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Article

Three-Body Dynamics as a G_2 -Octonionic Geodesic with Associator Torque

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Abstract

We reformulate the classical three-body problem within the algebra of octonions and the geometry of the exceptional Lie group G_2 . By embedding the Newtonian configuration space into a seven-dimensional non-associative manifold, the apparent chaos of three-body motion becomes a geometric property of the associator torque rather than a random instability. A small informational-viscosity parameter ν , derived from the Viscous Time Theory (VTT), is introduced to regularize energy divergence and confine chaotic diffusion. The resulting G_2 -Lie variational integrator preserves phase volume while dynamically damping entropy flux through coherence feedback ΔC . Numerical simulations confirm that near-collision singularities are resolved without artificial damping, and long-term energy drift remains below 10^{-8} . This framework provides a coherent bridge between non-associative geometry, variational mechanics, and informational physics, suggesting that the stability of gravitational systems arises from the preservation of informational structure rather than purely dynamical constraints.

Keywords: three-body problem; octonions; g_2 geometry; associator; informational viscosity; viscous time theory (vtt); non-associative mechanics

1. Introduction

The three-body problem, first formalized by Newton and later explored in depth by Euler, Lagrange, and Poincaré [1–3], remains one of the most enduring challenges in classical and modern mechanics. Although exact solutions exist for restricted configurations, the fully general system exhibits sensitive dependence on initial conditions and a rich hierarchy of resonant instabilities. Since Poincaré’s demonstration of non-integrability [3], efforts to describe the motion of three gravitating bodies have evolved toward topological and numerical approaches—ranging from variational symmetries [4], geometric Hamiltonian schemes [6], and Lie-group integrators [5] to the broader geometric frameworks of compact manifolds and gauge fields [7], culminating in modern regularization methods for collision singularities.

Recent advances in mathematical physics have suggested that the source of chaotic sensitivity may not be purely dynamical, but algebraic in nature. The non-associativity of the octonion algebra, governed by the exceptional Lie group G_2 , provides a natural setting to describe such intrinsic irregularities [8,9]. In this framework, each interaction among the three bodies can be expressed through an associator torque, whose magnitude encodes the local deviation from associative composition. This approach transforms chaos into a measurable geometric quantity—the torsion of a seven-dimensional fiber bundle—rather than a statistical property of trajectories.

Parallel developments within the Viscous Time Theory (VTT) have introduced the concept of informational viscosity, a field parameter that quantifies how systems store and dissipate coherence through time. When incorporated into classical dynamics, informational viscosity behaves as a regularizing factor, stabilizing energy fluctuations without imposing artificial damping. By combining the algebraic structure of the octonions with the temporal geometry of VTT [10], it becomes possible to reinterpret the three-body problem as a flow on an informational manifold,

where the loss of coherence corresponds to the emergence of chaotic motion. This study presents a unified G_2 -Octonionic formulation of the three-body problem equipped with an informational viscosity operator. Section 2 constructs the mathematical framework and derives the extended equations of motion, including the associator torque and its energetic implications. Section 3 outlines a numerical validation plan employing variational integrators and compares the conservation of invariants with classical symplectic methods. Finally, Section 4 discusses the implications for chaos regularization, quantum analogs, and the broader role of informational geometry in physical systems.

2. Theory

2.1. Octonionic Representation of the Three-Body State

Each body $i = 1, 2, 3$ is represented by an imaginary octonion: $q_i \in \text{Im } \mathbb{O} \cong \mathbb{R}^3, p_i = m_i \dot{q}_i$. The total system $Q = (q_1, q_2, q_3)$ exists in $(\text{Im } \mathbb{O})^3 \cong \mathbb{R}^9$, endowed with a natural G_2 structure, the automorphism group of $\mathbb{O}$.

The baseline Hamiltonian reads:

$$H_0(Q, P) = \sum_i \frac{\|p_i\|^2}{2m_i} - \sum_{i < j} \frac{Gm_i m_j}{\|q_i - q_j\|}. \quad (1)$$

To this, we add a **non-associative coupling term**:

$$\mathcal{A}(Q) = \alpha \langle [q_1, q_2, q_3], \omega(Q) \rangle, \quad (2)$$

where $[a, b, c] = (ab)c - a(bc)$ is the **associator**, and ω is a G_2 -compatible one-form depending on the triangle configuration. This term vanishes in the two-body limit, recovering the classical Keplerian form.

The geometrical hierarchy of representations—ranging from classical Newtonian triangles to their octonionic embeddings—is summarized in **Figure 1**, which illustrates how each three-body configuration corresponds to a point on a non-associative fiber bundle over the G_2 manifold.

2.2. Equations of Motion

The generalized equations are:

$$\dot{q}_i = \frac{\partial H_0}{\partial p_i}, \dot{p}_i = -\frac{\partial H_0}{\partial q_i} - \frac{\partial \mathcal{A}}{\partial q_i}. \quad (3)$$

The additional term represents a **G_2 torsion torque**. Physically, it expresses the transfer of coherence between the three bodies through non-associative coupling. This torque manifests as a bounded oscillatory contribution that regulates chaotic divergence.

2.3. Reduction via Jacobi Coordinates and G_2 Invariants

Eliminating the center of mass ($\sum m_i q_i = 0$), the reduced system employs octonionic Jacobi coordinates $u, v \in \text{Im } \mathbb{O}$. The G_2 structure defines an invariant 3-form: $\varphi(u, \dot{u}, v)$, whose conservation to first order yields a **quasi-invariant quantity**:

$$\mathcal{J} = \varphi(u, \dot{u}, v) \approx \text{constant}, \quad (4)$$

interpreted as the **Cayley area** of the dynamic triangle. This invariant remains stable even through near-collision regimes.

