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Article

Polyhedral Embeddings of Triangular Regular Maps of Genus g, $2 \leqslant g \leqslant 14$, and Neighborly Spatial Polyhedra

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† Dedicated to Prof. Dr. Dr. (h.c.) Jörg M. Wills, University Siegen, on the occasion of his 88th birthday, in recognition of his long commitment to polyhedral embeddings of regular maps.

Abstract: This article provides a survey about polyhedral embeddings of triangular regular maps of genus g, $2 \leqslant g \leqslant 14$, and of neighborly spatial polyhedra. An old conjecture of Grünbaum from 1967, although disproved in 2000, lies behind this investigation. We discuss all duals of these polyhedra as well, whereby we accept, e.g., the Szilassi torus with its non-convex faces to be a dual of the Möbius torus. A numerical optimization approach of the second author for finding such embeddings, was first applied to finding (unsuccessfully) a dual polyhedron of one of the 59 closed oriented surfaces with the complete graph of 12 vertices as their edge graph. The same method has been successfully applied for finding polyhedral embeddings of triangular regular maps of genus g, $2 \leqslant g \leqslant 14$. The effectiveness of the new method has led to ten additional new polyhedral embeddings of triangular regular maps and their duals. There does exist symmetrical polyhedral embeddings of all triangular regular maps with genus g, $2 \leqslant g \leqslant 14$, except a single undecided case of genus 13.

Keywords: Neighborly polyhedron, regular map; computational synthetic geometry

MSC: 52B70

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1. Introduction

This article provides a contribution to the field of Computational Synthetic Geometry, which explores methods for realizing abstract geometric objects in concrete vector spaces, [1]. Our focus lies in the construction and analysis of triangular regular maps of genus g, $2 \leqslant g \leqslant 14$ and of neighborly spatial polyhedra and of all these embeddings. These spatial polyhedra are all related to an old conjecture of Branko Grünbaum from 1967 [2], page 253, who conjectured that all oriented simplicial 2-manifolds have polyhedral embeddings in R^3. This conjecture was not believed by many experts, however, a proof of a counter example was given only first in 2000, [3], and additional counter-examples provided by Lars Schewe in his Ph.D. thesis, see his publication in 2010 [4]. When we study convex polytopes with their convex faces, we have the polar dual polytopes with convex faces again. When we have polyhedra with higher genus there are polyhedra for which we have in the dual face lattice embeddings with non-convex faces. A famous example for such an embedding is Szilassi's polyhedron with a dual face lattice compared with the 7 vertex torus of Möbius. We do allow non-convex faces when we use duality in this paper.

The investigation of the second author to study the dual cases, of what has been investigated by Bokowski, Guedes de Oliveira, and Schewe, could not find questionable embeddings in this area. The use of his numerical optimization approach has led to all but one embeddings of triangular regular maps of genus g, $2 \leqslant g \leqslant 14$. We recall that a regular map generalizes in a topological manner what we know from Platonic solids. A regular map is a decomposition of a two-dimensional manifold into topological disks such that every flag (an incident vertex-edge-face triple) can be transformed into any other flag by a combinatorial symmetry of the decomposition. We use the result of M. Conder [5] and his notation for triangular oriented regular maps of genus g, $2 \leqslant g \leqslant 14$ with multiplicity 1. These 14

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regular maps are listed in Table 1. For only five of them polyhedral embeddings were known. For the remaining cases, we have now eight new polyhedral embeddings of regular maps and for two former polyhedral embeddings, the symmetry properties have been improved. This has even led to detecting a fault in a previous paper, [6].

In the next two sections we provide tables with the list of triangular regular maps and neighborly polyhedra that were tested according to Grünbaum's conjecture.

Afterwards we describe the algorithm of the second author.

2. Polyhedral Embeddings of Triangulated Orientable Regular Maps with Genus g, $2 \leqslant g \leqslant 14$ and Some of Their Duals

We have listed in Table 1 all polyhedral embeddings of triangulated orientable regular maps with genus g, $2 \leqslant g \leqslant 14$ and some of their duals. All triangular polyhedral embeddings in Table 1 are new for genus g, $g \geqslant 8$. Geometric symmetries are listed with Schoenflies Notation. The embeddings of genus 6 and 7 have higher geometrical symmetries compared to those that were known before. A former embedding of R6.1 of Brehm has never been published. It had no geometric symmetries according to a private communication of J.M.Wills.

Table 1. Triangular regular maps of genus g, $2 \leqslant g \leqslant 14$ and their duals with polyhedral embeddings. Embeddings marked with a * were previously known, all others are new.

Conder Notation	Genus	Schläfli Type	f_0	f_1	f_2	Map Author	Comb. Sym.	Embedding Symmetries	Dual Embedding	Fig.
R3.1	3	$\{3,7\}_8$	24	84	56	Klein	336 PSL(2,7)×C_2	T*	T*	2
R3.2	3	$\{3,8\}_6$	12	48	32	Dyck	192	D3*, S2		5
R5.1	5	$\{3,8\}_{12}$	24	96	64	Fricke, Klein	384	O*, S2	D2, C3	10
R6.1	6	$\{3,10\}_6$	15	75	50	Coxeter, Moser	300	C3, C2, C1*		12
R7.1	7	$\{3,7\}_{18}$	72	252	168	Hurwitz, Macbeath	1008 PSL(2,8)×C_2	C3, C2, S2, C1*	C3, C2	13
R8.1	8	$\{3,8\}_8$	42	168	112		672 PSL(3,2)⋊C_2	D2, C4, C3, S2		19
R8.2	8	$\{3,8\}_{14}$	42	168	112		672 PSL(3,2)⋊C_2	D2, C4, C3		20
R10.1	10	$\{3,9\}_{12}$	36	162	108		648	D2		21
R10.2	10	$\{3,12\}_6$	18	108	72		432	C2		22
R13.1	13	$\{3,10\}_{30}$	36	180	120		720 $A_5 \times S_3$	C3, C2		24
R13.2	13	$\{3,12\}_{12}$	24	144	96		576			
R14.1	14	$\{3,7\}_{12}$	156	546	364		2184 PSL(2,13)	D2		25
R14.2	14	$\{3,7\}_{26}$	156	546	364		2184 PSL(2,13)	C2		26
R14.3	14	$\{3,7\}_{14}$	156	546	364		2184 PSL(2,13)	D2		27

3. Polyhedral Embeddings of Neighborly Spatial Polyhedra with Complete Graphs as Their Edge Graph and Their Duals

In Table 2 we have listed neighborly spatial polyhedra according to complete graph embeddings on 2-manifolds and pseudo-manifolds. For complete graphs with 9 and 10 vertices we have polyhedra with pseudo-manifolds as their boundaries: A vertex can have more than one circular sequence

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of incident faces. For the seven vertex torus of Möbius you can download a video with all four embeddings at http://science-to-touch.com/ForJB/MoebiusTorus.mov.

Table 2. Neighborly spatial polyhedra according to complete graph embeddings on 2-manifolds and pseudo-manifolds.

Graph	Genus	f_0	f_1	f_2	Number of embeddings	Combinatorial polyhedra, articles	Geometrical embeddings, articles
K_4	0	4	6	4	1		
K_7	1	7	21	14	4	[7]	[8], [9]
dual	1	14	21	7	1		[10], [11]
K_9	–	9	36	24	16	[12]	[12]
K_{10}	–	10	45	30	4	[13]	[13]
K_{12}	6	12	66	44	none	[14]	[3] , [4]
K_{15}	11	15	105	70	unknown	[15]	

4. Polyhedral Embeddings as an Optimization Problem

Finding a flat embedding of a simplicial complex without intersections is, in general, a difficult problem. While solutions can be easily verified in polynomial time, there are no efficient algorithms to generate them or prove their non-existence without a full search of the space. For small vertex counts, there has been some success using SAT solvers with oriented matroids [4] however, this can take significant computational resources and becomes intractable for larger complexes.

There are methods that are more efficient at solving NP problems of this sort if we introduce heuristics and non-determinism to our search. As a consequence, we will not be able to guarantee that a solution will be found for a given complex, and questions about which symmetries can be realized will remain open. This compromise is acceptable as we are looking for any embeddable examples where none exist currently.

We choose to focus on triangular regular maps specifically. This is because a polyhedron with all triangle faces can be completely defined by its set of 3-dimensional vertices and triangulation with no additional constraints. This greatly simplifies the optimization. By contrast, faces with polygons of more than 3 vertices need additional constraints to find embeddings since the points may be skew and not all lie on a common plane.

Duals of triangular maps are also equally efficient since these polyhedra can be entirely described by a set of planes and vertex adjacency list. Since the edge graph is cubic, the vertices of the dual are simply the intersection of the 3 planes that share the vertex.

Enforcing a geometric symmetry helps reduce the search space, speed up the computation, and in general seems to produce the best results when a suitable symmetry is used. All embeddings found so far have had some non-trivial geometric symmetry, and there have been no cases where an asymmetric solution has had fewer intersections than a symmetric one for a non-embeddable example. Therefore, we conjecture that all embeddable regular maps can be embedded with at least 1 non-trivial geometric symmetry.

Possible geometric symmetries can be inferred from the automorphisms of the regular map. Examples:

Permutation Group	Possible Geometric Symmetries
(a,b)(c,d)(e,f)	S2, C2, Cs
(a,b)(c,d)(e)(f)	C2, Cs
(a,b,c)(d,e,f)	C3
(a,b,c,d)(e,f,g,h)	C4, D2
(a,b,c,d)(e,f)	D2
(a,b,c,d)(e)(f)	C4

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To enforce a geometric symmetry, the first point of a permutation becomes the reference and the other points in the permutation get defined relative to the first one by the given symmetry type.

The solver works in 2 stages; First a large random search is conducted to find low-intersection candidates, then a second stage is used to refine each solution to both reduce the intersections if they are non-zero, and improve the aesthetics of the shape to eliminate things like large scale differences between edges, near-intersections, or very skinny polygons. This stage can also truncate the vertex positions to produce small integer coordinates.

The heuristic used for the large search is simply the number of edge-polygon intersections plus the number of self-crossings of each polygon as illustrated in Figure 1. For triangular maps, self-crossings are always zero, but duals may have crossings.

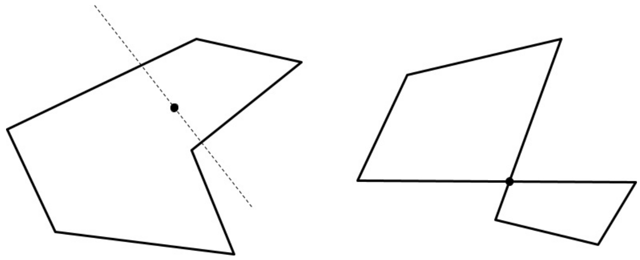

Figure 1. Left: An edge-polygon intersection. Right: A polygon with a self-crossing.

The second stage heuristic includes the penalty from the first stage plus one minus the minimum of the following metrics:

$$\begin{aligned}
&\text{Length Quality:} && \frac{\text{minimum side length}}{\text{maximum side length}} \text{ for each polygon} \\
&\text{Distance Quality:} && \frac{\text{closest distance between non-neighboring edges}}{\text{farthest distance between 2 points}} \\
&\text{Angle Quality:} && 1 - \cos(\text{smallest angle in any polygon}) \\
&\text{Plane Quality:} && 1 - \cos(\text{smallest angle between neighboring faces})
\end{aligned}$$

The general algorithm is listed in Algorithm 1

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Algorithm 1 Primary search for embeddings

Input: List of index triplets representing the triangles T

Procedure SEARCHEMBEDDINGS(T, $iters$, $clusters$, σ=1.0, β=0.997, γ=1.25)
for i in $clusters$ **do**
  $V_i \leftarrow$ APPLYSYMMETRY(RANDOMVERTICES())
  $p_i \leftarrow$ PENALTY(T, V_i)
end for
for j in $iters$ **do**
  $m \leftarrow$ ARGMINIMUM(p)
  for i in $clusters$ **do**
    if $p_i > p_m * \gamma$ **then**
      $r =$ RANDOMINDEX()
      $V_i \leftarrow V_r$
      $p_i \leftarrow p_r$
    end if
    $V_{new} \leftarrow \beta * (V_i +$ RANDOMNOISE(σ))
    $V_{new} \leftarrow$ APPLYSYMMETRY(V_{new})
    $p_{new} \leftarrow$ PENALTY(T, V_{new})
    if $p_{new} \leqslant p_i$ **or** $(i \neq m$ **and** $p_{new} \leqslant p_m * \gamma)$ **then**
      $V_i \leftarrow V_{new}$
      $p_i \leftarrow p_{new}$
    end if
  end for
end for
return V_m
End Procedure

Depending on the complexity of the problem, the optimizer can usually find solutions within only seconds or minutes for the smallest examples such as R3.2, and about a day for the largest ones like R14.2 on a standard desktop computer. Again for dual problems, the algorithm is nearly identical, but each point represents a plane instead, which has the same degrees of freedom. These are generally slower and harder to find since the placement of the planes is more sensitive than the vertex positions.

For any undecided cases, best results are Kepler–Poinsot-like with low intersection count. R13.2 has been realized with as few as 64 edge intersections with D2 symmetry.

5. Polyhedral Embeddings According to Table 1

5.1. Case R3.1

This regular map R3.1 is also called Felix Klein's quartic of Schläfli type $\{3,7\}_8$ and genus 3. It is the first element of the infinite sequence of Hurwitz of type $\{3,7\}$, [16]. Its abstract symmetry group has order 336. References are [17,18].

It is an example of a regular Leonardo polyhedron, i.e., it is a polyhedral embedding of a regular map with a geometrical symmetry group of the rotational symmetry group of a Platonic solid. This polyhedral embedding is due to Schulte and Wills [19]. For additional articles about the six regular Leonardo polyhedra, see [20–25].

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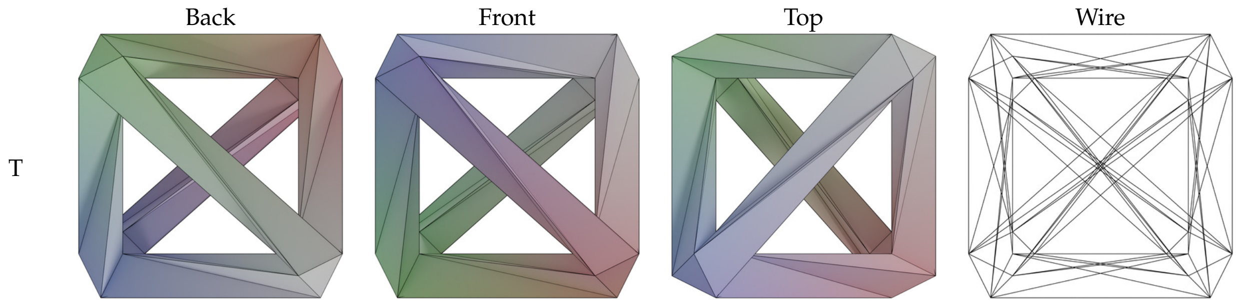

Figure 2. R3.1 with chiral tetrahedral symmetry

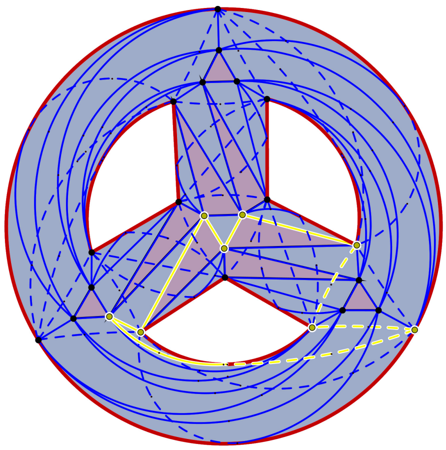

Figure 3. The combinatorial input for a polyhedron of Felix Klein's quartic of type $\{3,7\}_8$. Here we see an example of a Petrie polygon.

5.2. The Dual Case R3.1'

An embedding of the dual polyhedron of R3.1 is due to D. McCooey, [26].

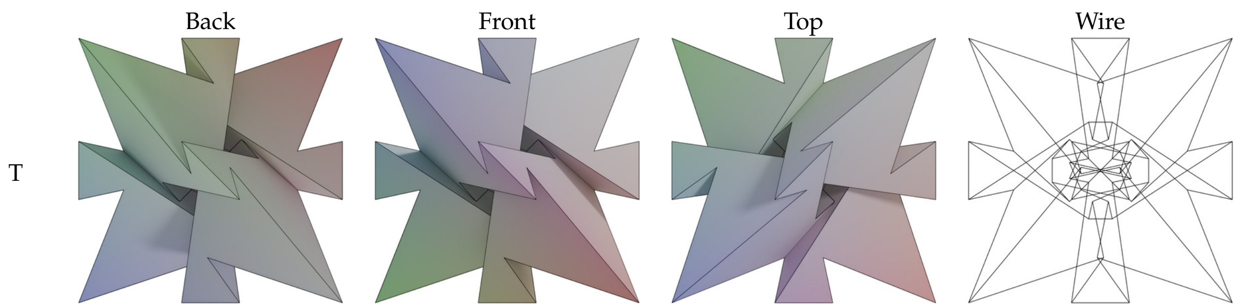

Figure 4. R3.1' with chiral tetrahedral symmetry

5.3. Case R3.2

The regular map of R3.2 is due to W. Dyck, [27] [28] of Schläfli type $\{3,8\}_6$ and genus 3. The combinatorial symmetry group has order 192.

A first embedding was found by J. Bokowski, [29]. An embedding with better geometrical symmetries, with the dihedral group D3, is due to U. Brehm, [30]. An additional embedding symmetry S2 and an alternative D3 embedding are new and due to the second author.

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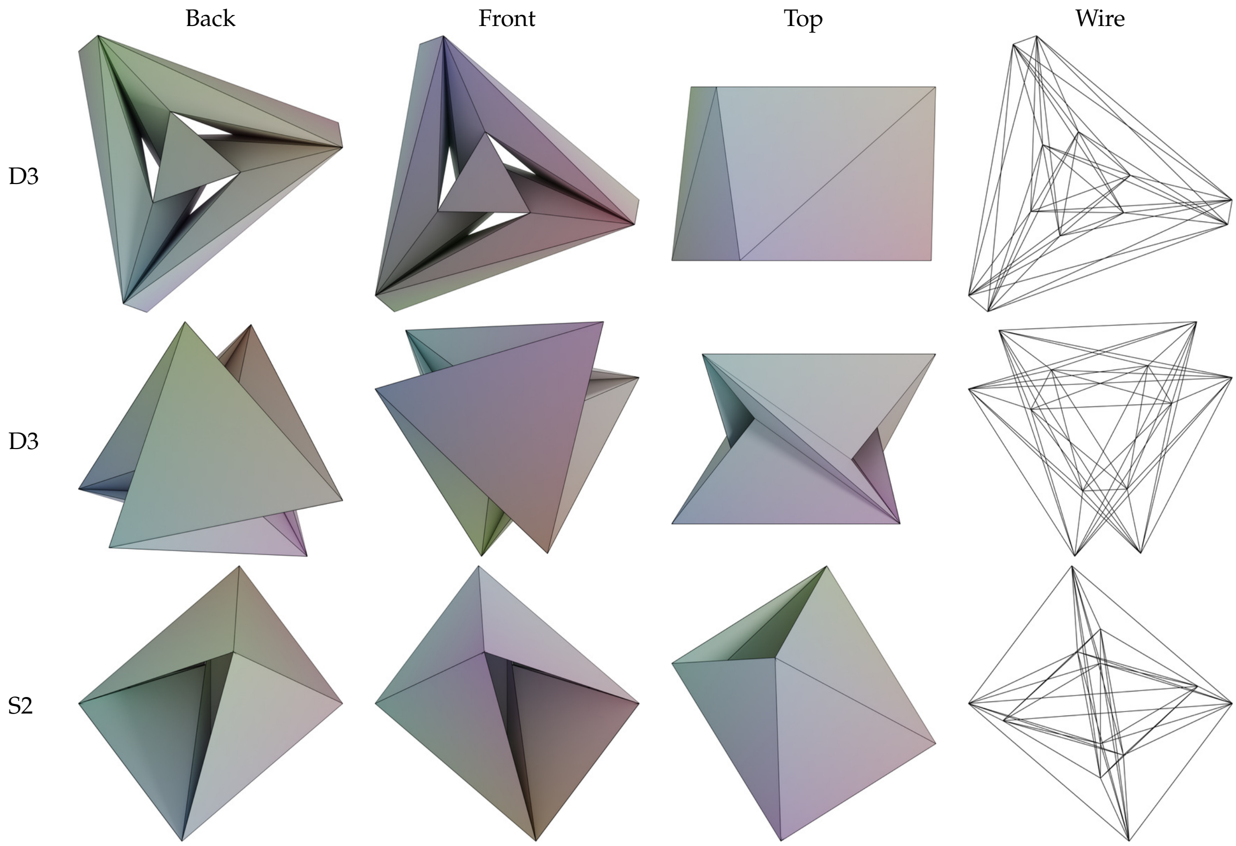

Figure 5. R3.2 with D3 and S2 symmetry

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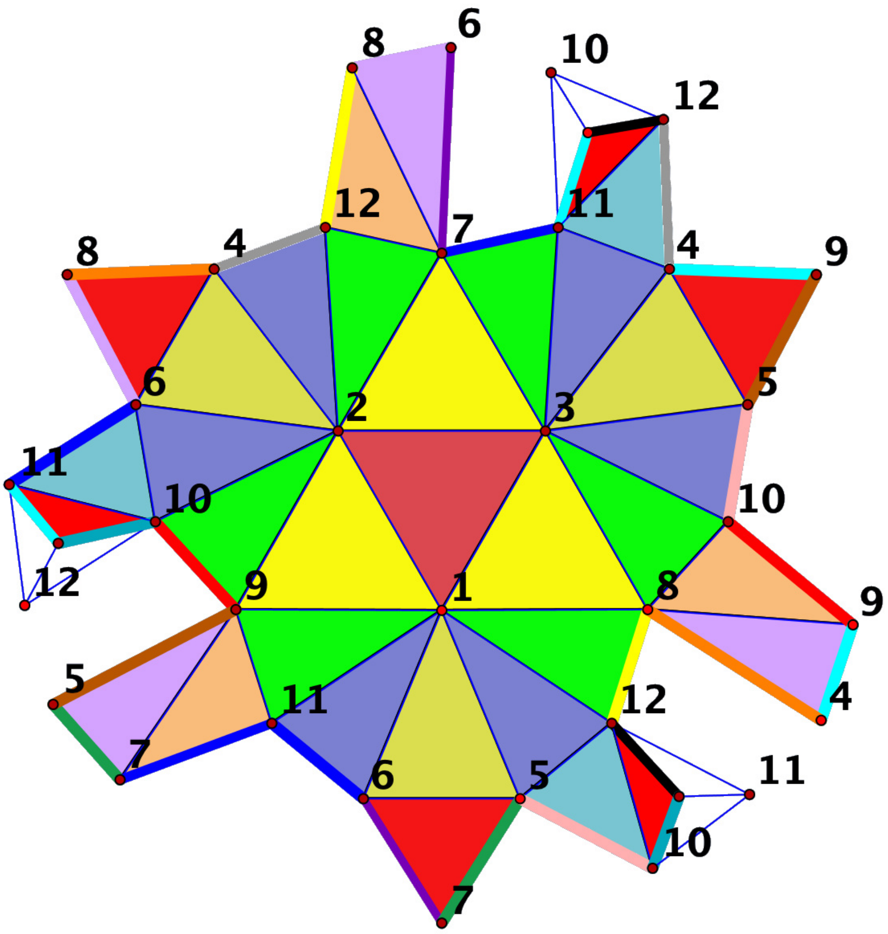

Figure 6. The triangles of Dyck's regular map R3.2 of type $\{3,8\}_6$ shown with a cyclic symmetry of order 3.

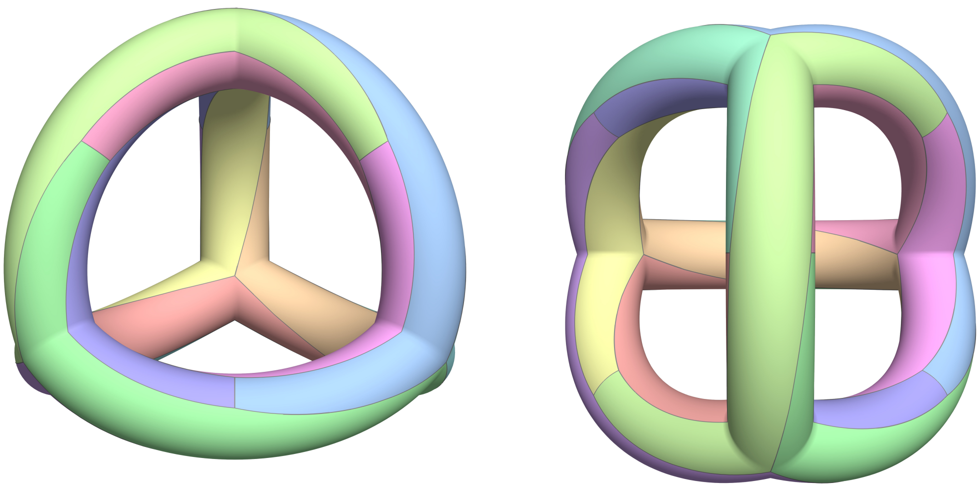

Figure 7. Two pictures of Jarke van Wijk's video with topological embeddings of regular maps, [31], [32]. Here we see Dyck's regular map R3.2′ of type $\{8,3\}_6$.

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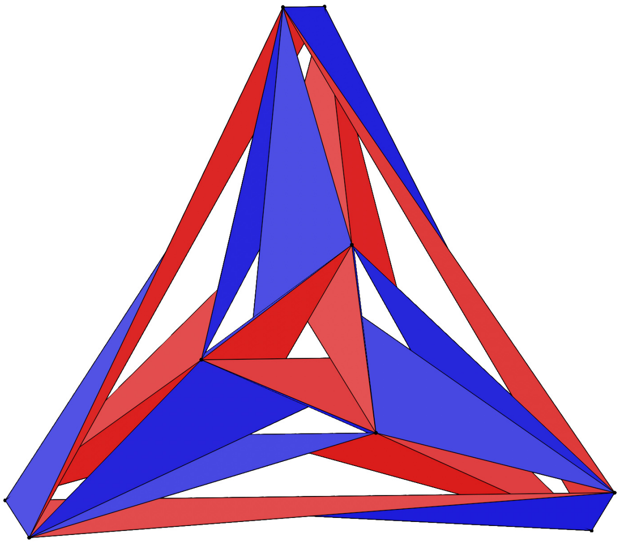

Figure 8. Here we see Brehm's polyhedral embedding of Dyck's regular map R3.2 of type $\{3.8\}_6$. The polyhedron is complete when the red parts cannot be seen. In the front and back parts, eight blue triangles have to be reinserted. The red triangles are the inner sides of the polyhedron. The polyhedron has a geometrical dihedral symmetry D3 of order 6.

