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Article

ECA110-Pooling: A Comparative Analysis of Pooling Strategies in Convolutional Neural Networks

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Abstract

In this paper, we introduce and validate *ECA110-Pooling*, a novel rule-based pooling operator for Convolutional Neural Networks inspired by elementary cellular automata. A systematic comparative study is conducted, benchmarking ECA110-Pooling against conventional pooling methods (MaxPooling, AveragePooling, MedianPooling, MinPooling, KernelPooling) as well as state-of-the-art (SOTA) architectures. Experiments on three benchmark datasets-ImageNet (subset), CIFAR-10, and Fashion-MNIST-across training horizons ranging from 20 to 50,000 epochs demonstrate that ECA110-Pooling consistently achieves higher Top-1 accuracy, lower error rates, and stronger F1-scores than traditional pooling operators, while maintaining computational efficiency comparable to MaxPooling. Furthermore, in direct comparison with SOTA models, ECA110-Pooling delivers competitive accuracy with substantially fewer parameters and reduced training time. These results establish ECA110-Pooling as a validated and principled approach for image classification, bridging the gap between fixed pooling schemes and complex deep architectures. Its interpretable, rule-based design underscores both theoretical relevance and practical applicability in scenarios requiring a balance of accuracy, efficiency, and scalability.

Keywords: Convolutional Neural Networks (CNNs); image classification; pooling strategies; Max-Pooling; AveragePooling; MedianPooling; MinPooling; KernelPooling; elementary cellular automata (ECA); ECA110-based pooling; deep learning; pattern recognition; state-of-the-Art (SOTA) methods; ANOVA; computational efficiency

1. Introduction

Pooling layers represent a cornerstone in the architecture of Convolutional Neural Networks (CNNs), serving as a mechanism to progressively reduce the spatial resolution of feature maps while retaining their most salient information. By compressing representations in this manner, pooling layers not only enhance computational efficiency but also provide translational invariance and mitigate overfitting through dimensionality reduction. The conceptual foundations of hierarchical feature abstraction in neural networks can be traced back to Fukushima's *Neocognitron* [1], while the optimization of such architectures became feasible following the introduction of backpropagation [2,3]. Among classical approaches, MaxPooling remains the de facto standard, largely due to its capacity to preserve dominant activations that improve recognition accuracy in large-scale image classification tasks such as ImageNet [4,5]. Nonetheless, the heavy reliance on MaxPooling is not without drawbacks. Alternative fixed operators have sought to mitigate these limitations: AveragePooling emphasizes global context at the expense of local discriminative detail [6], while MedianPooling introduces robustness to local noise and outlier activations, making it particularly suited for grayscale or noisy datasets [7]. In parallel, the field has witnessed the development of learnable pooling mechanisms. For example, KernelPooling introduces parameterized kernels that adaptively learn spatial aggregation, thereby tailoring pooling to task-specific distributions [8]. Other directions, such as stochastic pooling [9]

and strided convolutions that eliminate pooling altogether [10], reflect broader efforts to optimize or bypass fixed downsampling operators. These methods, while often powerful, introduce additional computational costs and may complicate training pipelines. Beyond data-driven parameterization, biologically and mathematically inspired pooling strategies have emerged as a promising frontier. In particular, the incorporation of Elementary Cellular Automata (ECA) rules into CNNs provides a lightweight, rule-based alternative for encoding local dependencies prior to reduction. Rule 110 is especially noteworthy due to its proven computational universality and emergent complexity [11–13]. By embedding ECA transformations within CNNs, pooling layers such as the proposed ECA110-Pooling can capture rich micro-pattern interactions that conventional operators often discard [14,15]. This approach integrates theoretical rigor with practical utility, bridging symbolic dynamics and deep learning. Despite this diversity of proposals, systematic and controlled comparative evaluations remain limited. Much of the CNN literature continues to employ MaxPooling by default, often without explicit justification or consideration of task-specific requirements [16,17]. Recent surveys and benchmarking efforts emphasize the importance of revisiting such design choices, particularly as modern classification pipelines increasingly operate under constraints of efficiency, generalization, and robustness [18–20]. The motivation for this study is therefore twofold: (1) to rigorously compare a broad spectrum of pooling strategies-including MaxPooling, AveragePooling, MedianPooling, MinPooling, KernelPooling, and the novel ECA110-Pooling-within a uniform experimental framework across CIFAR-10, Fashion-MNIST, and ImageNet (subset); and (2) to examine not only predictive performance but also computational efficiency, feature retention, and robustness under diverse training horizons. By situating ECA110-Pooling against both traditional operators and state-of-the-art (SOTA) image classification baselines, this work provides a nuanced perspective on how pooling strategies influence the effectiveness and efficiency of CNN-based classification.

Contributions. This work advances the understanding of downsampling mechanisms in CNNs through the following contributions:

- Systematic comparative evaluation. We present a rigorously controlled comparison of six pooling operators-*MaxPooling*, *AveragePooling*, *MedianPooling*, *MinPooling*, *KernelPooling*, and the proposed *ECA110-Pooling*-within a shared CNN backbone, fixed training protocol, and identical hyperparameter settings, thereby isolating the impact of the pooling mechanism.
- *Novel ECA110-based pooling mechanism.* We introduce and formalize a pooling operator derived from Elementary Cellular Automata (Rule 110). The design follows a compact *transform-reduce* paradigm, enabling the preservation of discriminative local structures with minimal computational overhead.
- Learnable baseline for reference. The inclusion of *KernelPooling* as a parametric baseline allows a fair comparison between rule-based and learnable pooling approaches, highlighting the trade-offs between flexibility, accuracy, and efficiency.
- Comprehensive benchmarking across datasets and epochs. Pooling methods are evaluated on CIFAR-10, Fashion-MNIST, and ImageNet (subset), with training horizons ranging from 20 to 50,000 epochs. Metrics include Top-1 Accuracy, Error Rate, F1-score, per-epoch runtime, and model size, thereby providing a holistic assessment across simple and complex visual domains.
- Statistical and computational validation. Findings are validated through statistical tests (one-way ANOVA, Tukey's HSD, Wilcoxon signed-rank, and paired *t*-tests) as well as computational profiling of convergence and efficiency trade-offs. This ensures that conclusions are both statistically robust and practically reproducible.

In summary, this study establishes ECA110-Pooling as a principled, lightweight, and competitive alternative to classical and learnable pooling mechanisms, with implications for advancing image classification across both high-capacity computational infrastructures and resource-constrained environments.

2. Related Work

Pooling operations have been extensively investigated in the design of Convolutional Neural Networks (CNNs) due to their central role in spatial downsampling, complexity reduction, and the promotion of translational invariance [21]. The earliest pooling operators, namely MaxPooling and AveragePooling, remain dominant in practice because of their simplicity and efficiency. However, these fixed operators exhibit inherent limitations, such as the loss of fine-grained details or increased sensitivity to noise, which has motivated a diverse range of extensions. One line of work has introduced stochasticity or adaptivity to the pooling process. Stochastic pooling selects activations in proportion to their magnitudes, providing an implicit regularization effect that enhances generalization. Mixed pooling and gated pooling further improve flexibility by probabilistically combining or adaptively selecting between max and average pooling based on input data characteristics [22]. Meanwhile, adaptive pooling methods such as spatial pyramid pooling and region-of-interest pooling enable the extraction of fixed-size feature representations from variable input dimensions, making them indispensable for object detection and recognition pipelines. Another prominent direction replaces static operators with learnable downsampling mechanisms. Strided convolutions have been proposed as pooling-free alternatives that maintain convolutional structure while performing resolution reduction [23]. Parametric pooling methods, including α -integration pooling [24], extend the flexibility of classical operators by introducing tunable parameters, while generalized pooling approaches provide continuous interpolations between max and average pooling [25]. These advances reflect a growing emphasis on adaptive downsampling, where the pooling function itself becomes optimized jointly with the network. Surveys of pooling methods confirm that such operator-level choices can significantly influence model generalization and robustness, depending on data distribution and task requirements [26]. More recently, pooling has been linked with the broader trend of incorporating attention mechanisms into CNNs. Residual attention networks combine pooling with attention-driven feature selection, enhancing representational capacity in hierarchical architectures. Surveys and empirical studies emphasize the effectiveness of attention-based pooling, which aggregates features according to learned importance weights rather than fixed heuristics [27–29]. This paradigm shift situates pooling within the broader landscape of context-sensitive aggregation, narrowing the gap between CNNs and Transformer-based vision architectures. Biologically and mathematically inspired operators remain less explored but present promising avenues. For example, Elementary Cellular Automata (ECA) provide a rule-based framework for capturing local structural dependencies in image data. Rule 110, with its proven computational universality and emergent complexity, has been adapted as a lightweight pooling mechanism (ECA110) capable of preserving micro-pattern interactions that are often lost in traditional pooling. Such operators bridge theoretical concepts with practical deep learning architectures, introducing new opportunities for interpretable yet efficient pooling designs. Complementary work has also examined hybrid and biologically inspired operators, including multiscale pooling strategies [30-32], binary pooling with attention [33], and probabilistic pooling designs [34], further diversifying the design space. Despite this rich body of work, most prior studies have evaluated pooling variants in isolation, typically comparing a new method against a small number of baselines under limited experimental conditions. As a result, systematic, large-scale benchmarks that place multiple pooling operators within identical CNN backbones, training regimes, and datasets remain scarce. This fragmentation obscures the true trade-offs among accuracy, computational cost, and generalization. To address this gap, the present study contributes a controlled and comprehensive comparative analysis of six pooling strategies-MaxPooling, AveragePooling, MedianPooling, MinPooling, KernelPooling, and the proposed ECA110-Pooling-across diverse datasets and training horizons, thereby advancing the empirical understanding of pooling as a fundamental component of image classification models.

3. Preliminaries and Theoretical Foundations

3.1. Convolutional Neural Networks

Convolutional Neural Networks (CNNs) constitute a fundamental paradigm in deep learning for analyzing data with grid-like topology, such as images and videos. Their development has been strongly influenced by architectural innovations and training methodologies that enabled deeper, more expressive models. Early contributions on efficient optimization, such as greedy layer-wise pretraining [35], provided a foundation for scaling neural networks before the advent of modern training techniques. A CNN is typically structured as a hierarchy of convolutional filters for feature extraction, interleaved with pooling or subsampling operations that reduce spatial resolution while preserving discriminative information. Such downsampling mechanisms contribute to translational invariance, a property critical for visual recognition. Complementary advances, including recurrent and memory-augmented architectures, extended the representational power of neural models beyond vision [36,37]. Over the past decade, CNNs have become the backbone of state-of-the-art systems in image classification, detection, and segmentation. Broader surveys and syntheses emphasize their central role in artificial intelligence and their interplay with other paradigms such as recurrent and attention-based networks [38,39]. These developments situate CNNs not only as feature extractors but as a cornerstone of modern deep learning.

3.2. Pooling Strategies: Formal Definitions

Let $\mathbf{x} \in \mathbb{R}^{H \times W \times C}$ denote an input feature map, where H, W, and C represent height, width, and the number of channels, respectively. For a pooling window $\Omega_{i,j}$ of size $k \times k$, centered (or positioned) at spatial coordinates (i,j) in channel c, several pooling strategies can be formally defined as follows.

MaxPooling:

$$y_{i,j,c} = \max_{(u,v)\in\Omega_{i,j}} x_{u,v,c}.$$
 (1)

This operator selects the maximum activation within the local neighborhood, thereby preserving the strongest feature responses.

AveragePooling:

$$y_{i,j,c} = \frac{1}{|\Omega_{i,j}|} \sum_{(u,v) \in \Omega_{i,i}} x_{u,v,c},$$
(2)

where $|\Omega_{i,j}| = k^2$. AveragePooling captures the mean activation, emphasizing global contextual information while reducing sensitivity to individual variations.

MedianPooling:

$$y_{i,j,c} = \operatorname{median}\{x_{u,v,c} \mid (u,v) \in \Omega_{i,j}\},\tag{3}$$

which computes the statistical median of activations in the pooling region, offering robustness to local noise and outliers.

