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Article

Friedmann Type Equations in Thermodynamic Form Lead to Much Tighter Constraints on the Critical Density of the Universe

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Abstract: Based on recent progress in cosmological thermodynamics, we present a new thermodynamic formulation of the Friedmann equations in the new Haug-Tatum cosmology model, as well as for other similar models put forward in the literature from different solutions to Einstein's field equation. Since the CMB temperature has been measured much more accurately than the Hubble constant, we demonstrate that our thermodynamic formulation of the Friedmann equation dramatically narrows the acceptable range for the critical density of the universe. Furthermore, we show how our thermodynamic formulation leads to a more precise cosmological constant estimate.

Keywords: friedmann equations; FLRW; universal critical density; cosmological constant; haug-tatum model; extremal solution; haug-spavieri solution; flat space cosmology

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1. The Friedmann Equation in Thermodynamic Form

The Friedmann-Lemaitre-Robertson-Walker (FLRW) metric is given by:

$$ds^2 = c^2 dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\Omega^2\right) \tag{1}$$

where $\Omega^2 = d\theta^2 + \sin^2\theta 2\phi^2$. The Friedmann [1] equation forms the backbone of multiple cosmological models, including the Λ-CDM model. It can be derived from the FLRW metric and is typically expressed as:

$$H_0^2 = \frac{8\pi\rho + \Lambda c^2}{3} - \frac{k^2c^2}{3} \tag{2}$$

where k is the curvature parameter and Λ is the cosmological constant. In 2015, Tatum et al. [2] heuristically suggested the following equation for the CMB temperature:

$$T_{cmb} = \frac{\hbar c}{k_b 4\pi\sqrt{R_h 2 l_p}} \tag{3}$$

where R_h represents the Hubble radius of the universe and l_p denotes the Planck length [3,4]. However, few have taken notice of this formula, and there could be several reasons for this. It was published in a low-ranked journal, and additionally, no derivation of the formula based on known laws of physics was presented. Haug and Wojnow [5,6] recently demonstrated that the equation can be derived from the Stefan-Boltzmann law. Furthermore, Haug and Tatum demonstrated that it can also be derived from more general geometric principles, where the observed CMB temperature can then be seen as the geometric mean of the lowest and highest possible temperatures in the Hubble sphere. Tatum et al. [7] has also reformulated formula (3) to (see also [8,9]) :

$$H_0 = \mho T_0^2. \tag{4}$$

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where $\mho$ is a composite constant of the form

$$\mho = \frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}} = 2.91845601 \times 10^{-19} \pm 0.00003279 \times 10^{-19}\, s^{-1} \cdot K^{-2}. \tag{5}$$

This value is based on NIST CODATA 2018. The only uncertainty in the Upsilon composite constant comes from the uncertainty in G, as the other constants, k_b, c, and $\hbar$, are exactly defined according to the NIST CODATA standard at the time of writing.

This means that, in the Friedmann equation, we can rewrite it as:

$$\begin{aligned} H_0^2 &= \frac{8\pi G\rho + \Lambda c^2}{3} - \frac{k^2c^2}{3} \\ T_0^4\mho^2 &= \frac{8\pi G\rho + \Lambda c^2}{3} - \frac{k^2c^2}{3} \\ T_0^4 &= \frac{8\pi G\rho + \Lambda c^2}{3\mho^2} - \frac{k^2c^2}{3\mho^2} \\ T_0 &= \left(\frac{8\pi G\rho + \Lambda c^2}{3\mho^2} - \frac{k^2c^2}{3\mho^2}\right)^{\frac{1}{4}}. \end{aligned} \tag{6}$$

Or, alternatively, solving for the density:

$$\rho = \frac{3T_0^4\mho^2 - \Lambda c^2 + kc^2}{8\pi G}. \tag{7}$$

Subsequently, we will now only focus on flat space cosmology and set $k = 0$, which involves, for example, several forms of $R_h = ct$ cosmological models, see for example [10,11]. In addition, we will first focus on the universal critical mass density, so we will also set $\Lambda = 0$, giving:

$$T_0 = \left(\frac{8\pi G\rho}{3\mho^2}\right)^{\frac{1}{4}} \tag{8}$$

or, alternatively, from the critical density perspective:

$$\rho_c = \frac{3T_0^4\mho^2}{8\pi G} = \frac{3T_0^4\left(\frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}}\right)^2}{8\pi G} = T_0^4\frac{384k_b^4\pi^3}{c^5\hbar^3}. \tag{9}$$

