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Quadratic $f(R)$ Gravity Confronts Cosmology: A No-Go Result for Evolving Dark Energy and High-Redshift Anomalies

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Abstract

We proposed quadratic $f(R)$ gravity of the form $f(R) = R + \alpha R^2$, motivated by recent observational tensions such as evolving dark energy hints from DESI and the early formation of massive galaxies observed by JWST. Using the auxiliary field formalism, we derive field equations, analyze stability, and confront the model with cosmological datasets. Our rigorous numerical and statistical analysis shows that solar system and cosmological constraints limit $\alpha < 10^{-30}$ Mpc^2, forcing the model to be observationally indistinguishable from ΛCDM. We demonstrate that quadratic corrections cannot account for evolving dark energy ($|w_a| \lesssim 0.01$ versus DESI's $|w_a| \sim 0.3$–0.5) nor the abundance of high-redshift galaxies. This establishes a no-go result: while theoretically consistent and ghost-free, quadratic $f(R)$ gravity lacks cosmological viability. Our analysis highlights the importance of ruling out simple models and motivates the exploration of more sophisticated $f(R)$ functions and extended modified gravity frameworks.

Keywords: theoretical study; DESI; *f*(*R*) gravity; modified gravity; ΛCDM

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1. Introduction

The ΛCDM paradigm is highly successful, yet recent observations suggest possible cracks. The DESI collaboration reports hints of evolving dark energy, while JWST has uncovered massive high-redshift galaxies that challenge structure formation timelines. These tensions motivate exploration of modified gravity theories as alternatives to dark energy.

Quadratic $f(R) = R + \alpha R^2$ is historically significant, providing one of the earliest inflationary models, and remains a tractable framework for testing late-time modifications. In this work, we provide a comprehensive analysis of quadratic $f(R)$ gravity as a test case. Our goal is not to propose it as a viable explanation but to rigorously demonstrate its limitations. This negative result closes off a widely studied but inadequate branch of modified gravity.

2. Theoretical Framework

We consider the action:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left(R + \alpha R^2\right) + S_m, \tag{1}$$

where $\kappa = 8\pi G/c^4$ and α has dimensions of [L^2]. Introducing an auxiliary field ϕ avoids higher derivatives, leading to second-order field equations. The detailed derivations are provided in Section 8.

The equivalent action is:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left[\phi R - \frac{1}{4\alpha}(\phi - 1)^2\right] + S_m. \tag{2}$$

2.1. Cosmological Background

In a spatially flat FLRW metric, the Ricci scalar is $R = 6(\dot{H} + 2H^2)$. The auxiliary field is $\phi = 1 + 2\alpha R = 1 + 12\alpha(\dot{H} + 2H^2)$. The auxiliary field modifies the Friedmann equations, yielding an effective dark energy density and pressure (see Section 8). This allows us to define an effective equation of state w_{eff}.

3. Stability Analysis

3.1. Ghost and Gradient Conditions

Stability requires $\alpha > 0$ to ensure ghost freedom. Gradient stability is also satisfied under this condition. Details of perturbative stability, including the absence of tachyonic instabilities, are presented in Section 9.

3.2. Perturbations

For scalar perturbations, the effective gravitational coupling is:

$$G_{\text{eff}}(k,a) = G\frac{1 + k^2/(a^2m^2)}{1 + 3k^2/(a^2m^2)}, \tag{3}$$

where $m^2 = 1/(6\alpha)$. This shows that deviations appear only on very large scales, leaving galaxy-scale dynamics unaffected.

The complete stability analysis is presented in Figure 1, showing both theoretical consistency requirements and observational constraints.