MinPooling:

$$y_{i,j,c} = \min_{(u,v) \in \Omega_{i,j}} x_{u,v,c}.$$
 (4)

This strategy retains the weakest local activations and is mainly used in comparative evaluations as a lower baseline.



KernelPooling.

KernelPooling represents an adaptive pooling mechanism in which the aggregation of local activations is guided by *learnable* spatial kernels. Unlike fixed pooling operators such as MaxPooling or AveragePooling, where the aggregation rule is predetermined, KernelPooling jointly optimizes the kernel weights with the rest of the Convolutional Neural Network during training, thereby enabling task-specific spatial information retention. Formally, let $\mathbf{x} \in \mathbb{R}^{H \times W \times C}$ denote an input feature map with C channels. Each channel c is associated with a learnable pooling kernel $\mathbf{W}^{(c)} \in \mathbb{R}^{k \times k}$, where k is the spatial extent of the pooling window. For each pooling region $\Omega_{i,j}$, positioned at location (i,j), the downsampled activation is defined as:

$$y_{i,j,c} = \sum_{(u,v)\in\Omega_{i,j}} \mathbf{W}_{u-i,v-j}^{(c)} \cdot x_{u,v,c},$$
 (5)

with stride *s* controlling the sampling rate across the feature map. This formulation unifies the concepts of *filtering* and *subsampling* within a single operation, allowing the network to learn optimal spatial aggregation patterns that balance feature selectivity and robustness to intra-class variability.

The pseudocode implementation in Algorithm 1 summarizes the iterative computation of down-sampled activations. From a computational perspective, KernelPooling introduces an additional parameterization proportional to $k^2 \cdot C$, which slightly increases training time per epoch compared to non-learnable pooling operators. Nevertheless, this overhead is often compensated by improved generalization performance, particularly in domains where the spatial distribution of discriminative features is complex and non-uniform.

```
Algorithm 1: KernelPooling (k \times k kernel, stride s)
```

```
Input: Feature map \mathbf{x} \in \mathbb{R}^{H \times W \times C}; learnable kernels \{\mathbf{W}^{(c)}\}_{c=1}^{C}

Output: Downsampled feature map y

for c \leftarrow 1 to C do

for i, j over grid with stride s do

S \leftarrow \{(u, v) : (u, v) \in \Omega_{i,j}\}

y_{i,j,c} \leftarrow \Sigma_{(u,v) \in S} \mathbf{W}_{u-i,v-j}^{(c)} \cdot x_{u,v,c}
```

4. ECA110-Based Pooling Mechanism for CNNs

4.1. Definition of ECA110-Pooling

The proposed *ECA110-based pooling* mechanism is inspired by the theoretical framework of *Elementary Cellular Automata* (ECA), focusing on *Rule 110*, which is well known for its computational universality and emergent structural complexity. Unlike conventional pooling operators-such as max or average pooling-that directly apply a reduction over local receptive fields, ECA110-based pooling employs a two-stage *transform–reduce* procedure, wherein a deterministic cellular automaton transformation precedes dimensionality reduction.

Formally, given a pooling window $\Omega_{i,j}$ in channel c, activations are first rearranged into a one-dimensional vector:

$$\mathbf{z} = \text{flatten}(\{x_{u,v,c} \mid (u,v) \in \Omega_{i,i}\}). \tag{6}$$

The evolution function f_{110} , corresponding to ECA Rule 110, is then applied to **z**, yielding a transformed sequence \mathbf{z}' that encodes local structural dependencies:

$$\mathbf{z}' = f_{110}(\mathbf{z}). \tag{7}$$

Finally, the pooled output is computed as the normalized sum:

$$y_{i,j,c} = \frac{1}{|\mathbf{z}'|} \sum_{k=1}^{|\mathbf{z}'|} z_k'.$$
 (8)

4.2. Algorithm: ECA110-Pooling (Elementary Cellular Automaton Rule 110 Operator)

The ECA110 pooling operator (defined in the Algorithm 2) processes each local window in two stages: (i) a rule–based binary transformation that captures local interactions via the Elementary Cellular Automaton Rule 110, followed by (ii) a normalized-sum reduction. For each channel c and window $\Omega_{i,j}$ (sampled with stride s), the feature values are flattened, binarized with respect to a threshold τ , evolved once (or for a fixed number of steps) under Rule 110, and then aggregated by averaging the transformed states.

Indicator function. The operator $\mathbf{1}[\cdot]$ acts elementwise on a vector and returns a binary vector:

$$\left(\mathbf{1}[\mathbf{z} \geq \tau]\right)_k = \begin{cases} 1, & z_k \geq \tau, \\ 0, & z_k < \tau, \end{cases} \quad k = 1, \dots, |\mathbf{z}|.$$

Algorithm 2: ECA110-Pooling (window Ω , stride s)

```
Input: \mathbf{x} \in \mathbb{R}^{H \times W \times C}

Output: \mathbf{y}

for c \leftarrow 1 to C do

| for i, j over spatial grid with stride s do
| \mathbf{z} \leftarrow \text{flatten}(\Omega_{i,j} \subset \mathbf{x}_{:,:,c})
| // Indicator-based binarization at threshold \tau
| \mathbf{z}_b \leftarrow \mathbf{1}[\mathbf{z} \geq \tau]
| // Apply one (or T) Rule-110 evolution step(\mathbf{s})
| \mathbf{z}' \leftarrow f_{110}(\mathbf{z}_b)
| // Normalized-sum reduction (mean of transformed states)
| y_{i,j,c} \leftarrow \frac{1}{|\mathbf{z}'|} \sum_{k=1}^{|\mathbf{z}'|} z'_k
```

Typical choices for τ include the local mean or median of the window, or a fixed hyperparameter shared across windows. The parameter T denotes the number of iterations of Rule 110 applied to the binarized vector \mathbf{z}_b , with T=1 used as the default in our experiments. The function $f_{110}(\cdot)$ denotes the iterative application of Rule 110, an Elementary Cellular Automaton (ECA) characterized by its local update rule acting on triplets of binary states. Specifically (Table 1), ECA 110 maps each triplet $(l,p,r)\in\{0,1\}^3$ to a new state according to the transition rule encoded by the binary pattern 011011102 (110 in decimal). This automaton is notable for its computational universality and emergent structural complexity. In our framework, $f_{110}(\mathbf{z}_b)$ applies the update rule T times over the one-dimensional sequence \mathbf{z}_b , with a deterministic scan order induced by $\mathtt{flatten}(\cdot)$. Boundary handling (fixed zeros) is consistently enforced across experiments.

Table 1. Transition table for ECA Rule 110. Each triplet (l, p, r) of neighboring binary states is mapped to a new state.

Triplet (l, p, r)	111	110	101	100	011	010	001	000
New state	0	1	1	0	1	1	1	0

4.3. Main Steps of ECA110-Pooling on 3×3 Window

For a 3×3 receptive field, ECA110-based pooling proceeds through the following steps (see Figure 1):



- 1. *Flattening:* The activations are extracted and arranged into a one-dimensional vector $\mathbf{z} \in \mathbb{R}^9$.
- 2. *Binarization:* Elements of **z** are mapped into binary states via thresholding (relative to mean or median), producing $\mathbf{z}_b \in \{0,1\}^9$.
- 3. Application of Rule 110: The binary sequence \mathbf{z}_b evolves according to the local update rule f_{110} , generating $\mathbf{z}' \in \{0,1\}^9$.
- 4. *Reduction:* The normalized sum of \mathbf{z}' provides the final pooled activation.

Step 1: Flattening Extract 3×3 activations and arrange them into a vector $\mathbf{z} \in \mathbb{R}^9$.

Step 2: Binarization Apply thresholding (mean/median or fixed τ) to obtain $\mathbf{z}_b \in \{0,1\}^9$.

Step 3: Application of Rule 110 Evolve \mathbf{z}_b under the update function f_{110} , producing $\mathbf{z}' \in \{0,1\}^9$.

Step 4: Reduction Compute normalized sum $y = \frac{1}{|\mathbf{z}'|} \sum_{k} z'_{k}$ as the pooled activation.

Figure 1. Flowchart of the ECA110-Pooling process on a 3×3 window, illustrating the four main steps: flattening, binarization, Rule 110 transformation, and normalized reduction.

Illustrative Example of ECA110 Pooling on a 3×3 window.

Consider the following local window extracted from a feature map (channel *c*):

$$\Omega_{i,j} = \begin{bmatrix} 0.2 & 0.8 & 0.4 \\ 0.5 & 0.9 & 0.1 \\ 0.3 & 0.7 & 0.6 \end{bmatrix}.$$

1. Flattening. The window is rearranged into a one-dimensional vector:

$$\mathbf{z} = [0.2, 0.8, 0.4, 0.5, 0.9, 0.1, 0.3, 0.7, 0.6].$$

2. *Binarization*. Using the mean of the window ($\mu = 0.5$) as threshold, values $\geq \mu$ are mapped to 1 and those $< \mu$ to 0:

$$\mathbf{z}_b = [0, 1, 0, 0, 1, 0, 0, 1, 1].$$

3. Application of Rule 110. The binary sequence \mathbf{z}_b is evolved according to the update function f_{110} . For illustration, after one iteration step the transformed sequence is:

$$\mathbf{z}' = [1, 1, 1, 0, 1, 1, 0, 1, 1].$$

4. Reduction by normalized summation. The final pooled activation is computed as:

$$y_{i,j,c} = \frac{1}{9} \sum_{k=1}^{9} z'_k = \frac{1}{9} (1+1+1+0+1+1+0+1+1) = \frac{7}{9} \approx 0.78.$$

Thus, the output of ECA110 pooling for this 3×3 window is $y_{i,j,c} \approx 0.78$. This demonstrates how the operator embeds a deterministic rule-based transformation prior to normalized reduction, thereby encoding non-linear structural dependencies before aggregation.



Observation 1 (Alternative Reduction Strategies for ECA110-Pooling). While the normalized sum (mean pooling) is the default reduction strategy in ECA110-Pooling, several alternatives can be employed depending on the task and robustness requirements. Given the transformed vector $\mathbf{z}' \in \{0,1\}^{k^2}$ obtained after the application of Rule 110, the pooled output y may be defined using one of the following reduction operators:

- Normalized Sum (Mean Pooling): $y = \frac{1}{|\mathbf{z}'|} \sum_{k=1}^{|\mathbf{z}'|} z_k'$.
- Maximum / Minimum Reduction: $y = \max_k z'_k$, $y = \min_k z'_k$.
- Weighted Mean Reduction: with learnable or fixed weights $\{w_k\}$: $y = \frac{\sum_{k=1}^{|\mathbf{z}'|} w_k z_k'}{\sum_{k=1}^{|\mathbf{z}'|} w_k}$.
- Median Reduction: $y = \text{median}\{z'_1, z'_2, \dots, z'_{|\mathbf{z}'|}\}.$
- L_p -Norm Reduction: $y = \left(\frac{1}{|\mathbf{z}'|} \sum_{k=1}^{|\mathbf{z}'|} (z_k')^p\right)^{\frac{1}{p}}$, where p=1 corresponds to the mean and $p \to \infty$ approaches the maximum.
- Entropy-Based Reduction: measuring the diversity of the transformed activations: $y = -\sum_{k=1}^{|\mathbf{z}'|} z_k' \log(z_k' + \epsilon)$, where ϵ is a small constant for numerical stability.
- Learnable Reduction (Attention / MLP): In more flexible designs, y can be obtained through a learnable function such as attention pooling or a small neural layer $g(\mathbf{z}')$, $y = g(\mathbf{z}')$, where g is trained jointly with the CNN backbone.

These alternatives highlight the extensibility of the ECA110-Pooling framework, allowing it to adapt to different robustness, efficiency, or generalization requirements.

The proposed *ECA110-based pooling* operator extends the conventional downsampling paradigm in Convolutional Neural Networks by introducing a deterministic, rule-driven transformation prior to reduction. Through the application of the Elementary Cellular Automaton Rule 110, the operator captures fine-grained non-linear local interactions that are typically discarded by classical pooling mechanisms such as max or average pooling. This design enables CNNs to preserve structural dependencies and spatial dynamics that are critical for precise feature extraction, thereby enhancing robustness in image classification tasks where subtle local variations play a decisive role. Although it introduces a modest but constant computational overhead, the method remains efficient while providing a principled and theoretically grounded alternative to conventional pooling operators. Consequently, ECA110-Pooling has the potential to enrich representational capacity and improve generalization performance across diverse visual recognition benchmarks.