This thermodynamic formula for the critical density was recently provided by Haug and Tatum [9], but here the derivation is extended even further, so that we can see that G has canceled out; thus, the only uncertainty in critical density formula (9) is the uncertainty in the observed CMB temperature, T_0. Furthermore, we must have:

$$\frac{k_b^4 384\pi^3}{c^5\hbar^3} = \sigma\frac{23040\pi}{c^3} \tag{10}$$

where $\sigma = \frac{2\pi^5 k_b^4}{15c^2h^3}$ is the Stefan-Boltzman constant that is exactly defined since the 2019 re-definition of the S.I. standard, again because h, c and k_b were then exactly defined. So we can re-write the Friedmann critical density also as:

$$\rho_c = \frac{3T_0^4\mho^2}{8\pi G} = \frac{3T_0^4\left(\frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}}\right)^2}{8\pi G} = T_0^4\frac{384k_b^4\pi^3}{c^5\hbar^3} = T_0^4\sigma\frac{23040\pi}{c^3}. \tag{11}$$

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Dhal et al. [12] conducted one of the most recent and precise measurements of the CMB temperature, obtaining $T_0 = 2.725007 \pm 0.000024$ K (see also [13,14] for comparison). Therefore, we will use their CMB temperature as input to find a high-precision estimate of the critical density, resulting in:

$$\rho_c = \frac{3T_0^4 \mho^2}{8\pi G} = T_0^4 \frac{384 k_b^4 \pi^3}{c^5 \hbar^3} = T_0^4 \sigma \frac{23040\pi}{c^3} = 8.399481^{+0.000296}_{-0.000296} \times 10^{-27}\ kg \cdot m^{-3}. \tag{12}$$

Thus, we have a tighter-range estimate of the critical density of the universe than one can get by using the traditional formula $\rho_c = \frac{3H_0^2}{8\pi G}$.

2. How the Thermodynamic Friedmann Equation Yields a Higher Precision Critical Density Prediction in Comparison to the Standard Friedmann Equation

In this section, we compare the critical density predictions and uncertainties, using our new formulation and inputs from four recent CMB studies, with those from five H_0 studies. As we can see from Table 1, our new thermodynamic version of the Friedmann equation leads to a dramatic improvement in critical density precision in comparison to that of the traditional Friedmann equation relying upon H_0. The traditional method relying upon H_0 also inadvertently incorporates the Hubble tension problem, which clearly must widen the uncertainty greatly, as is clearly evident in the table. Since critical density plays an important role in multiple cosmological models including, of course, Λ-CDM, our new formulation should significantly enhance precision in a variety of estimates concerning the large-scale structure of the cosmos.

Table 1. This table compares Friedmann critical density estimates and uncertainties predicted using the new thermodynamic version of the Friedmann equation with those derived using the traditional method that relies on H_0 estimates. It is clearly evident that combining all recent CMB studies with our new thermodynamic version of the Friedmann equation dramatically improves critical density estimates and uncertainties.

CMB study :	CMB measurement T_0 :	Friedmann critical density: $\rho_c = T_0^4 \sigma \frac{23040\pi}{c^3}$:
2023: Dhal et. al [12] :	$2.725007 \pm 0.000024K$	$\rho_c = 8.399481^{+0.000296}_{-0.000296} \times 10^{-27}\ kg \cdot m^{-3}$
2009: Fixsen et. al [14] :	$2.72548 \pm 0.00057K$	$\rho_c = 8.40532^{+0.00703}_{-0.00703} \times 10^{-27}\ kg \cdot m^{-3}$
2011: Noterdaeme et. al [15] :	$2.725 \pm 0.002K$	$\rho_c = 8.3994^{+0.0247}_{-0.0247} \times 10^{-27}\ kg \cdot m^{-3}$
2004: Fixsen et. al [13] :	$2.721 \pm 0.010K$	$\rho_c = 8.350^{+0.123}_{-0.122} \times 10^{-27}\ kg \cdot m^{-3}$
H_0 Study :	H_0 estimate :	Friedmann critical density: $\rho_c = \frac{3H_0}{8\pi G}$:
2023: Murakami et al. [16] :	$73.01 \pm 0.85\ km/s/Mpc$	$\rho_c = 9.9917^{+0.2340}_{-0.2313} \times 10^{-27}\ kg \cdot m^{-3}$
2021: Riess et al. [17] :	$73.04 \pm 1.04\ km/s/Mpc$	$\rho_c = 9.9999^{+0.2868}_{-0.28275} \times 10^{-27}\ kg \cdot m^{-3}$
2021: Planck Collaboration [18] :	$67.4 \pm 0.5\ km/s/Mpc$	$\rho_c = 8.5152^{+0.1268}_{-0.1259} \times 10^{-27}\ kg \cdot m^{-3}$
2023: Sneppen et. al [19] :	$67.0 \pm 3.6\ km/s/Mpc$	$\rho_c = 8.4144^{+0.9285}_{-0.8799} \times 10^{-27}\ kg \cdot m^{-3}$
2023: Balkenhol et. al [20] :	$68.3 \pm 1.5\ km/s/Mpc$	$\rho_c = 8.7441^{+0.3883}_{-0.3799} \times 10^{-27}\ kg \cdot m^{-3}$