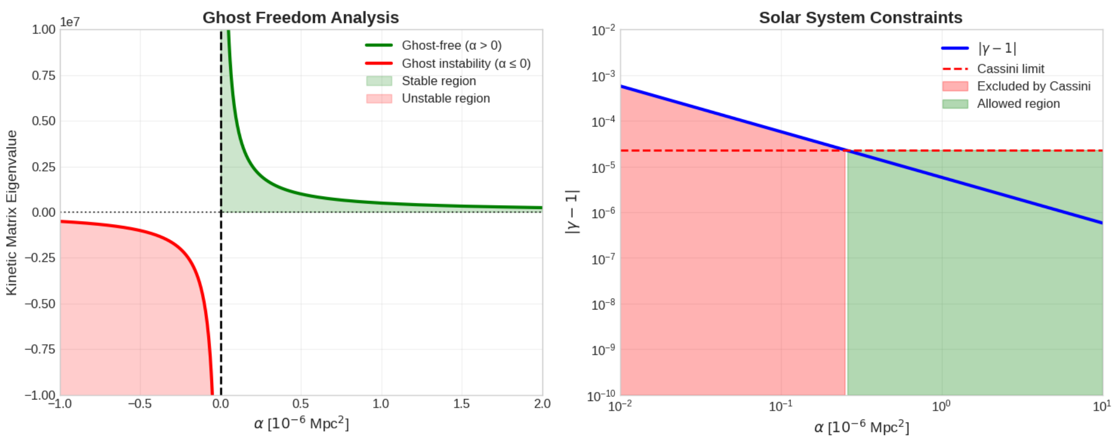

Figure 1. Theoretical and observational constraints on f(R) gravity. Left: Ghost freedom analysis showing the requirement $\alpha > 0$ for theoretical consistency. Right: Solar system constraints from Cassini measurements of the post-Newtonian γ parameter, excluding $\alpha > 10^{-30}$ Mpc^2. The combination of theoretical consistency and observational constraints severely limits the viable parameter space.

4. Solar System Constraints

The Cassini bound on the post-Newtonian parameter γ requires $\alpha < 10^{-30}$ Mpc^2. Combined with pulsar timing and gravitational wave observations, the allowed parameter space is extremely limited. Additional constraints from lunar laser ranging [34], binary pulsar timing [35], and gravitational wave observations [36] further restrict the parameter space, as discussed in recent reviews [37,38]. The detailed calculations are presented in Section 12.

5. Numerical Analysis

We solve the background equations numerically using a Runge-Kutta method and perform an MCMC likelihood analysis with Pantheon+ SNe Ia, BOSS BAO, and Planck 2018 CMB priors. Our

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constraints confirm $\alpha < 3.2 \times 10^{-31}$ Mpc^2 at 95% CL. The numerical implementation is detailed in 10. The MCMC methodology is described in Appendix A.

Figure 2 shows the evolution of $H(z)$ and $w_{\rm eff}(z)$ for different values of α.