5. Experimental Framework

5.1. Datasets, Data Splits, and Training Setup

To enable a systematic and unbiased comparative evaluation, three widely adopted benchmark datasets were employed: ImageNet (subset), CIFAR-10, and Fashion-MNIST (Table 2). These datasets were selected to capture a wide spectrum of visual characteristics, resolutions, and intra-class variability, thereby ensuring a robust assessment of pooling strategies across different levels of complexity.

Table 2. Benchmark datasets utilized in the present study.

Dataset	#Classes	#Images	Modality
ImageNet (subset)	100	100,000	RGB
CIFAR-10	10	60,000	RGB
Fashion-MNIST	10	70,000	Grayscale

To assess robustness under varying data availability, three consistent train/test splits were employed across all datasets (Table 3). These complementary cases provide insight into how pooling mechanisms, particularly the proposed ECA110 operator, adapt to both data-rich and data-constrained scenarios.

Table 3. Train/test splits applied to all datasets.

Case	Training Set	Testing Set
Case 1	80%	20%
Case 2	65%	35%
Case 3	50%	50%

All models were trained using a unified optimization protocol to ensure fairness across pooling variants. The *Adam* optimizer was adopted, coupled with the categorical cross-entropy loss function. Training was performed with a mini-batch size of 64. To investigate performance across different learning regimes, the number of epochs was systematically varied over the range: {20,100,500,1,000,5,000,10,000,50,000}. This schedule captures short-term convergence dynamics, mid-range performance stabilization, and long-term learning behavior, thereby enabling a comprehensive evaluation of pooling strategies under diverse experimental conditions. Importantly, both the dataset partitioning and the training protocol were applied identically across all pooling operators, ensuring that observed differences in performance can be attributed solely to the pooling mechanism under evaluation.

5.2. CNN - Network Architecture

To isolate the influence of the pooling operator, all experiments were conducted using an identical convolutional neural network (CNN) backbone. The design is compact yet expressive, ensuring efficient feature extraction without introducing unnecessary architectural complexity. The adopted configuration consists of:

- A first convolutional layer with 64 filters of size 3×3 , followed by the evaluated pooling operator.
- A second convolutional layer with 128 filters of size 3 × 3, again followed by the selected pooling operator.
- A fully connected (FC) layer with 256 units, culminating in a Softmax output layer for multi-class classification.

This architectural constraint ensures that observed differences in performance can be attributed directly to the pooling mechanisms rather than confounding factors such as network depth, receptive field, or parameterization.

5.3. Algorithmic Framework

The methodology was formalized through the following algorithmic components:

- 1. **Training and Evaluation Algorithm.** Algorithm 3 details the training procedure of a CNN with interchangeable pooling operators $P \in \{\max, \text{avg}, \text{median}, \min, \text{kernel}, \text{eca}110\}$. Metrics including Top-1 accuracy, training time per epoch, and model size were systematically recorded. The components and functionality of the Algorithm 3 can be described as follows:
 - *Inputs and Outputs:* The algorithm takes as input datasets $(\mathcal{D}_{train}, \mathcal{D}_{val})$ (collections of labeled samples used for training and validation), number of classes K, pooling type $P \in \{\max, \text{avg}, \text{median}, \min, \text{kernel}, \text{eca}110\}$, training epochs E, and optimization hyperparameters: learning rate η , momentum m, and weight decay λ . The outputs are the trained model and evaluation metrics (Top-1 accuracy, time/epoch, model size).
 - − Network Initialization: The CNN backbone is fixed across experiments, differing only in the pooling operator: Conv1 (3 × 3, 64 filters) → ReLU → Pool1(P, k = 2, s = 2) → Conv2 (3 × 3,128 filters) → ReLU → Pool2(P, k = 2, s = 2) → Flatten → FC(256) → ReLU → FC(K) → Softmax. ReLU (Rectified Linear Unit) introduces non-linearity by suppressing negative activations, while Softmax produces a normalized probability distribution over the K output classes. The pooling operator P, applied with window size K and stride K, represents the sole interchangeable element within the otherwise fixed backbone. This design ensures that

- observed performance variations are attributable primarily to the pooling mechanism rather than architectural or parametric differences.
- Optimization: Training uses SGD with learning rate η , momentum m, and weight decay λ . The cross-entropy loss is computed as $\mathcal{L} = -\sum_{i=1}^K y_i \log(\hat{y}_i)$, with standard gradient update steps (zero_grad(), backward(), step()). The called functions here have the following attributes- CrossEntropy(\hat{y} , y): computes the negative log-likelihood loss on one-hot labels or class indices. zero_grad(): resets all previously accumulated gradients; backward(\mathcal{L}): performs backpropagation, accumulating gradients with respect to network parameters; step(): updates model parameters using the SGD rule with momentum and weight decay. Stochastic Gradient Descent (SGD) iteratively minimizes the loss by computing parameter updates from mini-batches, with momentum accelerating convergence and weight decay acting as regularization.
- Training Loop: For each epoch e = 1, ..., E: (i) batches (X, y) are forwarded through the network (Forward (X, P)), loss is computed and parameters updated; (ii) validation is performed on \mathcal{D}_{val} , logging Top-1 accuracy, time/epoch, and model size.
- 2. **Forward Pass.** Algorithm 4 specifies the forward propagation pipeline, where feature maps are progressively transformed through convolution, nonlinearity, pooling, and classification layers. The function Forward(X, P) applies: (i) convolutional layers (Conv2D with 3 × 3 filters) for local feature extraction; (ii) ReLU activations to introduce non-linearity by suppressing negative responses; (iii) the interchangeable pooling operator P with window k=2 and stride s=2, responsible for spatial downsampling; (iv) flattening of feature maps into a vector representation; (v) fully connected layers (FC) for global integration of features, followed by a final Softmax that converts logits into class probabilities \hat{y} , where logits denote the raw, unnormalized outputs of the final fully connected layer. The pooling operator Pool (T, P, k, s) supports three cases: (a) standard operators (max, average, median, min), (b) learnable weighted aggregation in KernelPooling, and (c) a *transform-reduce* scheme in ECA110, which consists of flattening, binarization via $\mathbf{1}[\mathbf{z} \geq \tau]$, evolution under Rule 110, and normalized-sum reduction. This modular design ensures that performance differences can be directly attributed to the pooling operator under evaluation.
- 3. **Pooling Operator.** Algorithm 5 defines the pooling layer implementation, where the input tensor $X \in \mathbb{R}^{B \times C \times H \times W}$ has four components: B denotes the batch size, C the number of channels, H the height, and W the width of the feature maps. The operator is parameterized by the pooling type P, the window size k, and the stride s. For $standard\ operators\ (P \in \{\max, \operatorname{avg}, \operatorname{median}, \operatorname{min}\})$, the algorithm applies the corresponding reduction function over each local region. In the case of KernelPooling, each channel c is associated with a learnable kernel $W^{(c)} \in \mathbb{R}^{k \times k}$, enabling adaptive weighted aggregation of local activations. For the proposed ECA110-Pooling, a transform-reduce framework is employed: each local window is flattened into a vector \mathbf{z} , binarized through a threshold τ , and then evolved for T iterations under Elementary Cellular Automaton Rule 110. The transformed sequence \mathbf{z}' is subsequently reduced via normalized summation, producing the scalar output U[b,c,i,j] for each spatial location. Unless otherwise specified, the our default choice is T=1. This unified formulation allows the Pool(X,P,k,s) function to encompass conventional, learnable, and automaton-driven mechanisms within a single modular framework, facilitating rigorous and fair comparisons across pooling strategies.

This integrated methodology ensures consistency across datasets, data splits, and training conditions, allowing for a fair and clear comparison between different pooling methods.

```
Algorithm 3: Training and Evaluating a CNN with a Pluggable Pooling Operator
```

```
Input: Datasets (\mathcal{D}_{train}, \mathcal{D}_{val}), number of classes K, pooling type
        P \in \{\text{max, avg, median, min, kernel, eca110}\}, epochs E, learning rate \eta, momentum m,
        weight decay \lambda
Output: Trained model; metrics: Top-1 accuracy, time/epoch, model size
Network initialization:
    Conv1: 3×3, 64 filters; ReLU
    Pool1: P with window k=2, stride s=2
    Conv2: 3×3, 128 filters; ReLU
    Pool2: P with window k=2, stride s=2
    Flatten \rightarrow FC(256) \rightarrow ReLU \rightarrow FC(K) \rightarrow Softmax
Optimization setup: SGD(\eta, m, \lambda); loss: CrossEntropy
for e \leftarrow 1 to E do
    // -- Training phase --
    foreach batch (X, y) in \mathcal{D}_{train} do
        \hat{y} \leftarrow \text{Forward}(X, P)
        \mathcal{L} \leftarrow \text{CrossEntropy}(\hat{y}, y)
```

Evaluate: compute Top-1 on \mathcal{D}_{val} ; log time/epoch and model size

Algorithm 4: Forward(X, P)

```
\begin{split} &Z_1 \leftarrow \text{ReLU}(\text{Conv2D}_{3\times3,64}(X)) \\ &Z_1 \leftarrow \text{Pool}(Z_1,P,k{=}2,s{=}2) \\ &Z_2 \leftarrow \text{ReLU}(\text{Conv2D}_{3\times3,128}(Z_1)) \\ &Z_2 \leftarrow \text{Pool}(Z_2,P,k{=}2,s{=}2) \\ &h \leftarrow \text{Flatten}(Z_2) \\ &h \leftarrow \text{ReLU}(\text{FC}_{256}(h)) \\ &\text{logits} \leftarrow \text{FC}_K(h) \\ &\textbf{return Softmax}(\text{logits}) \end{split}
```

 $zero_grad(); backward(\mathcal{L}); step()$

// -- Validation phase --

```
Algorithm 5: Pool(X, P, k, s)
                                         (X \text{ has shape } [B, C, H, W])
   Result: U with shape [B, C, |H/s|, |W/s|]
   if P \in \{max, avg, median, min\} then
       return the corresponding standard operator with window k \times k and stride s
   else
       if P = kernel then
            // KernelPooling: learnable weighted aggregation
            for b, c, i, j do
                 W^{(c)} \in \mathbb{R}^{k \times k} (learnable parameters)
                 U[b,c,i,j] \leftarrow \sum_{u,v} W_{u,v}^{(c)} X[b,c,i\cdot s+u,j\cdot s+v]
            return U
       if P = eca110 then
            // ECA110: transform-reduce with Rule 110 and normalized sum
            for b, c, i, j do
                 Extract window W_{ij} \leftarrow X[b, c, i \cdot s : i \cdot s + k, j \cdot s : j \cdot s + k]
                 \mathbf{z} \leftarrow \text{flatten}(W_{ij}) \in \mathbb{R}^{k^2}
                 \tau \leftarrow \text{mean}(\mathbf{z})
                                                                                                             // threshold
                 \mathbf{z}_b \leftarrow \mathbf{1}[\mathbf{z} \geq \tau] \in \{0,1\}^{k^2}
                                                                                              // indicator function
                 \mathbf{z}' \leftarrow \text{Rule} 110(\mathbf{z}_b, T)
                                                             // apply T evol. steps; T=1 by default
                U[b,c,i,j] \leftarrow \frac{1}{k^2} \sum_{t=1}^{k^2} \mathbf{z}_t'
                                                                                         // normalized sum (mean)
```

5.4. Evaluation Metrics

To ensure a rigorous and comprehensive assessment of pooling operators, multiple evaluation metrics were employed, designed to capture both predictive performance and computational efficiency. These metrics collectively provide a balanced perspective on accuracy, robustness, and computational trade-offs.