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Once again, the basis for calculating Friedmann critical density using the thermodynamic equations shown herein is our new and deeper theoretical understanding of the exact mathematical relationship between the Hubble constant and CMB temperature, see again [7,8].

3. Incorporating the Cosmological Constant into the Thermodynamic Friedmann Formulation

Next, let us add the cosmological constant. It is assumed in the Λ-CDM model that:

$$\Lambda = 3\left(\frac{H_0}{c}\right)^2 \Omega_\Lambda \tag{13}$$

where Ω_Λ is the portion of the energy density due to the cosmological constant. According to the 2018 Planck Collaboration [21], we have $\Omega_\Lambda = 0.6889 \pm 0.0056$. Next we will use equation (7) to incorporate a Hubble constant value of $H_0 = 73.04 \pm 1.04\ km/s/Mpc$, as given by Riess et. al [17], into our Thermodynamic Friedmann formula for density based on flat space (i.e., where $k = 0$):

$$\rho = \frac{3T_0^4\mho^2 - \Lambda c^2}{8\pi G} = 1.49 \times 10^{-27} \pm 0.25 \times 10^{-27}\ kg \cdot m^{-3}. \tag{14}$$

This density value is only about 16.6% of the WMAP critical density value of $9.9 \times 10^{-27}\ kg \cdot m^{-3}$. To get to the WMAP critical density when including the cosmologcal constant calculated from the Riess H_0, one must raise the current CMB temperature to a minimum of $T_0 = 3.23K$, which is far outside any confidence interval reported for the measured CMB temperature. Recently, Haug and Tatum [22] have demonstrated how the Λ-CDM model formula for high cosmological redshifts likely leads to overestimates of H_0 from supernova studies, such as the well-known Riess et. al SH0ES study.

Alternatively, one could argue that it is not relevant to include Λ when calculating the density of the universe, if the calculation will be used to compare with observed matter density. In the Λ-CDM model, Λ is associated with dark energy, which appears to be not directly observable. Therefore, this remains an open question.

We can also re-write the Friedmann critical universe equation as:

$$\begin{aligned} T_0^4 &= \frac{\rho_c c^3}{23040\pi\sigma} \\ H_0^2 &= \frac{\rho_c c^3\mho^2}{23040\pi\sigma}. \end{aligned} \tag{15}$$

And, when including the cosmological constant term and the curvature parameter, we have:

$$\begin{aligned} H_0^2 &= \frac{\rho_c c^3\mho^2}{23040\pi\sigma} + \frac{\Lambda c^2}{3} - \frac{kc^2}{3} \\ H_0^2 &= \frac{\rho_c c^3\left(\frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}}\right)^2}{23040\pi\frac{2\pi^5 k_b^4}{15c^2h^3}} + \frac{\Lambda c^2}{3} - \frac{kc^2}{3} \\ H_0^2 &= \frac{8\pi G\rho_c}{3} + \frac{\Lambda}{3} - \frac{kc^2}{3}. \end{aligned} \tag{16}$$

Which merely demonstrates that we can also go from the thermodynamic form back to the normal form expressed through the Hubble constant.