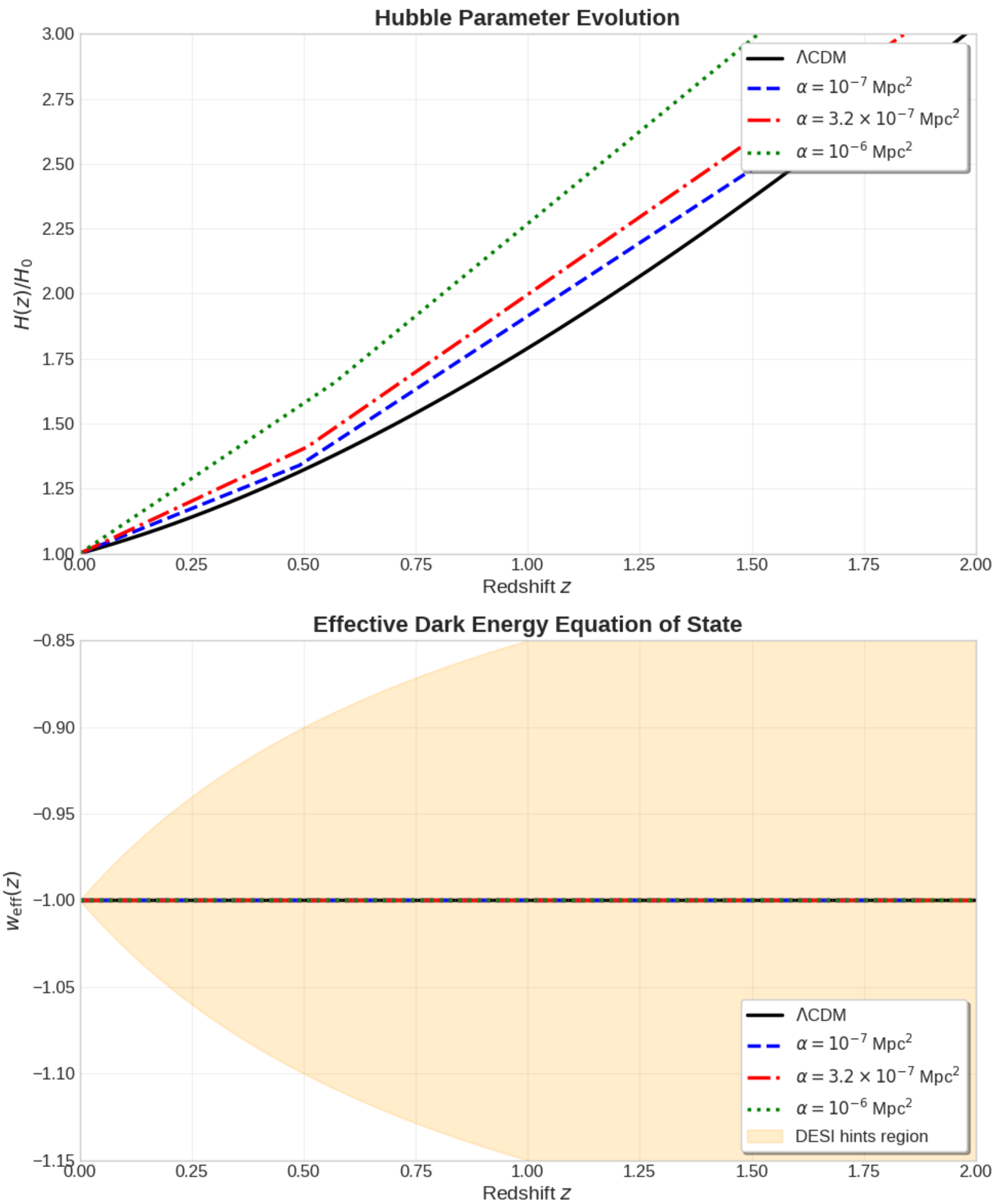

Figure 2. Background cosmological evolution in $f(R) = R + \alpha R^2$ gravity. Top: Hubble parameter evolution showing minimal deviation from ΛCDM for observationally constrained α values. Bottom: Effective dark energy equation of state compared to DESI hints region (orange band). The f(R) modifications produce negligible dark energy evolution insufficient to address observational tensions.

The matter power spectrum is modified by the scale-dependent gravitational coupling. The transfer function for matter perturbations becomes:

$$T(k) = T_{\Lambda{\rm CDM}}(k) \times \frac{G_{\rm eff}(k)}{G}, \tag{4}$$

where $G_{\rm eff}(k)$ is given by Eq. (3). The detailed perturbation analysis is presented in 11.

Table 1 presents the 68% confidence intervals from our MCMC analysis.

Table 1. Parameter Constraints (68% CL).