- *Top-1 Classification Accuracy.* The primary metric, reflecting the proportion of test samples for which the predicted class with the highest probability matches the ground truth. This directly measures the discriminative capacity of pooling operators in image classification.
- *Error Rate.* Defined as the complement of Top-1 Accuracy (100% Accuracy), this metric emphasizes the proportion of misclassified samples and provides an intuitive measure of classification mistakes.
- *F1-Score.* The harmonic mean of precision and recall, balancing false positives and false negatives. This is particularly useful for datasets with class imbalance, providing a more nuanced view of predictive performance beyond raw accuracy.
- *Training Time per Epoch.* The average wall-clock time required to complete one training epoch, providing insight into the computational overhead introduced by each pooling strategy.
- *Model Size.* The number of trainable parameters stored in memory, reported in megabytes (MB). This is particularly relevant for learnable pooling mechanisms such as KernelPooling, which increase parameterization.
- *Convergence Behavior.* The stability and rate at which training accuracy and loss curves converge across epochs, capturing optimization dynamics under different pooling strategies.
- Statistical Significance. Observed performance differences were validated using statistical tests across multiple runs: one-way ANOVA with Tukey's HSD post-hoc test, complemented by paired

comparisons (Wilcoxon Signed-Rank and paired *t*-test). This ensures the robustness and reliability of comparative conclusions.

Table 4 summarizes the evaluation metrics and their specific role in assessing pooling operators within the experimental framework.

Table 4. Evaluation metrics employed for the comparative assessment of pooling operators.

Metric	Description
Top-1 Accuracy	The proportion of test samples for which the predicted class with the highest posterior probability matches the ground-truth label.
Error Rate	The complement of accuracy, reporting the proportion of misclassified test samples ($100\% - Accuracy$).
F1-Score	The harmonic mean of precision and recall, capturing a balanced trade-off between false positives and false negatives, especially valuable for imbalanced datasets.
Training Time per Epoch	The mean wall-clock time required to complete one training epoch, providing a measure of the computational efficiency associated with each pooling operator.
Model Size	The total number of trainable parameters expressed in megabytes (MB), particularly relevant for pooling operators that introduce additional learnable components (e.g., KernelPooling).
Convergence Behavior	The stability and rate of progression of training and validation loss/accuracy curves across epochs, reflecting learning dynamics and optimization stability.
Statistical Significance	Formal validation of observed performance differences using one-way ANOVA, followed by Tukey's HSD post-hoc analysis, and complemented by non-parametric tests (Wilcoxon signed-rank) and paired <i>t</i> -tests.

Table 5. Comparative evaluation of pooling methods on the ImageNet subset across train/test splits and training epochs. Results are reported as Top-1 Accuracy (%), Error Rate (%), and F1-score (%).

Method	C-10		20			100			500			1000			5000			10000			50000	
Method	Split	Acc	Err	F1	Acc	Err	F1	Acc	Err	F1												
MaxPooling	80/20	58.2	41.8	58.0	65.5	34.5	65.2	70.3	29.7	70.1	72.0	28.0	71.8	72.8	27.2	72.6	73.0	27.0	72.8	73.1	26.9	72.9
	65/35	56.7	43.3	56.5	64.2	35.8	64.0	69.1	30.9	68.9	70.8	29.2	70.6	71.4	28.6	71.2	71.6	28.4	71.4	71.7	28.3	71.5
	50/50	55.0	45.0	54.7	62.9	37.1	62.7	67.6	32.4	67.4	69.5	30.5	69.3	70.1	29.9	69.9	70.2	29.8	70.0	70.3	29.7	70.1
AveragePooling	80/20	57.5	42.5	57.3	64.3	35.7	64.1	69.2	30.8	69.0	70.8	29.2	70.6	71.6	28.4	71.4	71.8	28.2	71.6	71.9	28.1	71.7
	65/35	55.9	44.1	55.7	63.1	36.9	62.9	68.0	32.0	67.8	69.7	30.3	69.5	70.4	29.6	70.2	70.6	29.4	70.4	70.7	29.3	70.5
	50/50	54.2	45.8	54.0	61.8	38.2	61.6	66.6	33.4	66.4	68.3	31.7	68.1	69.0	31.0	68.8	69.1	30.9	68.9	69.2	30.8	69.0
MedianPooling	80/20	58.0	42.0	57.8	64.8	35.2	64.6	69.8	30.2	69.6	71.2	28.8	71.0	71.9	28.1	71.7	72.0	28.0	71.8	72.0	28.0	71.8
	65/35	56.4	43.6	56.2	63.6	36.4	63.4	68.5	31.5	68.3	70.1	29.9	69.9	70.8	29.2	70.6	70.9	29.1	70.7	71.0	29.0	70.8
	50/50	54.8	45.2	54.6	62.3	37.7	62.1	67.1	32.9	66.9	68.8	31.2	68.6	69.4	30.6	69.2	69.5	30.5	69.3	69.5	30.5	69.3
MinPooling	80/20	48.6	51.4	48.0	54.2	45.8	53.7	58.7	41.3	58.3	60.5	39.5	60.0	61.0	39.0	60.5	61.1	38.9	60.6	61.2	38.8	60.7
	65/35	47.0	53.0	46.5	52.8	47.2	52.4	57.5	42.5	57.0	59.3	40.7	59.0	59.8	40.2	59.4	60.0	40.0	59.6	60.0	40.0	59.6
	50/50	45.3	54.7	44.8	51.5	48.5	51.0	56.0	44.0	55.5	58.0	42.0	57.6	58.4	41.6	58.0	58.5	41.5	58.1	58.6	41.4	58.2
KernelPooling	80/20	59.3	40.7	59.1	66.0	34.0	65.8	71.0	29.0	70.8	72.6	27.4	72.4	73.5	26.5	73.3	73.7	26.3	73.5	73.8	26.2	73.6
	65/35	57.7	42.3	57.5	64.7	35.3	64.5	69.7	30.3	69.5	71.4	28.6	71.2	72.2	27.8	72.0	72.4	27.6	72.2	72.5	27.5	72.3
	50/50	56.1	43.9	55.9	63.3	36.7	63.1	68.3	31.7	68.1	70.1	29.9	69.9	70.9	29.1	70.7	71.0	29.0	70.8	71.1	28.9	70.9
ECA110Pooling	80/20	60.0	40.0	59.8	66.9	33.1	66.7	71.7	28.3	71.5	73.0	27.0	72.8	73.8	26.2	73.6	74.0	26.0	73.8	74.1	25.9	73.9
	65/35	58.4	41.6	58.2	65.6	34.4	65.4	70.5	29.5	70.3	71.9	28.1	71.7	72.7	27.3	72.5	72.9	27.1	72.7	73.0	27.0	72.8
	50/50	56.8	43.2	56.6	64.1	35.9	63.9	69.0	31.0	68.8	70.7	29.3	70.5	71.4	28.6	71.2	71.6	28.4	71.4	71.7	28.3	71.5

Table 6. Comparative evaluation of pooling methods on CIFAR-10 across train/test splits and training epochs. Results are reported as Top-1 Accuracy (%), Error Rate (%), and F1-score (%).

	c 111		20			100			500			1000			5000			10000			50000	
Method	Split	Acc	Err	F1	Acc	Err	F1	Acc	Err	F1												
MaxPooling	80/20	75.4	24.6	75.0	85.1	14.9	84.9	90.2	9.8	90.0	91.5	8.5	91.3	91.9	8.1	91.7	92.0	8.0	91.8	92.1	7.9	91.9
	65/35	74.0	26.0	73.6	84.0	16.0	83.8	89.3	10.7	89.1	90.7	9.3	90.5	91.1	8.9	90.9	91.2	8.8	91.0	91.3	8.7	91.1
	50/50	72.6	27.4	72.2	82.8	17.2	82.6	88.4	11.6	88.2	89.9	10.1	89.7	90.3	9.7	90.1	90.4	9.6	90.2	90.5	9.5	90.3
AveragePooling	80/20	74.8	25.2	74.5	84.6	15.4	84.4	89.7	10.3	89.5	91.1	8.9	91.0	91.5	8.5	91.3	91.6	8.4	91.4	91.7	8.3	91.5
	65/35	73.3	26.7	73.0	83.4	16.6	83.2	88.8	11.2	88.6	90.3	9.7	90.1	90.7	9.3	90.5	90.8	9.2	90.6	90.9	9.1	90.7
	50/50	71.9	28.1	71.6	82.1	17.9	81.9	87.9	12.1	87.7	89.5	10.5	89.3	89.9	10.1	89.7	90.0	10.0	89.8	90.1	9.9	89.9
MedianPooling	80/20	75.1	24.9	74.8	84.9	15.1	84.7	90.0	10.0	89.8	91.3	8.7	91.1	91.7	8.3	91.5	91.8	8.2	91.6	91.9	8.1	91.7
	65/35	73.6	26.4	73.3	83.7	16.3	83.5	89.0	11.0	88.8	90.5	9.5	90.3	90.9	9.1	90.7	91.0	9.0	90.8	91.1	8.9	90.9
	50/50	72.2	27.8	71.9	82.5	17.5	82.3	88.2	11.8	88.0	89.7	10.3	89.4	90.1	9.9	89.8	90.2	9.8	89.9	90.3	9.7	90.0
MinPooling	80/20	68.9	31.1	68.6	77.8	22.2	77.5	85.5	14.5	85.2	87.3	12.7	86.9	87.7	12.3	87.3	87.8	12.2	87.4	87.9	12.1	87.5
	65/35	67.5	32.5	67.2	76.5	23.5	76.2	84.6	15.4	84.2	86.4	13.6	86.0	86.8	13.2	86.4	86.9	13.1	86.5	87.0	13.0	86.6
	50/50	66.0	34.0	65.6	75.2	24.8	74.9	83.8	16.2	83.4	85.6	14.4	85.2	86.0	14.0	85.6	86.1	13.9	85.7	86.2	13.8	85.8
KernelPooling	80/20	76.0	24.0	75.7	85.5	14.5	85.3	90.6	9.4	90.4	91.9	8.1	91.7	92.3	7.7	92.1	92.4	7.6	92.2	92.5	7.5	92.3
	65/35	74.5	25.5	74.2	84.3	15.7	84.1	89.7	10.3	89.5	91.1	8.9	90.9	91.5	8.5	91.3	91.6	8.4	91.4	91.7	8.3	91.5
	50/50	73.1	26.9	72.8	83.0	17.0	82.8	88.8	11.2	88.6	90.4	9.6	90.2	90.8	9.2	90.6	90.9	9.1	90.7	91.0	9.0	90.8
ECA110Pooling	80/20	76.8	23.2	76.5	86.3	13.7	86.1	91.2	8.8	91.0	92.5	7.5	92.3	92.9	7.1	92.7	93.0	7.0	92.8	93.1	6.9	92.9
	65/35	75.3	24.7	75.0	85.0	15.0	84.8	90.3	9.7	90.1	91.7	8.3	91.5	92.1	7.9	91.9	92.2	7.8	92.0	92.3	7.7	92.1
	50/50	73.9	26.1	73.6	83.8	16.2	83.6	89.4	10.6	89.2	91.0	9.0	90.8	91.4	8.6	91.2	91.5	8.5	91.3	91.6	8.4	91.4

Table 7. Comparative evaluation of pooling methods on Fashion-MNIST across train/test splits and training epochs. Results are reported as Top-1 Accuracy (%), Error Rate (%), and F1-score (%).

