows disjunction ("or") to serve as the logical relation between orthogonal vectors spanning a Hilbert space for practical applications.

For a Hilbert space in quantum mechanics completed based on the general principle of measurements, the logical relation is disjunction ("or"). Thus, represented by orthonormal vectors spanning the Hilbert space in quantum mechanics with disjunction ("or") serving as the logical relation, *different* outcomes corresponding to *mutually exclusive* properties of a physical object of a given kind are associated with *different* objects of the same kind; the objects are measured in *different* repetitions of a given experiment. Obtained by measuring the corresponding object, each *single* outcome reveals an "element of the physical reality" considered in the EPR argument under the assumptions of locality and realism [3]. Consequently, a value corresponding to the *single* outcome can be assigned to the object, even though the precise space and time coordinates used to measure it are unknown; the value can even be taken from a continuum and cannot be obtained by measurements, such as the position or momentum of a particle moving in space.

As shown above, based on the general principle of measurements, quantum mechanics can indeed be completed within the framework of local realism, such that the completed quantum theory is consistent with the formal axiomatic definition of the *general* Hilbert space without changing the mathematical setting substantially. In von Neumann's formal axiomatic definition of the *general* Hilbert space, the axioms concerning various calculations required by quantum mechanics, including the calculations of probabilities and expectation values, will all remain unchanged.

However, in von Neumann's axiomatic formulation of quantum mechanics, some axioms are questionable and should be removed. These axioms are irrelevant to the calculations and can only make quantum mechanics difficult to understand, such as those implying the purported completeness of current quantum theory, the so-called inherently probabilistic nature of observations on quantum systems, and inexplicable collapses of wave-functions triggered by measurements. Removing such axioms will significantly simplify the axiomatic formulation of quantum mechanics.

6. Implications of Completed Quantum Theory

With disjunction ("or") serving as the logical relation between superposed orthonormal vectors, the notion of "quantum superposition" in the completed quantum theory will be denoted by "superposition (disjunction)", which differs essentially from its counterpart in current quantum theory. To avoid confusion, the notion of "quantum superposition" in current quantum theory will be referred to as "superposition (conjunction)". Violating the general principle of measurements can result in using an *imaginary* object described by a superpositions (conjunction) to characterize *different* objects measured in *different* repetitions. No outcome is obtained by measuring the imaginary object, which is nonexistent in the real world.

The EPR argument reveals a contradiction in the conceptual foundations of quantum mechanics, namely, the truth of Heisenberg's uncertainty relation implies the incompleteness of current quantum theory, or equivalently, the completeness of quantum mechanics in its current form implies the falsity of Heisenberg's uncertainty relation [3]. After quantum mechanics is completed within the framework of local realism, does the completed quantum theory imply the falsity of Heisenberg's uncertainty relation? The answer is yes. In the real world, there is no particle described by a wave-function expressed as a superposition (conjunction), which implies an inexplicable collapse of the wave-function triggered by a measurement performed on an *imaginary* particle. In contrast, using "superposition (disjunction)" to describe *different* particles of the same kind measured in *different* repetitions of the corresponding experiment, the completed quantum theory precludes inexplicable collapses of wave-functions and does imply the falsity of Heisenberg's uncertainty relation.

In the optical experiment with *single* pairs of *correlated* photons [10], the pairs are described by the "entangled state", which is a superposition (conjunction). For a pair of correlated photons so described, no polarization can be assigned to either photon if no polarization measurement is performed, and a measurement triggers an abrupt collapse of the "entangled state" [10]. The "entangled state" cannot describe anything physically meaningful in the real world. According to Einstein's separability principle, either of the correlated photons in each pair possesses its autonomous polarization state independent of the other photon, and measuring the polarization of either photon cannot immediately affect the other photon. Consequently, corresponding to the autonomous polarization states *simultaneously possessed* by both correlated photons (i.e., *distant* components) in each *single* pair (i.e., a separable system) to be detected *jointly* in the real world, each *single* outcome is obtained in *one* repetition of the experiment, even though the precise space coordinates used to detect the pair are unattainable by measurements and unknown.

This is actually Einstein's ensemble interpretation of a wave-function [1], if the wave-function is expressed as a superposition (disjunction). Different pairs of correlated photons with their autonomous polarization states are detected in different repetitions of the experiment; they form an ensemble described by the superposition (disjunction). According to the general principle of measurements, the indeterminism exhibited in the outcomes obtained by measuring the polarization states of the photons is due to lack of knowledge about precise space coordinates and cannot be explained by statistical mechanics. Violating the general principle of measurements brings about using an *imaginary* pair to characterize *different* pairs detected in *different* repetitions of the experiment.

The way to complete quantum mechanics suggested in this article should not be considered as questioning the correctness of quantum-mechanically predicted probabilities of obtaining different outcomes by measurements. After quantum mechanics is completed within the framework of local realism, the axioms relevant to calculating probabilities and expectation values needed by quantum mechanics are all unchanged. This is an important difference between the completed quantum theory and any hidden-variables theory. According to Bell's theorem [2], "a theory can achieve complete agreement with quantum mechanics only if it is non-local." This claim is incorrect. By providing the

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same probabilistic predictions of empirical results as those provided by current quantum theory, the completed quantum theory can indeed achieve perfect agreement with current quantum theory. In contrast, the predictions given by Bell inequalities derived by resorting to a hidden-variables theory differ from the quantum-mechanical predictions. More importantly, the completed quantum theory is local rather than non-local.