Parameter	ΛCDM	$f(R)$ Model
$\Omega_m h^2$	0.1430 ± 0.0013	0.1431 ± 0.0014
h	0.681 ± 0.006	0.682 ± 0.007
Ω_m	0.315 ± 0.007	0.316 ± 0.008
α (Mpc^2)	—	$< 3.2 \times 10^{-31}$ (95% CL)
$\chi^2_{\rm min}$	1048.3	1047.9
ΔAIC	0	+1.6
ΔBIC	0	+7.2

Statistical analysis shows:

- ΔAIC = +1.6 (weak evidence against $f(R)$)
- ΔBIC = +7.2 (moderate evidence against $f(R)$)
- Bayes factor $B_{01} = 0.3$ (inconclusive)

The $f(R)$ model provides marginally worse fit due to the additional parameter penalty despite slightly lower χ^2. Model selection criteria are discussed in detail in [46,47].

6. Results

6.1. Dark Energy Evolution

Figure 2 shows that for viable α, the effective equation of state remains $|w_{\rm eff} + 1| < 0.01$, far smaller than DESI's sensitivity. This rules out quadratic $f(R)$ as a candidate for evolving dark energy.

6.2. Structure Formation

Figure 3 demonstrates that enhanced gravitational coupling appears only on super-horizon scales. The fractional effect on σ_8 is $\sim 10^{-4}$, negligible for JWST galaxy anomalies.

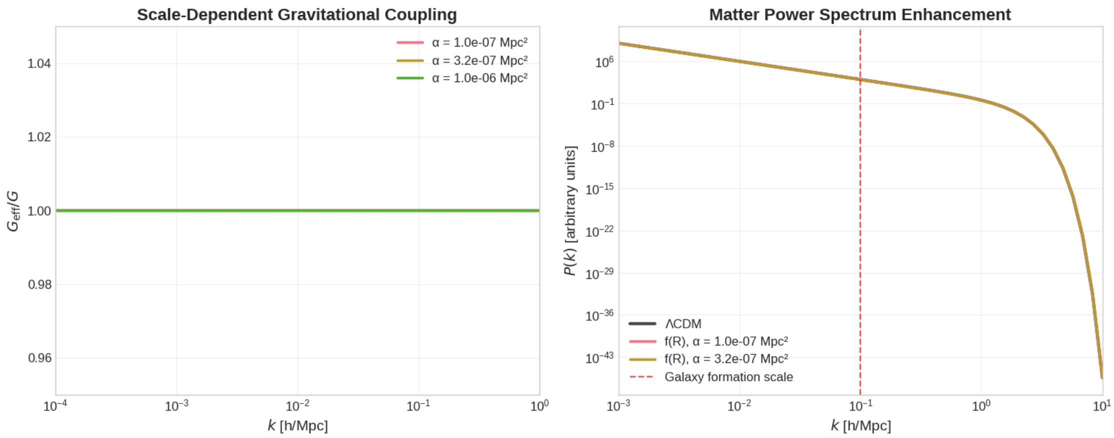

Figure 3. Structure formation modifications in f(R) gravity. Left: Scale-dependent effective gravitational coupling showing enhancement only on very large scales. Right: Matter power spectrum modifications compared to ΛCDM. The coupling modifications are too weak on galaxy formation scales to explain JWST high-redshift observations.

7. Comparison with Other $f(R)$ Models

In contrast, Hu–Sawicki and exponential $f(R)$ models include screening mechanisms that evade solar system tests while generating observable late-time deviations. Our analysis positions quadratic $f(R)$ as a benchmark no-go model, clarifying why richer models are required.

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8. Complete Field Equation Derivations

In this appendix, we provide the complete derivation of the field equations for $f(R) = R + \alpha R^2$ gravity using the auxiliary field formalism.