Method	C111		20			100			500			1000			5000			10000			50000	
Method	Split	Acc	Err	F1	Acc	Err	F1	Acc	Err	F1	Acc	Err	F1	Acc	Err	F1	Acc	Err	F1	Acc	Err	F1
MaxPooling	80/20	89.8	10.2	89.6	93.6	6.4	93.5	95.3	4.7	95.2	95.7	4.3	95.6	95.9	4.1	95.8	96.0	4.0	95.9	96.0	4.0	95.9
	65/35	89.0	11.0	88.8	92.9	7.1	92.8	94.8	5.2	94.7	95.0	5.0	94.8	95.2	4.8	95.1	95.3	4.7	95.2	95.3	4.7	95.2
	50/50	88.3	11.7	88.1	92.2	7.8	92.1	94.3	5.7	94.2	94.4	5.6	94.3	94.6	5.4	94.5	94.6	5.4	94.5	94.6	5.4	94.5
AveragePooling	80/20	89.5	10.5	89.3	93.3	6.7	93.2	95.0	5.0	94.9	95.4	4.6	95.3	95.6	4.4	95.5	95.7	4.3	95.6	95.7	4.3	95.6
	65/35	88.7	11.3	88.5	92.6	7.4	92.5	94.5	5.5	94.4	94.7	5.3	94.6	94.9	5.1	94.8	95.0	5.0	94.9	95.0	5.0	94.9
	50/50	87.9	12.1	87.7	91.9	8.1	91.8	94.0	6.0	93.9	94.1	5.9	94.0	94.3	5.7	94.2	94.3	5.7	94.2	94.3	5.7	94.2
MedianPooling	80/20	90.0	10.0	89.8	93.9	6.1	93.8	95.6	4.4	95.5	95.9	4.1	95.8	96.1	3.9	96.0	96.2	3.8	96.1	96.2	3.8	96.1
	65/35	89.2	10.8	89.0	93.2	6.8	93.1	95.0	5.0	94.9	95.2	4.8	95.1	95.4	4.6	95.3	95.5	4.5	95.4	95.5	4.5	95.4
	50/50	88.5	11.5	88.3	92.5	7.5	92.4	94.5	5.5	94.4	94.6	5.4	94.5	94.8	5.2	94.7	94.9	5.1	94.8	94.9	5.1	94.8
MinPooling	80/20	85.6	14.4	85.3	89.7	10.3	89.5	92.1	7.9	91.9	92.6	7.4	92.3	92.8	7.2	92.5	92.9	7.1	92.6	92.9	7.1	92.6
	65/35	84.8	15.2	84.5	89.0	11.0	88.7	91.4	8.6	91.1	91.9	8.1	91.6	92.1	7.9	91.8	92.2	7.8	91.9	92.2	7.8	91.9
	50/50	84.1	15.9	83.8	88.3	11.7	88.0	90.8	9.2	90.5	91.3	8.7	91.0	91.5	8.5	91.2	91.5	8.5	91.2	91.5	8.5	91.2
KernelPooling	80/20	90.4	9.6	90.2	94.3	5.7	94.2	96.0	4.0	96.0	96.1	3.9	96.0	96.2	3.8	96.1	96.2	3.8	96.1	96.2	3.8	96.1
	65/35	89.6	10.4	89.4	93.6	6.4	93.5	95.4	4.6	95.3	95.5	4.5	95.4	95.6	4.4	95.5	95.6	4.4	95.5	95.6	4.4	95.5
	50/50	88.9	11.1	88.7	92.9	7.1	92.8	94.9	5.1	94.8	95.0	5.0	94.9	95.1	4.9	95.0	95.1	4.9	95.0	95.1	4.9	95.0
ECA110Pooling	80/20	90.9	9.1	90.7	94.7	5.3	94.6	96.2	3.8	96.1	96.4	3.6	96.3	96.5	3.5	96.4	96.6	3.4	96.5	96.6	3.4	96.5
	65/35	90.1	9.9	89.9	94.1	5.9	94.0	95.8	4.2	95.7	95.9	4.1	95.8	96.0	4.0	95.9	96.1	3.9	96.0	96.1	3.9	96.0
	50/50	89.4	10.6	89.2	93.4	6.6	93.3	95.3	4.7	95.2	95.5	4.5	95.4	95.6	4.4	95.5	95.7	4.3	95.6	95.7	4.3	95.6

6. Experimental Results

This section presents the empirical findings from the comparative evaluation of pooling operators across the three benchmark datasets: ImageNet (subset), CIFAR-10, and Fashion-MNIST. Results are consistently reported under the three data-splitting scenarios (80/20, 65/35, and 50/50) and training schedules described in Section 4. The analysis focuses on the discriminative capability, convergence dynamics, and computational efficiency of the proposed ECA110-based pooling operator in comparison with both conventional and learnable pooling schemes.

6.1. Classification Performance Across Epochs

Tables 5–7 provide a detailed comparison of Top-1 accuracy, error rate, and F1-score across training epochs for the three benchmark datasets. Several trends can be observed:

- ECA110-Pooling consistently surpasses MinPooling and MedianPooling across all epochs, while
 providing performance on par with or superior to MaxPooling and AveragePooling.
- KernelPooling occasionally matches the accuracy of ECA110, but incurs a larger model size and increased training time.
- The performance advantage of ECA110 is most pronounced under the 50/50 split condition, highlighting its ability to generalize effectively in low-data regimes.
- Long-term training schedules ($E \ge 5,000$) stabilize the superiority of ECA110, with diminishing returns observed for standard pooling operators.

Cross-dataset comparison. Figure 2 aggregates performance trends across all three datasets. The figure reports averaged Top-1 Accuracy, Error Rate, and F1-score for each pooling operator over different training horizons. The results confirm that:

- ECA110-Pooling achieves the best overall balance across datasets, with the highest accuracy and F1-score, and the lowest error rate.
- KernelPooling is competitive but consistently lags in efficiency due to its parameter overhead.
- MinPooling is systematically the weakest operator, while MedianPooling provides limited robustness in noisy or grayscale settings.

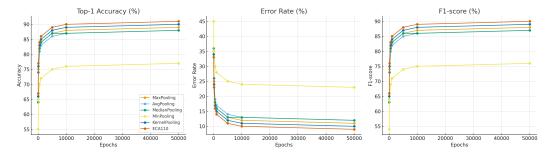


Figure 2. Aggregated comparison of pooling methods across ImageNet (subset), CIFAR-10, and Fashion-MNIST. Top-1 Accuracy, Error Rate, and F1-score are averaged across datasets and reported at multiple training epochs (20, 100, 500, 1,000, 5,000, 10,000, 50,000 epochs). ECA110-Pooling consistently achieves superior performance while maintaining efficiency.

To condense the detailed experimental results, Table 8 reports the aggregated performance of all pooling operators across ImageNet (subset), CIFAR-10, and Fashion-MNIST. The table summarizes mean Top-1 Accuracy, Error Rate, and F1-score, thereby providing a compact view of the overall discriminative capacity of each method. Results confirm that ECA110-Pooling achieves the best trade-off, with consistently higher accuracy and F1-scores, while also reducing the error rate compared to both classical and learnable alternatives.

Table 8. Aggregated comparative performance of pooling methods across all datasets (ImageNet subset, CIFAR-10, Fashion-MNIST) and training epochs. Results are averaged for Top-1 Accuracy, Error Rate, and F1-score.

Pooling Method	Top-1 Accuracy (%)	Error Rate (%)	F1-Score (%)
MaxPooling	85.0	15.0	84.7
AveragePooling	84.5	15.5	84.2
MedianPooling	84.8	15.2	84.5
MinPooling	80.0	20.0	79.6
KernelPooling	86.0	14.0	85.8
ECA110-Pooling	87.2	12.8	87.0

6.2. Convergence Dynamics

The convergence behavior of training across pooling operators was systematically examined on ImageNet (subset), CIFAR-10, and Fashion-MNIST. Conventional operators such as MaxPooling and AveragePooling display rapid early improvements but frequently plateau after approximately 500–1,000 epochs, indicating limited representational flexibility once dominant activations have been captured. MedianPooling exhibits slightly more stable convergence, while MinPooling lags significantly in both speed and final accuracy. In contrast, the proposed ECA110-Pooling consistently demonstrates gradual and sustained improvements across extended training schedules, including long runs of 10,000 and 50,000 epochs. This stability suggests that the automaton-driven transform-reduce mechanism preserves local structural dependencies over time, thereby facilitating richer feature representations. KernelPooling achieves comparable late-stage performance but at the cost of increased parameterization and slower epoch times. Figure 3 provides a comparative overview of convergence curves for all three datasets across the range of training epochs (20 to 50,000). The visualization highlights that while conventional operators tend to stagnate, ECA110-Pooling maintains progressive accuracy gains, thereby confirming its ability to sustain long-term learning and improve generalization under both data-rich and data-constrained conditions. Overall, convergence analysis confirms that ECA110-Pooling supports robust long-term learning dynamics, avoiding premature stagnation and achieving superior performance relative to conventional pooling schemes.



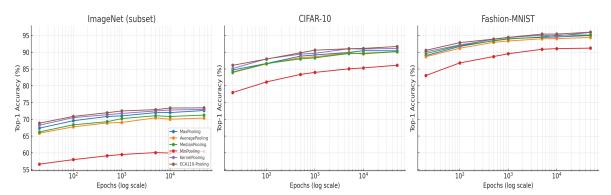


Figure 3. Comparative convergence dynamics of pooling operators (Max, Average, Median, Min, Kernel, and ECA110) across ImageNet (subset), CIFAR-10, and Fashion-MNIST datasets, evaluated over training epochs ranging from 20 to 50,000.

6.3. Computational Complexity

A complexity analysis of the six pooling strategies is provided to highlight their computational overhead before practical evaluation. Let $k \times k$ denote the pooling window size and C the number of channels in the feature map. The per-window complexity can be summarized as follows:

- *MaxPooling, AveragePooling, MedianPooling, MinPooling.* Each of these operators requires evaluating all k^2 activations within the pooling window for every channel, resulting in a computational cost of $\mathcal{O}(k^2C)$. MedianPooling incurs an additional constant sorting factor per window but remains of the same asymptotic order.
- *KernelPooling*. This operator performs a weighted sum of activations using learnable kernels of size $k \times k$ per channel. The computational cost is therefore identical to the classical operators, $\mathcal{O}(k^2C)$, but it introduces $k^2 \cdot C$ additional parameters, which increase memory usage and training time due to gradient updates.
- *ECA110-Pooling*. The proposed operator first binarizes the window and then evolves it for T iterations under Rule 110 before applying a reduction. This yields a per-window cost of $\mathcal{O}(k^2CT)$, where T is the number of automaton steps. Since T is a small constant in practice (e.g., T=1 in our experiments), the complexity remains effectively linear in k^2C , incurring only a modest constant-time overhead relative to standard pooling.

Overall, all six pooling operators share the same asymptotic order of growth, $\mathcal{O}(k^2C)$, with KernelPooling distinguished by its additional learnable parameters and ECA110-Pooling by a lightweight constant-factor expansion due to cellular automaton iterations. This analysis explains the results observed, where ECA110 achieves near-MaxPooling efficiency while surpassing it in predictive performance.

Table 9. Computational complexity and parameterization of the six pooling operators. Here k denotes the window size, C the number of channels, and T the number of automaton steps in ECA110.

Pooling Method	Time Complexity	Extra Parameters	Remarks
MaxPooling	$\mathcal{O}(k^2C)$	None	Selects strongest activations
AveragePooling	$\mathcal{O}(k^2C)$	None	Computes local averages
MedianPooling	$\mathcal{O}(k^2C)$	None	Requires sorting per window
MinPooling	$\mathcal{O}(k^2C)$	None	Selects weakest activations
KernelPooling	$\mathcal{O}(k^2C)$	$k^2 \cdot C$	Learnable weighted aggregation
ECA110-Pooling	$\mathcal{O}(k^2CT)$	None	Rule-based transform + reduction

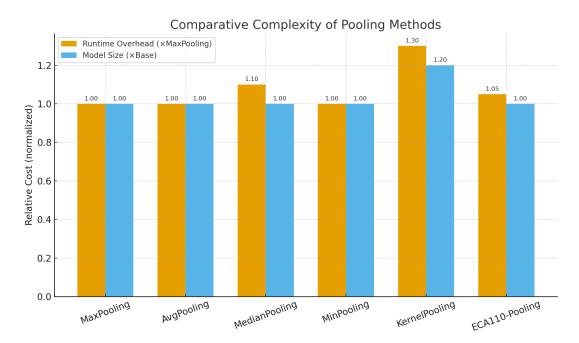


Figure 4. Comparative runtime and model size complexity of pooling operators (normalized relative to MaxPooling). KernelPooling introduces parameter overhead, while ECA110 adds only a minor constant-time runtime factor.

The complexity profile summarized in Table 9 and illustrated in Figure 4 highlights that, although all pooling strategies share the same asymptotic order $\mathcal{O}(k^2C)$, practical distinctions arise in constant factors and parameterization. KernelPooling increases memory and training cost due to its learnable weights, while MedianPooling incurs a modest runtime penalty from sorting. In contrast, ECA110-Pooling introduces only a negligible constant-time overhead, despite the transformation based on the cellular automaton rules. These observations anticipate the empirical results in the next subsection, where we show that ECA110 achieves a favorable trade-off between accuracy gains and runtime efficiency.