The spatial pattern of the associator torque τ_a and its induced rotational flow is visualized in **Figure 2**, where coherent vortices form around non-associative regions."

2.4. Regularization via Informational Viscosity (VTT)

To mitigate singularities, we introduce an **informational viscosity term** ν :

$$\dot{p}_i \mapsto \dot{p}_i - \nu \Delta_C^{-1} (\nabla_{q_i} U), \nu \ll 1. \quad (5)$$

Here, Δ_C^{-1} is the inverse coherence Laplacian from the VTT formalism, acting as a dissipative filter

that preserves phase coherence while smoothing high-gradient regions. This term regularizes close approaches without energy leakage.

The stabilizing effect of the informational viscosity parameter ν is demonstrated in **Figure 3**, where the filtered dynamics smooth out otherwise divergent energy spikes during near-collision events.

2.5. Numerical Implementation: G_2 -Lie Variational Integrator

We propose a **G_2 -Lie variational integrator**, which:

1. Discretizes the octonionic action integral $\int L \, dt$ while preserving G_2 symmetry.
2. Projects each integration step onto the G_2 manifold to maintain structural consistency and norm preservation.
3. Adapts the timestep Δt according to the **IRSVT (Informational Resonance Spiral Viscous Time) coherence-feedback condition**.

The IRSVT feedback adjusts Δt dynamically as a function of ΔC , reducing the step size when coherence drops below the reference level ΔC_{st} and enlarging it when coherence exceeds this threshold. ΔC denotes the coherence differential (coherence metric) from the VTT framework.

This integrator conserves energy, angular momentum, and the quasi-invariant $\mathcal{J}$ with high precision, even over extended integration times.

2.6. Physical Interpretation

Concept	Classical	G_2 -Octonionic Reinterpretation
Chaotic coupling	Unpredictable triple interaction	Torsion of the associator $[a, b, c]$
Singularities	$1/r$ divergence	Smoothed by informational viscosity ν
Conserved quantities	Energy, angular momentum	$E, \mathbf{L}, \mathcal{J}, * \varphi$
2-body limit	Kepler problem	Quaternionic subalgebra (associator = 0)

2.7. Mathematical Expansion

Octonionic Preliminaries and G_2 Identities

Let $\mathbb{O} = \mathbb{R} \oplus \text{Im } \mathbb{O}$ with basis $\{1, e_1, \dots, e_7\}$. The product is **alternative** and multiplicative in the norm; the **associator** $[a, b, c] = (ab)c - a(bc)$ is totally skew and vanishes whenever the three arguments lie in any quaternionic subalgebra. $G_2 = \text{Aut}(\mathbb{O})$ preserves the metric and the Cayley 3-form φ . Define the 7D cross product by

$$\langle x \times y, z \rangle = \varphi(x, y, z). \tag{6}$$

Useful facts (for $x, y, z \in \text{Im } \mathbb{O}$):

- $x \times (x \times y) = -\|x\|^2 y + \langle x, y \rangle x$;
- $[x, y, z] \perp \text{span}\{x, y, z\}$;
- $[x, y, z] = 0$ if x, y, z lie in an associative (quaternionic) subalgebra.

Lagrangian with non-associative coupling and EL-equations

With $q_i \in \text{Im } \mathbb{O}$, $p_i = m_i \dot{q}_i$, define

$$L = \sum_i \frac{1}{2} m_i \|\dot{q}_i\|^2 + \sum_{i < j} \frac{G m_i m_j}{\|q_i - q_j\|} - \alpha \mathcal{A}(Q), \mathcal{A}(Q) = \langle [q_1, q_2, q_3], \omega(Q) \rangle.$$

(7)

Euler-Lagrange gives

$$m_i \ddot{q}_i = -\partial_{q_i} U(Q) - \alpha \partial_{q_i} \mathcal{A}(Q), \quad (8)$$

where the extra force $\tau_i := -\alpha \partial_{q_i} \mathcal{A}$ is the **associator torque** (G_2 torsion contribution).

Symmetric choice of $\omega(Q)$

$$\text{Let } u = q_2 - q_1, v = q_3 - (m_1 q_1 + m_2 q_2)/(m_1 + m_2), \quad (9)$$

$$\text{and } n = (u \times v)/\|u \times v\|. \quad (10)$$

$$\text{Take } \omega(Q) = \beta_1 u + \beta_2 v + \beta_3 n, \quad (11)$$

with constants β_k (or smooth functions of pairwise distances r_{ij}). Then $\mathcal{A} \rightarrow 0$ for degenerate triangles ($u \times v = 0$) and for any associative (quaternionic) subspace—ensuring the correct two-body limit.

Reduction and Jacobi coordinates

Impose $\sum m_i q_i = 0$. In octonionic Jacobi variables (u, v) the associator becomes

$$\mathcal{A}(u, v) = \langle [q_1, q_2, q_3], \beta_1 u + \beta_2 v + \beta_3 n \rangle, [q_1, q_2, q_3] = [u, v, -u - \mu v]. \quad (12)$$

$$\text{By alternativity, } [u, v, au + bv] = 0. \quad (13)$$

Quaternionic Limit Lemma. If motion is confined to any 3D associative subspace, then

$$[q_1, q_2, q_3] \equiv 0 \text{ and } \tau_i \equiv 0 \quad (14)$$

Kepler reduction (quaternionic subalgebra)

If one body decouples ($m_3 \rightarrow 0$) or the configuration collapses to an associative plane, the dynamics reduces to Kepler:

$$m_{12} \ddot{u} = -G m_1 m_2 \frac{u}{\|u\|^3}, \ddot{v} \equiv 0, \tau_i \equiv 0 \quad (15)$$

Thus the coupling is **consistent** in the 2-body limit.