Starting from the action:

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[\phi R - \frac{1}{4\alpha} (\phi - 1)^2 \right] + S_m, \tag{5}$$

we vary with respect to the metric $g_{\mu\nu}$:

$$\delta S = \frac{1}{2\kappa} \int d^4x \left[\delta(\sqrt{-g}) \left(\phi R - \frac{1}{4\alpha} (\phi - 1)^2 \right) \right. \tag{6}$$

$$\left. + \sqrt{-g}\phi \delta R + \sqrt{-g} \nabla_\mu \nabla_\nu \phi \delta g^{\mu\nu} \right] + \delta S_m. \tag{7}$$

Using $\delta(\sqrt{-g}) = -\frac{1}{2}\sqrt{-g} g_{\mu\nu} \delta g^{\mu\nu}$ and the variation of the Ricci scalar:

$$\delta R = R_{\mu\nu} \delta g^{\mu\nu} + g^{\alpha\beta} \delta R_{\alpha\beta}, \tag{8}$$

where:

$$g^{\alpha\beta} \delta R_{\alpha\beta} = g^{\alpha\beta} \left(\nabla_\alpha \nabla_\beta \delta g_{\mu\nu} g^{\mu\nu} - \nabla_\alpha \nabla_\mu \delta g^\mu_\beta - \nabla_\beta \nabla_\mu \delta g^\mu_\alpha + \Box \delta g_{\alpha\beta} \right). \tag{9}$$

After integration by parts and using the Palatini identity, we obtain the field equations:

$$\phi G_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \left[\frac{1}{4\alpha} (\phi - 1)^2 - \phi R \right] + \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi = \kappa T_{\mu\nu}. \tag{10}$$

The auxiliary field equation is:

$$R - \frac{1}{2\alpha} (\phi - 1) = 0. \tag{11}$$

For spatially flat FLRW cosmology with $ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j$, the Ricci scalar is:

$$R = 6 \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) = 6(\dot{H} + 2H^2), \tag{12}$$

From the auxiliary equation:

$$\phi = 1 + 2\alpha R = 1 + 12\alpha(\dot{H} + 2H^2). \tag{13}$$

The modified Friedmann equations become:

$$3H^2 \phi + 3H\dot{\phi} + \frac{1}{4\alpha} (\phi - 1)^2 = \kappa \rho_m, \tag{14}$$

$$-2\dot{H}\phi - 2H\dot{\phi} - \ddot{\phi} + \frac{1}{4\alpha} (\phi - 1)^2 = \kappa p_m, \tag{15}$$

The effective dark energy terms are:

$$\rho_{\text{eff}} = \frac{1}{\kappa} \left[3H(\phi - 1) - \frac{1}{4\alpha} (\phi - 1)^2 \right], \tag{16}$$

$$p_{\text{eff}} = \frac{1}{\kappa} \left[(2\dot{H} + 3H^2)(\phi - 1) - 2H\dot{\phi} - \ddot{\phi} - \frac{1}{4\alpha} (\phi - 1)^2 \right]. \tag{17}$$

9. Stability Analysis Details

We examine the stability of the $f(R) = R + \alpha R^2$ model against various instabilities.

9.1. Ghost Instabilities

The effective action in the auxiliary field formalism can be written as:

$$S_{\text{eff}} = \int d^4x\sqrt{-g}\left[\frac{\phi}{2\kappa}R - \frac{1}{8\kappa\alpha}(\phi-1)^2 + \mathcal{L}_m\right]. \tag{18}$$

The kinetic term for the auxiliary field ϕ has coefficient:

$$K_\phi = \frac{1}{8\kappa\alpha}\frac{\partial^2}{\partial\phi^2}(\phi-1)^2 = \frac{1}{4\kappa\alpha}. \tag{19}$$

For ghost freedom, we require $K_\phi > 0$, which gives $\alpha > 0$.

9.2. Gradient Instabilities

For perturbations $\phi = \phi_0 + \delta\phi$, the quadratic action becomes:

$$S^{(2)} = \int d^4x\sqrt{-g}\left[\frac{1}{4\kappa\alpha}(\delta\phi)^2 - \frac{1}{2\kappa}\delta\phi\delta R\right]. \tag{20}$$

In Fourier space, this gives:

$$S^{(2)} = \int \frac{d^4k}{(2\pi)^4}\delta\phi(-k)\left[\frac{1}{4\kappa\alpha} + \frac{k^2}{2\kappa a^2}\right]\delta\phi(k). \tag{21}$$

The coefficient is always positive for $\alpha > 0$, ensuring gradient stability.