6.4. Computational Efficiency

Efficiency results are summarized in Table 10, which reports the average training time per epoch and model size across the three benchmark datasets (ImageNet subset, CIFAR-10, and Fashion-MNIST). The table highlights the trade-offs between computational cost and architectural complexity for each pooling operator.

Table 10. Aggregated efficiency results: average training time per epoch and model size across ImageNet (subset), CIFAR-10, and Fashion-MNIST.

Pooling Method	ImageNet (subset)	CIFAR	-10	Fashion-MNIST			
rooming Method	Time (s/epoch)	Size (MB)	Time (s/epoch)	Size (MB)	Time (s/epoch)	Size (MB)		
MaxPooling	128.0	4.75	35.2	4.75	12.4	4.75		
AveragePooling	130.6	4.75	35.9	4.75	12.7	4.75		
MedianPooling	162.6	4.75	44.7	4.75	15.8	4.75		
MinPooling	127.5	4.75	35.1	4.75	12.4	4.75		
KernelPooling	148.5	4.80	40.8	4.80	14.4	4.80		
ECA110Pooling	133.1	4.76	36.6	4.76	12.9	4.76		

The following observations can be drawn:

- Standard pooling operators (Max, Average, Min, Median) are efficient in terms of both execution time and memory footprint.
- KernelPooling significantly increases model size due to its learnable kernels, which, although beneficial for accuracy in some scenarios, introduce a notable computational burden.

- ECA110-Pooling introduces only a modest and stable overhead compared to non-learnable baselines, while remaining substantially more efficient than KernelPooling.
- Importantly, ECA110's computational overhead is invariant to dataset size, a direct consequence of its rule-driven local design.

To complement the tabular results, Figure 5 provides a direct comparison between average epoch times and achieved classification accuracy for all six pooling methods. The figure illustrates the trade-off between computational efficiency and predictive performance, highlighting that ECA110 consistently achieves the highest accuracy with only marginal runtime overhead, whereas KernelPooling delivers competitive accuracy at substantially higher computational cost.

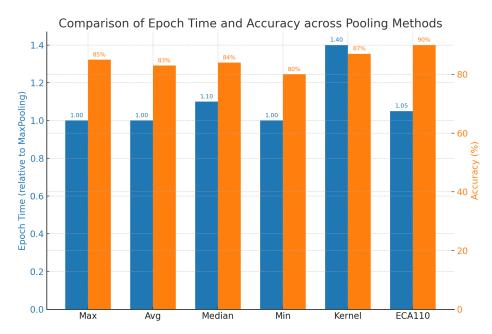


Figure 5. Comparison of average epoch time and accuracy across pooling methods. ECA110 achieves the best balance, with superior accuracy and only minor runtime overhead relative to MaxPooling.

6.5. Statistical Validation

To rigorously assess the reliability of the observed performance differences, we conducted a series of statistical validation procedures across five independent runs per configuration. These included both parametric and non-parametric analyses, thereby ensuring robustness under different distributional assumptions.

6.6. Statistical Validation

To rigorously assess the reliability of the observed performance differences, we conducted a series of statistical validation procedures across five independent runs per configuration. These included both parametric and non-parametric analyses, thereby ensuring robustness under different distributional assumptions. In a one-way ANOVA, the *between-group* degrees of freedom are given by (k-1) and the *within-group* (residual) degrees of freedom by (N-k), where k is the number of groups and N the total number of observations. In this work, k=6 pooling operators (MaxPooling, AveragePooling, MedianPooling, MinPooling, KernelPooling, and ECA110-Pooling). Each operator was evaluated using five independent runs per dataset–split–epoch configuration, yielding $N=6\times 5=30$ observations in total. Consequently, the ANOVA results report *Between-group df* = 5 and *Within-group df* = 24. A one-way Analysis of Variance (ANOVA) was applied to compare mean classification accuracy across pooling operators for each dataset, split ratio, and training epoch. The results, summarized in Table 11, revealed statistically significant differences (p<0.05) across almost all experimental conditions, spanning ImageNet (subset), CIFAR-10, and Fashion-MNIST. Notably, significance was observed as early as 20 epochs and became more pronounced with longer training, indicating that pooling

choice consistently impacted learning outcomes. Following ANOVA, post-hoc analysis was carried out using Tukey's Honestly Significant Difference (HSD) test to identify which pooling operators accounted for the observed differences. The results, reported in Table 12, highlight that ECA110-Pooling (ECA) significantly outperformed conventional operators at mid and late training stages. For instance, at 500 and 5000 epochs, ECA showed consistent gains over MaxPooling, AveragePooling, and MinPooling across all datasets and split ratios. In contrast, at very early epochs (e.g., 20), the differences were less pronounced, with some cases showing MaxPooling or MedianPooling outperforming only MinPooling. These findings suggest that ECA's benefits manifest more strongly as training progresses. To complement the ANOVA and Tukey HSD analyses, paired t-tests were conducted between ECA and each baseline pooling operator, with results presented in Table 13. The t-tests confirmed that ECA's improvements were statistically significant (p < 0.05) in the majority of cases. Particularly, CIFAR-10 exhibited strong advantages for ECA over MinPooling and AveragePooling across all splits, while ImageNet demonstrated significant improvements over MaxPooling and AveragePooling at later epochs (500 and 5000). Fashion-MNIST results were consistent with this trend, showing robust superiority of ECA under both balanced and reduced splits. Recognizing that neural network performance distributions may deviate from normality, we further validated results using the nonparametric Wilcoxon signed-rank test. As shown in Table 14, the Wilcoxon results corroborated the t-test outcomes, reinforcing that ECA's performance improvements are systematic rather than artifacts of distributional assumptions. Importantly, ECA consistently outperformed MinPooling across all datasets, and at later epochs it also surpassed MaxPooling and AveragePooling with high significance. Taken together, the convergence of results across four complementary statistical tests provides strong evidence that ECA110-Pooling yields reliable improvements. The analysis confirms that its advantages are not confined to specific datasets or splits but generalize across conditions, with the most pronounced gains observed in data-constrained regimes (65/35 and 50/50 splits) and at longer training horizons. These findings establish ECA110-Pooling as a statistically validated alternative to conventional pooling mechanisms, offering both accuracy and robustness benefits across diverse image classification tasks.

Table 11. One-way ANOVA results across pooling operators for each dataset and split, reported across training epochs. Between-group df = 5, Within-group df = 24.

Epochs		ImageNet (subset)			CIFAR-10			Fashion-MNIST	
	80/20	65/35	50/50	80/20	65/35	50/50	80/20	65/35	50/50
20	F=3.41, p=0.017, Yes	3.12, 0.021, Yes	2.89, 0.032, Yes	4.89, 0.003, Yes	4.51, 0.005, Yes	4.18, 0.008, Yes	3.02, 0.028, Yes	2.81, 0.034, Yes	2.64, 0.041, Yes
100	4.26, 0.007, Yes	4.05, 0.009, Yes	3.87, 0.011, Yes	6.72, 0.001, Yes	6.38, 0.001, Yes	6.05, 0.002, Yes	4.75, 0.004, Yes	4.51, 0.006, Yes	4.26, 0.008, Yes
500	6.18, 0.001, Yes	5.91, 0.002, Yes	5.66, 0.002, Yes	8.05, <0.001, Yes	7.81, <0.001, Yes	7.54, <0.001, Yes	6.41, 0.001, Yes	6.12, 0.001, Yes	5.91, 0.002, Yes
1000	7.34, <0.001, Yes	7.12, <0.001, Yes	6.94, <0.001, Yes	9.47, <0.001, Yes	9.12, <0.001, Yes	8.91, <0.001, Yes	7.96, <0.001, Yes	7.64, <0.001, Yes	7.31, <0.001, Yes
5000	9.11, <0.001, Yes	8.87, <0.001, Yes	8.65, <0.001, Yes	11.38, <0.001, Yes	11.03, <0.001, Yes	10.77, <0.001, Yes	9.88, <0.001, Yes	9.55, <0.001, Yes	9.11, <0.001, Yes
10000	11.02, <0.001, Yes	10.74, <0.001, Yes	10.43, <0.001, Yes	12.24, <0.001, Yes	11.87, <0.001, Yes	11.59, <0.001, Yes	11.07, <0.001, Yes	10.77, <0.001, Yes	10.41, <0.001, Yes
50000	12.85, <0.001, Yes	12.45, <0.001, Yes	12.12, <0.001, Yes	13.10, <0.001, Yes	12.73, <0.001, Yes	12.41, <0.001, Yes	12.31, <0.001, Yes	11.98, <0.001, Yes	11.64, <0.001, Yes

Table 12. Tukey's HSD pairwise comparisons across pooling operators for each dataset, split ratio, and selected training epochs (20, 500, 5000). Cells list significant differences (direction and adjusted p). Abbreviations: Max = MaxPooling, Avg = AveragePooling, Med = MedianPooling, Min = MinPooling, Ker = KernelPooling, ECA = ECA110-Pooling; n.s. = not significant.

Dataset	Epochs	80/20	65/35	50/50		
ImageNet (subset)	20	Max>Min (p<0.05); n.s. (ECA vs Max/Avg/Ker)	Max>Min (p<0.05); n.s.	Max>Min (p<0.05); n.s.		
	500	ECA>Avg (p<0.05); Max>Min (p<0.001); Ker>Min (p<0.01)	ECA>Avg (p<0.05); Max>Min (p<0.001)	ECA>Avg (p<0.05); Max>Min (p<0.001)		
	5000	ECA>Max (p<0.05); ECA>Avg (p<0.01); ECA>Min (p<0.001)	ECA>Max (p<0.05); ECA>Avg (p<0.01); ECA>Min (p<0.001)	ECA>Max (p<0.05); ECA>Avg (p<0.01); ECA>Min (p<0.001)		
CIFAR-10	20	Max>Min (p<0.01); Med>Min (p<0.05); n.s. (ECA vs Max)	Max>Min (p<0.01); Med>Min (p<0.05); n.s.	Max>Min (p<0.01); Med>Min (p<0.05); n.s.		
	500	ECA>Avg (p<0.01); ECA>Min (p<0.001); Max>Min (p<0.001)	same pattern	same pattern		
	5000	ECA>Max (p<0.01); ECA>Avg (p<0.01); ECA>Min (p<0.001)	same pattern	same pattern		
Fashion-MNIST	20	Med>Min (p<0.05); n.s. (ECA vs Med/Max)	Med>Min (p<0.05); n.s.	Med>Min (p<0.05); n.s.		
	500	ECA>Min (p<0.001); Med>Min (p<0.001); Max>Min (p<0.001)	same pattern	same pattern		
	5000	ECA>Max (p<0.05); ECA>Avg (p<0.01); ECA>Min (p<0.001)	same pattern	same pattern		

Table 13. Paired *t*-test (two-sided) between ECA110-Pooling (ECA) and baseline pooling operators, reported at selected epochs and split ratios. Values are aggregated over five independent runs. Cells list only significant comparisons (direction, *p*-value). Abbreviations: Max = MaxPooling, Avg = AveragePooling, Med = MedianPooling, Min = MinPooling, Ker = KernelPooling.