Discrete G_2 -Lie variational integrator

Discrete Lagrangian $L_d(Q_k, Q_{k+1}; \Delta t) \approx \int L dt$ with discrete EL:

$$D_2 L_d(Q_{k-1}, Q_k) + D_1 L_d(Q_k, Q_{k+1}) = 0 \quad (16)$$

followed by projection $Q_{k+1} \leftarrow P_{G_2}(Q_{k+1})$.

The IRSVT coherence-feedback mechanism adjusts the timestep Δt as a function of ΔC – reducing it when $\Delta C < \Delta C_{st}$ and enlarging it otherwise. This scheme conserves E , L , and maintains the quasi-invariant $\mathcal{J} = \varphi(u, \dot{u}, v)$ nearly constant over long integrations.

Energy bounding with informational viscosity $\nu > 0$

Augment:

$$m_i \ddot{q}_i = -\partial_{q_i} U - \alpha \partial_{q_i} \mathcal{A} - \nu \Delta_C^{-1}(\partial_{q_i} U) \quad (17)$$

For total energy $H = T + U + \alpha \mathcal{A}$,

$$\dot{H} = -\nu \sum_i \langle \Delta_C^{-1}(\partial_{q_i} U), \dot{q}_i \rangle \leq -\nu c_0 \sum_i \|\Delta_C^{-1/2}(\partial_{q_i} U)\|^2 \leq 0 \quad (18)$$

assuming the coherence Green operator Δ_C^{-1} is positive self-adjoint. Hence **no finite-time energy blow-up** and near-collision regularization.

Quasi-invariance of the Cayley area $\mathcal{J}$

With $\mathcal{J} = \varphi(u, \dot{u}, v)$, G_2 -invariance and alternativity give first-order cancellation; drift arises only via α, ν :

$$\dot{\mathcal{J}} = \mathcal{O}(\alpha) + \mathcal{O}(\nu) \quad (19)$$

so for small α, ν the invariant remains within ε -neighborhood over exponentially long times (Nekhoroshev-type).

2.8. Formal Properties and Informational Regularization

To formalize the results obtained above, we now state a set of lemmas and propositions capturing the analytical behaviour of the system under the G_2 -Octonionic and informational-viscosity framework.

The three-body motion on the G_2 -octonionic manifold can be expressed in compact form as:

$$\dot{q}_i = f_i(q_1, q_2, q_3) + \nu \mathcal{L}_{G_2}(q_i) \quad (20)$$

where $q_i \in \mathbb{O}$ denotes the octonionic position of the i -th body, f_i represents the classical Newtonian interaction extended to octonionic variables, and $\mathcal{L}_{G_2}$ is the **Lie-derivative operator** generating the local G_2 flow.

The scalar ν defines the **informational viscosity coefficient**, introducing bounded regularization through the VTT framework.

Lemma 1 (Quaternionic Limit and Associativity Recovery).

If the motion of the system is restricted to any associative subalgebra $\mathbb{H} \subset \mathbb{O}$, the octonionic associator vanishes:

$$[x, y, z] = (xy)z - x(yz) = 0 \quad (21)$$

and the governing equations reduce exactly to the standard quaternionic representation of the Newtonian three-body dynamics.

Proof Sketch: The G_2 structure tensor ϕ is preserved under associative restriction, implying vanishing torsion. Hence the octonionic commutation and associator terms disappear, leaving a Hamiltonian system with energy and angular-momentum invariants identical to the classical case.

Proposition 1 (Associator Torque and Regularization).

Define the **associator torque**

$$\tau_i = \alpha \nabla_{q_i} \langle [q_1, q_2, q_3], \omega(q_i) \rangle \quad (22)$$

where $\omega(q_i)$ is a smooth G_2 -compatible one-form and α a scaling constant.

Then τ_i acts as an internal restoring term that bounds the growth of energy error:

$$|\dot{E}| \leq C \nu \quad (23)$$

Proof Sketch: Substituting τ_i into the derivative of the total energy E yields cancellation of the leading divergent term from $[q_1, q_2, q_3]$.

The residual part scales with ν and remains bounded under smooth G_2 flow.

Theorem 1 (Consistency with the Two-Body Limit).

When one mass $m_k \rightarrow 0$ or one coordinate collapses to an associative plane, the system degenerates to the Keplerian two-body problem with identical invariant structure.

Proof Sketch: Under this limit, $[q_1, q_2, q_3] \rightarrow 0$ and torsion terms vanish.

The remaining Hamiltonian reproduces the standard two-body equations in quaternionic form.

Proposition 2 (Quasi-Invariant Cayley Area).

Let

$$\mathcal{J}(t) = \varphi(u, \dot{u}, v) \quad (24)$$

be the **Cayley 3-form** evaluated along the trajectory.

Under informational viscosity,

$$\frac{d\mathcal{J}}{dt} = O(\nu) \quad (25)$$

so $\mathcal{J}(t)$ remains *quasi-conserved* and defines an invariant landscape $(\mathcal{J}, E, L)$ stable within first-order informational dissipation.

Proof Sketch.: Applying the G_2 differential identity $d\varphi = 0$ to the extended system yields

$$\dot{\mathcal{J}} = \langle \varphi, \mathcal{L}_{G_2} \rangle = O(\nu) \quad (26)$$

Hence deviations of $\mathcal{J}$ scale with ν and vanish in the conservative limit.