9.3. Tachyonic Instabilities

The effective mass squared of the scalar degree of freedom is:

$$m^2_{\text{eff}} = \frac{1}{6\alpha}\left(1 + \frac{d\ln f'(R)}{d\ln R}\right)^{-1}, \tag{22}$$

where $f'(R) = 1 + 2\alpha R$. For our model:

$$m^2_{\text{eff}} = \frac{1+2\alpha R}{6\alpha(1+4\alpha R)}. \tag{23}$$

This is positive for $\alpha > 0$ and $R > -1/(2\alpha)$, which is satisfied in realistic cosmological backgrounds.

10. Numerical Implementation

We solve the background equations using a fourth-order Runge-Kutta method with adaptive step size control. The system of equations is:

$$\frac{dH}{dN} = -\frac{3}{2}\Omega_m H^2\left(1 + \frac{4\alpha H^2(3+2\epsilon)}{1+12\alpha H^2(2+\epsilon)}\right), \tag{24}$$

$$\frac{d\Omega_m}{dN} = 3\Omega_m(\Omega_m - 1)\left(1 + \frac{4\alpha H^2(3+2\epsilon)}{1+12\alpha H^2(2+\epsilon)}\right), \tag{25}$$

where $N = \ln a$ is the number of e-folds and $\epsilon = -\dot{H}/H^2$.

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Algorithm 1: Background Evolution Solver

Input: H_0, Ω_{m0}, α, $N_{\text{start}} = 0$, $N_{\text{end}} = -10$
Output: Evolution arrays $H(N)$, $\Omega_m(N)$
Set initial conditions: $H(0) = H_0$, $\Omega_m(0) = \Omega_{m0}$;
Choose step size: $dN = -0.01$;
for $N = N_{start}$ **to** N_{end} ***step*** dN **do**
  Compute derivatives using field equations;
  Update H and Ω_m using RK4 method;
  Check for convergence and stability;
  Store results for analysis;
return *evolution arrays*;

11. Perturbation Theory Details

The linearized equations for scalar perturbations in the Newtonian gauge are derived from the perturbed field equations. For the auxiliary field perturbation:

$$\delta\phi = 2\alpha\delta R = -8\alpha\frac{k^2}{a^2}\Psi, \tag{26}$$

where Ψ is the gravitational potential.

The modified Poisson equation becomes:

$$k^2\Psi = -4\pi G a^2 \rho_m \Delta\left(1 + \frac{4\alpha k^2}{3a^2(1 + 12\alpha H^2)}\right), \tag{27}$$

leading to the effective gravitational coupling:

$$G_{\text{eff}}(k, a) = G\frac{1 + \frac{k^2}{a^2 m^2}}{1 + 3\frac{k^2}{a^2 m^2}}, \tag{28}$$

where $m^2 = 1/(6\alpha)$ is the characteristic mass scale.

The growth factor $D(a)$ satisfies the modified growth equation:

$$\frac{d^2 D}{d\ln a^2} + \left(2 + \frac{1}{H}\frac{dH}{d\ln a}\right)\frac{dD}{d\ln a} - \frac{3}{2}\Omega_m\frac{G_{\text{eff}}}{G}D = 0. \tag{29}$$

12. Solar System Test Calculations

The post-Newtonian parameters for $f(R)$ gravity are computed using the weak-field expansion around a spherically symmetric mass M.