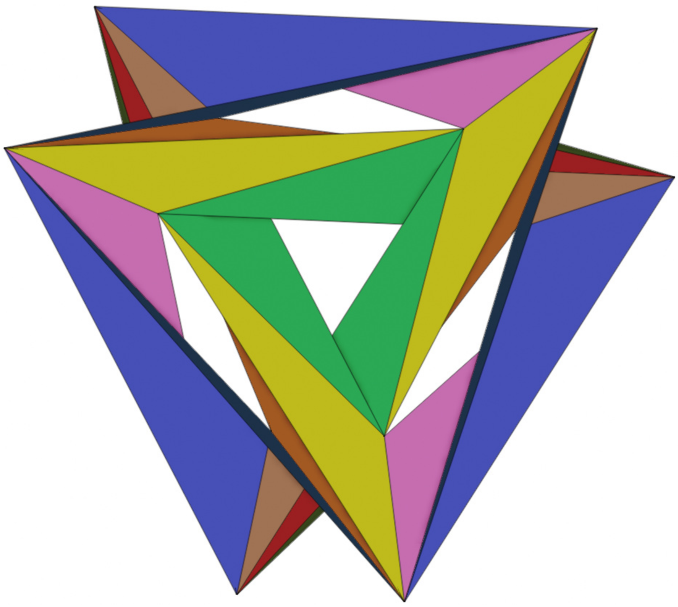

Figure 9. This embedding of Dyck's regular map R3.2 of type $\{3,8\}_6$ has the geometrical dihedral symmetry D3 like Behm's embedding. However, it is different. For a front triangle and for a triangle of the rear part we see only their boundaries, so that we can imagine the shape of this polyhedron.

5.4. Case R5.1

R5.1, due to Klein and Fricke [33] of type $\{3,8\}_{12}$ and genus 5, symmetry group of order 384.

The first embedding was found by B. Grünbaum, [34]. This is another example of the six regular Leonardo polyhedra. Later U. Brehm and J. M. Wills independently discovered this embedding again.

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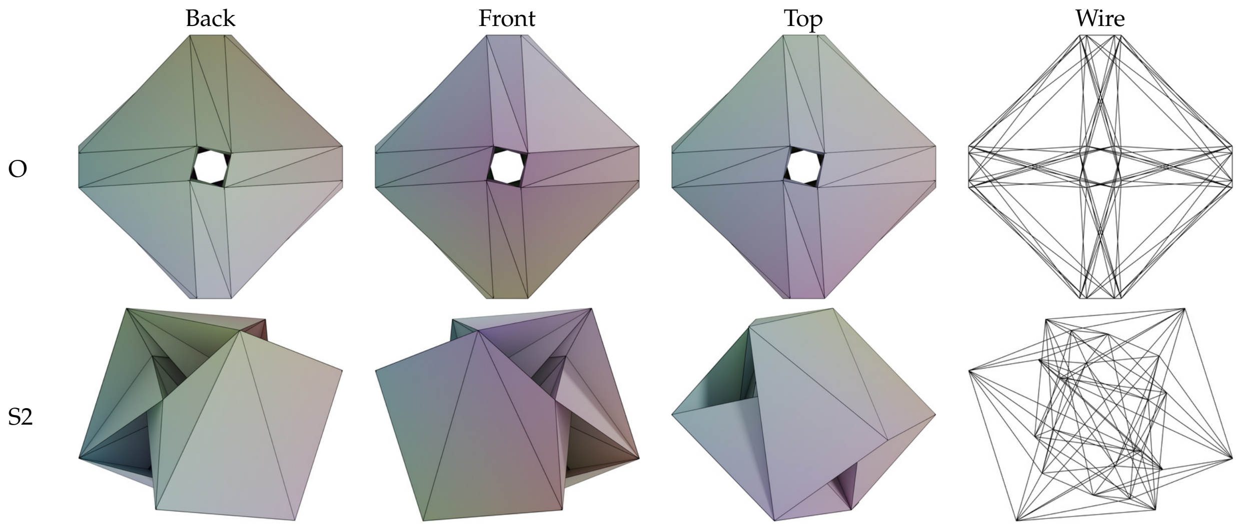

Figure 10. R5.1 with chiral octahedral and S2 symmetry

5.5. The Dual Case R5.1′

The embedding of the dual is a new result of the second author in this paper.

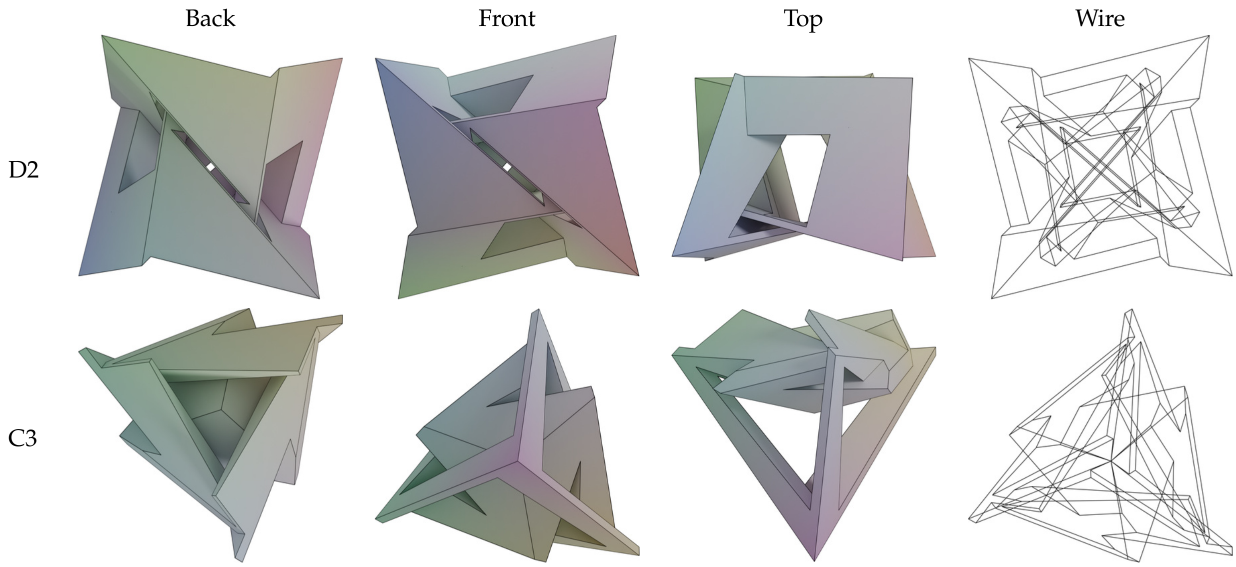

Figure 11. R5.1′ with D2 and C3 symmetry

5.6. Case R6.1

R6.1, due to Coxeter and Moser of type $\{3,10\}_6$ and genus 6, symmetry group of order 300.

According to a private communication by J. M. Wills: A former polyhedral embedding of this regular map without any symmetry was found by U. Brehm. However, It was never published.

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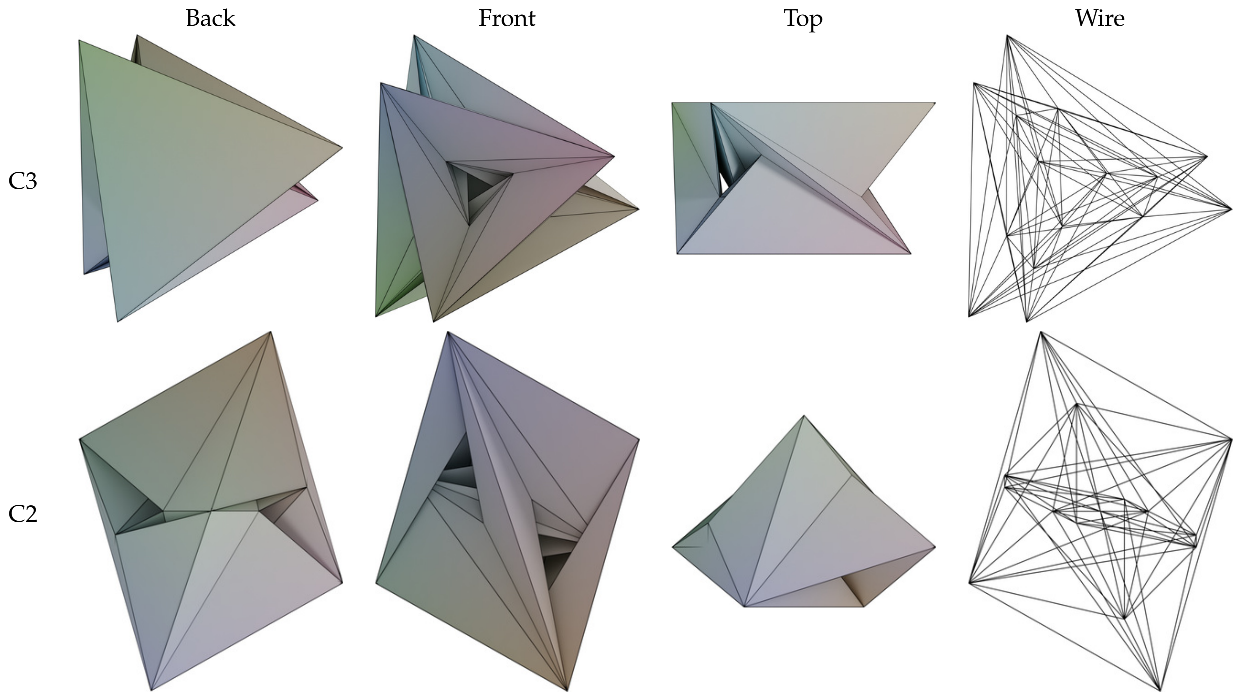

Figure 12. R6.1 with C3 and C2 symmetries

5.7. Case R7.1

R7.1, due to Hurwitz of type $\{3,7\}_{18}$ and genus 7, the second element of the infinite Hurwitz sequence with types $\{3,7\}$, also denoted as Macbeath surface as she rediscovered it, symmetry group of order 1008. A first embedding without geometrical symmetries was found in 2018, [35]. The symmetry of this new embedding has order 3, although we find in [6] the claim that such a symmetry is not possible. In other words, this example tells us that a fault in that paper has been detected.

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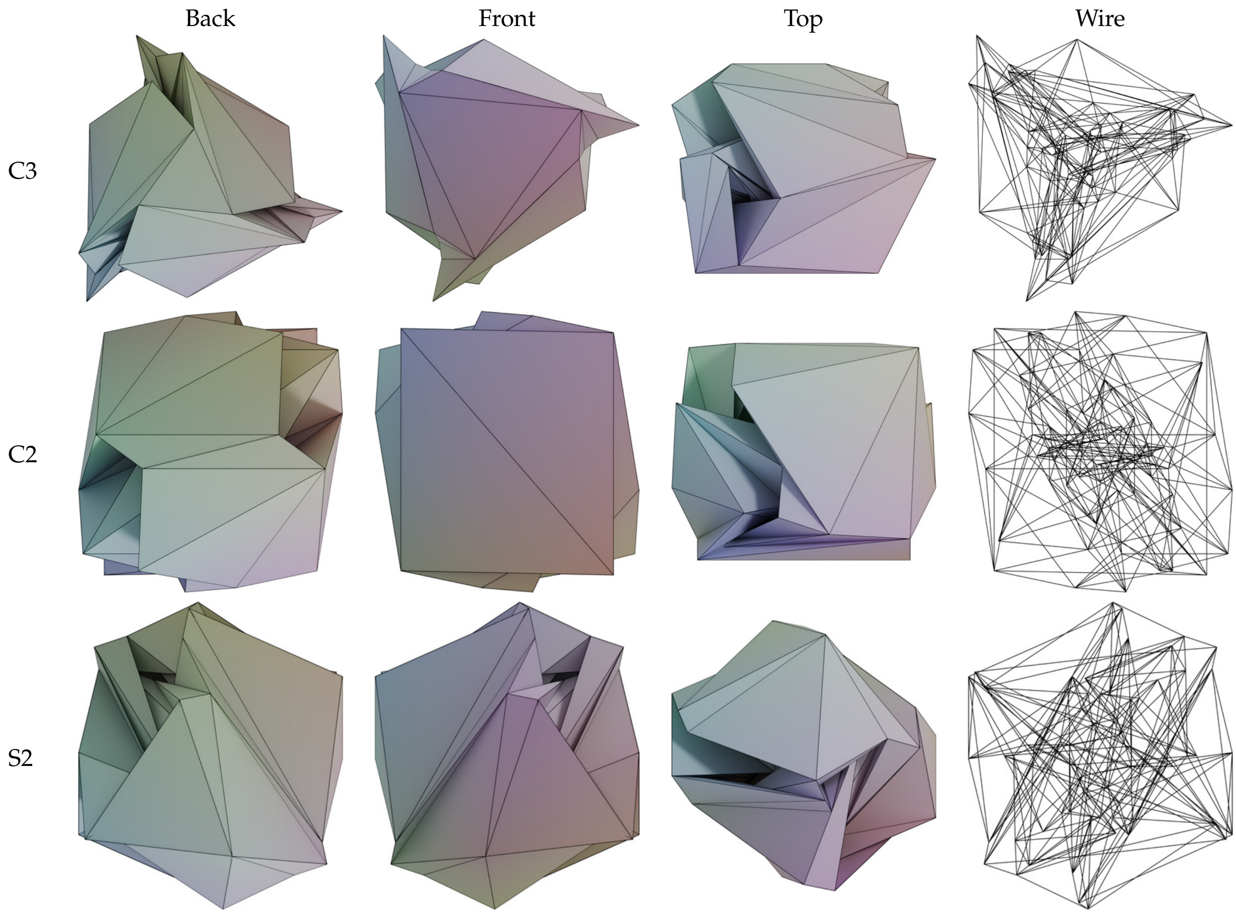

Figure 13. R7.1 with C3, C2, and S2 symmetry

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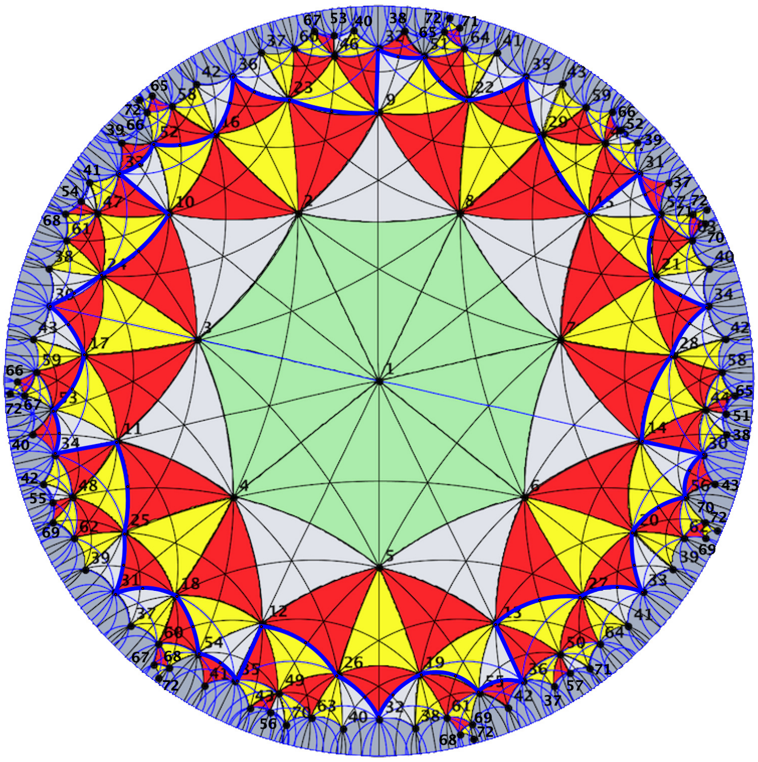

Figure 14. We have 72 vertices, 252 edges, and 168 triangles

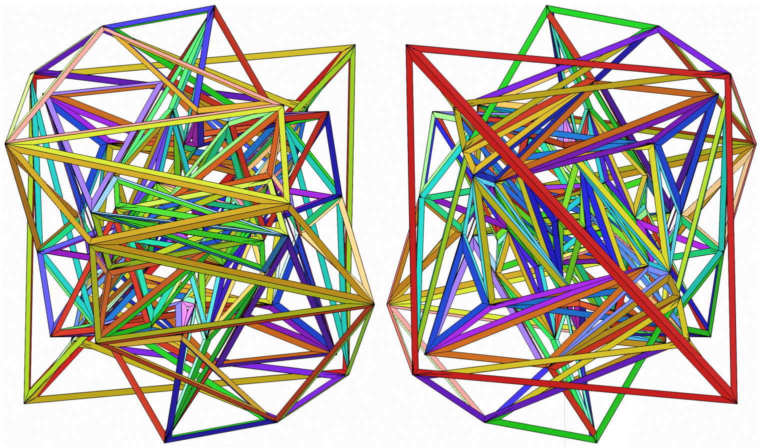

Figure 15. We see two orthogonal projections of the order 2 symmetric polyhedral embedding of Hurwitz's regular map $\{3,7\}_{18}$ of genus 7 through the axis of symmetry. The 168 triangles are marked as their boundaries.

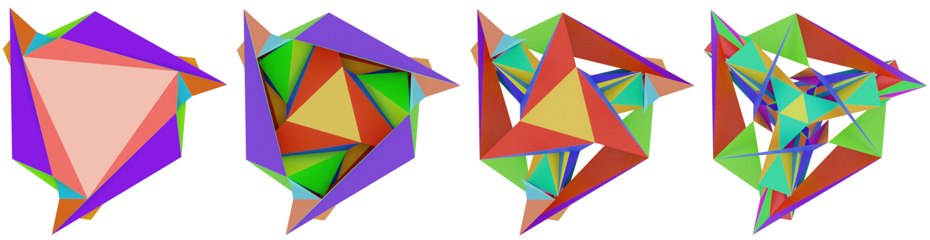

Figure 16. An order 3 symmetrical polyhedral embedding of Hurwitz's regular map $\{3,7\}_{18}$ of genus 7 with triangles progressively removed to show internal structure.

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5.8. The Dual Case R7.1′

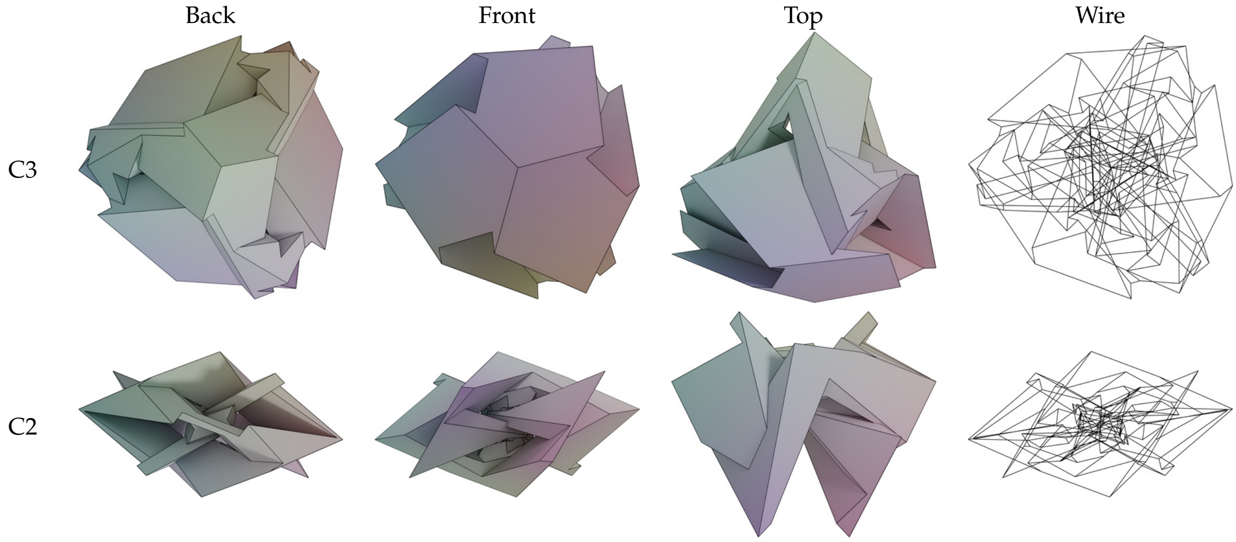

Figure 17. R7.1′ with C3 and C2 symmetry

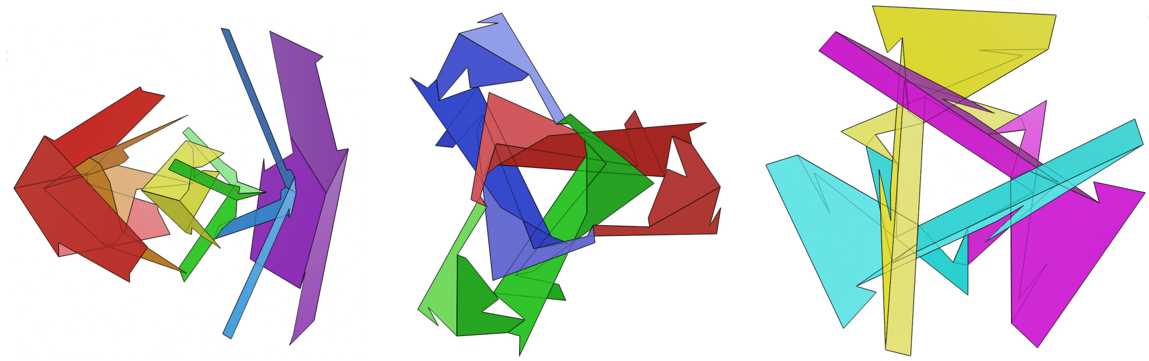

Figure 18. R7.1′ with C3 symmetry. Faces removed to show: central axis, knotted cycle, ring cycle

5.9. Case R8.1

The regular map R8.1 has Schläfli type $\{3,8\}_8$ and genus 8. Its combinatorial symmetry group has order 672. The regular map has 42 vertices, 112 faces.

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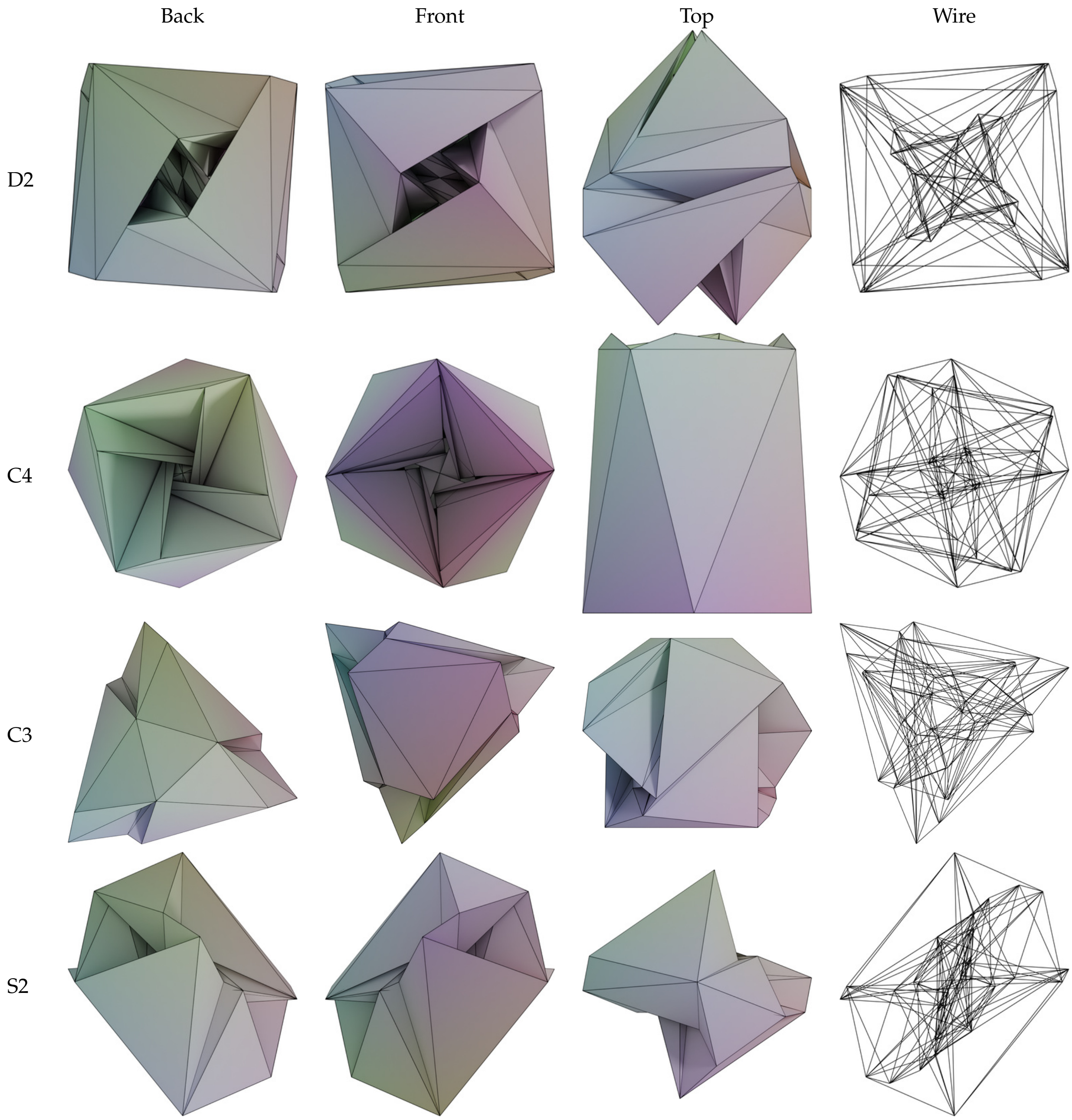

Figure 19. R8.1 with D2, C4, C3, and S2 symmetry

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5.10. Case R8.2

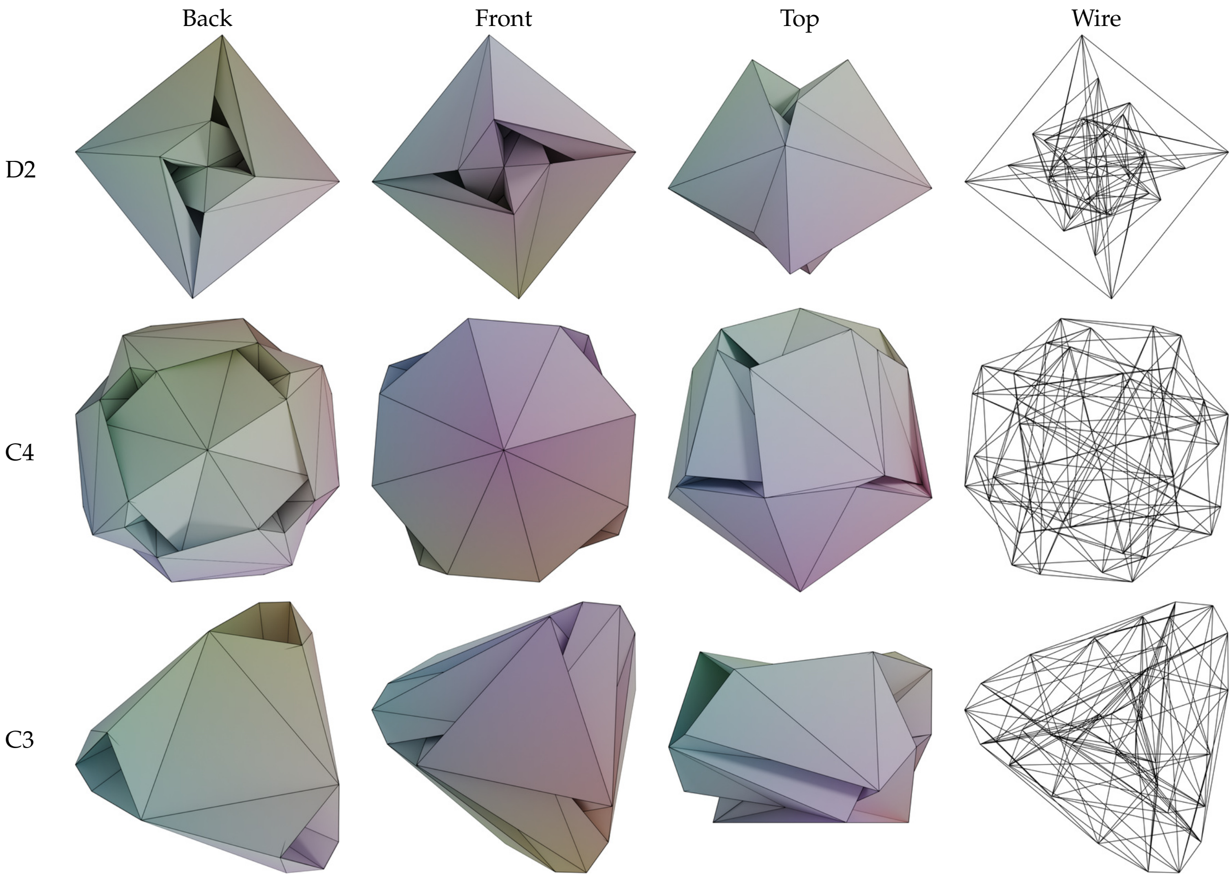

Figure 20. R8.2 with D2, C4, and C3 symmetry

The regular map R8.2 has Schläfli type $\{3,8\}_{14}$ and genus 8. Its combinatorial symmetry group has order 672. This regular map has 42 vertices and 112 faces.