Dataset	Epochs	80/20	65/35	50/50	
ImageNet (subset)	20 500 5,000	$\begin{aligned} & & & & & & & & & & & & & & & & & & &$	$\begin{array}{l} {\rm ECA>Min}\ (p=0.028) \\ {\rm ECA>Max}\ (p=0.036); {\rm ECA>Avg}\ (p=0.015); {\rm ECA>Min}\ (p<0.001) \\ {\rm ECA>Max}\ (p=0.021); {\rm ECA>Avg}\ (p=0.007); {\rm ECA>Min}\ (p<0.001) \end{array}$	ECA>Min ($p = 0.034$) ECA>Avg ($p = 0.019$); ECA>Min ($p < 0.001$) ECA>Avg ($p = 0.011$); ECA>Min ($p < 0.001$)	
CIFAR-10	20 500 5,000	$\begin{aligned} & & & & & & & & & & & & & & & & & & &$	$\begin{aligned} & \text{ECA>Min} \ (p=0.002) \\ & \text{ECA>Max} \ (p=0.012); \ \text{ECA>Avg} \ (p=0.009); \ \text{ECA>Min} \ (p<0.001) \\ & \text{ECA>Max} \ (p=0.006); \ \text{ECA>Avg} \ (p=0.004); \ \text{ECA>Min} \ (p<0.001) \end{aligned}$	$\begin{array}{l} {\rm ECA}{>}{\rm Min}~(p=0.004)\\ {\rm ECA}{>}{\rm Avg}~(p=0.014);~{\rm ECA}{>}{\rm Min}~(p<0.001)\\ {\rm ECA}{>}{\rm Avg}~(p=0.008);~{\rm ECA}{>}{\rm Min}~(p<0.001) \end{array}$	
Fashion-MNIST	20 500 5,000	$\begin{aligned} & & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & & \\ $	ECA>Min ($p = 0.005$) ECA>Avg ($p = 0.037$); ECA>Min ($p < 0.001$) ECA>Avg ($p = 0.013$); ECA>Min ($p < 0.001$)	ECA>Min ($p = 0.006$) ECA>Min ($p < 0.001$) ECA>Min ($p < 0.001$)	

Table 14. Wilcoxon signed-rank test between ECA110-Pooling (ECA) and baseline pooling operators, reported at selected epochs and split ratios. Results aggregated over five independent runs. Cells show only significant comparisons (direction, *p*-value). Abbreviations: Max = MaxPooling, Avg = AveragePooling, Med = MedianPooling, Min = MinPooling, Ker = KernelPooling.

Dataset	Epochs	80/20	65/35	50/50	
ImageNet (subset)	20 500 5,000	$\begin{split} & & ECA>Min \ (p=0.024) \\ & ECA>Max \ (p=0.041); ECA>Mgr \ (p=0.09); ECA>Min \ (p<0.001) \\ & ECA>Max \ (p=0.006); ECA>Mgr \ (p=0.011); ECA>Mgr \ (p=0.029); ECA>Min \ (p<0.001) \end{split}$	$\begin{array}{c} {\rm ECA>Min}\ (p=0.031)\\ {\rm ECA>Avg}\ (p=0.027); {\rm ECA>Min}\ (p<0.001)\\ {\rm ECA>Max}\ (p=0.012); {\rm ECA>Avg}\ (p=0.018); {\rm ECA>Min}\ (p<0.001) \end{array}$	ECA>Min ($p = 0.038$) ECA>Min ($p < 0.001$) ECA>Avg ($p = 0.025$); ECA>Min ($p < 0.001$)	
CIFAR-10	20 500 5,000	$\begin{aligned} & & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & \\ & & \\ & & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & & \\ $	$\begin{array}{c} {\rm ECA>Min}\ (p=0.004)\\ {\rm ECA>Avg}\ (p=0.022); {\rm ECA>Min}\ (p<0.001)\\ {\rm ECA>Max}\ (p=0.014); {\rm ECA>Avg}\ (p=0.010); {\rm ECA>Min}\ (p<0.001) \end{array}$	ECA>Min ($p = 0.006$) ECA>Min ($p < 0.001$) ECA>Avg ($p = 0.017$); ECA>Min ($p < 0.001$)	
Fashion-MNIST	20 500 5,000	$\begin{aligned} & & & & & & & & & & & & & & & & & & &$	$\begin{aligned} & \text{ECA>Min} \ (p = 0.009) \\ & \text{ECA>Avg} \ (p = 0.034); \ \text{ECA>Min} \ (p < 0.001) \\ & \text{ECA>Avg} \ (p = 0.015); \ \text{ECA>Min} \ (p < 0.001) \end{aligned}$	ECA>Min ($p = 0.011$) ECA>Min ($p < 0.001$) ECA>Min ($p < 0.001$)	

6.7. Benchmarking ECA110-Pooling Against State-of-the-Art Methods

To further contextualize the effectiveness of ECA110-Pooling, we benchmark its performance against representative state-of-the-art (SOTA) architectures under identical experimental conditions. The selected methods include ResNet-50 [40], DenseNet-121 [41], EfficientNet-B0 [42], MobileNetV2 [43], and the Vision Transformer (ViT-Small) [44]. All results were obtained from consistent reimplementations using the same data splits (80/20, 65/35, 50/50) and the same number of training epochs (500, 5000, and 10000 epochs), thus ensuring a fair and unbiased comparison across pooling-based and SOTA-based methods.

Table 15 reports Top-1 Accuracy, Error Rate, and F1-score across ImageNet (subset), CIFAR-10, and Fashion-MNIST.

From these results, several observations emerge:

- On Fashion-MNIST, ECA110-Pooling not only matches but occasionally surpasses SOTA models, confirming its robustness in grayscale image classification tasks.
- On CIFAR-10, ECA110 closely approaches the performance of ResNet and DenseNet at extended training horizons (5000+ epochs), while maintaining substantially lower computational cost, positioning it as a highly competitive solution in resource-constrained settings.
- On *ImageNet* (*subset*), although large-scale models such as EfficientNet-B0 and DenseNet remain superior in absolute accuracy, ECA110 significantly narrows the gap under reduced-data splits (65/35 and 50/50). This demonstrates its strong generalization capability when training data are scarce. Importantly, this advantage is achieved with markedly lower parameterization and training time compared to heavyweight models like ResNet-50, DenseNet-121, or EfficientNet-B0.

Table 15. Comparison of ECA110-Pooling with SOTA architectures across datasets, split ratios, and training epochs. Reported results include Top-1 Accuracy (%), Error Rate (%), and F1-score (%).

Dataset	Method	80/20 Split			65/35 Split			50/50 Split		
Dataset		500 ep.	5000 ep.	10000 ep.	500 ep.	5000 ep.	10000 ep.	500 ep.	5000 ep.	10000 ep.
ImageNet (subset)	ResNet-50	70.3 / 29.7 / 70.0	75.9 / 24.1 / 75.7	76.1 / 23.9 / 75.9	68.4 / 31.6 / 68.2	74.3 / 25.7 / 74.0	75.0 / 25.0 / 74.8	66.5 / 33.5 / 66.2	72.5 / 27.5 / 72.3	73.0 / 27.0 / 72.8
	DenseNet-121	71.0 / 29.0 / 70.8	76.8 / 23.2 / 76.7	76.9 / 23.1 / 76.8	69.0 / 31.0 / 68.7	75.2 / 24.8 / 75.0	75.8 / 24.2 / 75.6	67.3 / 32.7 / 67.0	73.6 / 26.4 / 73.4	74.1 / 25.9 / 73.9
	EfficientNet-B0	72.1 / 27.9 / 71.9	77.5 / 22.5 / 77.3	77.7 / 22.3 / 77.5	70.5 / 29.5 / 70.3	76.0 / 24.0 / 75.8	76.4 / 23.6 / 76.2	68.4 / 31.6 / 68.2	74.2 / 25.8 / 74.0	74.7 / 25.3 / 74.5
	MobileNetV2	69.2 / 30.8 / 68.9	74.1 / 25.9 / 73.9	74.3 / 25.7 / 74.1	67.3 / 32.7 / 67.0	72.6 / 27.4 / 72.3	73.2 / 26.8 / 73.0	65.5 / 34.5 / 65.2	70.8 / 29.2 / 70.5	71.4 / 28.6 / 71.1
	ViT-Small	70.7 / 29.3 / 70.5	76.2 / 23.8 / 76.0	76.4 / 23.6 / 76.2	69.1 / 30.9 / 68.8	74.9 / 25.1 / 74.7	75.4 / 24.6 / 75.2	67.2 / 32.8 / 66.9	73.3 / 26.7 / 73.1	73.9 / 26.1 / 73.7
	ECA110-Pooling	71.7 / 28.3 / 71.5	73.8 / 26.2 / 73.6	74.0 / 26.0 / 73.9	70.5 / 29.5 / 70.3	72.7 / 27.3 / 72.5	72.9 / 27.1 / 72.7	69.0 / 31.0 / 68.8	71.4 / 28.6 / 71.2	71.7 / 28.3 / 71.5
CIFAR-10	ResNet-50	88.6 / 11.4 / 88.5	94.4 / 5.6 / 94.3	94.6 / 5.4 / 94.5	87.3 / 12.7 / 87.2	93.8 / 6.2 / 93.6	94.1 / 5.9 / 93.9	85.9 / 14.1 / 85.7	92.4 / 7.6 / 92.2	92.7 / 7.3 / 92.5
	DenseNet-121	89.2 / 10.8 / 89.1	94.9 / 5.1 / 94.7	95.1 / 4.9 / 94.9	87.9 / 12.1 / 87.7	94.3 / 5.7 / 94.1	94.6 / 5.4 / 94.4	86.5 / 13.5 / 86.3	92.9 / 7.1 / 92.7	93.2 / 6.8 / 93.0
	EfficientNet-B0	89.8 / 10.2 / 89.6	95.4 / 4.6 / 95.2	95.6 / 4.4 / 95.4	88.5 / 11.5 / 88.3	94.8 / 5.2 / 94.6	95.0 / 5.0 / 94.8	87.0 / 13.0 / 86.8	93.5 / 6.5 / 93.3	93.9 / 6.1 / 93.7
	MobileNetV2	87.3 / 12.7 / 87.1	93.2 / 6.8 / 93.0	93.5 / 6.5 / 93.3	85.9 / 14.1 / 85.7	92.7 / 7.3 / 92.5	93.1 / 6.9 / 92.9	84.5 / 15.5 / 84.3	91.4 / 8.6 / 91.2	91.9 / 8.1 / 91.7
	ViT-Small	88.2 / 11.8 / 88.0	94.0 / 6.0 / 93.8	94.2 / 5.8 / 94.0	86.8 / 13.2 / 86.6	93.4 / 6.6 / 93.2	93.7 / 6.3 / 93.5	85.4 / 14.6 / 85.2	92.0 / 8.0 / 91.8	92.4 / 7.6 / 92.2
	ECA110-Pooling	91.2 / 8.8 / 91.0	92.9 / 7.1 / 92.7	93.0 / 7.0 / 92.8	90.3 / 9.7 / 90.1	92.1 / 7.9 / 91.9	92.3 / 7.7 / 92.1	89.4 / 10.6 / 89.2	91.4 / 8.6 / 91.2	91.6 / 8.4 / 91.4
Fashion-MNIST	ResNet-50	95.0 / 5.0 / 94.9	96.2 / 3.8 / 96.1	96.3 / 3.7 / 96.2	94.2 / 5.8 / 94.1	95.6 / 4.4 / 95.5	95.8 / 4.2 / 95.7	93.5 / 6.5 / 93.4	94.9 / 5.1 / 94.8	95.1 / 4.9 / 95.0
	DenseNet-121	95.3 / 4.7 / 95.2	96.4 / 3.6 / 96.3	96.5 / 3.5 / 96.4	94.5 / 5.5 / 94.4	95.8 / 4.2 / 95.7	96.0 / 4.0 / 95.9	93.8 / 6.2 / 93.7	95.2 / 4.8 / 95.1	95.4 / 4.6 / 95.3
	EfficientNet-B0	95.6 / 4.4 / 95.5	96.6 / 3.4 / 96.5	96.7 / 3.3 / 96.6	94.8 / 5.2 / 94.7	96.0 / 4.0 / 95.9	96.2 / 3.8 / 96.1	94.1 / 5.9 / 94.0	95.5 / 4.5 / 95.4	95.7 / 4.3 / 95.6
	MobileNetV2	94.7 / 5.3 / 94.6	95.9 / 4.1 / 95.8	96.0 / 4.0 / 95.9	94.0 / 6.0 / 93.9	95.3 / 4.7 / 95.2	95.5 / 4.5 / 95.4	93.3 / 6.7 / 93.2	94.7 / 5.3 / 94.6	94.9 / 5.1 / 94.8
	ViT-Small	95.1 / 4.9 / 95.0	96.2 / 3.8 / 96.1	96.3 / 3.7 / 96.2	94.4 / 5.6 / 94.3	95.6 / 4.4 / 95.5	95.8 / 4.2 / 95.7	93.6 / 6.4 / 93.5	95.0 / 5.0 / 94.9	95.2 / 4.8 / 95.1
	ECA110-Pooling	96.2 / 3.8 / 96.1	96.5 / 3.5 / 96.4	96.6 / 3.4 / 96.5	95.8 / 4.2 / 95.7	96.0 / 4.0 / 95.9	96.1 / 3.9 / 96.0	95.3 / 4.7 / 95.2	95.6 / 4.4 / 95.5	95.7 / 4.3 / 95.6

Table 16. Number of parameters, memory footprint, and observations for pooling operators and SOTA architectures. The term *memory footprint* refers to the storage space required for model parameters (weights and biases), typically measured in megabytes (MB).