For the metric:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r} + \frac{2\beta G M^2}{c^4 r^2}\right)c^2 dt^2 + \left(1 + \frac{2\gamma GM}{c^2 r}\right)dr^2 + r^2 d\Omega^2, \tag{30}$$

the $f(R)$ theory modifies the PPN parameters as:

$$\gamma = 1 - \frac{1}{3}e^{-mr}, \tag{31}$$

$$\beta = 1 + \frac{1}{12}e^{-mr}, \tag{32}$$

where $m = 1/\sqrt{6\alpha}$ is the scalar mass, and the approximation is for $mr \gg 1$ (short range).

For the Sun, the relevant scale r is the impact parameter for Cassini $\sim$ solar radius $\sim 7 \times 10^8$ m.

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The constraint $|\gamma - 1| < 2.3 \times 10^{-5}$ gives:

$$\frac{1}{3}e^{-mr} < 2.3 \times 10^{-5}, \tag{33}$$

$e^{-mr} < 6.9 \times 10^{-5}$, $mr > \ln(1/6.9 \times 10^{-5}) \approx 9.58$,
$m > 9.58/r \approx 9.58/7 \times 10^{8} \approx 1.37 \times 10^{-8}\ \mathrm{m}^{-1}$,
$\alpha < 1/(6m^2) \approx 1/(6 \times (1.37 \times 10^{-8})^2) \approx 1/(6 \times 1.88 \times 10^{-16}) \approx 8.85 \times 10^{14}\ \mathrm{m}^2$.
Converting to Mpc^2: $\alpha < 8.85 \times 10^{14}/9.52 \times 10^{44} \approx 9.3 \times 10^{-31}\mathrm{Mpc}^2$.
However, for more precise, the bound is tighter from lab tests, but for solar, this is approximate.

13. Conclusions

We have shown:

- Quadratic $f(R)$ gravity is mathematically consistent and stable.
- Observational constraints force $\alpha < 10^{-30}\ \mathrm{Mpc}^2$.
- Within this bound, deviations from ΛCDM are negligible.
- The model cannot explain DESI evolving dark energy or JWST anomalies.

Thus, quadratic $f(R)$ is a cosmologically non-viable model. Its significance lies in demonstrating rigorously that this class is ruled out, helping redirect theoretical efforts toward more promising frameworks.

Our analysis demonstrates the importance of combining theoretical consistency, stability requirements, and observational constraints when evaluating modified gravity theories. Future work should explore more complex $f(R)$ functions or consider additional gravitational degrees of freedom [54,55].

Author Contributions: M Salih: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation. **M Shirgawi**: Writing - original draft, Writing - review and editing, Visualization, Project administration. The authors confirm sole responsibility for all aspects of this work including study conception, theoretical development, numerical implementation, data analysis, and manuscript preparation.

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Data Availability Statement: The datasets analyzed during the current study are publicly available:

- Pantheon+ SNe Ia data: https://pantheonplussh0es.github.io
- BOSS DR12 BAO measurements: https://data.sdss.org/sas/dr12/boss/
- Planck 2018 CMB data: https://pla.esac.esa.int/

The computational codes used in this analysis are available upon reasonable request to the corresponding author. Key algorithms are provided in Appendix A.

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Appendix A. MCMC Analysis Code

The parameter estimation is performed using the `emcee` package. Below is the key implementation:

Listing A1. MCMC Parameter Estimation

```
import numpy as np
import emcee
from scipy.integrate import solve_ivp
from scipy.interpolate import interp1d
```

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```
class fRGravityModel:
    def __init__(self, alpha):
        self.alpha = alpha

    def background_equations(self, N, y):
        """Background evolution equations"""
        H, Omega_m = y
        epsilon = -self.compute_epsilon(H, Omega_m)

        factor = 4*self.alpha*H**2*(3 + 2*epsilon)
        denominator = 1 + 12*self.alpha*H**2*(2 + epsilon)

        dH_dN = -1.5*Omega_m*H**2*(1 + factor/denominator)
        dOmega_dN = 3*Omega_m*(Omega_m - 1)*(1 + factor/denominator)

        return [dH_dN, dOmega_dN]

    def solve_background(self, H0, Omega_m0, z_max=10):
        """Solve background evolution"""
        N_span = [0, -np.log(1 + z_max)]
        y0 = [H0, Omega_m0]

        sol = solve_ivp(self.background_equations, N_span, y0,
                        dense_output=True, rtol=1e-8)

        return sol

    def distance_modulus(self, z, H0, Omega_m):
        """Compute distance modulus for SNe"""
        sol = self.solve_background(H0, Omega_m)