5.11. Case R10.1

The regular map R10.1 has Schläfli type $\{3,9\}_{12}$ and genus 10. Its combinatorial symmetry group has order 648.

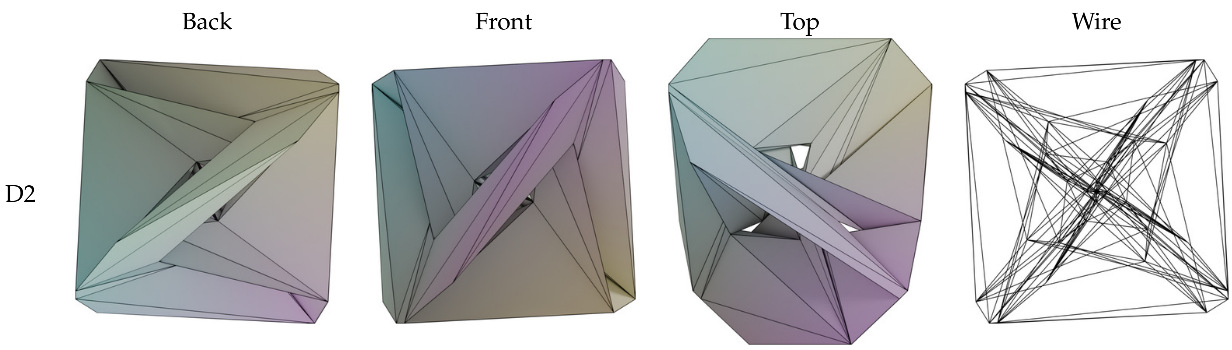

Figure 21. R10.1 with D2 symmetry

5.12. Case R10.2

R10.2, of type $\{3,12\}_6$ and genus 10, symmetry group of order 432.

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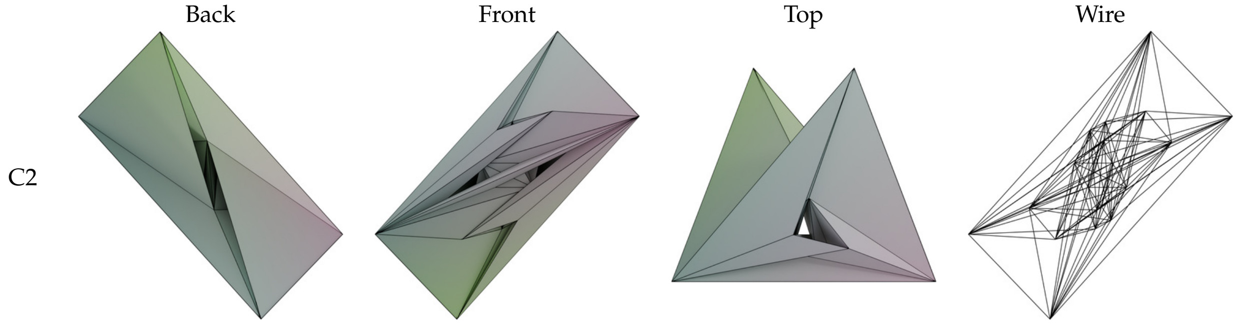

Figure 22. R10.2 with C2 symmetry

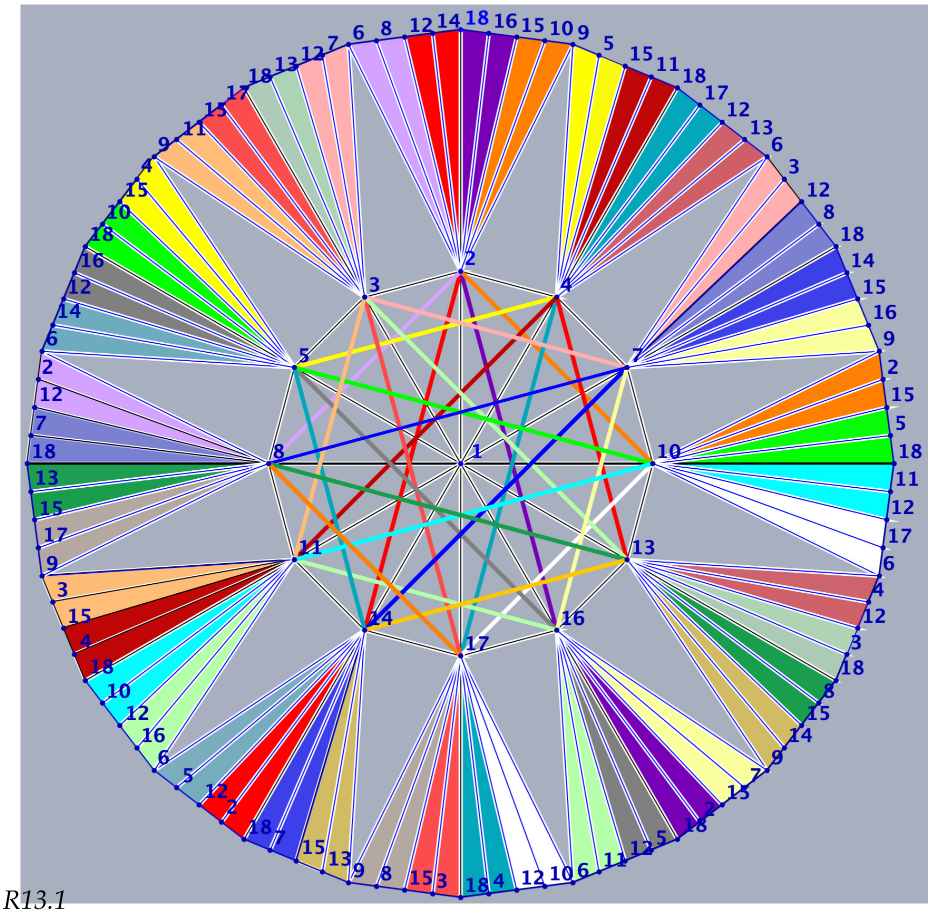

Figure 23. The triangles in the outer circular sequence appear twice. The colored line segments indicate these pairs.

5.13. Case R13.1

R13.1, of type $\{3,10\}_{30}$ and genus 13, symmetry group of order 720.

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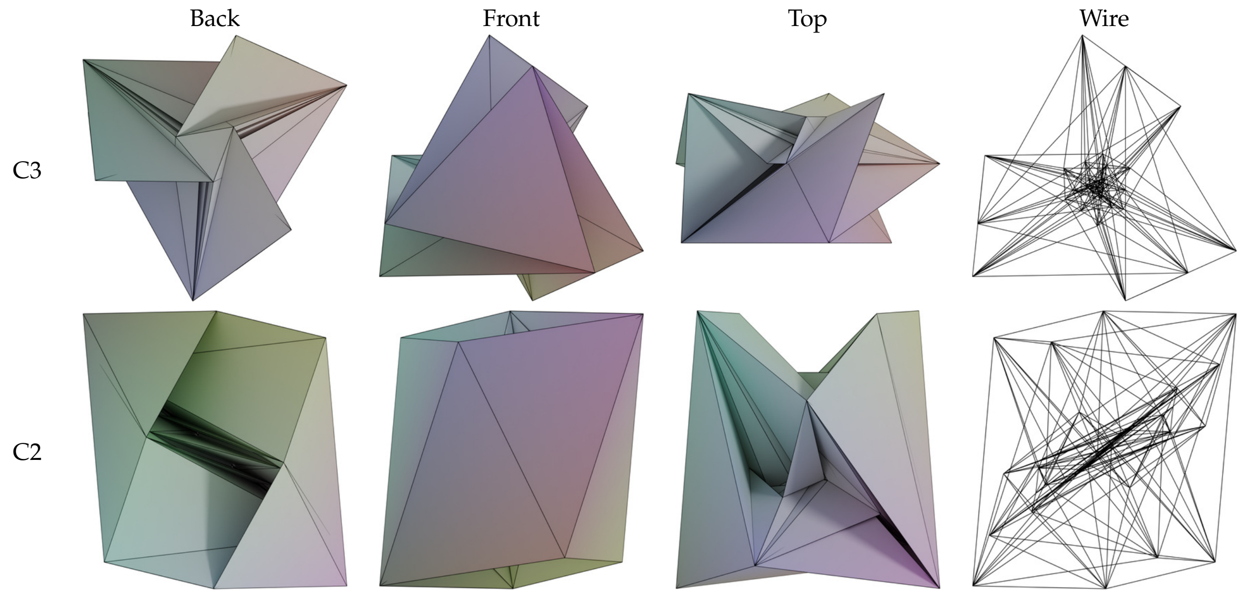

Figure 24. R13.1 with C3 and C2 symmetry

5.14. *Case R14.1*

R14.1 due to Hurwitz of type $\{3,7\}_{12}$ of genus 14, symmetry group of order 2184.

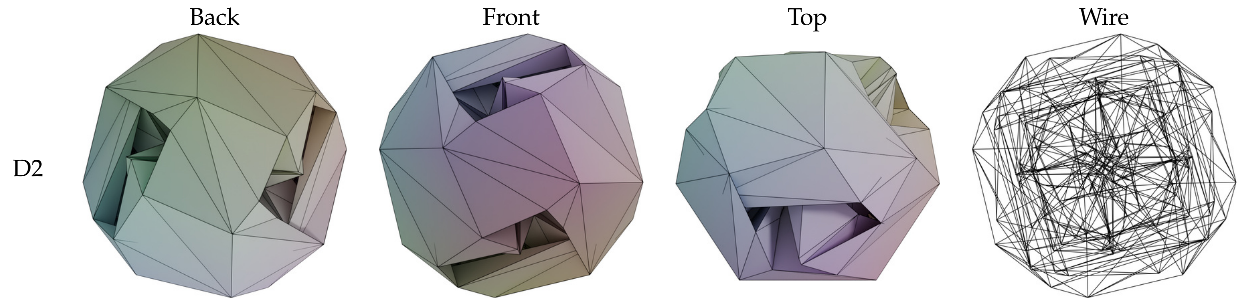

Figure 25. R14.1 with D2 symmetry

In the theory of Riemann surfaces, the first Hurwitz triplet is a triple of distinct Hurwitz surfaces (R14.1, R14.2, and R14.3) with the identical automorphism group of the lowest possible genus, here 14.

5.15. *Case R14.2*

R14.2 due to Hurwitz of type $\{3,7\}_{26}$ of genus 14, symmetry group of order 2184.

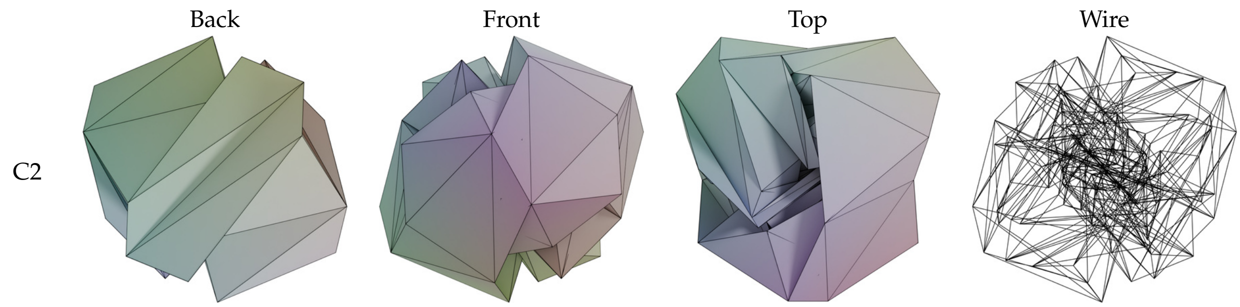

Figure 26. R14.2 with C2 symmetry

5.16. *Case R14.3*

R14.3 due to Hurwitz of type $\{3,7\}_{14}$ of genus 14, symmetry group of order 2184.

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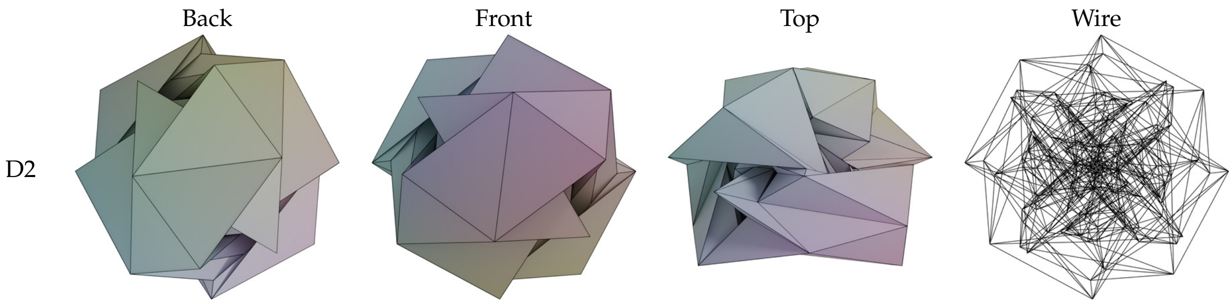

Figure 27. R14.3 with D2 symmetry

5.17. Case R13.2

The regular map R13.2 has Schläfli type $\{3, 12\}_{12}$ and genus 13. Its combinatorial symmetry group has order 576.

It is the only triangular regular map of genus g, $2 \leqslant g \leqslant 14$ that is not likely to be embeddable.

6. Complete Graphs with 4, 7, and 12 Vertices on Closed Oriented 2-Manifolds, No Diagonals

In general, a k-neighborly polytope is a convex polytope where any k or fewer vertices form a face. We concentrate on the spatial case (k=1), where a polyhedron must not be convex and it is considered neighborly if either its edge graph is complete (no diagonals) or each face shares exactly one edge with every other face (face-sharing property). These two properties are linked through duality: The dual of a neighborly polyhedron with a complete edge graph is a neighborly polyhedron with the face-sharing property, and vice versa.

Crucially, we are interested in embeddings of these neighborly polyhedra that are free of self-intersections. This requires us to consider only oriented closed surfaces for our polyhedra, as non-orientable surfaces cannot be embedded in R^3. Furthermore, neighborly polyhedra with complete edge graphs necessarily have triangular faces. Therefore, our investigation centers on embeddings of triangular complete graphs on surfaces and their duals. These embeddings have played a significant role in results like the map color theorem, [15].

While we do not assume familiarity with oriented matroids, it is worth noting that all such triangular complete graph embeddings can be obtained by naturally extending the concept of pseudoline arrangements to curve arrangements on surfaces, [36].

Computations of all combinatorially possible embeddings is a second step before we discuss possible polyhedral embeddings.

We find in this section the solution of a long standing conjecture of Branco Grünbaum whether a cell decomposition of a triangulated orientable surface exists that cannot have an embedding in R^3.

6.1. The Tetrahedron

The Tetrahedron has the complete graph with 4 vertices as its edge graph. This is the easiest example of a neighborly polyhedron.

6.2. The Seven Vertex Torus of Möbius

For the seven vertex torus of Möbius in his collected works, [7], Császár in [8] was the first to find an embedding for Möbius's combinatorial description, although he was not aware of this former reference. Here the edge graph has seven vertices. In 1991 Bokowski and Eggert [9] have found via oriented matroid techniques additional three symmetrical polyhedral embeddings of this seven vertex torus of Möbius.

In this video we see another embedding as a YouTube video: https://youtu.be/LGUyT6xfTFs We find all four symmetric embeddings of this neighborly seven vertex torus also in [37]. The best version might be a video from 1986 that can be downloaded from http://science-to-touch.com/ForJB/MoebiusTorus.mov.

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Figure 28. A symmetric Möbius torus embedding of Bokowski and Eggert [9] as a 3D-print, https://youtu.be/6GhtRzemOwU

6.3. The 59 Examples of the Complete Graph with 12 Vertices

The 59 combinatorial different examples of candidates for a triangular embedding with the complete graph with 12 vertices have been published in 1994, [12]. These 59 surfaces can be drawn topologically on this ceramic model in Figure 29. It is a shape that is topologically a sphere with six handles.

Figure 29. A genus 6 surface for all 59 topological embeddings of the complete graph with 12 vertices. Even finding just one such embedding is a challenge.

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A crucial first proof that an oriented closed triangular 2-manifold does not allow an embedding in 3-space was provided by Bokowski and Guedes de Oliveira in 2000, [3]. The theory of oriented matroids helped decisively: The set of possible oriented matroids that are not forbidden because of edge-face intersections turned out (after long computations) to be the empty set. Several models of the corresponding topological embedding of the corresponding surface have been produced by Bokowski. See: Die Geschichte eines Modells, in [37], p. 88ff.

For the topological embedding of this surface we have the attempt to visualize it in Figure 30.

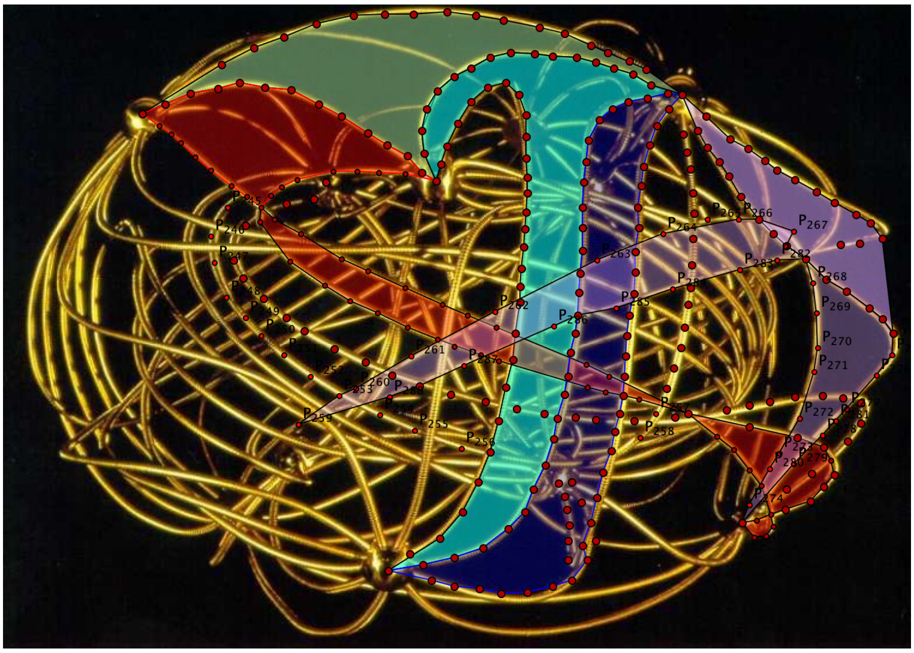

Figure 30. Here you see the model for the topological shape of an example surface from the 59 surfaces that was used in [3]. Some topological triangles are drawn. The others are clear when you go along adjacent topological edges and take as a third edge the one connecting the other endpoints. A membrane with these three edges forms a triangle of the surface.

6.4. Neighborly Spatial Pseudo-Manifolds with 9 and 10 Vertices

We have additional spatial polyhedra without diagonals with 9 vertices in [12]. In Figure 31 we see an example of this paper.

Figure 31. A neighborly pseudomanifold with 9 vertices. Computer graphics and model.

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We have additional spatial polyhedra without diagonals with 10 vertices in [13]. Among these examples are embeddings of four pinched spheres. The polyhedra in all these cases are orientable neighborly 2-pseudomanifolds. There are several circular sequences of triangles around a vertex.

7. The Dual Case of the Former Section

7.1. The Tetrahedron

The tetrahedron has also the face-sharing property. It is the only dual case with convex faces.

7.2. Szilassi's Polyhedron

The combinatorial property of Szilassi's polyhedron is dual to the seven vertex torus of Möbius with non-convex faces.

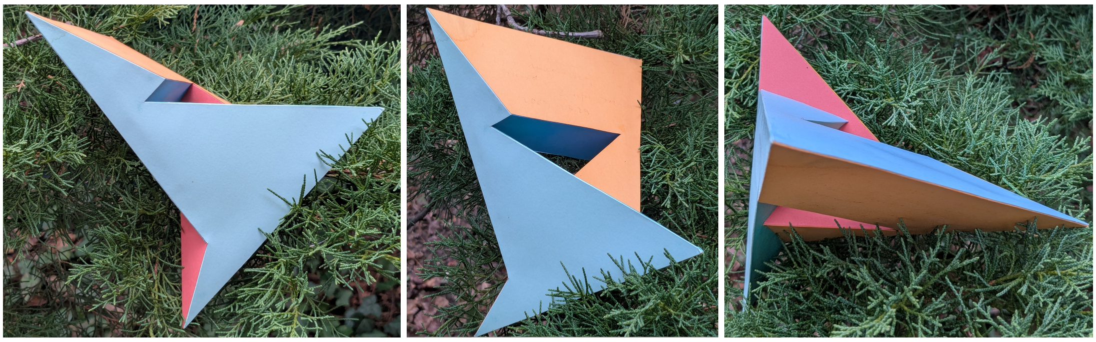

Figure 32. Szilassi's polyhedron with seven hexagons and the face-sharing property

That this embedding of Szilassi is essentially unique, was shown via oriented matroid techniques in [11].

7.3. The 59 Examples of the Complete Graph with 12 Vertices Used for Its 59 Duals

No embeddings of the 59 dual abstract polyhedra with 12 eleven-gons and 44 vertices was found with methods of the second author. Therefore a Kepler-Poinsot version of an embedding with a low number of intersections is of interest. Such a polyhedron has been depicted in Figure 33. The two orthogonal projections along the x- ,y- and z-axis are shown in the four columns. The lower part shows an exploded view of all 12 eleven-gons.

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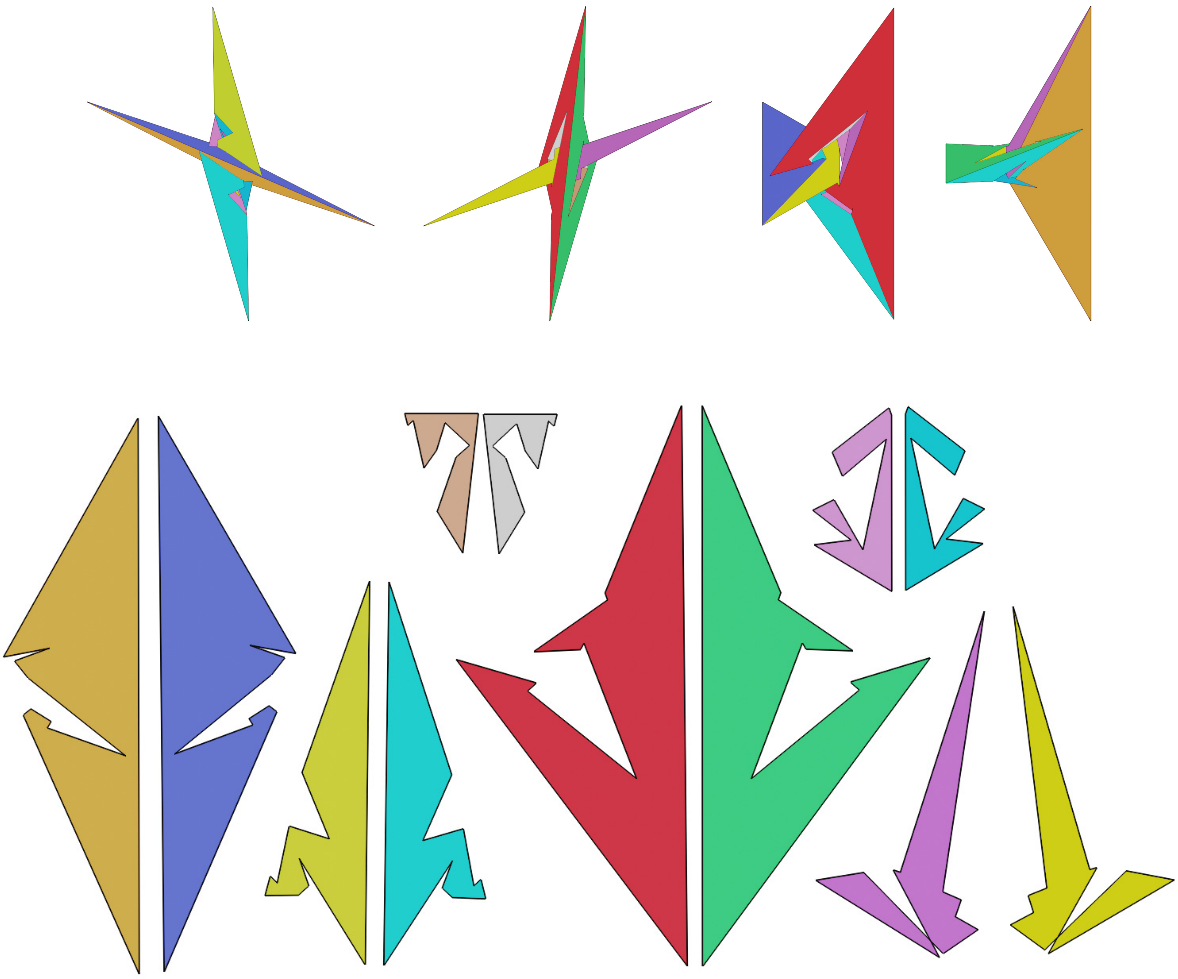

Figure 33. A Kepler-Poinsot model with only 2 crossed polygons. Back, front, side, top, and faces to construct the polyhedron.

8. Conclusions

Compared with former progress in finding polyhedral embeddings of regular maps, these new results provide a huge step forward. Finding the maximal order of symmetry is in many cases still open. So far we have proofs for non-embeddable cases when the number of vertices is 12. For the open case R13.2, we have 24 vertices. We cannot hope that this open case is easy to tackle. What can still be done is the non-triangular case of regular maps. Whether we can still find an additional regular Leonardo polyhedron remains an interesting open problem.

The paper of A. Altshuler and U. Brehm [38] has additional neighborly pseudo-manifolds with 11 vertices. An investigation of their polyhedral embeddings is still open.

We do not have a neighborly spatial polyhedron according to a complete graph embedding with 12 vertices on a 2-manifold, however, we have seen the attempt to show such a topological embedding in Figure 30. Another topological embedding for the example with the highest symmetry has been constructed by Carlo Séquin and Ling Xiao, see Figure 34 and the article [39]. When you insert in each topological triangle a vertex and you construct topological eleven-gons around former vertices, you obtain a dual topological embedding.

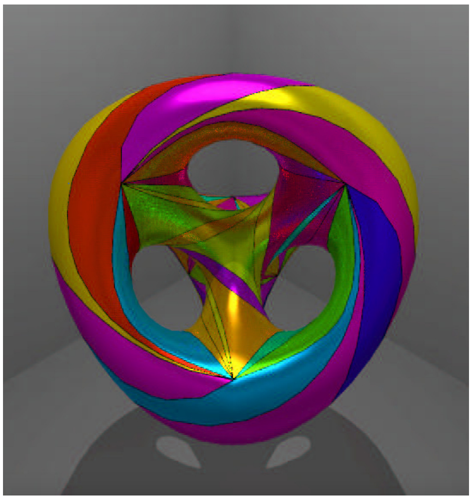

Figure 34. A topological embedding of the complete graph with 12 vertices on a surface of genus 6 with the highest symmetry by Carlo Séquin and Ling Xiao.