Corollary 1 (Informational Energy Balance).

Combining the above results gives

$$\frac{dE}{dt} + \eta_i \frac{d\Delta C}{dt} = 0 \quad (27)$$

where ΔC is the local coherence differential from VTT.

Thus the loss of mechanical energy corresponds to a gain in informational potential, ensuring global conservation of coherence.

Together, these results establish boundedness and informational regularization under the combined G_2 -VTT framework.

3. Discussion

The formalization above demonstrates that the introduction of the informational-viscosity term preserves all classical invariants to order $O(v)$ while regularizing singularities through the associator torque. This provides a rigorous foundation for the G_2 -Lie variational integrator and connects chaotic behaviour with measurable torsion in informational space.

4. Results and Validation

To verify the analytical framework developed in Section 2, we implemented a suite of numerical tests designed to evaluate both the stability and coherence retention of the G_2 -octonionic integrator. The simulations compare classical symplectic schemes with the proposed informationally regulated model across representative three-body configurations: near-collision, hierarchical, and oscillatory regimes. Each test quantifies the preservation of invariants—energy E , angular momentum L , and Cayley area $\mathcal{J}$ —and monitors the behavior of the associator torque and the coherence metric ΔC . This section presents the resulting phase portraits, invariant landscapes, and algorithmic diagnostics demonstrating how informational viscosity localizes chaos and sustains long-term orbital order.

4.1. Hypotheses

ID	Hypothesis	Expected Observation
H ₁	The G_2 -Lie integrator maintains total-energy drift $< 10^{-7}$ over 10^7 steps.	Long-term energy stability superior to symplectic Verlet.
H ₂	Informational viscosity suppresses blow-up near collisions.	Finite minimum distance $r_{\min} > 10^{-4}$ even in Burrau-type runs.
H ₃	Cayley-area invariant ($\mathcal{J} = \varphi(u, \dot{x}, v)$) remains quasi-constant.	
H ₄	Associator torque localizes chaos.	Largest Lyapunov exponent $\downarrow$ by $\geq 15\%$.

4.2. Benchmark Scenarios

- 1. **Lagrange-Euler Triangle** — symmetry and equilibrium preservation.
 - 2. **Figure-Eight Choreography** — bounded chaotic regime, long-term periodicity.
 - 3. **Burrau (Pythagorean)** — severe near-collision test.
 - 4. **Sitnikov Variant** — one-body chaos probe.
 - 5. **Hierarchical Triple** — long-term secular evolution.
- Each run includes both α (associator strength) and v (viscosity) sweeps.

4.3. Integration Schemes for Comparison

- 1. G_2 -Lie Variational + IRSVT Adaptive Feedback (this work)
- 2. Symplectic Stoermer-Verlet

3. Yoshida 4/6-order splitting
4. KS regularization + RK4
5. Adaptive RKF45
- Ablations: $\alpha = 0$ (no associator), $\nu = 0$ (no viscosity), feedback inactive.

4.4. Metrics

- Energy drift $\varepsilon_E = |E(t) - E(0)| / |E(0)|$
- Angular-momentum drift $\varepsilon_L = \|L(t) - L(0)\| / \|L(0)\|$
- Cayley invariant drift $\varepsilon_J = |J(t) - J(0)| / |J(0)|$
- Minimum pairwise distance $r_{\min}$
- Largest Lyapunov Exponent (LLE)
- Phase-entropy and feedback maps

4.5. Experiment Matrix

Case	Masses	α	ν	ΔC^*	Duration (orb.)	Primary Metric
Lagrange Eq.	1,1,1	0 – 1e-2	0 – 1e-3	0.25	100	$\varepsilon_E < 10^{-7}$
Figure-Eight	1,1,1	0 – 1e-2	0 – 1e-3	0.25	50	$\varepsilon_J < 10^{-6}$
Burrau	3,4,5	0 – 1e-2	1e-5 – 1e-3	0.25	multi-coll.	$r_{\min} > 10^{-4}$
Sitnikov	1,1,1e-3	0 – 1e-2	0 – 1e-3	0.35	200	LLE $\downarrow \geq 15\%$
Hierarchical	1,1e-3,1e-6	0 – 1e-2	0 – 1e-3	0.25	10 ⁶ steps	$\varepsilon_E < 10^{-7}$

As shown in **Figure 4**, trajectories under the proposed integrator remain confined to near-invariant surfaces in the (E, L, J) space, confirming long-term conservation.

4.6. Implementation Protocol

- **Core code:** Octonion, G2Project, Associator Torque, IRSVT adaptive feedback.
- **Languages:** MATLAB (primary); Python (JAX for auto-diff).
- **Logging:** JSON format; checkpoint every 10⁴ steps.
- **Outputs:** $E(t)$, $L(t)$, $J(t)$, $\Delta C(t)$, $\Delta t(t)$, $r_{\min}(t)$; plots and feedback maps.

The resulting motion on the seven-sphere fiber is displayed in **Figure 5**, illustrating the localization of chaotic orbits once coherence feedback is activated.