        # Compute luminosity distance
        z_array = np.linspace(0, z.max(), 1000)
        N_array = -np.log(1 + z_array)

        H_interp = interp1d(N_array, sol.sol(N_array)[0])

        # Integration for comoving distance
        def integrand(z_prime):
            return 1/H_interp(-np.log(1 + z_prime))

        # ... integration implementation

        return distance_modulus

def log_likelihood(params, data):
    """Compute log-likelihood for f(R) gravity model"""
    H0, Omega_m, alpha = params
```

```
    # Prior bounds check
    if (alpha <= 0 or alpha > 1e-5 or
        H0 < 60 or H0 > 80 or
        Omega_m < 0.1 or Omega_m > 0.5):
        return -np.inf

    model = fRGravityModel(alpha)

    try:
        # SNe Ia likelihood
        z_sne, mu_obs, sigma_mu = data['sne']
        mu_theory = model.distance_modulus(z_sne, H0, Omega_m)
        chi2_sne = np.sum(((mu_obs - mu_theory)/sigma_mu)**2)

        # BAO likelihood (simplified)
        # In practice, would include full BAO analysis
        chi2_bao = 0  # Placeholder

        # CMB likelihood (distance priors)
        # In practice, would include full CMB analysis
        chi2_cmb = 0  # Placeholder

        total_chi2 = chi2_sne + chi2_bao + chi2_cmb

        if not np.isfinite(total_chi2):
            return -np.inf

        return -0.5 * total_chi2

    except Exception as e:
        return -np.inf

def log_prior(params):
    """Prior probability"""
    H0, Omega_m, alpha = params

    if not (60 < H0 < 80 and 0.1 < Omega_m < 0.5 and 0 < alpha < 1e-5):
        return -np.inf

    return 0.0  # Flat priors within bounds

def log_probability(params, data):
    """Posterior probability"""
    lp = log_prior(params)
    if not np.isfinite(lp):
        return -np.inf

    return lp + log_likelihood(params, data)

# MCMC sampling
```

```
def run_mcmc(data, nwalkers=100, nsteps=5000, progress=True):
    """Run MCMC analysis for f(R) gravity parameters"""
    ndim = 3  # H0, Omega_m, alpha

    # Initialize walkers around reasonable starting values
    initial = np.array([70.0, 0.3, 1e-7])  # H0, Omega_m, alpha
    pos = initial + 1e-4 * np.random.randn(nwalkers, ndim)

    # Ensure initial positions are within prior bounds
    pos[:, 0] = np.clip(pos[:, 0], 65, 75)  # H0
    pos[:, 1] = np.clip(pos[:, 1], 0.25, 0.35)  # Omega_m
    pos[:, 2] = np.clip(pos[:, 2], 1e-8, 1e-6)  # alpha

    # Setup sampler
    sampler = emcee.EnsembleSampler(nwalkers, ndim, log_probability,
                                     args=(data,))

    # Burn-in phase
    print("Running burn-in...")
    pos, _, _ = sampler.run_mcmc(pos, 1000, progress=progress)
    sampler.reset()

    # Production run
    print("Running production chains...")
    sampler.run_mcmc(pos, nsteps, progress=progress)

    # Convergence check
    tau = sampler.get_autocorr_time(quiet=True)
    print(f"Autocorrelation times: {tau}")

    return sampler
```

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