Acknowledgments: We wish to thank Jörg M. Wills for his advice and stimulating questions for finding polyhedral embeddings of regular maps. We wish to thank Alice Niemeyer and Reymond Akpanya for their support to find for some regular maps their detailed information.

Appendix A. Vertex Tables

Table A1. R3.1 with T Symmetry

Vertex	X	Y	Z	Vertex	X	Y	Z	Triangles		
1	4	5	-6	13	4	3	-4	(1,5,23)	(1,21,22)	(2,6,20)
2	-4	5	6	14	-4	3	4	(2,22,21)	(3,7,21)	(3,23,20)
3	-4	-5	-6	15	-4	-3	-4	(4,8,13)	(4,12,14)	(5,9,12)
4	6	-4	5	16	4	-4	3	(5,13,15)	(6,10,15)	(6,14,12)
5	6	4	-5	17	4	4	-3	(7,11,14)	(7,15,13)	(8,4,24)
6	-6	4	5	18	-4	4	3	(8,16,17)	(8,24,19)	(9,1,18)
7	-6	-4	-5	19	-4	-4	-3	(9,5,1)	(9,17,16)	(10,2,17)
8	5	-6	4	20	3	-4	4	(10,6,2)	(10,18,19)	(11,3,16)
9	5	6	-4	21	3	4	-4	(11,7,3)	(11,19,18)	(12,4,5)
10	-5	6	4	22	-3	4	4	(12,16,20)	(12,20,6)	(13,5,4)
11	-5	-6	-4	23	-3	-4	-4	(13,17,21)	(13,21,7)	(14,6,7)
12	4	-3	4	24	4	-5	6	(14,18,22)	(14,22,4)	(15,7,6)
								(15,19,23)	(15,23,5)	(16,8,11)
								(16,12,9)	(17,9,10)	(17,13,8)
								(18,10,9)	(18,14,11)	(19,11,8)
								(19,15,10)	(20,16,3)	(20,24,2)
								(21,1,3)	(21,17,2)	(22,2,24)
								(22,18,1)	(23,3,1)	(23,19,24)
								(24,4,22)	(24,20,23)	
Vertex Permutation Groups										
(1,24)(2,3)(4,5)(6,7)(8,9)(10,11)(12,13)(14,15)(16,17)(18,19)(20,21)(22,23)										
(1,3)(2,24)(4,6)(5,7)(8,10)(9,11)(12,14)(13,15)(16,18)(17,19)(20,22)(21,23)										

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Table A2. R3.1′ with T Symmetry.

Vertex	X	Y	Z	Vertex	X	Y	Z	Polygons
1	513	513	-2337	29	209	-209	-855	(12,16,42,30,11,8,26)
2	-513	513	2337	30	-209	-209	855	(12,21,44,32,2,56,16)
3	-513	-513	-2337	31	-209	209	-855	(12,26,49,37,5,4,21)
4	2337	-513	513	32	855	209	209	(13,17,43,31,10,9,27)
5	2337	513	-513	33	855	-209	-209	(13,20,45,33,3,1,17)
6	-2337	513	513	34	-855	-209	209	(13,27,48,36,4,5,20)
7	-2337	-513	-513	35	-855	209	-209	(14,18,40,28,9,10,24)
8	513	-2337	513	36	209	855	209	(14,23,46,34,56,2,18)
9	513	2337	-513	37	209	-855	-209	(14,24,51,39,7,6,23)
10	-513	2337	513	38	-209	-855	209	(15,19,41,29,8,11,25)
11	-513	-2337	-513	39	-209	855	-209	(15,22,47,35,1,3,19)
12	2337	-2337	2337	40	549	141	549	(15,25,50,38,6,7,22)
13	2337	2337	-2337	41	549	-141	-549	(52,40,18,2,32,33,45)
14	-2337	2337	2337	42	-549	-141	549	(52,45,20,5,37,38,50)
15	-2337	-2337	-2337	43	-549	141	-549	(52,50,25,11,30,28,40)
16	342	57	1539	44	549	549	141	(53,41,19,3,33,32,44)
17	342	-57	-1539	45	549	-549	-141	(53,44,21,4,36,39,51)
18	-342	-57	1539	46	-549	-549	141	(53,51,24,10,31,29,41)
19	-342	57	-1539	47	-549	549	-141	(54,42,16,56,34,35,47)
20	1539	342	57	48	141	549	549	(54,47,22,7,39,36,48)
21	1539	-342	-57	49	141	-549	-549	(54,48,27,9,28,30,42)
22	-1539	-342	57	50	-141	-549	549	(55,43,17,1,35,34,46)
23	-1539	342	-57	51	-141	549	-549	(55,46,23,6,38,37,49)
24	57	1539	342	52	342	-342	342	(55,49,26,8,29,31,43)
25	57	-1539	-342	53	342	342	-342	
26	-57	-1539	342	54	-342	342	342	
27	-57	1539	-342	55	-342	-342	-342	
28	209	209	855	56	513	-513	2337	
Face Permutation Groups								
(1,23)(2,10)(3,5)(4,11)(6,17)(7,13)(8,14)(9,20)(12,16)(15,21)(18,24)(19,22)								
(1,21)(2,23)(3,22)(4,16)(5,17)(6,19)(7,18)(8,12)(9,13)(10,15)(11,14)(20,24)								

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Table A3. R3.2 with D3 Symmetry

Vertex	X	Y	Z	Triangles		
1	3.50807	-2.08966	4.12484	(1,2,3)	(1,3,5)	(1,4,2)
2	-1.34797	-3.85437	-4.12484	(1,5,8)	(1,7,4)	(1,8,11)
3	-8.28589	-8.38412	6.58506	(1,10,7)	(1,11,10)	(2,4,9)
4	11.73538	-1.10915	-6.58506	(2,6,3)	(2,8,6)	(2,9,10)
5	11.40380	-2.98373	6.58506	(2,10,12)	(2,12,8)	(3,6,7)
6	4.01197	0.75981	-4.12484	(3,7,12)	(3,9,5)	(3,11,9)
7	-3.56373	-1.99324	4.12484	(3,12,11)	(4,5,9)	(4,6,11)
8	-2.66400	3.09456	-4.12484	(4,7,6)	(4,11,12)	(4,12,5)
9	-6.82824	-9.60857	-6.58506	(5,6,8)	(5,10,6)	(5,12,10)
10	0.05567	4.08290	4.12484	(6,10,11)	(7,8,12)	(7,9,8)
11	-4.90714	10.71771	-6.58506	(7,10,9)	(8,9,11)	
12	-3.11791	11.36785	6.58506			
Vertex Permutation Groups						
(1,2,10,6,7,8)(3,9,5,4,12,11)						

Table A4. R3.2 with S2 Symmetry

Vertex	X	Y	Z	Triangles		
1	0	-7	0	(1,2,3)	(1,3,5)	(1,4,2)
2	5	-24	8	(1,5,8)	(1,7,4)	(1,8,11)
3	0	13	7	(1,10,7)	(1,11,10)	(2,4,9)
4	0	-13	-7	(2,6,3)	(2,8,6)	(2,9,10)
5	-17	-3	-4	(2,10,12)	(2,12,8)	(3,6,7)
6	0	7	0	(3,7,12)	(3,9,5)	(3,11,9)
7	-5	24	-8	(3,12,11)	(4,5,9)	(4,6,11)
8	23	0	7	(4,7,6)	(4,11,12)	(4,12,5)
9	-4	9	24	(5,6,8)	(5,10,6)	(5,12,10)
10	-23	0	-7	(6,10,11)	(7,8,12)	(7,9,8)
11	17	3	4	(7,10,9)	(8,9,11)	
12	4	-9	-24			
Vertex Permutation Groups						
(1,6)(2,7)(3,4)(5,11)(8,10)(9,12)						

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Table A5. R5.1 with O Symmetry

Vertex	X	Y	Z	Triangles		
1	-3	19	2	(1,2,3)	(1,2,4)	(1,3,5)
2	-2	3	19	(1,4,7)	(1,5,8)	(1,7,12)
3	-19	2	3	(1,8,14)	(1,12,14)	(2,3,6)
4	3	2	19	(2,4,10)	(2,6,11)	(2,10,20)
5	-2	19	-3	(2,11,18)	(2,20,18)	(3,5,9)
6	-19	-3	2	(3,6,13)	(3,9,16)	(3,13,21)
7	2	19	3	(3,16,21)	(4,7,15)	(4,10,17)
8	3	-2	-19	(4,15,16)	(4,16,21)	(4,17,21)
9	-19	3	-2	(5,8,15)	(5,9,19)	(5,15,20)
10	19	-3	-2	(5,19,18)	(5,20,18)	(6,11,17)
11	-3	-2	19	(6,12,14)	(6,13,19)	(6,14,19)
12	-19	-2	-3	(6,17,12)	(7,9,11)	(7,11,18)
13	2	-19	-3	(7,12,9)	(7,15,22)	(7,18,22)
14	2	3	-19	(8,10,13)	(8,13,21)	(8,14,10)
15	19	3	2	(8,15,23)	(8,21,23)	(9,12,23)
16	2	-3	19	(9,16,11)	(9,19,23)	(10,14,22)
17	-2	-19	3	(10,17,22)	(10,20,13)	(11,16,24)
18	3	19	-2	(11,17,24)	(12,17,21)	(12,23,21)
19	-3	2	-19	(13,19,24)	(13,20,24)	(14,19,18)
20	19	-2	3	(14,22,18)	(15,16,20)	(15,22,23)
21	-3	-19	-2	(16,24,20)	(17,22,24)	(19,23,24)
22	19	2	-3	(22,23,24)		
23	-2	-3	-19			
24	3	-19	2			

Vertex Permutation Groups

(1,6,24,15)(2,11,16,4)(3,17,20,7)(5,12,13,22)
(8,14,19,23)(9,21,10,18)

(1,21)(2,23)(3,12)(4,8)
(5,17)(6,9)(7,13)(10,15)
(11,19)(14,16)(18,24)(20,22)

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Table A6. R5.1 with S2 Symmetry

Vertex	X	Y	Z	Triangles		
1	-16	-16	-2	(1,2,3)	(1,3,5)	(1,4,2)
2	-9	10	2	(1,5,8)	(1,7,4)	(1,8,14)
3	-2	-22	8	(1,12,7)	(1,14,12)	(2,4,10)
4	23	29	7	(2,6,3)	(2,10,20)	(2,11,6)
5	4	-14	5	(2,18,11)	(2,20,18)	(3,6,13)
6	-18	-6	2	(3,9,5)	(3,13,21)	(3,16,9)
7	-4	14	-5	(3,21,16)	(4,7,15)	(4,15,16)
8	-23	-29	-7	(4,16,21)	(4,17,10)	(4,21,17)
9	17	-15	-12	(5,9,19)	(5,15,8)	(5,18,20)
10	-8	23	29	(5,19,18)	(5,20,15)	(6,11,17)
11	-11	10	-20	(6,12,14)	(6,14,19)	(6,17,12)
12	-13	-4	-26	(6,19,13)	(7,9,11)	(7,11,18)
13	15	-26	24	(7,12,9)	(7,18,22)	(7,22,15)
14	-36	12	0	(8,10,14)	(8,13,10)	(8,15,23)
15	16	16	2	(8,21,13)	(8,23,21)	(9,12,23)
16	36	-12	0	(9,16,11)	(9,23,19)	(10,13,20)
17	-15	26	-24	(10,17,22)	(10,22,14)	(11,16,24)
18	-17	15	12	(11,24,17)	(12,17,21)	(12,21,23)
19	11	-10	20	(13,19,24)	(13,24,20)	(14,18,19)
20	13	4	26	(14,22,18)	(15,20,16)	(15,22,23)
21	8	-23	-29	(16,20,24)	(17,24,22)	(19,23,24)
22	2	22	-8	(22,24,23)		
23	9	-10	-2			
24	18	6	-2			

Vertex Permutation Groups

(1,15)(2,23)(3,22)(4,8)
(5,7)(6,24)(9,18)(10,21)
(11,19)(12,20)(13,17)(14,16)

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Table A7. R5.1′ with D2 Symmetry

Vertex	X	Y	Z	Vertex	X	Y	Z	Polygons
1	20.65379	-21.80366	30.77811	33	-21.71225	-46.69755	-23.86363	(1,2,4,6,8,7,5,3)
2	12.74359	19.76101	29.74751	34	-23.71552	-42.38214	-21.10591	(1,3,15,17,19,18,16,9)
3	22.41124	-22.24871	26.29238	35	21.80365	20.65380	-30.77811	(1,9,11,13,14,12,10,2)
4	29.87010	38.88395	-26.55009	36	-19.76099	12.74359	-29.74751	(2,10,21,24,23,22,20,4)
5	17.18350	5.22081	25.61127	37	-60.30722	-73.01170	-49.44046	(3,5,25,27,29,28,26,15)
6	24.93641	54.96569	-21.91335	38	38.09246	-54.72530	-51.99704	(4,20,38,39,37,35,36,6)
7	33.96277	41.71029	-39.06823	39	33.11198	-49.63967	-50.27328	(5,7,41,40,42,44,43,25)
8	33.11615	50.33837	-41.42038	40	-21.80365	-20.65380	-30.77811	(6,36,46,54,53,32,31,8)
9	-73.01172	60.30723	49.44046	41	-22.24871	-22.41124	-26.29238	(7,8,31,34,57,58,49,41)
10	54.96571	-24.93641	21.91335	42	60.30722	73.01170	-49.44046	(9,16,33,34,31,32,30,11)
11	-49.63968	-33.11198	50.27328	43	49.63968	33.11198	50.27328	(10,12,48,40,41,49,50,21)
12	38.88394	-29.87010	26.55009	44	54.72530	38.09245	51.99703	(11,30,52,51,45,35,37,13)
13	-54.72530	-38.09245	51.99704	45	22.24871	22.41125	-26.29238	(12,14,29,27,59,61,56,48)
14	46.69755	-21.71224	23.86363	46	-38.88394	29.87010	26.55009	(13,37,39,58,57,28,29,14)
15	-41.71029	33.96277	39.06823	47	-54.96571	24.93641	21.91335	(15,26,47,46,36,35,45,17)
16	-38.09246	54.72530	-51.99703	48	19.76099	-12.74359	-29.74751	(16,18,42,40,48,56,55,33)
17	-5.22081	17.18350	-25.61127	49	5.22081	-17.18350	-25.61127	(17,45,51,61,59,22,23,19)
18	-33.11197	49.63968	-50.27328	50	41.71029	-33.96277	39.06823	(18,19,23,24,53,54,44,42)
19	-31.31434	39.35709	-17.76609	51	-33.96277	-41.71029	-39.06823	(20,22,59,27,25,43,60,38)
20	21.71225	46.69755	-23.86363	52	-17.18350	-5.22081	25.61127	(21,50,62,52,30,32,53,24)
21	50.33838	-33.11615	41.42038	53	-42.38214	23.71552	21.10591	(26,28,57,34,33,55,63,47)
22	23.71552	42.38214	-21.10591	54	-46.69755	21.71224	23.86363	(38,60,64,62,50,49,58,39)
23	14.61668	51.09704	-18.10958	55	-29.87010	-38.88395	-26.55009	(43,44,54,46,47,63,64,60)
24	1.40164	30.78678	33.25093	56	-24.93641	-54.96569	-21.91335	(51,52,62,64,63,55,56,61)
25	39.35710	31.31435	17.76608	57	-14.61668	-51.09704	-18.10958	
26	-50.33838	33.11615	41.42038	58	31.31434	-39.35709	-17.76609	
27	51.09705	-14.61667	18.10958	59	30.78679	-1.40162	-33.25093	
28	-1.40164	-30.78678	33.25093	60	73.01172	-60.30723	49.44046	
29	42.38214	-23.71552	21.10591	61	-33.11615	-50.33837	-41.42038	
30	-39.35710	-31.31435	17.76608	62	-22.41124	22.24871	26.29238	
31	-30.78679	1.40162	-33.25093	63	-12.74359	-19.76102	29.74751	
32	-51.09705	14.61667	18.10958	64	-20.65379	21.80365	30.77811	
Face Permutation Groups								
(1,9,24,10)(2,7,23,13)(3,11,22,8)(4,12,19,20)(5,16,17,14)(6,18,15,21)								

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Table A8. R5.1′ with C3 Symmetry

Vertex	X	Y	Z	Vertex	X	Y	Z	Polygons
1	43.46366	-21.64751	-61.72107	33	-39.16046	-28.68813	-56.03963	(1,2,4,6,8,7,5,3)
2	44.42489	-19.56988	-56.03962	34	9.96950	45.84838	-20.81775	(1,3,15,17,19,18,16,9)
3	53.86739	-46.58218	-39.71776	35	11.90551	85.75792	-36.03035	(1,9,11,13,14,12,10,2)
4	52.86341	-40.45273	-38.74069	36	38.96070	16.42411	-33.31914	(2,10,21,24,23,22,20,4)
5	55.43655	-42.78937	-30.10898	37	7.80724	83.27623	-40.60529	(3,5,25,27,29,28,26,15)
6	42.06058	26.82332	-27.12557	38	-2.98453	48.46441	-61.72105	(4,20,38,39,37,35,36,6)
7	43.23307	35.96650	-14.69191	39	13.40766	69.94163	-39.71775	(5,7,41,40,42,44,43,25)
8	41.34849	42.18495	-17.26068	40	9.53136	-55.42417	-14.69193	(6,36,46,54,53,32,31,8)
9	-50.16896	-22.72487	-43.52481	41	0	0	-3.48967	(7,8,31,34,57,58,49,41)
10	-44.69060	-14.29033	-20.81776	42	15.85899	-56.90131	-17.26070	(9,16,33,34,31,32,30,11)
11	-66.69357	-19.12815	-29.56035	43	34.72110	-31.55802	-20.81777	(10,12,48,40,41,49,50,21)
12	-54.41173	-16.29590	-24.30598	44	35.27372	-0.02124	-16.67734	(11,30,52,51,45,35,37,13)
13	-2.37166	14.42542	51.11662	45	0	0	79.84747	(12,14,29,27,59,61,56,48)
14	1.26806	12.52908	44.90553	46	27.61252	32.62213	-21.48363	(13,37,39,58,57,28,29,14)
15	68.21570	-48.39938	-40.60532	47	16.78131	67.32243	-29.56033	(15,26,47,46,36,35,45,17)
16	-61.46479	-25.55469	-38.74069	48	-64.77494	-26.61476	-30.10897	(16,18,42,40,48,56,55,33)
17	68.31578	-53.18943	-36.03039	49	-52.76442	19.45769	-14.69192	(17,45,51,61,59,22,23,19)
18	2.19938	-49.83717	-27.12558	50	-57.20747	14.71636	-17.26069	(18,19,23,24,53,54,44,42)
19	-5.25665	-41.95300	-33.31916	51	-80.22128	-32.56847	-36.03038	(20,22,59,27,25,43,60,38)
20	44.76481	-32.08516	-43.52481	52	-33.70404	25.52891	-33.31914	(21,50,62,52,30,32,53,24)
21	-17.65526	-30.53731	-16.67734	53	0	0	-38.49300	(26,28,57,34,33,55,63,47)
22	49.91226	-48.19425	-29.56035	54	22.32151	20.10489	-28.72768	(38,60,64,62,50,49,58,39)
23	14.44533	-40.22420	-21.48364	55	-40.47914	-26.81687	-61.72107	(43,44,54,46,47,63,64,60)
24	6.25059	-29.38343	-28.72769	56	-67.27505	-23.35943	-39.71776	(51,52,62,64,63,55,56,61)
25	41.31855	-38.97399	-24.30600	57	13.09319	55.26989	-24.30597	
26	13.67862	-5.15880	51.11662	58	9.33840	69.40414	-30.10896	
27	-11.48454	-5.16638	44.90554	59	-11.30695	-9.26664	51.11662	
28	10.21648	-7.36273	44.90552	60	-5.26442	48.25803	-56.03961	
29	0	0	60.29438	61	-76.02295	-34.87683	-40.60531	
30	-42.05785	7.60208	-21.48363	62	-44.25996	23.01389	-27.12557	
31	-17.61847	30.55856	-16.67733	63	5.40416	54.81006	-43.52480	
32	-28.57210	9.27856	-28.72769	64	8.60138	66.00742	-38.74067	

Face Permutation Groups

(1,22,13)(2,15,19)(3,7,24)(4,23,6)(5,18,20)(8,14,10)(9,11,16)(12,17,21)

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Table A9. R6.1 with C2 Symmetry

Vertex	X	Y	Z	Triangles		
1	0	0	-11	(1,2,3)	(1,2,4)	(1,3,5)
2	8	-3	0	(1,4,7)	(1,5,8)	(1,7,10)
3	8	-2	-2	(1,8,11)	(1,10,13)	(1,11,14)
4	2	-9	2	(1,13,14)	(2,3,6)	(2,4,9)
5	5	-15	5	(2,6,8)	(2,8,12)	(2,9,10)
6	-11	-6	0	(2,10,15)	(2,12,14)	(2,15,14)
7	4	0	-6	(3,5,9)	(3,6,7)	(3,7,12)
8	-4	0	-6	(3,9,11)	(3,11,15)	(3,12,13)
9	11	6	0	(3,15,13)	(4,5,15)	(4,6,13)
10	-5	15	5	(4,7,6)	(4,9,5)	(4,11,12)
11	-2	9	2	(4,13,12)	(4,15,11)	(5,6,14)
12	-2	-1	1	(5,8,6)	(5,10,12)	(5,14,12)
13	-8	2	-2	(5,15,10)	(6,10,11)	(6,13,10)
14	-8	3	0	(6,14,11)	(7,8,15)	(7,9,14)
15	2	1	1	(7,10,9)	(7,12,8)	(7,15,14)
				(8,9,13)	(8,11,9)	(8,15,13)
				(9,14,13)	(10,12,11)	
Vertex Permutation Groups						
(2,14)(3,13)(4,11)(5,10)(6,9)(7,8)(12,15)						

Table A10. R6.1 with C3 Symmetry

Vertex	X	Y	Z	Triangles		
1	-5.99383	5.01307	1.24410	(1,2,3)	(1,2,4)	(1,3,5)
2	-18.71430	20.44390	9.90603	(1,4,7)	(1,5,8)	(1,7,10)
3	-1.34453	-7.69735	1.24410	(1,8,11)	(1,10,13)	(1,11,14)
4	27.06208	5.98511	9.90603	(1,13,14)	(2,3,6)	(2,4,9)
5	-12.16698	29.76071	-19.66275	(2,6,8)	(2,8,12)	(2,9,10)
6	-8.34778	-26.42901	9.90603	(2,10,15)	(2,12,14)	(2,15,14)
7	7.33837	2.68427	1.24410	(3,5,9)	(3,6,7)	(3,7,12)
8	-12.24958	-9.58165	-9.48644	(3,9,11)	(3,11,15)	(3,12,13)
9	-2.17316	15.39927	-9.48644	(3,15,13)	(4,5,15)	(4,6,13)
10	31.85702	-4.34345	-19.66276	(4,7,6)	(4,9,5)	(4,11,12)
11	-6.93376	-15.98406	-1.21418	(4,13,12)	(4,15,11)	(5,6,14)
12	-19.69004	-25.41726	-19.66275	(5,8,6)	(5,10,12)	(5,14,12)
13	14.42274	-5.81762	-9.48644	(5,15,10)	(6,10,11)	(6,13,10)
14	-10.37572	13.99685	-1.21417	(6,14,11)	(7,8,15)	(7,9,14)
15	17.30948	1.98721	-1.21418	(7,10,9)	(7,12,8)	(7,15,14)
				(8,9,13)	(8,11,9)	(8,15,13)
				(9,14,13)	(10,12,11)	
Vertex Permutation Groups						
(1,3,7)(2,6,4)(5,12,10)(8,13,9)(11,15,14)						

Table A11. R7.1 with C2 Symmetry

Vertex	X	Y	Z	Vertex	X	Y	Z	Triangles			
1	25	-29	20	37	-16	4	-31	(1,2,3)	(1,3,4)	(1,4,5)	(1,5,6)
2	25	28	15	38	0	7	0	(1,6,7)	(1,7,8)	(1,8,2)	(2,8,9)
3	-25	29	20	39	-12	6	5	(2,9,23)	(2,10,3)	(2,16,10)	(2,23,16)
4	-25	-28	15	40	11	2	10	(3,10,24)	(3,11,4)	(3,17,11)	(3,24,17)
5	19	-18	17	41	0	-3	-9	(4,25,18)	(8,7,15)	(8,15,29)	(10,52,33)
6	4	-4	0	42	-5	24	-2	(11,17,53)	(11,25,4)	(11,53,34)	(12,4,18)
7	8	-10	10	43	0	-13	4	(12,5,4)	(12,18,54)	(12,26,5)	(12,54,35)
8	14	-28	9	44	1	17	1	(13,5,19)	(13,6,5)	(13,27,6)	(14,7,6)
9	31	-9	3	45	-11	-2	10	(14,28,7)	(16,52,10)	(19,5,26)	(20,6,27)
10	-19	18	17	46	17	1	7	(20,14,6)	(20,27,33)	(21,7,28)	(21,15,7)
11	-14	28	9	47	-5	-6	-10	(21,28,34)	(21,34,40)	(21,40,63)	(21,57,15)
12	-6	-32	14	48	-30	24	-17	(21,63,57)	(22,8,29)	(22,9,8)	(22,29,35)
13	5	6	-10	49	6	-33	10	(22,35,41)	(22,51,9)	(30,14,56)	(30,17,24)
14	8	8	6	50	-13	3	-19	(30,24,38)	(30,43,17)	(30,56,43)	(31,15,57)
15	-3	-15	6	51	10	0	1	(31,18,25)	(31,37,18)	(31,57,37)	(32,9,51)
16	6	32	14	52	-17	10	15	(32,19,26)	(34,28,42)	(35,29,43)	(36,16,23)
17	-8	10	10	53	3	15	6	(36,23,37)	(36,42,16)	(37,23,60)	(37,60,18)
18	-27	-4	-30	54	17	-28	-21	(38,19,32)	(38,24,61)	(38,32,51)	(38,61,19)
19	1	3	-4	55	0	3	-9	(39,20,33)	(39,25,62)	(39,31,25)	(39,33,52)
20	0	-7	0	56	-1	-17	1	(39,62,20)	(40,26,63)	(40,32,26)	(40,34,53)
21	3	-6	11	57	-24	-21	9	(41,33,27)	(44,14,30)	(44,28,14)	(44,30,38)
22	30	-24	-17	58	-6	33	10	(44,38,51)	(44,51,65)	(44,58,28)	(44,65,58)
23	27	4	-30	59	-3	6	11	(45,15,31)	(45,29,15)	(45,31,39)	(45,39,52)
24	-4	4	0	60	16	-4	-31	(45,52,66)	(45,59,29)	(45,66,59)	(46,9,32)
25	-31	9	3	61	-10	1	-5	(46,23,9)	(46,32,40)	(46,40,53)	(46,53,67)
26	17	-10	15	62	-10	0	1	(46,60,23)	(46,67,60)	(47,10,33)	(47,24,10)
27	10	-1	-5	63	-3	-23	12	(47,33,41)	(47,41,54)	(47,54,68)	(47,61,24)
28	0	13	4	64	15	1	-8	(47,68,61)	(48,11,34)	(48,25,11)	(48,34,42)
29	4	-9	7	65	12	7	-4	(48,42,55)	(48,55,69)	(48,62,25)	(48,69,62)
30	-8	-8	6	66	3	23	12	(49,12,35)	(49,26,12)	(49,35,43)	(49,43,56)
31	-17	-1	7	67	24	21	9	(49,56,70)	(49,63,26)	(49,70,63)	(50,13,36)
32	12	-6	5	68	13	-3	-19	(50,27,13)	(50,36,37)	(50,37,57)	(50,57,71)
33	-1	-3	-4	69	-15	-1	-8	(50,64,27)	(50,71,64)	(54,18,60)	(54,41,35)
34	-4	9	7	70	-12	-7	-4	(55,13,19)	(55,19,61)	(55,36,13)	(55,42,36)
35	5	-24	-2	71	-12	1	-30	(55,61,69)	(56,14,20)	(56,20,62)	(56,62,70)
36	-17	28	-21	72	12	-1	-30	(58,16,42)	(58,42,28)	(58,52,16)	(58,65,66)
								(58,66,52)	(59,17,43)	(59,43,29)	(59,53,17)
								(63,70,71)	(63,71,57)	(64,22,41)	(64,41,27)
								(64,51,22)	(64,65,51)	(64,71,65)	(65,71,72)
								(66,65,72)	(67,53,59)	(67,59,66)	(67,66,72)
								(67,68,60)	(67,72,68)	(68,54,60)	(68,69,61)
								(68,72,69)	(69,70,62)	(69,72,70)	(71,70,72)