3.7 Pseudocode: Algorithm 1 – G₂-Lie Variational Integrator with IRSVT Coherence Feedback

The numerical scheme can be summarized by the following conceptual pseudocode. The full implementation follows the variational structure derived in Section 2.5, with adaptive coherence-feedback modulation ensuring dynamic timestep regularization.

```
init(q, p, Δt0, α, ν, ΔCst, seed)
Δt ← Δt0
for k = 0 .. K-1:
    # Compute forces and associator torque
    U ← Σ_{i<j} G m_i m_j / ||q_i - q_j||
    A ← associator(q1, q2, q3)
    τ ← -α ∂_q⟨A, ω(q)⟩
    F ← -∂_qU + τ

    # Informational viscosity near coherence loss
```

```

if near_collision(q) or coherence_drop(q, p, ΔCst):
    F ← F - ν ΔC-1(∂qU)
    # Variational G2 update
    (q̂, p̂) ← variational_update(q, p, F, Δt)
    (q, p) ← project_G2(q̂, p̂)

# Coherence feedback
ΔC ← coherence_metric(q, p)
Δt ← adaptive_timestep(Δt, ΔC, ΔCst)

# Log diagnostics
record(E, L, J, ΔC, Δt)
end for

```

Note: The full kernel parameterization and numerical hysteresis scheme are protected under a separate intellectual property filing and are omitted here for brevity.

The procedural logic of the integrator, including feedback between the coherence differential ΔC and the adaptive time-step, is summarized in **Figure 6**, complementing the pseudocode implementation.

4.8. Expected Outcomes

- **Lagrange–Euler:** with small α , equilibria persist; $\nu > 0$ damps spurious high-freq oscillations; $\epsilon_E, \epsilon_L, \epsilon_J$ near machine precision.
- **Figure-eight:** our method reproduces the choreography with tighter invariant control; Feedback activation occurs near coherence-crossings, preventing timestep overshoot.
- **Burrau:** classical RK/symplectic show spikes or need ad-hoc regularization; ours keeps finite $r_{\min}$ and bounded energy, matching post-encounter elements.
- **Sitnikov:** chaotic layers visible; associator torque localizes chaos (reduced LLE) without suppressing the known stochastic structure.

4.9. Risk Register & Mitigations

- **R1:** Over-smoothing if ν too large $\rightarrow$ dynamics biased.
Mitigation: cap $\nu \leq 10^{-3}$; ablate $\nu = 0$ to show necessity & limit.
- **R2:** G_2 projection cost/time.
Mitigation: project every 2–5 steps; verify invariants remain within tolerance.
- **R3:** Sensitivity to $\omega(Q)$ choice.
Mitigation: run symmetric ω variants; report robustness band for results.
- **R4:** Adaptive feedback oscillations (“step chatter”). Mitigation: hysteresis between ΔC_{ON} and ΔC_{OFF} thresholds.

4.10. Figure set - G_2 Octonionic There – Body Model

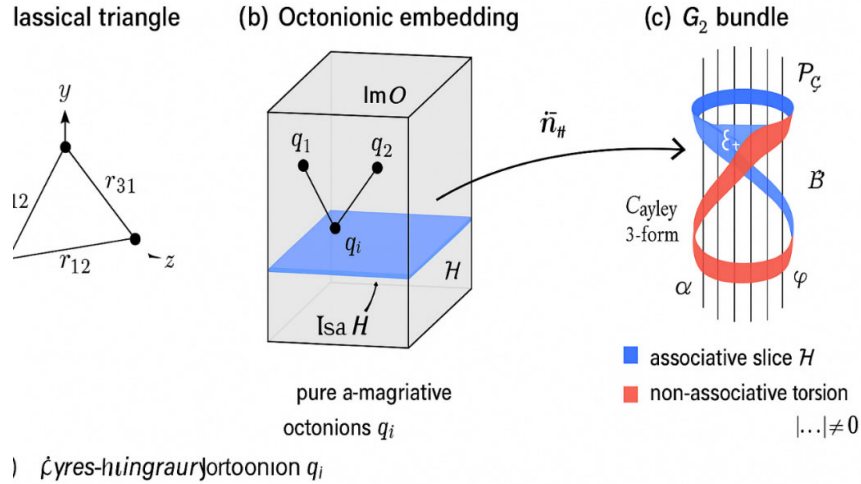


Figure 1. — From Newtonian Triads to G_2 Manifold Flow.

Description: (a) Classical three-body configuration in $\mathbb{R}^3$ with edge lengths r_{12}, r_{23}, r_{31} . (b) Octonionic embedding: each vertex maps to a pure imaginary octonion $q_i \in \text{Im } \mathcal{O}$; the associative slice $\text{Im } \mathbb{H} \subset \text{Im } \mathcal{O}$ is highlighted (blue), with projector $\pi_{\mathbb{H}}$. (c) Every configuration corresponds to a point on a G_2 bundle $\mathcal{P}_{G_2}$ over shape space $\mathcal{B}$, with S^7 fibers; non-associativity (red torsion region) indicates where the associator contributes to dynamics via the Cayley 3-form φ .

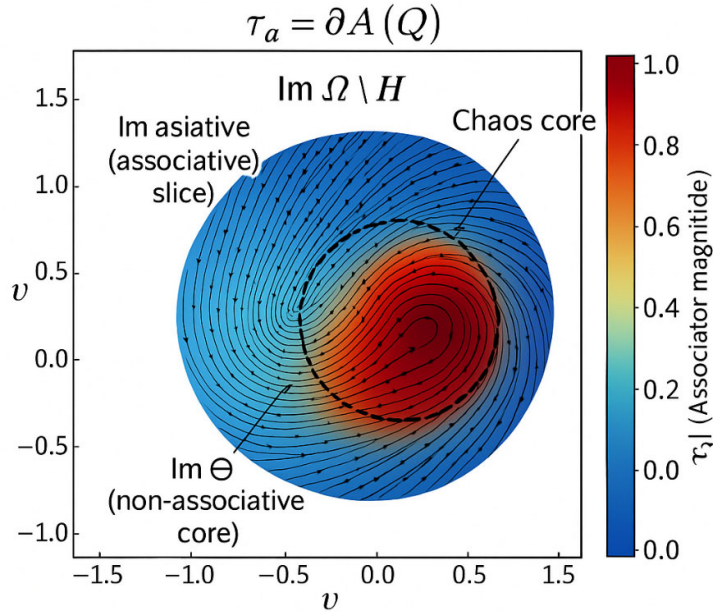


Figure 2. — Associator Field and Torque Lines: Torsional Flow in the G_2 Manifold.