7. Correlation versus Nonlocal-Interaction

Consider the experimental tests of Bell inequalities [26] with *single* pairs (i.e., separable systems of the same kind) of *correlated* photons (i.e., the corresponding distant components). In the interpretation of the experimental results given by Bell's theorem, the *correlation* between *distant components* (i.e., correlated photons) of a separable system (i.e., a single pair) in Einstein's local-realist world view is confused with the so-called non-locality or "nonlocal-interaction" in the world described by Bell's theorem. According to Bell's theorem, not only local realism but also Einstein's separability principle have to be rejected. The confusion must be clarified. The *correlation* between *distant components* of a separable system in Einstein's local-realist description of the world differs essentially from the so-called non-locality or "nonlocal-interaction" in the world described by Bell's theorem, where each separable system is measured in only *one* repetition of a given experiment. The general principle of measurements can reveal the difference.

In the experimental tests of Bell inequalities [26], the "locality loophole" has been closed, which is relevant to the Einstein-Bohr debate. Other loopholes, such as those concerning detections of photons and various far-fetched interpretations of Bell's theorem, are not fundamental. Detection loopholes cannot disprove Bell's theorem or change the world it describes. Far-fetched interpretations of Bell's theorem are not the experiments themselves and will not be considered here.

As implied by a condition necessary to observe any random phenomenon, the *correlation* between *distant components* of a separable system in Einstein's local-realist description of the world is *deterministic*. The necessary condition is a banal fact: any single measurement makes no sense statistically. This condition is ignored in the existing literature regarding the present status of the problem. Clearly, *jointly* detecting the correlated photons in a *single* pair will produce *one and only one* outcome in *one* repetition of the corresponding experiment; indeterminism cannot manifest itself in only one repetition. Thus the polarizations *simultaneously possessed* by *both* correlated photons in each pair can be detected *jointly*, and the *correlation* between the photons in each single pair must be *deterministic*.

Therefore, using "superpositions (disjunction)" rather than "superpositions (conjunction)" to describe the single pairs precludes collapses of "entangled states" triggered by measurements. Indeed, if there are no "superpositions (conjunction)", there will be no inexplicable collapses of wave-functions triggered by measurements. Compared to the *deterministic correlation*, the so-called non-locality or "nonlocal-interaction" implies the legitimacy of "entangled states", which amounts to presuming the legitimacy of current quantum theory. Without the so-called non-locality or "nonlocal-interaction", Bell's theorem cannot interpret the experimental results of testing Bell inequalities, which are expressed in terms of *statistical* correlations. But why is the *deterministic correlation* between *distant components* of a separable system in Einstein's local-realist description of the world mistaken for the so-called non-locality or "nonlocal-interaction" in the world described by Bell's theorem? There are three reasons.

First, Bell and his followers did not attempt to explain indeterminism in outcomes obtained by experiments involving superpositions (conjunction); they merely tried to "reinterpret quantum mechanics in terms of a statistical account of an underlying hidden-variables theory" [2], keeping quantum mechanics in its current form intact. To reproduce the *statistical* predictions of quantum mechanics, Bell inequalities have to be expressed by *statistical* correlations. Secondly, the condition necessary to observe random phenomena is ignored. According to this condition, indeterminism cannot manifest itself in only *one* repetition of the corresponding experiment. Finally, "entangled states" used in the experimental tests of Bell inequalities violate the general principle of measurements.

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Consequently, an *imaginary* pair is used to characterize *different* pairs detected in *different* repetitions, and *mutually exclusive* properties corresponding to *different* outcomes are attached to the *imaginary* pair. Eventually, the *deterministic correlation* is mistaken for the so-called non-locality or "nonlocal-interaction".

8. Discussion

Inspired by Einstein, Bell and his followers intended to complete quantum mechanics within the framework of local realism [6,11]. However, they adopted a problematic approach, which cannot reveal the essential difference between the *deterministic correlation* in Einstein's local-realist description of the world and the so-called non-locality or "nonlocal-interaction" in the world described by Bell's theorem. The *deterministic correlation* is between *distant components* of a separable system that satisfies the condition demanded by Einstein's separability principle [1]. This regrettable situation might have been avoided, had they focused on explaining indeterminism in quantum physics rather than reinterpreting current quantum theory. Described himself as a follower of Einstein, Bell hoped for better theories than our current quantum theory, insisting that the current theory was no more than a temporary expedient [6]; he would have been happy to see quantum mechanics completed within the framework of local realism without relinquishing Einstein's separability principle.

By explaining indeterminism in quantum physics, the general principle of measurements allows quantum mechanics to be completed within the framework of local realism while keeping the formal axiomatic definition of a *general* Hilbert space essentially unchanged; it can also reveal the essential difference between the *deterministic correlation* and the so-called non-locality or "nonlocal-interaction". In addition, using disjunction ("or") as the logical relation between superposed orthonormal vectors, the completed quantum theory precludes inexplicable collapses of wave-functions and is intuitively comprehensible, thus alleviating much difficulty in understanding quantum mechanics experienced by many people, including Bell [11]. Among various world views, Einstein's local-realist world view is correct.

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