Vertex Permutation Groups

(1,3)(2,4)(5,10)(6,24)(7,17)(8,11)(9,25)(12,16)(13,47)(14,30)(15,53)(18,23)
(19,33)(20,38)(21,59)(22,48)(26,52)(27,61)(28,43)(29,34)(31,46)(32,39)(35,42)(36,54)
(37,60)(40,45)(41,55)(44,56)(49,58)(50,68)(51,62)(57,67)(63,66)(64,69)(65,70)(71,72)

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Table A12. R7.1 with S2 Symmetry

Vertex	X	Y	Z	Vertex	X	Y	Z	Triangles			
1	54	77	-11	37	-62	0	-1	(1,2,8)	(1,3,2)	(1,4,3)	(1,5,4)
2	26	-9	45	38	25	37	7	(1,6,5)	(1,7,6)	(1,8,7)	(2,3,10)
3	55	55	-24	39	-5	20	8	(2,9,8)	(2,10,16)	(2,16,23)	(2,23,9)
4	-10	50	23	40	5	-20	-31	(3,4,11)	(3,11,17)	(3,17,24)	(3,24,10)
5	6	40	22	41	20	-24	68	(4,18,25)	(8,15,7)	(8,29,15)	(10,33,52)
6	8	99	16	42	-85	-2	-40	(11,4,25)	(11,34,53)	(11,53,17)	(12,4,5)
7	14	65	-78	43	85	-51	-13	(12,5,26)	(12,18,4)	(12,35,54)	(12,54,18)
8	75	70	-5	44	-19	-6	-16	(13,5,6)	(13,6,27)	(13,19,5)	(14,6,7)
9	18	-51	35	45	-18	-76	-36	(14,7,28)	(16,10,52)	(19,26,5)	(20,6,14)
10	12	13	52	46	8	-40	-3	(20,27,6)	(20,33,27)	(21,7,15)	(21,15,57)
11	46	71	-32	47	20	36	87	(21,28,7)	(21,34,28)	(21,40,34)	(21,57,63)
12	47	-7	10	48	-21	43	-52	(21,63,40)	(22,8,9)	(22,9,51)	(22,29,8)
13	14	95	29	49	18	-7	-11	(22,35,29)	(22,41,35)	(30,17,43)	(30,24,17)
14	-75	44	-15	50	-56	0	14	(30,38,24)	(30,43,56)	(30,56,14)	(31,18,37)
15	0	-40	-83	51	-10	-24	-13	(31,25,18)	(31,37,57)	(31,57,15)	(32,26,19)
16	-18	7	11	52	-20	7	1	(32,51,9)	(34,42,28)	(35,43,29)	(36,16,42)
17	56	0	-14	53	43	-6	-26	(36,23,16)	(36,37,23)	(37,18,60)	(37,60,23)
18	19	6	16	54	81	-54	29	(38,19,61)	(38,32,19)	(38,51,32)	(38,61,24)
19	18	76	36	55	-72	-56	31	(39,20,62)	(39,25,31)	(39,33,20)	(39,52,33)
20	-8	40	3	56	-12	30	-10	(39,62,25)	(40,26,32)	(40,53,34)	(40,63,26)
21	-20	-36	-87	57	-37	-49	14	(41,27,33)	(44,14,28)	(44,28,58)	(44,30,14)
22	21	-43	52	58	-47	7	-10	(44,38,30)	(44,51,38)	(44,58,65)	(44,65,51)
23	12	-30	10	59	-14	-95	-29	(45,15,29)	(45,29,59)	(45,31,15)	(45,39,31)
24	37	49	-14	60	75	-44	15	(45,52,39)	(45,59,66)	(45,66,52)	(46,9,23)
25	10	24	13	61	0	40	83	(46,23,60)	(46,32,9)	(46,40,32)	(46,53,40)
26	20	-7	-1	62	-18	51	-35	(46,60,67)	(46,67,53)	(47,10,24)	(47,24,61)
27	-43	6	26	63	-12	-13	-52	(47,33,10)	(47,41,33)	(47,54,41)	(47,61,68)
28	-81	54	-29	64	-46	-71	32	(47,68,54)	(48,11,25)	(48,25,62)	(48,34,11)
29	72	56	-31	65	10	-50	-23	(48,42,34)	(48,55,42)	(48,62,69)	(48,69,55)
30	62	0	1	66	-6	-40	-22	(49,12,26)	(49,26,63)	(49,35,12)	(49,43,35)
31	-25	-37	-7	67	-8	-99	-16	(49,56,43)	(49,63,70)	(49,70,56)	(50,13,27)
32	5	-20	-8	68	-14	-65	78	(50,27,64)	(50,36,13)	(50,37,36)	(50,57,37)
33	-5	20	31	69	-75	-70	5	(50,64,71)	(50,71,57)	(54,35,41)	(54,60,18)
34	-20	24	-68	70	-26	9	-45	(55,13,36)	(55,19,13)	(55,36,42)	(55,61,19)
35	85	2	40	71	-55	-55	24	(55,69,61)	(56,20,14)	(56,62,20)	(56,70,62)
36	-85	51	13	72	-54	-77	11	(58,16,52)	(58,28,42)	(58,42,16)	(58,52,66)
								(58,66,65)	(59,17,53)	(59,29,43)	(59,43,17)
								(63,57,71)	(63,71,70)	(64,22,51)	(64,27,41)
								(64,41,22)	(64,51,65)	(64,65,71)	(65,72,71)
								(66,72,65)	(67,59,53)	(67,60,68)	(67,66,59)
								(67,68,72)	(67,72,66)	(68,60,54)	(68,61,69)
								(68,69,72)	(69,62,70)	(69,70,72)	(71,72,70)

Vertex Permutation Groups

(1,72)(2,70)(3,71)(4,65)(5,66)(6,67)(7,68)(8,69)(9,62)(10,63)(11,64)(12,58)
(13,59)(14,60)(15,61)(16,49)(17,50)(18,44)(19,45)(20,46)(21,47)(22,48)(23,56)(24,57)
(25,51)(26,52)(27,53)(28,54)(29,55)(30,37)(31,38)(32,39)(33,40)(34,41)(35,42)(36,43)

Table A13. R7.1 with C3 Symmetry

Vertex	X	Y	Z	Vertex	X	Y	Z	Triangles			
1	-7.56801	10.70538	5.21656	37	-8.08031	1.28652	13.15790	(1,2,3)	(1,2,4)	(1,3,5)	(1,4,7)
2	-20.81640	20.48934	-0.01571	38	8.38186	-2.47831	-7.09248	(1,5,8)	(1,7,12)	(1,8,12)	(2,3,6)
3	-24.55717	15.79025	-11.94582	39	-1.13204	-10.49296	1.61630	(2,4,10)	(2,6,11)	(2,10,18)	(2,11,18)
4	5.08631	0.37848	-2.07012	40	-9.68365	-23.53667	-1.22782	(3,5,9)	(3,6,13)	(3,9,16)	(3,13,16)
5	-7.29889	9.52212	8.19785	41	11.89584	1.55996	8.19785	(4,7,15)	(4,10,17)	(4,15,26)	(4,17,26)
6	-8.13559	-22.80113	-17.04527	42	-0.29098	25.70097	8.86883	(5,8,14)	(5,9,19)	(5,14,24)	(5,19,24)
7	4.05356	3.57506	-6.10364	43	-9.62736	18.72593	23.87503	(6,11,22)	(6,13,20)	(6,20,34)	(6,22,34)
8	-4.68011	-0.83056	16.84014	44	4.66235	1.38416	13.35325	(7,12,23)	(7,15,25)	(7,23,38)	(7,25,38)
9	-8.52114	6.22686	1.61630	45	21.50829	9.85368	25.64286	(8,12,21)	(8,14,27)	(8,21,35)	(8,27,35)
10	-2.87093	4.21563	-2.07012	46	13.05513	1.20140	5.21656	(9,16,31)	(9,19,28)	(9,28,47)	(9,31,47)
11	-22.11221	-13.10248	8.86883	47	2.92600	-7.64101	13.15790	(10,17,32)	(10,18,29)	(10,29,48)	(10,32,48)
12	-3.52990	3.34564	13.35325	48	-5.48712	-11.90677	5.21656	(11,18,33)	(11,22,36)	(11,33,53)	(11,36,53)
13	1.27231	-12.11945	-21.10028	49	-1.39617	-29.16226	-11.94582	(12,21,40)	(12,23,37)	(12,40,37)	(13,16,30)
14	1.62077	4.46838	16.84014	50	-15.67856	18.44619	-17.04527	(13,20,39)	(13,30,49)	(13,39,49)	(14,24,44)
15	34.45072	6.38996	0.12212	51	-11.69149	-33.03018	0.12212	(14,27,41)	(14,41,46)	(14,44,46)	(15,25,45)
16	-11.13191	4.95787	-21.10028	52	-19.28768	13.69988	25.64286	(15,26,42)	(15,42,43)	(15,45,43)	(16,30,56)
17	-2.21538	-4.59411	-2.07012	53	4.40467	-10.68459	10.20493	(16,31,50)	(16,56,50)	(17,26,46)	(17,32,51)
18	-22.75923	26.64022	0.12212	54	-5.92856	-0.30062	9.70240	(17,46,61)	(17,51,61)	(18,29,55)	(18,33,52)
19	-18.93555	12.65559	4.40191	55	-15.54153	20.15462	-1.22782	(18,55,52)	(19,24,43)	(19,28,54)	(19,43,59)
20	-0.66200	-10.32516	-6.69882	56	25.95334	13.37201	-11.94582	(19,54,59)	(20,34,61)	(20,39,57)	(20,57,44)
21	-10.25955	-19.67105	1.72717	57	5.15431	6.35450	13.15790	(20,61,44)	(21,33,58)	(21,33,62)	(21,35,62)
22	-11.53865	-2.84358	18.63150	58	-2.22060	-23.55356	25.64286	(21,40,58)	(22,34,59)	(22,36,63)	(22,59,42)
23	9.27285	4.58927	-6.69882	59	-11.45546	1.52774	10.20493	(22,63,42)	(23,30,60)	(23,30,64)	(23,37,64)
24	-11.90586	18.72056	1.72717	60	23.81415	4.35494	-17.04527	(23,38,60)	(24,43,52)	(24,44,55)	(24,52,55)
25	25.22518	3.38205	-1.22782	61	1.06931	-5.29802	-6.10364	(25,38,47)	(25,45,65)	(25,47,68)	(25,65,68)
26	28.15249	7.78286	-0.01571	62	-1.49228	-22.72646	4.40191	(26,42,50)	(26,46,56)	(26,50,56)	(27,35,48)
27	3.05935	-3.63782	16.84014	63	3.30671	11.41455	18.63150	(27,41,65)	(27,48,68)	(27,65,68)	(28,47,53)
28	3.22463	-4.98398	9.70240	64	9.65319	4.26610	1.61630	(28,53,62)	(28,54,66)	(28,62,66)	(29,31,67)
29	-5.12287	1.72296	-6.10364	65	22.16541	0.95049	1.72717	(29,31,68)	(29,48,68)	(29,55,67)	(30,49,60)
30	9.85960	7.16158	-21.10028	66	2.70393	5.28460	9.70240	(30,56,64)	(31,47,68)	(31,50,67)	(32,48,49)
31	-8.61085	5.73589	-6.69882	67	-2.04465	8.49806	-7.09248	(32,49,60)	(32,51,69)	(32,60,69)	(33,52,58)
32	-7.33609	-28.27221	-0.01571	68	-1.13246	-4.72980	13.35325	(33,53,62)	(34,37,40)	(34,59,37)	(34,61,40)
33	-11.40346	-17.70050	23.87503	69	22.40318	-12.59849	8.86883	(35,39,49)	(35,48,49)	(35,62,39)	(36,38,60)
34	-6.33721	-6.01975	-7.09248	70	20.42783	10.07087	4.40191	(36,53,38)	(36,60,69)	(36,63,69)	(37,59,54)
35	-4.59695	-11.08208	8.19785	71	21.03082	-1.02543	23.87503	(37,64,54)	(38,47,53)	(39,57,66)	(39,62,66)
36	8.23194	-8.57097	18.63150	72	7.05079	9.15685	10.20493	(40,58,51)	(40,61,51)	(41,46,56)	(41,56,64)
								(41,64,70)	(41,65,70)	(42,43,59)	(42,63,50)
								(43,45,52)	(44,46,61)	(44,57,55)	(45,52,58)
								(45,58,71)	(45,65,71)	(50,67,63)	(51,58,71)
								(51,69,71)	(54,64,70)	(54,66,70)	(55,67,57)
								(57,66,72)	(57,67,72)	(63,67,72)	(63,69,72)
								(65,70,71)	(66,70,72)	(69,71,72)	(70,71,72)

Vertex Permutation Groups

(1,48,46)(2,32,26)(3,49,56)(4,10,17)(5,35,41)(6,60,50)(7,29,61)(8,27,14)
(9,39,64)(11,69,42)(12,68,44)(13,30,16)(15,18,51)(19,62,70)(20,23,31)(21,65,24)
(22,36,63)(25,55,40)(28,66,54)(33,71,43)(34,38,67)(37,47,57)(45,52,58)(53,72,59)

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Table A14. R7.1′ with C2 Symmetry

Vertex	X	Y	Z	Vertex	X	Y	Z	Polygons
1	-0.77228	-2.89729	35.97290	85	24.28991	13.49551	0.78380	(1,2,10,23,30,16,9)
2	0.77228	2.89729	35.97290	86	-52.70324	43.45797	167.66333	(1,7,6,5,4,3,2)
3	-23.96945	-5.27541	10.77695	87	-60.68767	-14.97272	-38.97069	(1,7,8,15,29,22,9)
4	-25.77635	-3.46354	8.21991	88	21.02101	9.79577	-3.24991	(2,3,11,24,31,17,10)
5	-23.37394	12.51777	6.14574	89	-18.71064	10.40841	3.84855	(3,4,12,25,32,18,11)
6	73.24480	-1.39085	118.17785	90	40.91615	-42.85596	-7.93866	(4,5,13,26,33,19,12)
7	48.91850	-17.87175	95.91875	91	-1.61397	-19.14422	11.99876	(5,6,14,27,34,20,13)
8	50.93260	0.10177	32.90858	92	60.68767	14.97272	-38.97069	(6,7,8,28,35,21,14)
9	23.96945	5.27541	10.77695	93	23.75477	-4.36734	3.29944	(8,15,43,50,57,36,28)
10	-48.91850	17.87175	95.91875	94	-67.66644	40.04232	-57.43208	(9,22,37,58,51,44,16)
11	-17.15787	-7.00599	3.69405	95	85.50433	-47.97673	-79.33041	(10,23,38,59,52,45,17)
12	-24.72954	-2.71453	3.80330	96	-21.02101	-9.79577	-3.24991	(11,24,39,60,53,46,18)
13	-22.49006	14.75382	3.32295	97	81.47538	25.48256	35.81934	(12,25,40,61,54,47,19)
14	37.93365	-42.26521	152.05721	98	-81.47539	-25.48256	35.81934	(13,26,41,62,55,48,20)
15	147.24025	17.12645	-13.08525	99	32.71715	13.79608	-47.99250	(14,27,42,63,56,49,21)
16	25.77635	3.46354	8.21991	100	21.88915	-13.32737	-0.26171	(15,29,64,92,99,106,43)
17	-50.93260	-0.10177	32.90858	101	-22.92542	28.83418	0.45938	(16,30,65,93,100,107,44)
18	-14.63621	-8.87724	5.10708	102	74.80479	-59.26288	-93.28657	(17,31,66,94,101,108,45)
19	-22.76907	5.54786	-0.59071	103	-30.63692	-29.02187	0.23133	(18,32,67,95,102,109,46)
20	0.31618	16.21817	22.03817	104	91.06590	10.89486	32.17862	(19,33,68,96,103,110,47)
21	-29.47967	-60.64068	-16.21246	105	-88.51120	-25.46504	36.73831	(20,34,69,97,104,111,48)
22	17.15787	7.00599	3.69405	106	43.83405	0.37877	-63.37598	(21,35,70,98,105,112,49)
23	-73.24480	1.39085	118.17785	107	22.76907	-5.54786	-0.59071	(22,29,64,71,78,85,37)
24	-83.46900	-18.69271	-27.84975	108	0.06085	33.59378	10.10680	(23,30,65,72,79,86,38)
25	26.14064	-21.56823	-29.65050	109	-11.08264	-13.51310	-2.71609	(24,31,66,73,80,87,39)
26	-23.75477	4.36734	3.29944	110	-27.26375	-29.41883	-1.75801	(25,32,67,74,81,88,40)
27	52.70324	-43.45797	167.66333	111	1.07018	8.93212	19.68921	(26,33,68,75,82,89,41)
28	22.31771	-21.42973	15.10144	112	-63.88404	17.99086	89.72525	(27,34,69,76,83,90,42)
29	83.46900	18.69271	-27.84975	113	8.30972	-2.76753	16.22901	(28,35,70,77,84,91,36)
30	23.37394	-12.51777	6.14574	114	-70.79834	52.51220	-57.49750	(36,57,122,128,134,148,91)
31	-147.24025	-17.12645	-13.08525	115	70.79834	-52.51220	-57.49750	(37,58,123,129,135,149,85)
32	27.68394	-23.17566	-27.27019	116	-18.90873	5.47599	-29.52697	(38,59,124,130,136,150,86)
33	-21.88915	13.32737	-0.26171	117	26.76078	44.16088	4.72524	(39,60,125,131,137,151,87)
34	113.29918	61.87540	52.23039	118	-91.06589	-10.89486	32.17861	(40,61,126,132,138,152,88)
35	3.99798	-36.88117	10.20856	119	-15.48267	-13.41034	-42.14531	(41,62,120,133,139,153,89)
36	-0.06085	-33.59378	10.10680	120	1.52311	-13.27359	20.80703	(42,63,121,127,140,154,90)
37	14.63621	8.87724	5.10708	121	-84.16370	-44.94635	-39.95758	(43,50,115,130,136,143,106)
38	-37.93365	42.26521	152.05721	122	15.61599	-12.63363	5.01436	(44,51,116,131,137,144,107)
39	-70.04882	-26.66724	-20.51257	123	-28.22441	12.97841	-12.26003	(45,52,117,132,138,145,108)
40	18.90873	-5.47599	-29.52698	124	118.67790	12.13467	57.79802	(46,53,118,133,139,146,109)
41	-8.30972	2.76753	16.22901	125	-25.67421	-9.29414	-24.20695	(47,54,119,127,140,147,110)
42	10.62177	-84.19041	-56.58475	126	25.67421	9.29414	-24.20695	(48,55,113,128,134,141,111)
43	146.46402	15.58591	-15.39566	127	-46.64407	-42.53370	-28.56465	(49,56,114,129,135,142,112)
44	24.72954	2.71453	3.80330	128	18.71064	-10.40841	3.84855	(50,57,122,74,67,95,115)
45	-22.31771	21.42973	15.10144	129	-85.50433	47.97673	-79.33041	(51,58,123,75,68,96,116)
46	-24.28991	-13.49551	0.78380	130	153.01996	17.63692	-12.20300	(52,59,124,76,69,97,117)
47	-0.82271	-4.55665	-14.34265	131	-25.48331	-3.11637	-37.65998	(53,60,125,77,70,98,118)
48	-1.52311	13.27359	20.80703	132	33.81064	36.84145	4.78407	(54,61,126,71,64,92,119)
49	-118.67792	-12.13467	57.79802	133	-1.07018	-8.93212	19.68921	(55,62,120,72,65,93,113)
50	67.66644	-40.04232	-57.43208	134	13.84846	-3.06746	6.59005	(56,63,121,73,66,94,114)
51	-26.14064	21.56823	-29.65050	135	-74.80478	59.26288	-93.28657	(71,78,104,97,117,132,126)
52	-3.99798	36.88117	10.20856	136	68.16949	54.79069	-45.99051	(72,79,105,98,118,133,120)
53	-40.21312	-15.64774	-6.85107	137	-6.64113	-3.16744	-22.87441	(73,80,99,92,119,127,121)
54	6.64113	3.16744	-22.87441	138	26.78432	25.07733	7.51519	(74,81,100,93,113,128,122)
55	0.63497	5.33467	23.37304	139	-8.10494	-12.02365	4.64034	(75,82,101,94,114,129,123)
56	-153.01996	-17.63692	-12.20300	140	-33.04987	-32.54760	-15.31074	(76,83,102,95,115,130,124)
57	22.92542	-28.83418	0.45938	141	8.10494	12.02365	4.64034	(77,84,103,96,116,131,125)
58	-27.68394	23.17566	-27.27019	142	-42.68237	45.53736	42.70621	(78,85,149,155,141,111,104)
59	29.47967	60.64068	-16.21246	143	84.16370	44.94635	-39.95758	(79,86,150,156,142,112,105)
60	-36.75237	-22.14047	-4.54998	144	0.82271	4.55665	-14.34265	(80,87,151,157,143,106,99)
61	25.48331	3.11637	-37.65998	145	1.61397	19.14422	11.99876	(81,88,152,158,144,107,100)
62	-0.63497	-5.33467	23.37305	146	-13.68730	-12.27985	4.68166	(82,89,153,159,145,108,101)
63	-68.16949	-54.79071	-45.99051	147	-46.60689	-43.02608	-24.86275	(83,90,154,160,146,109,102)

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Vertex	X	Y	Z	Vertex	X	Y	Z	Polygons
64	70.04882	26.66724	-20.51257	148	6.23564	-19.53143	8.53529	(84,91,148,161,147,110,103)
65	22.49006	-14.75382	3.32295	149	11.08264	13.51310	-2.71609	(134,141,155,162,168,161,148)
66	-146.46400	-15.58591	-15.39566	150	-10.62177	84.19041	-56.58475	(135,142,156,163,162,155,149)
67	28.22441	-12.97841	-12.26003	151	15.48267	13.41034	-42.14531	(136,143,157,164,163,156,150)
68	-27.79708	1.31883	1.85603	152	30.63692	29.02187	0.23133	(137,144,158,165,164,157,151)
69	88.51120	25.46504	36.73831	153	-13.84846	3.06746	6.59005	(138,145,159,166,165,158,152)
70	-26.76078	-44.16088	4.72524	154	-3.47818	-18.12546	6.20864	(139,146,160,167,166,159,153)
71	36.75237	22.14047	-4.54998	155	13.68730	12.27985	4.68166	(140,147,161,168,167,160,154)
72	-0.31618	-16.21817	22.03817	156	-40.91615	42.85596	-7.93866	(162,168,167,166,165,164,163)
73	-43.83405	-0.37877	-63.37598	157	46.64407	42.53370	-28.56465	
74	32.41477	-23.82625	-14.33892	158	27.26375	29.41883	-1.75801	
75	-32.41477	23.82625	-14.33892	159	-6.23564	19.53143	8.53529	
76	63.88404	-17.99086	89.72525	160	-6.12312	-16.55262	4.05183	
77	-33.81063	-36.84145	4.78407	161	-0.65333	-20.57255	8.57434	
78	40.21312	15.64774	-6.85107	162	6.12312	16.55262	4.05183	
79	-113.29918	-61.87540	52.23039	163	3.47818	18.12546	6.20864	
80	-32.71715	-13.79608	-47.99250	164	33.04987	32.54760	-15.31074	
81	27.79708	-1.31883	1.85603	165	46.60689	43.02608	-24.86275	
82	-15.61599	12.63363	5.01436	166	0.65333	20.57255	8.57434	
83	42.68237	-45.53736	42.70619	167	1.18737	11.23758	7.40785	
84	-26.78432	-25.07733	7.51519	168	-1.18737	-11.23758	7.40785	

Face Permutation Groups

(1,3)(2,4)(5,10)(6,24)(7,17)(8,11)(9,25)(12,16)(13,47)(14,30)(15,53)(18,23)
(19,33)(20,38)(21,59)(22,48)(26,52)(27,61)(28,43)(29,34)(31,46)(32,39)(35,42)(36,54)
(37,60)(40,45)(41,55)(44,56)(49,58)(50,68)(51,62)(57,67)(63,66)(64,69)(65,70)(71,72)

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Table A15. R7.1′ with C3 Symmetry