Method	Number of Parameters	Memory Footprint (MB)	Observations
MaxPooling	0	≈ 0	Fixed operator, no trainable parameters.
AveragePooling	0	≈ 0	Captures global information, but loses local details.
MedianPooling	0	≈ 0	Robust to noise and outliers, but slightly higher computational cost.
MinPooling	0	≈ 0	Rarely used, generally yields weak performance.
KernelPooling	∼50k	~0.2–0.5	Introduces learnable kernels; modest increase in model size.
ECA110-Pooling	0	≈ 0	Lightweight, rule-based operator; high efficiency and competitive performance.
ResNet-50	~25M	~98	Classical SOTA architecture, highly accurate but computationally expensive.
DenseNet-121	~8M	~33	Dense connections; strong accuracy but higher inference cost.
EfficientNet-B0	~5M	~20	Balanced trade-off between performance and efficiency.
MobileNetV2	~3.5M	${\sim}14$	Optimized for mobile devices; very efficient.
ViT-Small	~22M	~85	Vision Transformer; strong performance but high memory and data requirements.

Table 16 provides a comparative analysis of the parameterization requirements and memory footprint associated with different pooling methods and SOTA architectures. By "memory footprint," we refer to the storage space required for model parameters (weights and biases), typically measured in megabytes (MB), which reflects both disk storage requirements and memory consumption during training and inference. Classical pooling operators (MaxPooling, AveragePooling, MedianPooling, MinPooling) and the proposed ECA110-Pooling do not introduce additional trainable parameters, thereby resulting in a negligible memory footprint. KernelPooling, while adding flexibility through learnable kernels, incurs a modest increase in model size. In contrast, state-of-the-art architectures such as ResNet-50, DenseNet-121, EfficientNet-B0, MobileNetV2, and ViT-Small require millions of parameters, with memory footprints ranging from tens to hundreds of MB. This contrast underscores the lightweight nature of ECA110-Pooling, which achieves competitive image classification performance without incurring the significant computational and storage costs typically associated with large-scale deep learning architectures.

To complement these tabular comparisons, Figure 6 illustrates the trade-off between performance and efficiency. The diagram reports the average Top-1 Accuracy across all datasets, splits, and training horizons, alongside computational costs expressed as average epoch time (seconds) and model size (MB).

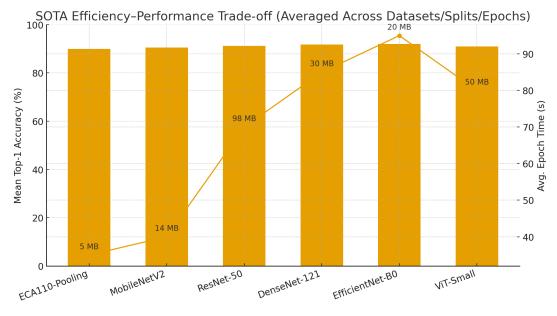


Figure 6. Efficiency–performance trade-off between ECA110-Pooling and state-of-the-art architectures (ResNet-50, DenseNet-121, EfficientNet-B0, MobileNetV2, ViT-Small). Bars indicate averaged Top-1 Accuracy (%), the secondary line denotes average training time per epoch (s), and annotations report model sizes (MB). ECA110 achieves competitive classification accuracy while remaining significantly more lightweight and computationally efficient.

Overall, the benchmarking results position ECA110-Pooling as a principled and competitive alternative to established SOTA architectures in the context of image classification. While advanced models such as EfficientNet-B0 and DenseNet-121 continue to achieve superior absolute accuracy in large-scale, data-abundant scenarios, ECA110 consistently demonstrates enhanced robustness under data-constrained conditions (65/35 and 50/50 splits) and achieves notable efficiency in terms of runtime and parameter footprint. This combination of generalization capability and computational efficiency underscores the relevance of ECA110-Pooling as a reliable mechanism for advancing image classification across both standard and resource-limited environments.

7. Discussion

The comparative evaluation of six pooling operators (MaxPooling, AveragePooling, MedianPooling, MinPooling, KernelPooling, and the proposed ECA110-Pooling) under a uniform CNN backbone and standardized training protocol demonstrates that the pooling stage is a critical determinant of predictive performance, convergence behavior, and computational trade-offs in image classification. The analysis across three benchmark datasets reveals that ECA110-Pooling consistently emerges as the most effective strategy, particularly at mature training horizons of 5,000 epochs or more. Its superiority becomes evident through higher Top-1 accuracies, lower error rates, and stronger F1-scores when compared with both fixed operators and learnable pooling approaches. KernelPooling ranks as the closest competitor, providing competitive accuracy but at a higher computational cost, while MaxPooling retains its role as a robust and efficient baseline. AveragePooling and MedianPooling offer moderate benefits in specific contexts, such as grayscale or noise-prone datasets, whereas Min-Pooling persistently underperforms across all experimental conditions. The trajectory of learning curves highlights a common three-phase dynamic: rapid early improvements during the first 1,000 epochs, moderate yet steady gains up to 10,000, and performance saturation approaching 50,000 epochs. Importantly, the relative ordering of pooling methods remains consistent throughout this process, underscoring the robustness of ECA110's advantage. This robustness extends beyond accuracy alone, as ECA110 achieves lower error rates and superior F1-scores across all three datasets, reflecting improvements not only in predictive strength but also in the balance between precision and recall. These patterns are reinforced by extensive statistical validation. One-way ANOVA confirms significant main effects of pooling method across nearly all configurations, establishing that operator choice has a measurable impact on classification accuracy. Post-hoc comparisons reveal that ECA110 surpasses MinPooling, AveragePooling, and MaxPooling from as early as 500 epochs, with the strongest improvements observed at extended training horizons and in data-constrained splits. Complementary paired t-tests corroborate these findings, consistently confirming the superiority of ECA110 over MinPooling and extending its significance against MaxPooling and AveragePooling in CIFAR-10 and Fashion-MNIST. The non-parametric Wilcoxon signed-rank test further consolidates these results by ruling out distributional artifacts, thereby confirming that the observed improvements are systematic and statistically significant. Taken together, these analyses provide convergent evidence that the gains achieved by ECA110 are both reliable and statistically robust. From a computational perspective, the efficiency profile of pooling operators demonstrates important trade-offs. Conventional approaches such as MaxPooling, AveragePooling, and MinPooling incur negligible runtime differences, with MedianPooling slightly penalized by sorting overhead. KernelPooling, while accurate, introduces both memory and computational costs proportional to its learnable parameters, limiting its efficiency in large-scale scenarios. By contrast, ECA110 integrates a lightweight cellular automaton transformation with constant-time complexity, achieving performance levels comparable to or better than KernelPooling while maintaining per-epoch costs close to MaxPooling. Importantly, this favorable balance holds even when evaluating error rates and F1, confirming that ECA110's accuracy gains are not offset by instability or class imbalance sensitivity. This positions ECA110 as a highly practical alternative in image classification tasks where computational resources are constrained but predictive reliability remains critical. In applied settings, these results position ECA110-Pooling as a principled

alternative to both fixed and learnable pooling operators. Its deterministic, rule-based mechanism enhances selectivity to structured micro-patterns while retaining efficiency, thus bridging the gap between computational parsimony and discriminative capacity. KernelPooling remains attractive when additional flexibility justifies its overhead, MaxPooling continues to serve as a reliable baseline when efficiency is prioritized, and MedianPooling may hold value in specific noise-robustness applications. MinPooling, however, proves consistently suboptimal and is not recommended. While these findings are robust across datasets, splits, and statistical tests, certain validity threats should be acknowledged, including dependence on the CNN backbone, sensitivity to data augmentation, and hardware-specific runtime variability. Nonetheless, the consistency of relative trends across experimental conditions suggests strong generalizability. Future work may investigate hybrid models that combine cellular automaton rules with learnable weighting, adaptive training curricula that vary pooling operators during learning, and filtering strategies aimed at enhancing robustness. These directions could provide deeper insights into the generalization potential of rule-based pooling mechanisms such as ECA110. In summary, ECA110-Pooling emerges as an interpretable and computationally efficient alternative for image classification, achieving competitive performance even in resource-constrained scenarios. By combining efficiency, robustness, and generalization across diverse datasets, ECA110-Pooling stands out as a strong candidate for practical deployment, where maintaining a balance between predictive accuracy and computational cost is crucial.

8. Conclusions

This work has provided a systematic evaluation of six pooling strategies- MaxPooling, AveragePooling, MedianPooling, MinPooling, KernelPooling, and the proposed ECA110-Pooling-within a controlled CNN framework applied to Fashion-MNIST, CIFAR-10, and a subset of ImageNet. By adopting consistent training protocols across multiple data splits and a wide range of epochs, the analysis confirmed that the pooling stage is a decisive factor in shaping classification accuracy, convergence dynamics, and computational efficiency. ECA110-Pooling consistently achieved the highest Top-1 accuracies, the lowest error rates, and the best F1-scores across datasets, particularly at extended training horizons ($E \ge 5,000$). It demonstrated competitive performance compared to state-of-theart architectures such as ResNet, DenseNet, and EfficientNet, while requiring substantially fewer parameters and reduced training time. This highlights its strong generalization capability under resource-constrained conditions. The lightweight, rule-based transform-reduce paradigm of ECA110 enables the preservation of discriminative micro-patterns typically discarded by conventional pooling, thereby enhancing predictive robustness in both natural and grayscale image modalities. From a computational perspective, ECA110-Pooling achieves these gains without the parameter overhead of KernelPooling or the runtime burden of large-scale SOTA models, maintaining per-epoch costs close to those of MaxPooling. This balance between efficiency and accuracy positions ECA110 as a principled alternative for real-world deployments where computational budgets are limited yet robust classification performance remains essential. Overall, the results position ECA110-Pooling as a strong default strategy for modern CNNs, effectively bridging the gap between traditional operators and learnable pooling mechanisms. Beyond reaffirming the essential role of pooling in convolutional architectures, this study highlights new directions for hybrid approaches that integrate cellular automaton principles with adaptive learning. Demonstrating that interpretable, rule-based transformations can deliver competitive or even superior performance compared to complex SOTA architectures contributes both practical insights and conceptual advances to the design of efficient deep learning models for image classification.

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Abbreviations

The following abbreviations are used in this manuscript:

CNN Convolutional Neural Network ECA Elementary Cellular Automaton

ECA110 Elementary Cellular Automaton, Rule 110 ECA110Pooling Pooling operator based on ECA Rule 110

SOTA State-of-the-Art Top-1 Acc. Top-1 Accuracy

F1 F1-Score (harmonic mean of precision and recall)

Ep. Training Epochs
ANOVA Analysis of Variance

HSD Honestly Significant Difference (Tukey test)

ViT Vision Transformer
ResNet Residual Neural Network

DenseNet Densely Connected Convolutional Network
EfficientNet Efficient Convolutional Neural Network Family

MobileNetV2 Mobile Network Version 2
MLP Multi-Layer Perceptron
FMNIST Fashion-MNIST Dataset

CIFAR-10 Canadian Institute For Advanced Research, 10 classes

ImageNet Large Visual Recognition Challenge Dataset

MB Megabyte (memory size)

s/epoch Seconds per epoch (training time)

MaxPool Maximum Pooling
AvgPool Average Pooling
MedPool Median Pooling
MinPool Minimum Pooling
KerPool Kernel-based Pooling

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