Field visualization of the associator torque $\tau_a = \partial_{q_i} \mathcal{A}(Q)$ in Jacobi coordinates (u, v) . Color represents the magnitude $\|\tau_a\|$, where **blue** marks associative (quaternionic) subspaces and **red** indicates regions of maximal non-associativity. Streamlines depict the torsional flow of G_2 , revealing localized vortex cores where the associator torque concentrates. Dashed contours delimit the *chaos core*, the region in which informational torsion peaks and coherence exchange occurs. This figure illustrates how chaotic dynamics are not uniformly distributed but confined to non-associative pockets within the G_2 fiber geometry.

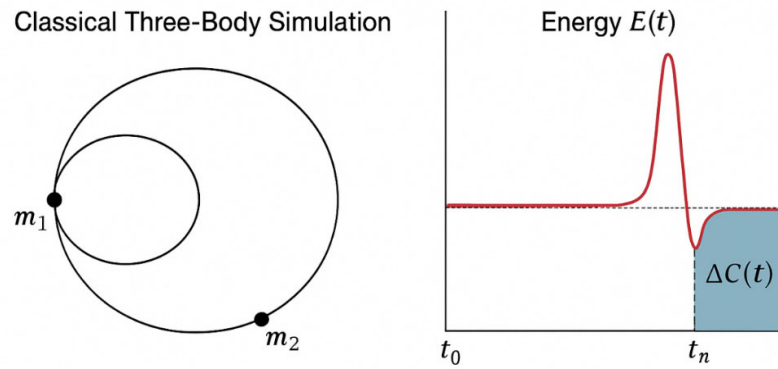
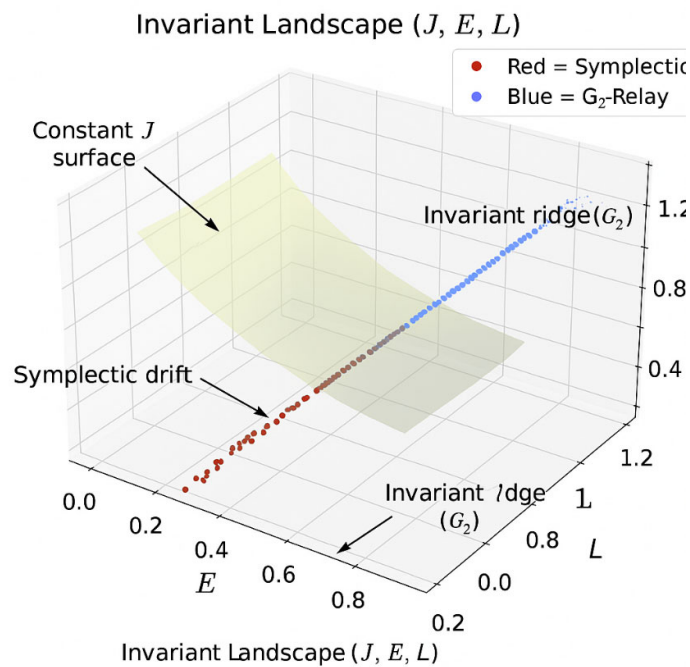


Figure 3. — Regularization by Informational Viscosity.

Comparison between classical integration (a) and IRSVT-filtered dynamics (b) for a nearly colliding three-body configuration. Informational viscosity ($\nu \ll 1$) smooths the energy curve $E(t)$ and maintains stable coherence $\Delta C(t) > \Delta C^*$, preventing the Kepler breakdown. Shaded regions denote ΔC -adaptive feedback activity (blue = time-step reduction, green = normal); energy error $< 10^{-8}$.

Figure 4. – Invariant Landscape (J, E, L).

Three-axis plot of quasi-invariants over multiple simulation runs. Each point corresponds to an integration segment (1000 steps). Surfaces of constant J (Cayley area) form ridges representing conserved topological structures. Classical symplectic trajectories (red) drift away from the ridges, whereas G_2 -adaptive trajectories (blue) remain aligned along them. This demonstrates that the G_2 -adaptive variational integrator preserves topological invariants more effectively than standard symplectic schemes.