Vertex	X	Y	Z	Vertex	X	Y	Z	Polygons
1	-11.66809	2.49525	4.08575	85	16.46599	56.92002	8.46202	(1,2,9,11,12,10,8)
2	-18.45931	10.01290	-4.68825	86	55.43167	-8.22392	32.09068	(1,3,5,7,6,4,2)
3	-28.24467	11.32991	-8.45819	87	0	0	47.85097	(1,8,14,16,15,13,3)
4	-6.54753	12.68733	-4.08775	88	31.13094	-57.31091	25.77840	(2,4,17,19,20,18,9)
5	19.70385	32.62620	-15.86063	89	-10.80701	-39.12773	40.70345	(3,13,22,24,23,21,5)
6	26.34879	34.11556	-15.52326	90	14.74078	6.16394	8.21624	(4,6,29,31,32,30,17)
7	8.16186	36.83691	-22.78513	91	38.49994	13.96717	-7.56511	(5,21,34,36,35,33,7)
8	-11.47521	-12.91115	7.11907	92	23.50046	34.67405	-6.87747	(6,7,33,49,51,50,29)
9	-21.47845	7.49906	-7.51574	93	16.69732	10.07417	5.73427	(8,10,25,28,27,26,14)
10	20.97795	6.43444	38.91895	94	37.45202	-49.28170	5.82672	(9,18,41,44,43,42,11)
11	-12.29819	-18.37750	7.22446	95	31.03196	-52.83044	-4.11475	(10,12,45,47,48,46,25)
12	21.39051	3.41731	39.92117	96	41.06120	-42.71998	8.46202	(11,42,71,73,72,45,12)
13	-32.52810	6.64744	-10.38541	97	-50.21369	18.66139	-10.80719	(13,15,37,40,39,38,22)
14	-1.34547	-10.74567	13.35736	98	-57.52719	-14.20004	8.46202	(14,26,53,55,54,52,16)
15	-27.70766	21.46598	-10.01821	99	-51.27690	14.42504	-8.40311	(15,16,52,64,66,65,37)
16	0.83487	-4.24459	13.56147	100	-61.26849	-0.45924	-4.11475	(17,30,60,63,62,61,19)
17	-9.76829	14.68989	-0.27841	101	-4.91659	-21.38465	38.91894	(18,20,67,69,70,68,41)
18	0	0	-24.38698	102	7.99500	8.85724	4.08575	(19,61,101,103,102,67,20)
19	-9.76628	19.83930	7.22447	103	-5.44378	16.39341	7.11907	(21,23,56,59,58,57,34)
20	17.23360	14.85135	-7.51574	104	-38.10704	0.75093	-15.86063	(22,38,75,77,76,74,24)
21	26.08258	-4.97616	-54.49568	105	-16.71866	-13.92984	-53.64426	(23,24,74,94,96,95,56)
22	17.22153	14.62318	-31.21380	106	-35.98262	-11.35010	-22.78514	(25,46,87,89,88,86,28)
23	20.42293	-7.51387	-53.64426	107	-36.46960	-14.52402	-22.97034	(26,27,78,81,80,79,53)
24	22.77604	-3.82443	-51.54537	108	-1.15610	17.41107	-29.86531	(27,28,86,127,126,128,78)
25	31.29230	4.38350	41.21877	109	-5.06434	31.72013	-5.23190	(29,50,92,91,90,93,31)
26	-2.03227	-15.84786	8.21625	110	0	0	-63.53915	(30,32,97,99,100,98,60)
27	0.37582	-19.49739	5.73427	111	-11.91701	18.47532	-38.79691	(31,93,133,132,138,97,32)
28	49.78823	-13.00933	32.28438	112	-20.28708	-28.84959	15.00294	(33,35,84,83,82,85,49)
29	27.27638	41.49163	-11.40372	113	-49.57100	2.87622	-11.40373	(34,57,105,107,106,104,36)
30	-10.00394	28.30738	8.83657	114	-42.71934	5.76094	-15.52326	(35,36,104,131,129,130,84)
31	-14.84093	31.99392	15.00294	115	-15.25329	-23.87951	9.12835	(37,65,119,112,113,118,40)
32	-13.05362	25.14949	9.12836	116	9.97876	4.20762	13.35736	(38,39,108,109,111,110,75)
33	5.65664	38.84562	-22.97032	117	32.44392	13.26254	-10.01821	(39,40,118,99,97,138,108)
34	0	0	-75.60766	118	-44.33855	17.39310	-12.56723	(41,68,122,123,121,120,44)
35	-3.70425	21.44370	-53.64426	119	-17.07315	9.42322	5.73427	(42,43,114,113,112,115,71)
36	-8.73181	25.07626	-54.49568	120	3.67309	-11.35248	4.08575	(43,44,120,131,104,106,114)
37	-31.34590	26.35835	-7.56511	121	16.91899	-3.48224	7.11907	(45,72,124,82,83,125,47)
38	21.43705	3.23293	-35.02782	122	-13.65472	16.81607	39.92115	(46,48,132,133,134,135,87)
39	16.52827	7.18121	-32.16852	123	-16.06135	14.95022	38.91894	(47,125,109,108,138,132,48)
40	-41.77884	3.01498	-6.87747	124	8.14308	-1.65676	34.09456	(49,85,141,142,128,126,51)
41	4.24484	-22.35041	-7.51574	125	7.82292	52.34404	26.16505	(50,51,126,127,136,137,92)
42	-7.83767	-15.80453	-0.27840	126	41.26808	34.15564	-10.80719	(52,54,116,90,91,117,64)
43	-7.71379	-12.01399	-4.08775	127	47.33844	37.88400	-9.88364	(53,79,139,140,130,129,55)
44	0.55823	-20.99269	-4.68825	128	28.30691	-1.26998	9.12835	(54,55,129,131,120,121,116)
45	3.64823	-7.59125	40.47314	129	10.50718	-31.49387	-10.38541	(56,95,151,80,81,150,59)
46	39.28909	10.20470	40.70346	130	-21.27481	7.60270	-31.21380	(57,58,143,144,145,146,105)
47	10.28827	50.99398	23.37592	131	4.31035	-30.12555	-8.45819	(58,59,150,69,67,102,143)
48	34.06721	55.61563	25.77840	132	-20.59369	52.11719	32.09068	(60,98,154,153,152,149,63)
49	30.23653	53.28968	-4.11475	133	-13.62771	49.62254	32.28438	(61,62,147,88,89,148,101)
50	37.23215	29.70176	-12.56723	134	-19.44236	24.90819	41.21877	(62,63,149,94,74,76,147)
51	38.13091	37.19458	-8.40311	135	-28.48206	28.92301	40.70346	(64,117,144,143,102,103,66)
52	0	0	24.67595	136	15.65648	-7.70432	-29.86531	(65,66,103,101,148,155,119)
53	-7.15405	-40.32552	-7.56511	137	-2.04502	-17.90451	-32.16852	(68,70,142,141,156,157,122)
54	3.25848	2.84532	13.56147	138	-56.47773	22.05429	-9.88364	(69,150,81,78,128,142,70)
55	-4.73626	-34.72852	-10.01821	139	-14.48324	10.72329	-32.16853	(71,115,160,151,95,96,73)
56	30.81298	-24.32162	-22.97033	140	-13.51832	16.94857	-35.02782	(72,73,96,94,149,152,124)
57	-17.35077	-20.10010	-54.49568	141	2.45729	8.39389	32.14307	(75,110,159,158,137,136,77)
58	18.40319	-33.37714	-15.86063	142	29.51688	-5.49003	8.83657	(76,77,136,127,86,88,147)
59	27.82077	-25.48685	-22.78513	143	23.93432	18.79565	-8.45819	(79,80,151,160,162,161,139)
60	-8.49797	-2.06886	32.14307	144	22.02090	24.84643	-10.38540	(82,124,152,153,156,141,85)
61	-7.73578	-20.23337	39.92115	145	4.05328	-22.22587	-31.21379	(83,84,130,140,111,109,125)
62	-8.39833	0.63616	40.47312	146	-14.70007	-17.81242	-51.54537	(87,135,164,163,155,148,89)
63	-8.34952	-6.30249	32.65667	147	39.01795	-34.40689	23.37591	(90,116,121,123,134,133,93)

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Vertex	X	Y	Z	Vertex	X	Y	Z	Polygons
64	-4.09336	1.39927	13.56146	148	-11.84993	-29.29166	41.21876	(91,92,137,158,145,144,117)
65	-12.70852	9.68392	8.21625	149	-5.50633	-6.22374	34.09457	(98,100,107,105,146,165,154)
66	-8.63329	6.53805	13.35736	150	16.37055	-39.87651	-15.52326	(99,118,113,114,106,107,100)
67	17.90109	10.97978	-4.68825	151	13.14600	-51.61961	-8.40311	(110,111,140,139,161,166,159)
68	22.06448	-1.46180	7.22447	152	0	0	36.25107	(112,119,155,163,162,160,115)
69	14.26131	-0.67334	-4.08775	153	-2.63675	7.88049	34.09457	(122,157,167,164,135,134,123)
70	17.60596	1.11464	-0.27841	154	-61.40520	-7.79354	5.82673	(145,158,159,166,168,165,146)
71	-19.51295	-22.81736	8.83657	155	-36.16052	-36.61320	32.28437	(153,154,165,168,167,157,156)
72	9.63288	-4.07964	32.65667	156	-1.28336	10.38214	32.65667	(161,162,163,164,167,168,166)
73	6.04069	-6.32503	32.14307	157	4.75009	6.95509	40.47312	
74	44.69453	-43.44576	15.85656	158	-7.91873	-20.18149	-35.02782	
75	21.95861	1.08277	-38.79692	159	-10.04160	-19.55810	-38.79691	
76	41.41981	-32.94687	26.16506	160	8.94561	-52.81702	-10.80719	
77	30.00261	-11.47423	-5.23191	161	-14.50038	-9.70675	-29.86531	
78	35.12801	-3.14433	15.00294	162	9.13930	-59.93830	-9.88364	
79	18.27838	-37.68902	-6.87747	163	-34.83796	-43.89327	32.09066	
80	7.10640	-47.09486	-12.56723	164	-65.19814	1.69525	25.77840	
81	22.29462	-44.36786	-11.40373	165	-59.97240	-16.98372	15.85657	
82	23.95319	57.07524	5.82673	166	-24.93826	-20.24592	-5.23191	
83	15.27787	60.42947	15.85656	167	-49.30622	-16.58709	23.37591	
84	-8.07597	21.63684	-51.54537	168	-49.24272	-19.39717	26.16505	

Face Permutation Groups

(1,48,46)(2,32,26)(3,49,56)(4,10,17)(5,35,41)(6,60,50)(7,29,61)(8,27,14)
(9,39,64)(11,69,42)(12,68,44)(13,30,16)(15,18,51)(19,62,70)(20,23,31)(21,65,24)
(22,36,63)(25,55,40)(28,66,54)(33,71,43)(34,38,67)(37,47,57)(45,52,58)(53,72,59)

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Table A16. R8.1 with S2 Symmetry

Vertex	X	Y	Z	Triangles		
1	5	8	17	(1,2,4)	(1,3,2)	(1,4,7)
2	-16	-40	-4	(1,5,3)	(1,7,12)	(1,8,5)
3	6	-20	11	(1,12,14)	(1,14,8)	(2,3,6)
4	-7	-24	-1	(2,6,11)	(2,10,4)	(2,11,19)
5	9	33	38	(2,19,20)	(2,20,10)	(3,5,9)
6	19	-1	-11	(3,9,17)	(3,13,6)	(3,17,21)
7	8	13	16	(3,21,13)	(4,10,18)	(4,16,7)
8	41	43	15	(4,18,25)	(4,25,26)	(4,26,16)
9	-8	22	53	(5,8,15)	(5,15,24)	(5,18,9)
10	-8	-3	-6	(5,24,27)	(5,27,18)	(6,13,16)
11	28	3	-13	(6,15,11)	(6,16,30)	(6,30,33)
12	9	26	20	(6,33,15)	(7,13,32)	(7,16,13)
13	-1	-17	-21	(7,23,12)	(7,31,23)	(7,32,31)
14	30	46	13	(8,11,15)	(8,14,22)	(8,22,31)
15	0	61	-1	(8,31,34)	(8,34,11)	(9,10,37)
16	-3	-2	6	(9,18,10)	(9,22,17)	(9,37,39)
17	17	0	11	(9,39,22)	(10,20,23)	(10,23,38)
18	-3	-4	0	(10,38,37)	(11,28,19)	(11,34,38)
19	-30	-46	-13	(11,38,28)	(12,20,40)	(12,23,20)
20	-17	0	-11	(12,24,29)	(12,29,14)	(12,40,24)
21	-9	-26	-20	(13,21,28)	(13,28,39)	(13,39,32)
22	50	1	13	(14,17,22)	(14,26,41)	(14,29,26)
23	4	10	0	(14,41,17)	(15,29,24)	(15,33,36)
24	1	17	21	(15,36,29)	(16,26,29)	(16,29,35)
25	-41	-43	-15	(16,35,30)	(17,30,35)	(17,35,21)
26	-28	-3	13	(17,41,30)	(18,27,35)	(18,35,36)
27	-8	-13	-16	(18,36,25)	(19,21,42)	(19,25,36)
28	53	7	-3	(19,28,21)	(19,36,20)	(19,42,25)
29	-53	-7	3	(20,33,40)	(20,36,33)	(21,27,42)
30	8	3	6	(21,35,27)	(22,23,31)	(22,28,23)
31	3	4	0	(22,39,28)	(23,28,38)	(24,37,38)
32	-9	-33	-38	(24,38,27)	(24,40,37)	(25,32,39)
33	8	-22	-53	(25,39,26)	(25,42,32)	(26,37,41)
34	7	24	1	(26,39,37)	(27,34,42)	(27,38,34)
35	-4	-10	0	(29,36,35)	(30,31,33)	(30,34,31)
36	-50	-1	-13	(30,41,34)	(31,32,33)	(32,40,33)
37	-19	1	11	(32,42,40)	(34,41,42)	(37,40,41)
38	3	2	-6	(40,42,41)		
39	0	-61	1			
40	-6	20	-11			
41	16	40	4			
42	-5	-8	-17			

Vertex Permutation Groups

(1,42)(2,41)(3,40)(4,34)(5,32)(6,37)(7,27)
(8,25)(9,33)(10,30)(11,26)(12,21)(13,24)(14,19)
(15,39)(16,38)(17,20)(18,31)(22,36)(23,35)(28,29)

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Table A17. R8.1 with C3 Symmetry

Vertex	X	Y	Z	Triangles		
1	-2.20394	-3.33551	-11.69687	(1,2,3)	(1,3,5)	(1,4,2)
2	17.78823	-3.93397	12.12479	(1,5,8)	(1,7,4)	(1,8,14)
3	6.06313	-5.63495	3.91730	(1,12,7)	(1,14,12)	(2,4,10)
4	17.52774	-0.64880	9.89485	(2,6,3)	(2,10,20)	(2,11,6)
5	-4.80849	-4.53634	-18.63129	(2,19,11)	(2,20,19)	(3,6,13)
6	9.25037	-2.26494	-18.86588	(3,9,5)	(3,13,21)	(3,17,9)
7	3.99060	-0.24091	-11.69687	(3,21,17)	(4,7,16)	(4,16,26)
8	-4.92751	-25.57663	-1.02392	(4,18,10)	(4,25,18)	(4,26,25)
9	-0.74958	-4.70524	-15.12677	(5,9,18)	(5,15,8)	(5,18,27)
10	-9.81494	4.80626	-0.23560	(5,24,15)	(5,27,24)	(6,11,15)
11	-0.52731	-21.65107	-10.27507	(6,15,33)	(6,16,13)	(6,30,16)
12	-1.78667	3.57642	-11.69687	(6,33,30)	(7,12,23)	(7,13,16)
13	6.33283	-1.89610	-18.63129	(7,23,31)	(7,31,32)	(7,32,13)
14	-9.32575	-14.85507	9.89485	(8,11,34)	(8,15,11)	(8,22,14)
15	-6.58668	-6.87859	-18.86588	(8,31,22)	(8,34,31)	(9,10,18)
16	24.61376	8.52097	-1.02392	(9,17,22)	(9,22,39)	(9,37,10)
17	1.16061	-10.87326	10.05103	(9,39,37)	(10,23,20)	(10,37,38)
18	-14.92760	7.96358	-9.57584	(10,38,23)	(11,19,28)	(11,28,38)
19	16.34658	-0.10121	15.41202	(11,38,34)	(12,14,29)	(12,20,23)
20	-19.68626	17.05566	-1.02392	(12,24,40)	(12,29,24)	(12,40,20)
21	1.21049	-17.08186	17.65667	(13,28,21)	(13,32,39)	(13,39,28)
22	0.74512	-10.90311	-0.23560	(14,17,41)	(14,22,17)	(14,26,29)
23	-8.20199	15.50387	9.89485	(14,41,26)	(15,24,29)	(15,29,36)
24	-7.91158	-2.43335	3.91730	(15,36,33)	(16,29,26)	(16,30,35)
25	8.83621	6.44175	10.05103	(16,35,29)	(17,21,35)	(17,30,41)
26	9.06981	6.09685	-0.23560	(17,35,30)	(18,25,36)	(18,35,27)
27	-15.39857	7.49262	17.65667	(18,36,35)	(19,20,36)	(19,21,28)
28	0.56714	-16.90947	-9.57584	(19,25,42)	(19,36,25)	(19,42,21)
29	-12.30103	-13.43807	12.12479	(20,33,36)	(20,40,33)	(21,27,35)
30	19.01403	10.36887	-10.27507	(21,42,27)	(22,23,28)	(22,28,39)
31	-5.48720	17.37204	12.12479	(22,31,23)	(23,38,28)	(24,27,38)
32	1.84845	8.06830	3.91730	(24,37,40)	(24,38,37)	(25,26,39)
33	-2.66369	9.14353	-18.86588	(25,32,42)	(25,39,32)	(26,37,39)
34	-8.08564	14.20716	15.41202	(26,41,37)	(27,34,38)	(27,42,34)
35	-8.26094	-14.10595	15.41202	(29,35,36)	(30,31,34)	(30,33,31)
36	-18.48672	11.28220	-10.27507	(30,34,41)	(31,33,32)	(32,33,40)
37	-3.70007	3.00177	-15.12677	(32,40,42)	(34,42,41)	(37,41,40)
38	-9.99683	4.43151	10.05103	(40,41,42)		
39	4.44964	1.70347	-15.12677			
40	-1.52434	6.43244	-18.63129			
41	14.36046	8.94589	-9.57584			
42	14.18808	9.58924	17.65667			

Vertex Permutation Groups

(1,7,12)(2,31,29)(3,32,24)(4,23,14)(5,13,40)(6,33,15)(8,16,20)
(9,39,37)(10,22,26)(11,30,36)(17,25,38)(18,28,41)(19,34,35)(21,42,27)

Table A18. R8.1 with D2 Symmetry

Vertex	X	Y	Z	Triangles		
1	-35	4	132	(1,2,3)	(1,3,5)	(1,4,2)
2	-43	-54	36	(1,5,8)	(1,7,4)	(1,8,14)
3	-21	-57	-93	(1,12,7)	(1,14,12)	(2,4,10)
4	-102	80	53	(2,6,3)	(2,10,20)	(2,11,6)
5	-29	-40	-48	(2,19,11)	(2,20,19)	(3,6,13)
6	5	-16	-26	(3,9,5)	(3,13,21)	(3,17,9)
7	91	78	4	(3,21,17)	(4,7,16)	(4,16,26)
8	-1	-22	11	(4,18,10)	(4,25,18)	(4,26,25)
9	-38	4	-21	(5,9,18)	(5,15,8)	(5,18,27)
10	-91	-78	4	(5,24,15)	(5,27,24)	(6,11,15)
11	-4	-38	21	(6,15,33)	(6,16,13)	(6,30,16)
12	43	54	36	(6,33,30)	(7,12,23)	(7,13,16)
13	22	-1	-11	(7,23,31)	(7,31,32)	(7,32,13)
14	-57	21	93	(8,11,34)	(8,15,11)	(8,22,14)
15	0	0	-31	(8,31,22)	(8,34,31)	(9,10,18)
16	84	102	-10	(9,17,22)	(9,22,39)	(9,37,10)
17	-54	43	-36	(9,39,37)	(10,23,20)	(10,37,38)
18	-102	84	10	(10,38,23)	(11,19,28)	(11,28,38)
19	57	-21	93	(11,38,34)	(12,14,29)	(12,20,23)
20	35	-4	132	(12,24,40)	(12,29,24)	(12,40,20)
21	-4	-35	-132	(13,28,21)	(13,32,39)	(13,39,28)
22	-16	-5	26	(14,17,41)	(14,22,17)	(14,26,29)
23	102	-80	53	(14,41,26)	(15,24,29)	(15,29,36)
24	-5	16	-26	(15,36,33)	(16,29,26)	(16,30,35)
25	16	5	26	(16,35,29)	(17,21,35)	(17,30,41)
26	-40	29	48	(17,35,30)	(18,25,36)	(18,35,27)
27	-80	-102	-53	(18,36,35)	(19,20,36)	(19,21,28)
28	40	-29	48	(19,25,42)	(19,36,25)	(19,42,21)
29	4	38	21	(20,33,36)	(20,40,33)	(21,27,35)
30	80	102	-53	(21,42,27)	(22,23,28)	(22,28,39)
31	102	-84	10	(22,31,23)	(23,38,28)	(24,27,38)
32	38	-4	-21	(24,37,40)	(24,38,37)	(25,26,39)
33	29	40	-48	(25,32,42)	(25,39,32)	(26,37,39)
34	78	-91	-4	(26,41,37)	(27,34,38)	(27,42,34)
35	-78	91	-4	(29,35,36)	(30,31,34)	(30,33,31)
36	1	22	11	(30,34,41)	(31,33,32)	(32,33,40)
37	-22	1	-11	(32,40,42)	(34,42,41)	(37,41,40)
38	-84	-102	-10	(40,41,42)		
39	0	0	31			
40	21	57	-93			
41	4	35	-132			
42	54	-43	-36			

Vertex Permutation Groups

(1,21,20,41)(2,42,12,17)(3,19,40,14)(4,27,23,30)
(5,28,33,26)(6,25,24,22)(7,35,10,34)(8,13,36,37)
(9,11,32,29)(15,39)(16,18,38,31)

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Table A19. R8.1 with C4 Symmetry

Vertex	X	Y	Z	Triangles		
1	21	1	22	(1,2,4)	(1,3,2)	(1,4,7)
2	31	18	-29	(1,5,3)	(1,7,12)	(1,8,5)
3	-1	21	22	(1,12,14)	(1,14,8)	(2,3,6)
4	18	-31	-29	(2,6,11)	(2,10,4)	(2,11,19)
5	31	0	42	(2,19,20)	(2,20,10)	(3,5,9)
6	-18	31	-29	(3,9,17)	(3,13,6)	(3,17,21)
7	1	-21	22	(3,21,13)	(4,10,18)	(4,16,7)
8	-5	-7	3	(4,18,25)	(4,25,26)	(4,26,16)
9	3	8	10	(5,8,15)	(5,15,24)	(5,18,9)
10	27	7	-43	(5,24,27)	(5,27,18)	(6,13,16)
11	-7	27	-43	(6,15,11)	(6,16,30)	(6,30,33)
12	0	-31	42	(6,33,15)	(7,13,32)	(7,16,13)
13	-21	-1	22	(7,23,12)	(7,31,23)	(7,32,31)
14	8	-3	10	(8,11,15)	(8,14,22)	(8,22,31)
15	-2	-5	-1	(8,31,34)	(8,34,11)	(9,10,37)
16	-31	-18	-29	(9,18,10)	(9,22,17)	(9,37,39)
17	7	-5	3	(9,39,22)	(10,20,23)	(10,23,38)
18	11	-1	13	(10,38,37)	(11,28,19)	(11,34,38)
19	1	11	13	(11,38,28)	(12,20,40)	(12,23,20)
20	-5	2	-1	(12,24,29)	(12,29,14)	(12,40,24)
21	0	31	42	(13,21,28)	(13,28,39)	(13,39,32)
22	0	0	4	(14,17,22)	(14,26,41)	(14,29,26)
23	-7	5	3	(14,41,17)	(15,29,24)	(15,33,36)
24	36	1	45	(15,36,29)	(16,26,29)	(16,29,35)
25	2	5	-1	(16,35,30)	(17,30,35)	(17,35,21)
26	7	-27	-43	(17,41,30)	(18,27,35)	(18,35,36)
27	-1	36	45	(18,36,25)	(19,21,42)	(19,25,36)
28	-8	3	10	(19,28,21)	(19,36,20)	(19,42,25)
29	-1	-11	13	(20,33,40)	(20,36,33)	(21,27,42)
30	-27	-7	-43	(21,35,27)	(22,23,31)	(22,28,23)
31	-3	-8	10	(22,39,28)	(23,28,38)	(24,37,38)
32	-31	0	42	(24,38,27)	(24,40,37)	(25,32,39)
33	-11	1	13	(25,39,26)	(25,42,32)	(26,37,41)
34	-21	31	-38	(26,39,37)	(27,34,42)	(27,38,34)
35	5	-2	-1	(29,36,35)	(30,31,33)	(30,34,31)
36	0	0	-11	(30,41,34)	(31,32,33)	(32,40,33)
37	21	-31	-38	(32,42,40)	(34,41,42)	(37,40,41)
38	31	21	-38	(40,42,41)		
39	5	7	3			
40	1	-36	45			
41	-31	-21	-38			
42	-36	-1	45			
Vertex Permutation Groups						
(1,3,13,7)(2,6,16,4)(5,21,32,12)(8,17,39,23) (9,28,31,14)(10,11,30,26)(15,35,25,20)(18,19,33,29) (24,27,42,40)(34,41,37,38)						

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Table A20. R8.2 with C3 Symmetry

Vertex	X	Y	Z	Triangles		
1	14.16807	5.08777	-8.95402	(1,2,6)	(1,6,13)	(1,8,17)
2	3.78487	8.02386	0.86412	(1,12,2)	(1,13,25)	(1,17,19)
3	2.19958	-0.19126	4.98653	(1,19,12)	(1,25,8)	(2,10,20)
4	-1.26543	-1.80926	4.98653	(2,12,26)	(2,14,10)	(2,18,6)
5	2.11976	-4.30032	8.94061	(2,20,18)	(2,26,14)	(3,4,18)
6	14.11143	12.53091	-0.51153	(3,5,4)	(3,12,5)	(3,18,32)
7	-12.55265	-0.06217	4.39903	(3,21,34)	(3,29,21)	(3,32,29)
8	-2.67789	-14.81380	-8.95402	(3,34,12)	(4,5,13)	(4,13,35)
9	-3.16223	11.87491	-6.27571	(4,22,36)	(4,23,22)	(4,35,23)
10	-6.11647	-1.97390	-0.39480	(4,36,18)	(5,12,31)	(5,24,28)
11	12.18826	16.06597	1.10914	(5,28,33)	(5,31,24)	(5,33,13)
12	2.34960	1.11712	-0.96729	(6,7,11)	(6,11,27)	(6,18,30)
13	10.66116	-14.61757	6.85814	(6,27,13)	(6,30,7)	(7,9,11)
14	-2.50975	2.03888	2.22025	(7,21,33)	(7,26,9)	(7,28,26)
15	-15.92490	-1.64829	-11.19981	(7,30,21)	(7,33,28)	(8,15,17)
16	-8.70286	-8.67603	-6.27571	(8,21,30)	(8,22,34)	(8,25,22)
17	-11.49018	9.72602	-8.95402	(8,30,15)	(8,34,21)	(9,15,16)
18	-0.93415	2.00052	4.98653	(9,16,37)	(9,19,11)	(9,26,38)
19	6.53499	14.61551	-11.19981	(9,37,19)	(9,38,15)	(10,14,16)
20	-2.14225	1.47625	-0.96729	(10,16,27)	(10,23,20)	(10,24,31)
21	3.79637	-18.48632	-0.51153	(10,27,24)	(10,31,23)	(11,19,29)
22	-0.20735	-2.59337	-0.96729	(11,24,27)	(11,29,32)	(11,32,24)
23	-8.84130	-0.73414	0.86412	(12,19,31)	(12,34,26)	(13,27,35)
24	11.14989	-9.99959	10.89373	(13,33,25)	(14,22,25)	(14,25,16)
25	9.38991	-12.96722	-11.19981	(14,26,28)	(14,28,36)	(14,36,22)
26	1.34879	6.28397	-0.39480	(15,20,17)	(15,30,39)	(15,38,20)
27	6.33016	-10.83983	4.39903	(15,39,16)	(16,25,37)	(16,39,27)
28	-14.23484	-4.65630	10.89373	(17,20,23)	(17,23,35)	(17,29,19)
29	7.32861	16.54162	6.85814	(17,35,29)	(18,20,32)	(18,36,30)
30	-17.98977	-1.92405	6.85814	(19,37,31)	(20,38,32)	(21,29,41)
31	-0.51085	-3.19295	2.22025	(21,41,33)	(22,23,42)	(22,42,34)
32	2.66430	3.98593	8.94061	(23,31,42)	(24,32,40)	(24,40,28)
33	7.81941	-18.58833	1.10914	(25,33,37)	(26,34,38)	(27,39,35)
34	5.05643	-7.28972	0.86412	(28,40,36)	(29,35,41)	(30,36,39)
35	-17.90781	5.95540	-0.51153	(31,37,42)	(32,38,40)	(33,41,37)
36	-4.78406	0.31439	8.94061	(34,42,38)	(35,39,41)	(36,40,39)
37	11.86509	-3.19889	-6.27571	(37,41,42)	(38,42,40)	(39,40,41)
38	3.02060	1.15407	2.22025	(40,42,41)		
39	-20.00767	2.52236	1.10914			
40	3.08495	14.65588	10.89373			
41	6.22249	10.90200	4.39903			
42	4.76768	-4.31007	-0.39480			

Vertex Permutation Groups

(1,17,8)(2,23,34)(3,18,4)(5,32,36)(6,35,21)(7,27,41)(9,16,37)
(10,42,26)(11,39,33)(12,20,22)(13,29,30)(14,31,38)(15,25,19)(24,40,28)