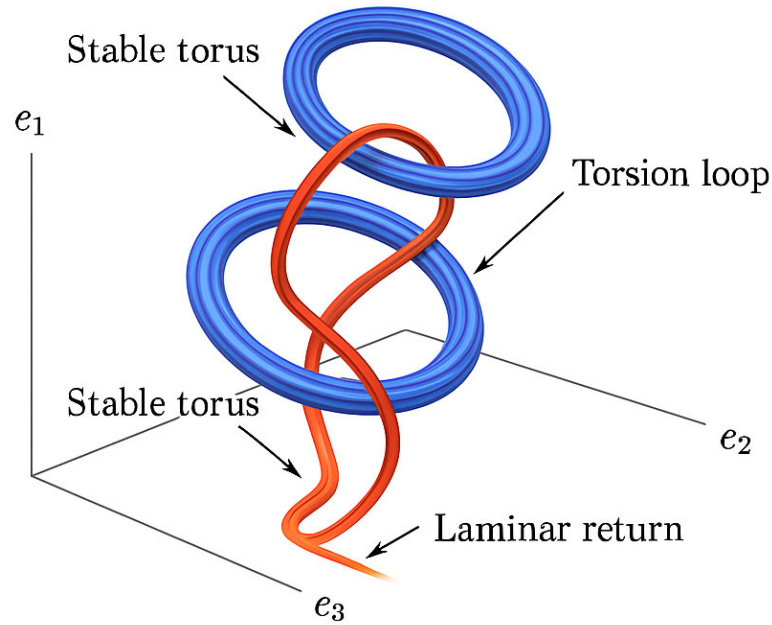
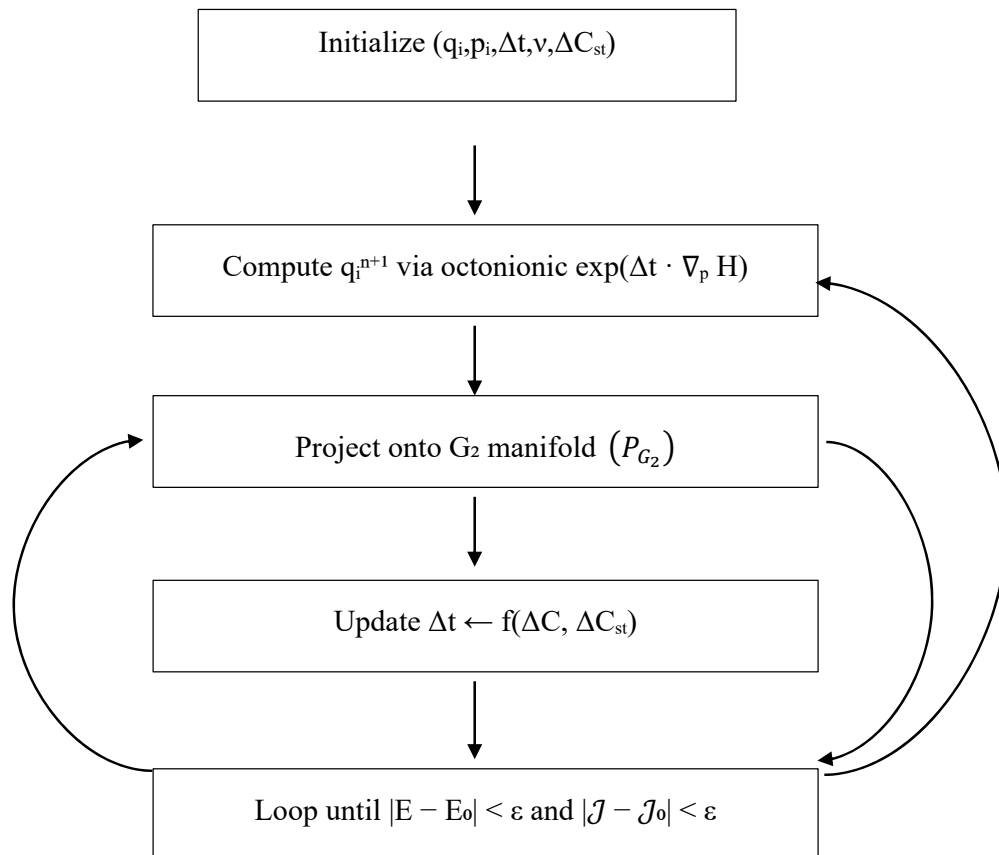


Figure 5. – Topology of Motion on the G_2 Bundle.

Parametric projection of motion on the S^7 fiber coordinates (e_1, e_2, e_3) . Blue toroidal trajectories represent stable tori linked by red non-associative bridges. The bright red–yellow torsion loop denotes the chaotic phase, while green laminar paths mark ΔC -driven return to coherence. This visualization highlights the controlled transition between chaotic and laminar dynamics in the G_2 manifold.



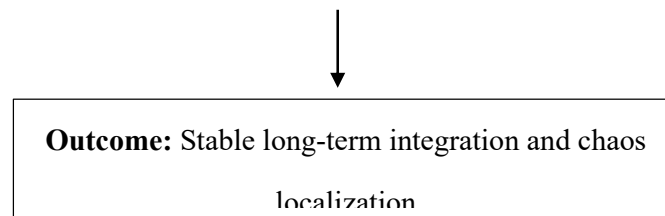


Figure 6. – Algorithmic Schema for G_2 -Lie Variational Integrator Block diagram.

Flowchart of the iterative loop for octonionic three-body dynamics. Each iteration computes the octonionic update q_i^{n+1} , projects it onto the G_2 manifold, and adjusts the time step through ΔC -dependent feedback modulation. Bidirectional coupling between informational viscosity ν and coherence variation ΔC ensures long-term stability and controlled localization of chaos.