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Table A21. R8.2 with D2 Symmetry

Vertex	X	Y	Z	Triangles		
1	29	9	-49	(1,2,12)	(1,6,2)	(1,8,25)
2	29	6	-65	(1,12,19)	(1,13,6)	(1,17,8)
3	6	-29	65	(1,19,17)	(1,25,13)	(2,6,18)
4	9	-29	49	(2,10,14)	(2,14,26)	(2,18,20)
5	-80	-9	13	(2,20,10)	(2,26,12)	(3,4,5)
6	-3	6	-15	(3,5,12)	(3,12,34)	(3,18,4)
7	-6	29	65	(3,21,29)	(3,29,32)	(3,32,18)
8	16	32	5	(3,34,21)	(4,13,5)	(4,18,36)
9	-9	29	49	(4,22,23)	(4,23,35)	(4,35,13)
10	0	0	-47	(4,36,22)	(5,13,33)	(5,24,31)
11	-6	-3	15	(5,28,24)	(5,31,12)	(5,33,28)
12	9	-80	-13	(6,7,30)	(6,11,7)	(6,13,27)
13	-54	1	18	(6,27,11)	(6,30,18)	(7,9,26)
14	-2	28	-40	(7,11,9)	(7,21,30)	(7,26,28)
15	20	39	19	(7,28,33)	(7,33,21)	(8,15,30)
16	-32	16	-5	(8,17,15)	(8,21,34)	(8,22,25)
17	39	-20	-19	(8,30,21)	(8,34,22)	(9,11,19)
18	6	3	15	(9,15,38)	(9,16,15)	(9,19,37)
19	-1	-54	-18	(9,37,16)	(9,38,26)	(10,16,14)
20	17	0	-13	(10,20,23)	(10,23,31)	(10,24,27)
21	0	0	47	(10,27,16)	(10,31,24)	(11,24,32)
22	20	-7	31	(11,27,24)	(11,29,19)	(11,32,29)
23	32	-16	-5	(12,26,34)	(12,31,19)	(13,25,33)
24	-29	-6	-65	(13,35,27)	(14,16,25)	(14,22,36)
25	7	20	-31	(14,25,22)	(14,28,26)	(14,36,28)
26	80	9	13	(15,16,39)	(15,17,20)	(15,20,38)
27	-17	0	-13	(15,39,30)	(16,27,39)	(16,37,25)
28	-9	80	-13	(17,19,29)	(17,23,20)	(17,29,35)
29	0	-17	13	(17,35,23)	(18,30,36)	(18,32,20)
30	0	17	13	(19,31,37)	(20,32,38)	(21,33,41)
31	2	-28	-40	(21,41,29)	(22,34,42)	(22,42,23)
32	3	-6	-15	(23,42,31)	(24,28,40)	(24,40,32)
33	-28	-2	40	(25,37,33)	(26,38,34)	(27,35,39)
34	28	2	40	(28,36,40)	(29,41,35)	(30,39,36)
35	-20	-39	19	(31,42,37)	(32,40,38)	(33,37,41)
36	1	54	-18	(34,38,42)	(35,41,39)	(36,39,40)
37	-20	7	31	(37,42,41)	(38,40,42)	(39,41,40)
38	54	-1	18	(40,41,42)		
39	-39	20	-19			
40	-29	-9	-49			
41	-16	-32	5			
42	-7	-20	-31			

Vertex Permutation Groups

(1,9,40,4)(2,7,24,3)(5,12,26,28)(6,11,32,18)
(8,16,41,23)(10,21)(13,19,38,36)(14,33,31,34)
(15,39,35,17)(20,30,27,29)(22,25,37,42)

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Table A22. R8.2 with C4 Symmetry

Vertex	X	Y	Z	Triangles		
1	0	0	-63	(1,2,6)	(1,6,13)	(1,8,17)
2	42	-27	-52	(1,12,2)	(1,13,25)	(1,17,19)
3	-3	65	31	(1,19,12)	(1,25,8)	(2,10,20)
4	-41	-44	-41	(2,12,26)	(2,14,10)	(2,18,6)
5	18	38	25	(2,20,18)	(2,26,14)	(3,4,18)
6	14	-55	-26	(3,5,4)	(3,12,5)	(3,18,32)
7	65	3	31	(3,21,34)	(3,29,21)	(3,32,29)
8	-42	27	-52	(3,34,12)	(4,5,13)	(4,13,35)
9	41	44	-41	(4,22,36)	(4,23,22)	(4,35,23)
10	44	-41	-41	(4,36,18)	(5,12,31)	(5,24,28)
11	45	61	26	(5,28,33)	(5,31,24)	(5,33,13)
12	55	14	-26	(6,7,11)	(6,11,27)	(6,18,30)
13	-27	-42	-52	(6,27,13)	(6,30,7)	(7,9,11)
14	61	-45	26	(7,21,33)	(7,26,9)	(7,28,26)
15	14	41	-28	(7,30,21)	(7,33,28)	(8,15,17)
16	3	-65	31	(8,21,30)	(8,22,34)	(8,25,22)
17	-14	55	-26	(8,30,15)	(8,34,21)	(9,15,16)
18	-14	-41	-28	(9,16,37)	(9,19,11)	(9,26,38)
19	27	42	-52	(9,37,19)	(9,38,15)	(10,14,16)
20	-38	18	25	(10,16,27)	(10,23,20)	(10,24,31)
21	-44	41	-41	(10,27,24)	(10,31,23)	(11,19,29)
22	-57	-27	-31	(11,24,27)	(11,29,32)	(11,32,24)
23	-65	-3	31	(12,19,31)	(12,34,26)	(13,27,35)
24	30	66	32	(13,33,25)	(14,22,25)	(14,25,16)
25	-55	-14	-26	(14,26,28)	(14,28,36)	(14,36,22)
26	57	27	-31	(15,20,17)	(15,30,39)	(15,38,20)
27	27	-57	-31	(15,39,16)	(16,25,37)	(16,39,27)
28	70	19	38	(17,20,23)	(17,23,35)	(17,29,19)
29	-27	57	-31	(17,35,29)	(18,20,32)	(18,36,30)
30	38	-18	25	(19,37,31)	(20,38,32)	(21,29,41)
31	41	-14	-28	(21,41,33)	(22,23,42)	(22,42,34)
32	-19	70	38	(23,31,42)	(24,32,40)	(24,40,28)
33	-41	14	-28	(25,33,37)	(26,34,38)	(27,39,35)
34	-61	45	26	(28,40,36)	(29,35,41)	(30,36,39)
35	-45	-61	26	(31,37,42)	(32,38,40)	(33,41,37)
36	66	-30	32	(34,42,38)	(35,39,41)	(36,40,39)
37	-18	-38	25	(37,41,42)	(38,42,40)	(39,40,41)
38	-66	30	32	(40,42,41)		
39	19	-70	38			
40	0	0	88			
41	-30	-66	32			
42	-70	-19	38			

Vertex Permutation Groups

(2,19,8,13)(3,23,16,7)(4,10,9,21)(5,20,37,30)
(6,12,17,25)(11,34,35,14)(15,33,18,31)(22,27,26,29)
(24,38,41,36)(28,32,42,39)

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Table A23. R10.1 with D2 Symmetry

Vertex	X	Y	Z	Triangles		
1	18	-26	-16	(1,2,3)	(1,3,5)	(1,4,2)
2	41	-45	-11	(1,5,8)	(1,7,4)	(1,8,14)
3	-4	0	-18	(1,12,7)	(1,14,20)	(1,20,12)
4	26	18	16	(2,4,10)	(2,6,3)	(2,10,22)
5	-34	19	-57	(2,11,6)	(2,19,11)	(2,22,29)
6	4	0	-18	(2,29,19)	(3,6,13)	(3,9,5)
7	37	49	40	(3,13,23)	(3,17,9)	(3,23,28)
8	49	-37	-40	(3,28,17)	(4,7,16)	(4,16,21)
9	-1	-1	-2	(4,18,10)	(4,21,34)	(4,24,18)
10	14	19	11	(4,34,24)	(5,9,21)	(5,15,8)
11	49	-40	-37	(5,18,15)	(5,21,26)	(5,26,33)
12	19	-14	-11	(5,33,18)	(6,11,26)	(6,15,24)
13	-41	45	-11	(6,16,36)	(6,24,13)	(6,26,16)
14	-26	-18	16	(6,36,15)	(7,12,27)	(7,13,33)
15	34	-19	-57	(7,17,16)	(7,27,13)	(7,30,17)
16	-19	-34	57	(7,33,30)	(8,11,25)	(8,15,22)
17	19	34	57	(8,22,30)	(8,25,14)	(8,30,34)
18	-49	37	-40	(8,34,11)	(9,10,36)	(9,17,30)
19	-40	-49	37	(9,19,21)	(9,25,19)	(9,30,10)
20	-45	-41	11	(9,36,25)	(10,18,23)	(10,23,27)
21	0	-4	18	(10,27,36)	(10,30,22)	(11,12,26)
22	11	-4	0	(11,19,25)	(11,27,12)	(11,34,27)
23	-49	40	-37	(12,20,32)	(12,22,28)	(12,28,26)
24	-18	26	-16	(12,32,22)	(13,14,25)	(13,24,14)
25	-14	-19	11	(13,25,33)	(13,27,23)	(14,17,31)
26	1	-1	2	(14,24,29)	(14,29,17)	(14,31,20)
27	40	49	37	(15,18,24)	(15,28,22)	(15,31,28)
28	-1	1	2	(15,36,31)	(16,17,29)	(16,26,21)
29	-37	-49	40	(16,29,32)	(16,32,36)	(17,28,31)
30	4	11	0	(18,20,23)	(18,32,20)	(18,33,32)
31	0	4	18	(19,20,21)	(19,23,20)	(19,29,35)
32	-4	-11	0	(19,35,23)	(20,31,21)	(21,31,34)
33	-11	4	0	(22,32,29)	(23,35,28)	(24,34,35)
34	45	41	11	(24,35,29)	(25,32,33)	(25,36,32)
35	-19	14	-11	(26,28,35)	(26,35,33)	(27,31,36)
36	1	1	-2	(27,34,31)	(30,33,35)	(30,35,34)
Vertex Permutation Groups						
(1,4,24,14)(2,34,13,20)(3,21,6,31)(5,16,15,17)(7,18,29,8) (9,26,36,28)(10,35,25,12)(11,27,23,19)(22,30,33,32)						

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Table A24. R10.2 with C2 Symmetry

Vertex	X	Y	Z	Triangles		
1	-29	-83	-105	(1,2,3)	(1,3,5)	(1,4,2)
2	-2	-20	-42	(1,5,8)	(1,7,4)	(1,8,11)
3	76	34	18	(1,10,7)	(1,11,14)	(1,13,10)
4	-41	-19	7	(1,14,17)	(1,16,13)	(1,17,16)
5	6	-26	-9	(2,4,9)	(2,6,3)	(2,8,6)
6	29	83	-105	(2,9,10)	(2,10,15)	(2,12,8)
7	3	31	-30	(2,14,12)	(2,15,16)	(2,16,18)
8	15	-7	0	(2,18,14)	(3,6,7)	(3,7,12)
9	-26	-37	-1	(3,9,5)	(3,11,9)	(3,12,13)
10	-6	26	-9	(3,13,18)	(3,15,11)	(3,17,15)
11	41	19	7	(3,18,17)	(4,5,9)	(4,6,13)
12	26	37	-1	(4,7,6)	(4,11,15)	(4,12,17)
13	-15	7	0	(4,13,12)	(4,15,5)	(4,17,18)
14	-3	-31	-30	(4,18,11)	(5,6,8)	(5,10,18)
15	-7	-7	2	(5,12,14)	(5,14,6)	(5,15,10)
16	2	20	-42	(5,16,12)	(5,18,16)	(6,10,13)
17	-76	-34	18	(6,11,16)	(6,14,11)	(6,16,17)
18	7	7	2	(6,17,10)	(7,8,12)	(7,9,16)
				(7,10,9)	(7,14,18)	(7,15,14)
				(7,16,15)	(7,18,8)	(8,9,11)
				(8,13,15)	(8,15,17)	(8,17,9)
				(8,18,13)	(9,13,16)	(9,14,13)
				(9,17,14)	(10,11,18)	(10,12,11)
				(10,17,12)	(11,12,16)	(13,14,15)
Vertex Permutation Groups						
(1,6)(2,16)(3,17)(4,11)(5,10)(7,14)(8,13)(9,12)(15,18)						

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Table A25. R13.1 with C2 Symmetry

Vertex	X	Y	Z	Triangles			
1	-18	27	8	(1,2,3)	(1,3,5)	(1,4,2)	(1,5,8)
2	79	-80	-125	(1,7,4)	(1,8,14)	(1,12,7)	(1,14,24)
3	12	10	-5	(1,20,12)	(1,24,20)	(2,4,10)	(2,6,3)
4	5	4	-6	(2,10,22)	(2,11,6)	(2,19,11)	(2,22,32)
5	-1	99	-61	(2,30,19)	(2,32,30)	(3,6,13)	(3,9,5)
6	27	23	5	(3,13,23)	(3,17,9)	(3,23,33)	(3,28,17)
7	-38	77	57	(3,33,28)	(4,7,16)	(4,16,31)	(4,18,10)
8	-5	-4	-6	(4,28,33)	(4,29,18)	(4,31,28)	(4,33,29)
9	-47	-13	-81	(5,9,21)	(5,15,8)	(5,21,29)	(5,27,15)
10	1	-99	-61	(5,29,30)	(5,30,32)	(5,32,27)	(6,11,26)
11	42	25	-13	(6,20,24)	(6,24,31)	(6,25,13)	(6,26,27)
12	-42	-25	-13	(6,27,20)	(6,31,25)	(7,12,15)	(7,15,35)
13	-27	-23	5	(7,19,30)	(7,21,16)	(7,23,21)	(7,30,23)
14	-72	-17	42	(7,35,19)	(8,15,18)	(8,16,14)	(8,18,19)
15	-79	80	-125	(8,19,33)	(8,23,34)	(8,33,23)	(8,34,16)
16	14	13	-4	(9,12,20)	(9,17,25)	(9,20,22)	(9,22,26)
17	47	13	-81	(9,25,34)	(9,26,21)	(9,34,12)	(10,14,35)
18	18	-27	8	(10,17,24)	(10,18,25)	(10,24,14)	(10,25,17)
19	38	-77	57	(10,26,22)	(10,35,26)	(11,14,16)	(11,16,26)
20	-49	-31	2	(11,17,28)	(11,18,36)	(11,19,18)	(11,28,14)
21	52	44	20	(11,36,17)	(12,13,15)	(12,29,33)	(12,32,13)
22	-82	-61	10	(12,33,32)	(12,34,29)	(13,21,23)	(13,22,36)
23	37	29	9	(13,25,15)	(13,32,22)	(13,36,21)	(14,28,30)
24	-52	-44	20	(14,29,35)	(14,30,29)	(15,25,18)	(15,26,35)
25	-12	-10	-5	(15,27,26)	(16,21,26)	(16,25,31)	(16,34,25)
26	-49	-9	-123	(17,27,32)	(17,32,24)	(17,36,27)	(18,21,36)
27	82	61	10	(18,29,21)	(19,24,33)	(19,31,24)	(19,35,31)
28	16	14	-3	(20,23,30)	(20,27,23)	(20,28,22)	(20,30,28)
29	72	17	42	(22,28,31)	(22,31,36)	(23,27,34)	(24,32,33)
30	94	97	73	(27,36,34)	(29,34,35)	(31,35,36)	(34,36,35)
31	-37	-29	9				
32	49	9	-123				
33	-14	-13	-4				
34	-16	-14	-3				
35	-94	-97	73				
36	49	31	2				

Vertex Permutation Groups

(1,18)(2,15)(3,25)(4,8)(5,10)(6,13)(7,19)(9,17)(11,12)
(14,29)(16,33)(20,36)(21,24)(22,27)(23,31)(26,32)(28,34)(30,35)

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Table A26. R13.1 with C3 Symmetry

Vertex	X	Y	Z	Triangles			
1	-29.01689	-20.72304	4.74648	(1,2,3)	(1,2,4)	(1,3,5)	(1,4,7)
2	6.54041	-0.52815	4.75682	(1,5,8)	(1,7,12)	(1,8,14)	(1,12,20)
3	0.01759	-8.95913	4.72149	(1,14,24)	(1,20,24)	(2,3,6)	(2,4,10)
4	21.20078	-19.86469	23.14094	(2,6,11)	(2,10,22)	(2,11,19)	(2,19,30)
5	0.51110	-1.53234	-0.04966	(2,22,32)	(2,30,32)	(3,5,9)	(3,6,13)
6	2.28051	5.20475	4.44815	(3,9,17)	(3,13,23)	(3,17,28)	(3,23,33)
7	-26.13677	7.45144	-11.59738	(3,28,33)	(4,7,16)	(4,10,18)	(4,16,31)
8	-3.72759	-5.40009	4.75682	(4,18,29)	(4,28,33)	(4,29,33)	(4,31,28)
9	6.07036	-5.06800	-6.16980	(5,8,15)	(5,9,21)	(5,15,27)	(5,21,29)
10	0.51354	-0.95603	-1.14829	(5,27,32)	(5,29,30)	(5,32,30)	(6,11,26)
11	-2.64214	0.41735	1.85202	(6,13,25)	(6,20,24)	(6,20,27)	(6,24,31)
12	1.35383	7.79109	-6.16980	(6,25,31)	(6,26,27)	(7,12,15)	(7,15,35)
13	7.75004	4.49480	4.72149	(7,16,21)	(7,19,30)	(7,21,23)	(7,30,23)
14	-1.08471	0.03327	-1.14829	(7,35,19)	(8,14,16)	(8,15,18)	(8,16,34)
15	1.53282	-0.40228	-0.82216	(8,18,19)	(8,19,33)	(8,33,23)	(8,34,23)
16	-27.80371	-8.42807	23.14094	(9,12,20)	(9,12,34)	(9,17,25)	(9,20,22)
17	-1.11479	-1.12632	-0.82216	(9,21,26)	(9,22,26)	(9,34,25)	(10,14,24)
18	0.95963	-2.49683	1.85202	(10,14,35)	(10,17,25)	(10,18,25)	(10,22,26)
19	1.68250	2.07948	1.85202	(10,24,17)	(10,35,26)	(11,14,16)	(11,14,28)
20	-7.42419	-2.72309	-6.16980	(11,17,36)	(11,19,18)	(11,26,16)	(11,28,17)
21	-5.64770	-0.62739	4.44815	(11,36,18)	(12,13,32)	(12,15,13)	(12,29,33)
22	32.45512	-14.76784	4.74648	(12,32,33)	(12,34,29)	(13,21,36)	(13,23,21)
23	-7.76763	4.46433	4.72149	(13,25,15)	(13,32,22)	(13,36,22)	(14,28,30)
24	-0.63254	1.02010	-0.26769	(14,30,29)	(14,35,29)	(15,18,25)	(15,27,26)
25	19.52152	18.90939	-11.59738	(15,35,26)	(16,21,26)	(16,31,25)	(16,34,25)
26	-0.56716	-1.05785	-0.26769	(17,24,32)	(17,32,27)	(17,36,27)	(18,29,21)
27	-1.58260	0.32354	-0.04966	(18,36,21)	(19,24,31)	(19,33,24)	(19,35,31)
28	6.61525	-26.36083	-11.59738	(20,22,28)	(20,23,30)	(20,27,23)	(20,28,30)
29	1.19971	0.03775	-0.26769	(22,28,31)	(22,36,31)	(23,34,27)	(24,32,33)
30	-0.41803	1.52860	-0.82216	(27,36,34)	(29,34,35)	(31,35,36)	(34,36,35)
31	6.60293	28.29276	23.14094				
32	1.07150	1.20880	-0.04966				
33	3.36719	-4.57736	4.44815				
34	-3.43823	35.49088	4.74648				
35	0.57117	0.92275	-1.14829				
36	-2.81282	5.92824	4.75682				

Vertex Permutation Groups

(1,22,34)(2,36,8)(3,13,23)(4,31,16)(5,32,27)(6,21,33)
(7,28,25)(9,12,20)(10,35,14)(11,18,19)(15,30,17)(24,26,29)

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Table A27. R14.1 with D2 Symmetry

Vertex	X	Y	Z	Vertex	X	Y	Z	Triangles			
1	2	67	-15	79	82	-58	-9	(1,2,3)	(1,3,5)	(1,4,2)	(1,5,8)
2	-87	28	13	80	3	0	-37	(1,7,4)	(1,8,12)	(1,12,7)	(2,4,10)
3	-2	55	1	81	-80	58	-45	(2,6,3)	(2,10,18)	(2,11,6)	(2,18,11)
4	-84	39	4	82	-67	62	22	(3,6,13)	(3,9,5)	(3,13,16)	(3,16,9)
5	36	46	54	83	-13	-7	69	(4,7,15)	(4,15,26)	(4,17,10)	(4,26,17)
6	-43	85	-10	84	55	2	-1	(5,9,19)	(5,14,8)	(5,19,24)	(5,24,14)
7	-15	51	-12	85	-57	-5	25	(6,11,22)	(6,20,13)	(6,22,34)	(6,34,20)
8	34	62	4	86	-13	67	-47	(7,12,23)	(7,23,38)	(7,25,15)	(7,38,25)
9	40	67	38	87	84	-39	4	(8,14,27)	(8,21,12)	(8,27,35)	(8,35,21)
10	-58	-82	9	88	57	-23	-86	(9,16,31)	(9,28,19)	(9,31,47)	(9,47,28)
11	-82	58	-9	89	12	22	-41	(10,17,32)	(10,29,18)	(10,32,48)	(10,48,29)
12	-5	57	-25	90	87	-28	13	(11,18,33)	(11,33,53)	(11,36,22)	(11,53,36)
13	-18	39	23	91	28	-35	82	(12,21,40)	(12,37,23)	(12,40,37)	(13,20,39)
14	57	5	25	92	17	-19	0	(13,30,16)	(13,39,49)	(13,49,30)	(14,24,44)
15	-67	-13	47	93	-56	-6	57	(14,41,27)	(14,44,64)	(14,64,41)	(15,25,45)
16	-7	70	73	94	-14	-54	-90	(15,42,26)	(15,45,65)	(15,65,42)	(16,30,55)
17	-42	-96	29	95	62	-86	6	(16,50,31)	(16,55,50)	(17,26,46)	(17,46,69)
18	-85	-43	10	96	-70	-7	-73	(17,51,32)	(17,69,51)	(18,29,54)	(18,52,33)
19	56	6	57	97	0	3	37	(18,54,52)	(19,28,51)	(19,43,24)	(19,51,66)
20	-49	57	33	98	46	-36	-54	(19,66,43)	(20,34,60)	(20,45,39)	(20,60,82)
21	13	7	69	99	7	-70	73	(20,82,45)	(21,35,61)	(21,56,40)	(21,61,83)
22	28	87	-13	100	22	-12	41	(21,83,56)	(22,36,41)	(22,41,84)	(22,57,34)
23	-62	34	-4	101	6	-25	0	(22,84,57)	(23,37,62)	(23,58,38)	(23,62,85)
24	24	-4	-13	102	-39	-84	-4	(23,85,58)	(24,43,71)	(24,67,44)	(24,71,67)
25	-18	16	5	103	-22	12	41	(25,38,63)	(25,39,45)	(25,63,89)	(25,89,39)
26	-58	-80	45	104	23	57	86	(26,42,70)	(26,68,46)	(26,70,68)	(27,36,86)
27	51	15	12	105	-14	-57	-45	(27,41,36)	(27,59,35)	(27,86,59)	(28,32,51)
28	57	-14	45	106	10	14	-4	(28,47,78)	(28,78,101)	(28,101,32)	(29,48,79)
29	-28	-87	-13	107	15	-51	-12	(29,62,54)	(29,79,102)	(29,102,62)	(30,49,80)
30	-6	6	-2	108	-62	86	6	(30,72,55)	(30,80,103)	(30,103,72)	(31,50,56)
31	-5	66	78	109	-67	40	-38	(31,56,104)	(31,73,47)	(31,104,73)	(32,74,48)
32	49	-57	33	110	2	-55	1	(32,101,74)	(33,52,72)	(33,72,105)	(33,75,53)
33	-57	-49	-33	111	70	7	-73	(33,105,75)	(34,57,90)	(34,87,60)	(34,90,87)
34	58	82	9	112	13	104	-14	(35,59,91)	(35,70,61)	(35,91,70)	(36,53,81)
35	19	17	0	113	-4	-24	13	(36,81,86)	(37,40,77)	(37,54,62)	(37,77,109)
36	39	84	-4	114	-51	-15	12	(37,109,54)	(38,58,67)	(38,67,88)	(38,88,63)
37	-46	36	-54	115	49	41	-52	(39,76,49)	(39,89,76)	(40,50,106)	(40,56,50)
38	-17	19	0	116	-54	14	90	(40,106,77)	(41,64,98)	(41,98,84)	(42,61,70)
39	-6	25	0	117	66	5	-78	(42,65,99)	(42,99,122)	(42,122,61)	(43,66,100)
40	4	24	13	118	-19	-17	0	(43,80,71)	(43,100,123)	(43,123,80)	(44,58,124)
41	67	-2	15	119	39	18	-23	(44,67,58)	(44,92,64)	(44,124,92)	(45,82,93)
42	5	-66	78	120	-24	4	-13	(45,93,65)	(46,68,75)	(46,75,125)	(46,94,69)
43	14	-10	4	121	14	57	-45	(46,125,94)	(47,73,87)	(47,87,107)	(47,107,78)
44	7	-13	-69	122	-2	-18	2	(48,74,110)	(48,90,79)	(48,110,90)	(49,71,80)
45	-57	14	45	123	-3	0	-37	(49,76,111)	(49,111,71)	(50,55,97)	(50,97,106)
46	-86	-62	-6	124	-10	0	-32	(51,69,95)	(51,95,66)	(52,54,96)	(52,55,72)
47	67	13	47	125	-49	-41	-52	(52,96,126)	(52,126,55)	(53,68,108)	(53,75,68)
48	43	-85	-10	126	-6	-6	2	(53,108,81)	(54,109,96)	(55,126,97)	(56,83,118)
49	0	7	-17	127	5	-57	-25	(56,118,104)	(57,79,90)	(57,84,119)	(57,119,141)
50	2	18	2	128	80	-58	-45	(57,141,79)	(58,85,120)	(58,120,124)	(59,86,121)
51	67	-62	22	129	-36	-46	54	(59,100,91)	(59,121,142)	(59,142,100)	(60,73,143)
52	-39	-18	-23	130	54	-14	90	(60,87,73)	(60,112,82)	(60,143,112)	(61,113,83)
53	-96	42	-29	131	-12	-22	-41	(61,122,113)	(62,102,114)	(62,114,85)	(63,88,94)
54	-55	-2	-1	132	96	-42	-29	(63,94,144)	(63,115,89)	(63,144,115)	(64,92,107)
55	-7	0	17	133	-14	10	4	(64,107,127)	(64,127,98)	(65,93,129)	(65,110,99)
56	0	10	32	134	-16	-18	-5	(65,129,110)	(66,91,100)	(66,95,130)	(66,130,91)
57	85	43	10	135	-66	-5	-78	(67,71,117)	(67,117,88)	(68,70,116)	(68,116,108)
58	-7	13	-69	136	-2	-67	-15	(69,88,128)	(69,94,88)	(69,128,95)	(70,91,116)
59	16	18	-5	137	104	-13	14	(71,111,117)	(72,103,134)	(72,134,105)	(73,104,137)
60	42	96	29	138	0	-7	-17	(73,137,143)	(74,99,110)	(74,101,138)	(74,138,149)
61	0	-10	32	139	6	-56	-57	(74,149,99)	(75,105,139)	(75,139,125)	(76,89,140)
62	-67	2	15	140	0	-3	37	(76,119,111)	(76,140,148)	(76,148,119)	(77,106,115)
63	35	28	-82	141	57	49	-33	(77,115,152)	(77,121,109)	(77,152,121)	(78,92,151)