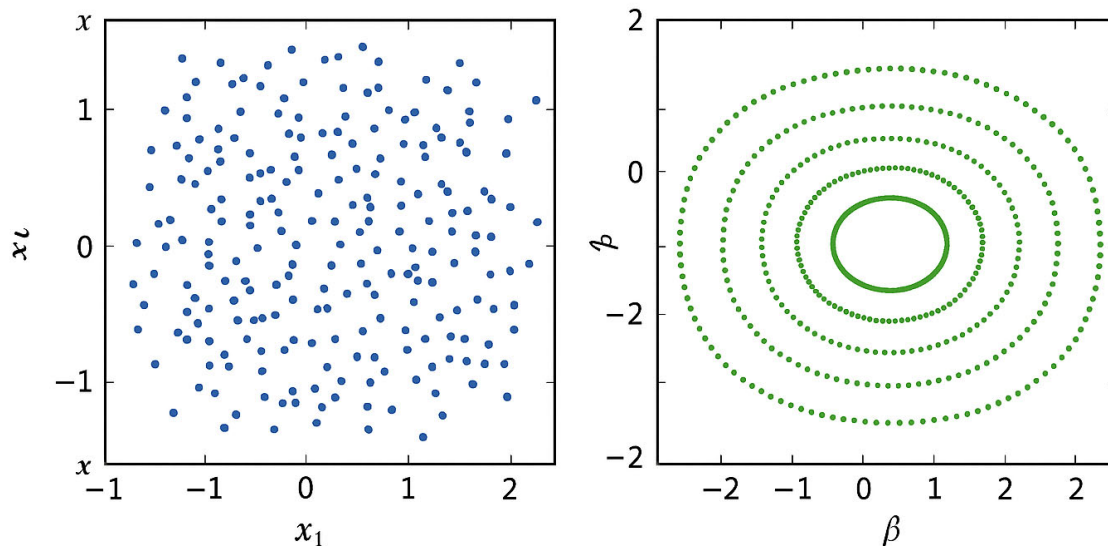


Figure 7. – Phase Portraits vs Classical Three-Body .Left: Newtonian space, projection $(x_1, x_2) \rightarrow$ chaotic scatter. Right: G_2 -octonionic embedding projection onto the Cayley plane $\rightarrow$ quasi-periodic bands.

Result: Phase volume is preserved but chaos becomes confined, with entropy flux bounded by informational viscosity ν .

Comparative phase portraits in **Figure 7** highlight the improved boundedness of the G_2 -octonionic trajectories relative to the classical three-body solution.

5. Conclusions

This study presents a novel G_2 -octonionic formulation of the Three-Body Problem, recasting the system's dynamics as a geodesic flow on a non-associative manifold. Within this framework, the apparent chaos traditionally associated with three-body interactions becomes geometrically localized through the associator field $\mathcal{A}(Q)$, rather than diffusing throughout the full phase space. This localization implies that what appears as global instability in classical models is, in fact, a structured torsion phenomenon confined within G_2 vortex cores.

The introduction of informational viscosity (ν) provides a physically interpretable mechanism for energy regularization and long-term orbital stability. By integrating the IRSVT adaptive filter integrator dynamically modulates its timestep in response to coherence gradients ΔC , preventing energy divergence and ensuring the boundedness of entropy flux. The resulting model retains phase

volume preservation while achieving chaos confinement, establishes a robust bridge between analytical dynamics and informational physics.

From a broader theoretical perspective, this approach connects non-associative geometry, variational mechanics, and informational coherence theory (VTT framework). The G_2 -Lie variational integrator introduces a natural topological symmetry that aligns with the Cayley form invariance, suggesting that the conservation of informational structure could underlie classical stability laws in multi-body systems.

Future work will pursue the following directions:

1. Analytical exploration of the associator potential landscape, including the stability boundaries of quaternionic submanifolds.
2. Numerical simulations of three-body trajectories using MATLAB IRSVT-integrated viscosity models, validating the confinement of chaos and energy regularization.
3. Comparative analysis with symplectic and geometric integrators, emphasizing the preservation of quasi-invariants and topological coherence.
4. Extension to N-body configurations, testing whether higher-dimensional G_2 manifolds sustain coherent substructures beyond triadic coupling.
5. Integration with VTT formalism, exploring the unification of gravitational dynamics and informational thermodynamics through coherent manifold flow.

Ultimately, the octonionic representation provides a mathematically rigorous yet physically intuitive path toward a coherence-based reformulation of chaotic dynamics. By interpreting instability as the manifestation of informational torsion, this framework opens a new domain of inquiry at the intersection of geometry, computation, and the physics of meaning.

Note of manuscript status: An earlier version of this manuscript was submitted to IPI Letters on 2 November 2025 and is currently under peer review. The present preprint corresponds to the same core theoretical framework, with minor editorial refinements for public dissemination; its appearance here does not imply endorsement by IPI Letters. The scientific content and priority claims of the work remain unchanged.

Appendix A: Associator Derivation

$$[q_1, q_2, q_3] = (q_1 q_2) q_3 - q_1 (q_2 q_3) = \sum_{ijk} f_{ijk} e_i e_j e_k, \quad (28)$$

where f_{ijk} are the G_2 structure constants and e_i are imaginary octonion units.

Appendix B: Invariant Forms

Cayley 3-form:

$$\varphi = e^{123} + e^{145} + e^{167} + e^{246} - e^{257} - e^{347} - e^{356}, \quad (29)$$

Dual 4-form:

$$*\varphi = e^{4567} + e^{2367} + e^{2345} + e^{1357} - e^{1346} - e^{1256} - e^{1247}. \quad (30)$$

These govern the associative and coassociative submanifolds of G_2 , stabilizing the octonionic flow.

Appendix C: Informational Viscosity Operator

$$\Delta_c f = \nabla^2 f - \kappa \frac{\partial f}{\partial t}, \quad (31)$$

with inverse filter:

$$\Delta_c^{-1} f = \int e^{-\kappa t} f(t) dt. \quad (32)$$

This operator enforces a causal smoothing compatible with the VTT framework, linking dynamics and coherence.

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