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Continued from previous page

Vertex	X	Y	Z	Vertex	X	Y	Z	Triangles			
64	62	-34	-4	142	25	6	0	(78,107,92)	(78,131,101)	(78,151,131)	(79,132,102)
65	-40	-67	38	143	86	62	-6	(79,141,132)	(80,123,133)	(80,133,103)	(81,108,112)
66	41	-49	52	144	14	54	-90	(81,112,150)	(81,135,86)	(81,150,135)	(82,108,145)
67	10	0	-32	145	-41	49	52	(82,112,108)	(82,145,93)	(83,113,127)	(83,127,146)
68	-104	13	14	146	-34	-62	4	(83,146,118)	(84,98,147)	(84,111,119)	(84,147,111)
69	-13	-104	-14	147	67	-40	-38	(85,114,146)	(85,129,120)	(85,146,129)	(86,109,121)
70	-23	-57	86	148	7	0	17	(86,135,109)	(87,90,136)	(87,136,107)	(88,117,128)
71	18	-2	-2	149	6	-6	-2	(89,106,140)	(89,115,106)	(90,110,136)	(91,130,116)
72	-25	-6	0	150	-57	23	-86	(92,124,150)	(92,150,151)	(93,120,129)	(93,133,120)
73	58	80	45	151	-35	-28	-82	(93,145,133)	(94,125,151)	(94,151,144)	(95,128,132)
74	18	-39	23	152	62	67	-22	(95,132,137)	(95,137,130)	(96,109,135)	(96,135,155)
75	-62	-67	-22	153	13	-67	-47	(96,138,126)	(96,155,138)	(97,126,131)	(97,131,156)
76	6	6	2	154	-28	35	82	(97,140,106)	(97,156,140)	(98,113,139)	(98,127,113)
77	-6	56	-57	155	-18	2	-2	(98,139,147)	(99,148,122)	(99,149,148)	(100,142,149)
78	18	-16	5	156	-10	-14	-4	(100,149,123)	(101,126,138)	(101,131,126)	(102,128,153)
								(102,132,128)	(102,153,114)	(103,133,145)	(103,145,154)
								(103,154,134)	(104,118,154)	(104,130,137)	(104,154,130)
								(105,134,153)	(105,147,139)	(105,153,147)	(107,136,127)
								(108,116,145)	(110,129,136)	(111,147,117)	(112,143,144)
								(112,144,150)	(113,122,156)	(113,156,139)	(114,118,146)
								(114,134,118)	(114,153,134)	(115,143,152)	(115,144,143)
								(116,130,154)	(116,154,145)	(117,147,153)	(117,153,128)
								(118,134,154)	(119,142,141)	(119,148,142)	(120,133,155)
								(120,155,124)	(121,141,142)	(121,152,141)	(122,140,156)
								(122,148,140)	(123,138,155)	(123,149,138)	(123,155,133)
								(124,135,150)	(124,155,135)	(125,131,151)	(125,139,156)
								(125,156,131)	(127,136,146)	(129,146,136)	(132,141,152)
								(132,152,137)	(137,152,143)	(142,148,149)	(144,151,150)

Vertex Permutation Groups

(1,62,136,41)(2,29,90,22)(3,54,110,84)(4,102,87,36)(5,37,129,98)(6,18,48,57)
(7,114,107,27)(8,23,146,64)(9,109,65,147)(10,79,34,11)(12,85,127,14)(13,52,74,119)
(15,153,47,86)(16,96,99,111)(17,132,60,53)(19,77,93,139)(20,33,32,141)(21,58,83,44)
(24,40,120,113)(25,134,78,59)(26,128,73,81)(28,121,45,105)(30,126,149,76)(31,135,42,117)
(35,38,118,92)(39,72,101,142)(43,106,133,156)(46,95,143,108)(49,55,138,148)(50,155,122,71)
(51,152,82,75)(56,124,61,67)(63,154,151,91)(66,115,145,125)(68,69,137,112)(70,88,104,150)
(80,97,123,140)(89,103,131,100)(94,130,144,116)

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Table A28. R14.2 with C2 Symmetry

Vertex	X	Y	Z	Vertex	X	Y	Z	Triangles			
1	-68	2	-198	79	74	41	-91	(1,2,3)	(1,3,5)	(1,4,2)	(1,5,13)
2	20	9	-15	80	61	-69	-13	(1,8,10)	(1,10,4)	(1,13,8)	(2,4,14)
3	21	8	3	81	-173	-67	-60	(2,6,11)	(2,9,3)	(2,11,9)	(2,14,6)
4	14	34	-118	82	-61	69	-13	(3,7,15)	(3,9,18)	(3,15,5)	(3,18,7)
5	-25	27	-98	83	212	60	-118	(4,10,21)	(4,12,30)	(4,21,12)	(4,30,14)
6	-38	30	31	84	-143	140	-172	(5,15,31)	(5,16,36)	(5,31,16)	(5,36,13)
7	-21	24	5	85	-20	-90	142	(6,14,43)	(6,28,49)	(6,32,28)	(6,43,32)
8	-21	-88	0	86	-41	51	166	(6,49,11)	(7,18,55)	(7,19,29)	(7,29,60)
9	33	33	70	87	47	-72	113	(7,55,19)	(7,60,15)	(8,13,42)	(8,20,38)
10	-14	-34	-118	88	27	183	91	(8,38,48)	(8,42,20)	(8,48,10)	(9,11,33)
11	15	29	11	89	-101	4	118	(9,17,39)	(9,33,17)	(9,39,18)	(10,22,40)
12	21	88	0	90	41	-51	166	(10,40,21)	(10,48,22)	(11,23,41)	(11,41,33)
13	-74	-41	-91	91	80	-176	98	(11,49,23)	(12,21,79)	(12,26,68)	(12,27,26)
14	15	21	0	92	-116	-156	1	(12,68,30)	(12,79,27)	(13,24,42)	(13,36,51)
15	-29	-4	-24	93	-66	125	103	(13,51,24)	(14,25,43)	(14,30,52)	(14,52,25)
16	50	155	-105	94	69	-114	-19	(15,34,53)	(15,53,31)	(15,60,34)	(16,26,37)
17	-2	89	90	95	152	21	14	(16,31,78)	(16,37,88)	(16,78,26)	(16,88,36)
18	39	0	22	96	8	2	45	(17,27,69)	(17,33,93)	(17,37,27)	(17,69,39)
19	54	-57	18	97	-33	-33	70	(17,93,37)	(18,35,55)	(18,39,63)	(18,63,35)
20	7	-94	-56	98	29	4	-24	(19,22,50)	(19,50,29)	(19,55,77)	(19,66,22)
21	68	-2	-198	99	77	-15	124	(19,77,66)	(20,23,62)	(20,42,92)	(20,62,38)
22	-15	-21	0	100	-27	-183	91	(20,67,23)	(20,92,67)	(21,40,64)	(21,45,79)
23	69	3	-40	101	9	-65	-17	(21,64,45)	(22,48,101)	(22,66,40)	(22,101,50)
24	-150	-81	-31	102	60	26	-34	(23,49,102)	(23,67,41)	(23,102,62)	(24,28,54)
25	-26	29	13	103	-90	-30	-36	(24,51,81)	(24,54,107)	(24,81,28)	(24,107,42)
26	-7	94	-56	104	136	88	-49	(25,29,50)	(25,50,108)	(25,52,82)	(25,82,29)
27	159	147	-68	105	-60	-26	-34	(25,108,43)	(26,27,37)	(26,59,68)	(26,78,59)
28	-77	15	124	106	23	-116	-152	(27,56,69)	(27,79,56)	(28,32,54)	(28,81,89)
29	-21	29	9	107	-19	-74	70	(28,89,49)	(29,70,60)	(29,82,70)	(30,46,84)
30	0	74	4	108	21	-29	9	(30,68,46)	(30,84,52)	(31,44,78)	(31,53,85)
31	-32	-6	-32	109	-170	-88	21	(31,85,44)	(32,34,54)	(32,43,76)	(32,76,86)
32	-38	46	143	110	-69	114	-19	(32,86,34)	(33,41,65)	(33,57,93)	(33,65,57)
33	20	90	142	111	140	-54	-209	(34,60,115)	(34,86,53)	(34,115,54)	(35,38,62)
34	-66	-57	52	112	-75	137	39	(35,62,118)	(35,63,95)	(35,95,38)	(35,118,55)
35	90	30	-36	113	-17	-47	174	(36,58,83)	(36,83,51)	(36,88,58)	(37,61,88)
36	4	214	-148	114	-90	44	14	(37,93,61)	(38,71,48)	(38,95,71)	(39,47,96)
37	116	156	1	115	69	-93	85	(39,69,47)	(39,96,63)	(40,44,97)	(40,66,44)
38	12	-135	-113	116	32	-57	216	(40,97,64)	(41,45,98)	(41,67,45)	(41,98,65)
39	5	3	77	117	-177	40	122	(42,72,92)	(42,107,72)	(43,73,76)	(43,108,73)
40	-20	-9	-15	118	90	-44	14	(44,66,123)	(44,85,97)	(44,123,78)	(45,64,98)
41	32	6	-32	119	173	67	-60	(45,67,124)	(45,124,79)	(46,51,111)	(46,68,127)
42	-159	-147	-68	120	176	-79	3	(46,81,51)	(46,111,84)	(46,127,81)	(47,52,112)
43	-54	57	18	121	-39	0	22	(47,69,128)	(47,82,52)	(47,112,96)	(47,128,82)
44	-15	-29	11	122	66	57	52	(48,71,74)	(48,74,101)	(49,75,102)	(49,89,75)
45	25	-27	-98	123	6	-35	16	(50,80,108)	(50,101,80)	(51,83,111)	(52,84,112)
46	-43	181	-180	124	-4	-214	-148	(53,57,113)	(53,86,57)	(53,113,85)	(54,87,107)
47	-26	28	75	125	26	-28	75	(54,115,87)	(55,90,77)	(55,118,90)	(56,79,106)
48	0	-74	4	126	67	25	-15	(56,99,104)	(56,104,69)	(56,106,119)	(56,119,99)
49	-6	35	16	127	-152	-21	14	(57,65,113)	(57,86,132)	(57,132,93)	(58,63,120)
50	26	-29	13	128	-47	72	113	(58,88,134)	(58,95,63)	(58,120,83)	(58,134,95)
51	-23	116	-152	129	91	-137	82	(59,78,105)	(59,103,68)	(59,105,110)	(59,110,114)
52	-9	65	-17	130	-140	54	-209	(59,114,103)	(60,70,91)	(60,91,115)	(61,93,117)
53	-38	-66	91	131	66	-125	103	(61,100,116)	(61,109,100)	(61,116,88)	(61,117,109)
54	-136	-88	-49	132	-26	56	202	(62,94,118)	(62,102,94)	(63,96,120)	(64,73,98)
55	64	-28	7	133	83	-71	42	(64,97,121)	(64,121,73)	(65,90,113)	(65,98,122)
56	150	81	-31	134	170	88	21	(65,122,90)	(66,77,99)	(66,99,123)	(67,92,100)
57	17	47	174	135	-44	20	55	(67,100,124)	(68,103,127)	(69,104,128)	(70,82,110)
58	194	134	-48	136	-5	-3	77	(70,105,129)	(70,110,105)	(70,129,91)	(71,95,119)
59	-83	56	-36	137	62	24	-6	(71,106,130)	(71,119,106)	(71,130,74)	(72,85,131)
60	-62	-24	-6	138	-194	-134	-48	(72,97,85)	(72,107,136)	(72,131,92)	(72,136,97)
61	-32	57	216	139	75	-137	39	(73,108,137)	(73,121,76)	(73,137,98)	(74,84,111)
62	83	-56	-36	140	-91	137	82	(74,111,139)	(74,130,84)	(74,139,101)	(75,89,135)
63	156	-53	26	141	26	-56	202	(75,96,112)	(75,112,140)	(75,135,96)	(75,140,102)

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Continued from previous page

Vertex	X	Y	Z	Vertex	X	Y	Z	Triangles			
64	-21	-8	3	142	-83	71	42	(76,103,114)	(76,114,86)	(76,121,103)	(77,90,122)
65	38	66	91	143	101	-4	118	(77,104,99)	(77,122,104)	(78,123,105)	(79,124,106)
66	38	-30	31	144	111	-39	49	(80,87,94)	(80,94,126)	(80,101,125)	(80,125,87)
67	-50	-155	-105	145	-156	53	26	(80,126,108)	(81,109,89)	(81,127,109)	(82,128,110)
68	-12	135	-113	146	-69	93	85	(83,91,129)	(83,120,91)	(83,129,111)	(84,130,112)
69	19	74	70	147	0	-33	18	(85,113,131)	(86,114,132)	(87,115,133)	(87,125,107)
70	-67	-25	-15	148	-212	-60	-118	(87,133,94)	(88,116,134)	(89,109,117)	(89,117,135)
71	43	-181	-180	149	177	-40	122	(90,118,141)	(90,141,113)	(91,120,144)	(91,144,115)
72	2	-89	90	150	-95	34	54	(92,116,100)	(92,131,116)	(93,132,117)	(94,102,126)
73	21	-24	5	151	-8	-2	45	(94,133,118)	(95,134,119)	(96,135,120)	(97,136,121)
74	143	-140	-172	152	-80	176	98	(98,137,122)	(99,119,143)	(99,143,123)	(100,109,138)
75	0	33	18	153	95	-34	54	(100,138,124)	(101,139,125)	(102,140,126)	(103,121,145)
76	-64	28	7	154	-111	39	49	(103,145,127)	(104,122,146)	(104,146,128)	(105,123,147)
77	38	-46	143	155	44	-20	55	(105,147,129)	(106,124,148)	(106,148,130)	(107,125,136)
78	-69	-3	-40	156	-176	79	3	(108,126,137)	(109,127,138)	(110,128,142)	(110,142,114)
								(111,129,139)	(112,130,140)	(113,141,131)	(114,142,132)
								(115,144,133)	(116,131,149)	(116,149,134)	(117,132,150)
								(117,150,135)	(118,133,141)	(119,134,143)	(120,135,144)
								(121,136,145)	(122,137,146)	(123,143,147)	(124,138,148)
								(125,139,151)	(125,151,136)	(126,140,152)	(126,152,137)
								(127,145,138)	(128,146,142)	(129,147,139)	(130,148,140)
								(131,141,149)	(132,142,150)	(133,144,153)	(133,153,141)
								(134,149,143)	(135,150,144)	(136,151,145)	(137,152,146)
								(138,145,156)	(138,156,148)	(139,147,151)	(140,148,152)
								(141,153,149)	(142,146,154)	(142,154,150)	(143,149,155)
								(143,155,147)	(144,150,153)	(145,151,156)	(146,152,154)
								(147,155,151)	(148,156,152)	(149,153,155)	(150,154,153)
								(151,155,156)	(152,156,154)	(153,154,155)	(154,156,155)

Vertex Permutation Groups

(1,21)(2,40)(3,64)(4,10)(5,45)(6,66)(7,73)(8,12)(9,97)(11,44)
(13,79)(14,22)(15,98)(16,67)(17,72)(18,121)(19,43)(20,26)(23,78)(24,56)
(25,50)(27,42)(28,99)(29,108)(30,48)(31,41)(32,77)(33,85)(34,122)(35,103)
(36,124)(37,92)(38,68)(39,136)(46,71)(47,125)(49,123)(51,106)(52,101)(53,65)
(54,104)(55,76)(57,113)(58,138)(59,62)(60,137)(61,116)(63,145)(69,107)(70,126)
(74,84)(75,147)(80,82)(81,119)(83,148)(86,90)(87,128)(88,100)(89,143)(91,152)
(93,131)(94,110)(95,127)(96,151)(102,105)(109,134)(111,130)(112,139)(114,118)(115,146)
(117,149)(120,156)(129,140)(132,141)(133,142)(135,155)(144,154)(150,153)

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Table A29. R14.3 with D2 Symmetry

Vertex	X	Y	Z	Vertex	X	Y	Z	Triangles			
1	35	62	32	79	0	-71	3	(1,2,21)	(1,3,2)	(1,13,43)	(1,20,3)
2	-35	23	35	80	58	1	65	(1,21,51)	(1,43,20)	(1,51,13)	(2,3,30)
3	-58	-1	65	81	-67	-83	-24	(2,6,42)	(2,30,34)	(2,34,6)	(2,42,21)
4	-9	-72	83	82	2	10	4	(3,12,60)	(3,20,33)	(3,33,12)	(3,60,30)
5	-13	3	10	83	77	-74	-44	(4,10,19)	(4,12,14)	(4,14,10)	(4,17,66)
6	-16	24	35	84	-3	-13	-10	(4,19,17)	(4,50,12)	(4,66,50)	(5,6,7)
7	-2	6	20	85	-1	-20	9	(5,7,15)	(5,15,29)	(5,18,67)	(5,29,18)
8	-58	-41	-34	86	-74	-77	44	(5,31,6)	(5,67,31)	(6,31,42)	(6,34,45)
9	-40	-22	-6	87	-53	71	-17	(6,45,7)	(7,17,22)	(7,22,15)	(7,45,80)
10	41	-58	34	88	-41	58	34	(7,80,17)	(8,9,11)	(8,11,13)	(8,13,61)
11	-127	4	-9	89	12	51	-48	(8,16,9)	(8,18,16)	(8,61,81)	(8,81,18)
12	-98	-7	95	90	-27	58	-40	(9,16,38)	(9,23,35)	(9,25,11)	(9,35,25)
13	-74	103	-57	91	13	-3	10	(9,38,23)	(10,14,24)	(10,24,39)	(10,26,36)
14	-103	-74	57	92	-6	-10	16	(10,36,19)	(10,39,26)	(11,25,27)	(11,27,87)
15	-9	-2	-3	93	7	-98	-95	(11,32,13)	(11,87,32)	(12,33,44)	(12,44,14)
16	6	-19	-3	94	-77	74	-44	(12,50,60)	(13,32,43)	(13,51,61)	(14,27,24)
17	51	-12	48	95	-72	-41	28	(14,44,94)	(14,94,27)	(15,22,57)	(15,28,54)
18	-18	-24	-2	96	-30	16	-39	(15,54,29)	(15,57,28)	(16,18,48)	(16,40,38)
19	83	-67	24	97	47	63	-2	(16,48,100)	(16,100,40)	(17,19,47)	(17,47,22)
20	-47	-19	81	98	2	-9	3	(17,80,66)	(18,29,48)	(18,81,67)	(19,36,40)
21	0	71	3	99	-2	9	3	(19,40,109)	(19,109,47)	(20,41,75)	(20,43,72)
22	-1	2	6	100	63	-47	2	(20,72,41)	(20,75,33)	(21,42,90)	(21,52,74)
23	-46	17	-3	101	62	-35	-32	(21,74,51)	(21,90,52)	(22,47,99)	(22,53,57)
24	-4	-127	9	102	-47	-63	-2	(22,99,53)	(23,38,70)	(23,70,112)	(23,76,35)
25	-55	-28	2	103	17	46	3	(23,112,113)	(23,113,76)	(24,27,63)	(24,58,39)
26	19	6	3	104	71	53	17	(24,63,114)	(24,114,58)	(25,35,53)	(25,53,119)
27	-109	6	-2	105	-23	-35	-35	(25,63,27)	(25,119,63)	(26,39,71)	(26,71,120)
28	-2	-1	-6	106	-30	-61	21	(26,77,36)	(26,97,77)	(26,120,97)	(27,94,87)
29	-4	-21	3	107	47	19	81	(28,57,89)	(28,89,111)	(28,92,54)	(28,98,92)
30	2	-6	20	108	10	-6	-16	(28,111,98)	(29,54,58)	(29,58,110)	(29,110,48)
31	-19	14	16	109	16	30	39	(30,59,91)	(30,60,73)	(30,73,59)	(30,91,34)
32	6	109	2	110	53	-71	-17	(31,56,68)	(31,67,56)	(31,68,103)	(31,103,42)
33	-71	0	-3	111	-6	-2	-20	(32,49,69)	(32,69,104)	(32,87,49)	(32,104,43)
34	16	-24	35	112	-43	-27	40	(33,70,105)	(33,75,70)	(33,105,44)	(34,71,106)
35	-20	1	-9	113	-66	15	24	(34,91,71)	(34,106,45)	(35,76,121)	(35,96,53)
36	24	-18	2	114	-6	-109	2	(35,121,96)	(36,77,122)	(36,84,40)	(36,122,84)
37	103	74	57	115	4	21	3	(37,50,66)	(37,66,101)	(37,69,88)	(37,83,128)
38	-14	-19	-16	116	-83	67	24	(37,88,50)	(37,101,83)	(37,128,69)	(38,40,84)
39	22	-40	6	117	72	-9	-83	(38,84,129)	(38,129,70)	(39,58,85)	(39,85,130)
40	44	-29	0	118	-37	64	-55	(39,130,71)	(40,100,109)	(41,72,107)	(41,86,95)
41	-64	-37	55	119	-21	4	-3	(41,95,102)	(41,102,75)	(41,107,86)	(42,76,90)
42	30	61	21	120	29	44	0	(42,103,76)	(43,77,72)	(43,104,77)	(44,62,94)
43	74	77	44	121	15	66	-24	(44,78,62)	(44,105,78)	(45,55,80)	(45,79,55)
44	-62	35	-32	122	21	-4	-3	(45,106,79)	(46,61,93)	(46,64,108)	(46,78,123)
45	35	-23	35	123	6	2	-20	(46,81,61)	(46,93,78)	(46,108,81)	(46,123,64)
46	-12	-51	-48	124	27	-58	-40	(47,64,99)	(47,82,64)	(47,109,82)	(48,65,100)
47	6	10	16	125	71	0	-3	(48,83,65)	(48,110,83)	(49,68,69)	(49,82,127)
48	41	-72	-28	126	30	-16	-39	(49,87,115)	(49,115,82)	(49,127,68)	(50,88,116)
49	-28	55	-2	127	1	20	9	(50,116,60)	(51,74,89)	(51,89,117)	(51,117,61)
50	9	72	83	128	109	-6	-2	(52,65,83)	(52,83,101)	(52,90,118)	(52,101,74)
51	-7	98	-95	129	-24	-16	-35	(52,118,65)	(53,96,57)	(53,99,119)	(54,85,58)
52	-19	47	-81	130	-17	-46	3	(54,92,131)	(54,131,85)	(55,79,93)	(55,86,107)
53	-10	2	-4	131	-16	-30	39	(55,93,132)	(55,107,80)	(55,132,86)	(56,67,102)
54	-2	-10	4	132	74	-103	-57	(56,88,68)	(56,95,134)	(56,102,95)	(56,134,88)
55	-35	-62	32	133	-41	72	-28	(57,96,135)	(57,135,89)	(58,114,110)	(59,64,82)
56	-19	-6	3	134	-24	18	2	(59,73,108)	(59,82,115)	(59,108,64)	(59,115,91)
57	-10	6	-16	135	67	83	-24	(60,92,73)	(60,116,92)	(61,117,93)	(62,65,118)
58	28	-55	-2	136	58	27	40	(62,78,79)	(62,79,124)	(62,118,94)	(62,124,65)
59	9	2	-3	137	10	-2	-4	(63,86,114)	(63,95,86)	(63,119,95)	(64,123,99)
60	-51	12	48	138	66	-15	24	(65,124,100)	(66,80,125)	(66,125,101)	(67,81,126)
61	-72	9	-83	139	23	35	-35	(67,126,102)	(68,88,69)	(68,127,103)	(69,128,104)
62	19	-47	-81	140	-15	-66	-24	(70,75,112)	(70,129,105)	(71,91,120)	(71,130,106)
63	-71	-53	17	141	-63	47	2	(72,77,97)	(72,97,136)	(72,136,107)	(73,92,98)

Continued on next page

Continued from previous page

Vertex	X	Y	Z	Vertex	X	Y	Z	Triangles			
64	2	1	-6	142	127	-4	-9	(73,98,137)	(73,137,108)	(74,101,139)	(74,111,89)
65	37	-64	-55	143	3	13	-10	(74,139,111)	(75,102,140)	(75,140,112)	(76,103,121)
66	98	7	95	144	18	24	-2	(76,113,90)	(77,104,122)	(78,93,79)	(78,105,123)
67	-29	-44	0	145	43	27	40	(79,106,124)	(80,107,125)	(81,108,126)	(82,109,127)
68	-22	40	6	146	55	28	2	(83,110,128)	(84,98,111)	(84,111,129)	(84,122,98)
69	4	127	9	147	24	16	-35	(85,112,130)	(85,113,112)	(85,131,113)	(86,132,114)
70	-61	30	-21	148	27	-43	-40	(87,94,133)	(87,133,115)	(88,134,116)	(89,135,117)
71	19	-14	16	149	-44	29	0	(90,113,141)	(90,141,118)	(91,115,144)	(91,144,120)
72	64	37	55	150	58	41	-34	(92,116,131)	(93,117,132)	(94,118,133)	(95,119,134)
73	1	-2	6	151	61	-30	-21	(96,97,120)	(96,120,135)	(96,121,97)	(97,121,136)
74	-1	58	-65	152	20	-1	-9	(98,122,137)	(99,123,143)	(99,143,119)	(100,124,138)
75	-58	-27	40	153	-6	19	-3	(100,138,109)	(101,125,139)	(102,126,140)	(103,127,145)
76	-27	43	-40	154	40	22	-6	(103,145,121)	(104,128,146)	(104,146,122)	(105,129,147)
77	72	41	28	155	14	19	-16	(105,147,123)	(106,130,148)	(106,148,124)	(107,136,125)
78	1	-58	-65	156	46	-17	-3	(108,137,126)	(109,138,127)	(110,114,142)	(110,142,128)
								(111,139,129)	(112,140,130)	(113,131,141)	(114,132,142)
								(115,133,144)	(116,134,149)	(116,149,131)	(117,135,150)
								(117,150,132)	(118,141,133)	(119,143,134)	(120,144,135)
								(121,145,136)	(122,146,137)	(123,147,143)	(124,148,138)
								(125,136,151)	(125,151,139)	(126,137,152)	(126,152,140)
								(127,138,145)	(128,142,146)	(129,139,147)	(130,140,148)
								(131,149,141)	(132,150,142)	(133,141,153)	(133,153,144)
								(134,143,149)	(135,144,150)	(136,145,151)	(137,146,152)
								(138,148,156)	(138,156,145)	(139,151,147)	(140,152,148)
								(141,149,153)	(142,150,154)	(142,154,146)	(143,147,155)
								(143,155,149)	(144,153,150)	(145,156,151)	(146,154,152)
								(147,151,155)	(148,152,156)	(149,155,153)	(150,153,154)
								(151,156,155)	(152,154,156)	(153,155,154)	(154,155,156)

Vertex Permutation Groups

(1,44,55,101)(2,105,45,139)(3,78,80,74)(4,117,50,61)(5,84,91,143)(6,129,34,147)
(7,111,30,123)(8,10,150,88)(9,39,154,68)(11,24,142,69)(12,93,66,51)(13,14,132,37)
(15,98,59,99)(16,26,153,56)(17,89,60,46)(18,36,144,134)(19,135,116,81)(20,62,107,52)
(21,33,79,125)(22,28,73,64)(23,130,156,103)(25,58,146,49)(27,114,128,32)(29,122,115,119)
(31,38,71,155)(35,85,152,127)(40,120,149,67)(41,65,72,118)(42,70,106,151)(43,94,86,83)
(47,57,92,108)(48,77,133,95)(53,54,137,82)(63,110,104,87)(75,124,136,90)(76,112,148,145)
(96,131,126,109)(97,141,102,100)(113,140,138,121)

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