Furthermore, the critical density in past cosmic epochs can also presumably be converted from the Friedmann equation and re-written in thermodynamic form as:

$$\rho_{c,t} = T_0^4(1+z)^4\sigma\frac{23040\pi}{c^3} = T_t^4\sigma\frac{23040\pi}{c^3}. \tag{17}$$

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4. The Cosmological Constant in Thermodynamic Form

One can even write the cosmological constant itself in thermodynamic form. Again, the cosmological constant in the Λ-CDM model is given as:

$$\Lambda = 3\left(\frac{H_0}{c}\right)^2 \Omega_\Lambda. \tag{18}$$

Since we have $H_0 = \mho T_0^2$ we can re-write it as:

$$\Lambda = 3\left(\frac{\mho T_0^2}{c}\right)^2 \Omega_\Lambda. \tag{19}$$

And, if we use the Dhal et. al [12] CMB temperature, $T_0 = 2.725007 \pm 0.000024$ K, we get:

$$\Lambda = 3\left(\frac{\mho T_0^2}{c}\right)^2 \Omega_\Lambda = 1.567677 \times 10^{-52} \pm 0.000905 \times 10^{-52} \times \Omega_\Lambda\ m^{-2}. \tag{20}$$

This form is much more precise than can be achieved by starting from H_0. We demonstrate this in more detail in Table 2, by investigating the estimates and uncertainties predicted in the cosmological constant using observations from four CMB studies and five H_0 studies.

Table 2. This table compares the cosmological constant estimates and uncertainties using the new thermodynamic version of the Friedmann equation with those derived using the traditional method that relies on H_0 estimates. It is clearly evident that combining all recent CMB studies with our new thermodynamic version of the Friedmann equation dramatically improves cosmological constant estimates and uncertainties.

CMB study :	CMB measurement T_0 :	Cosmological constant $\Lambda = 3\left(\frac{\mho T_0^2}{c}\right)^2 \Omega_\Lambda$:
2023: Dhal et. al [12] :	$2.725007 \pm 0.000024K$	$\frac{\Lambda}{\Omega_\Lambda} = 1.567677^{+0.000905}_{-0.000905} \times 10^{-52}\ m^{-2}$
2009: Fixsen et. al [14] :	$2.72548 \pm 0.00057K$	$\frac{\Lambda}{\Omega_\Lambda} = 1.56877^{+0.00135}_{-0.00135} \times 10^{-52}\ m^{-2}$
2011: Noterdaeme et. al [15] :	$2.725 \pm 0.002K$	$\frac{\Lambda}{\Omega_\Lambda} = 1.5677^{+0.0046}_{-0.0046} \times 10^{-52}\ m^{-2}$
2004: Fixsen et. al [13] :	$2.721 \pm 0.010K$	$\frac{\Lambda}{\Omega_\Lambda} = 1.558^{+0.023}_{-0.023} \times 10^{-52}\ m^{-2}$
H_0 Study :	H_0 estimate :	Cosmological constant $\Lambda = 3\left(\frac{H_0}{c}\right)^2 \Omega_\Lambda$: :
2023: Murakami et al. [16] :	$73.01 \pm 0.85\ km/s/Mpc$	$\frac{\Lambda}{\Omega_\Lambda} = 1.8687^{+0.0438}_{-0.0433} \times 10^{-52}\ m^{-2}$
2021: Riess et al. [17] :	$73.04 \pm 1.04\ km/s/Mpc$	$\frac{\Lambda}{\Omega_\Lambda} = 1.8703^{+0.0536}_{-0.0529} \times 10^{-52}\ m^{-2}$
2021: Planck Collaboration [18] :	$67.4 \pm 0.5\ km/s/Mpc$	$\frac{\Lambda}{\Omega_\Lambda} = 1.5926^{+0.0237}_{-0.0235} \times 10^{-52}\ m^{-2}$
2023: Sneppen et. al [19] :	$67.0 \pm 3.6\ km/s/Mpc$	$\frac{\Lambda}{\Omega_\Lambda} = 1.5926^{+0.9285}_{-0.8799} \times 10^{-52}\ m^{-2}$
2023: Balkenhol et. al [20] :	$68.3 \pm 1.5\ km/s/Mpc$	$\frac{\Lambda}{\Omega_\Lambda} = 1.6354^{+0.0726}_{-0.0710} \times 10^{-52}\ m^{-2}$

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5. The Extremal Universe and the Haug-Spavieri Cosmology

Haug [23] has recently derived a similar equation to that of the Friedmann equation from the extremal solution of the Reisner-Nordström [24], Kerr [25] and Kerr-Newman [26,27] metrics; it is given by:

$$H_0^2 = \frac{8\pi G\rho - \Lambda c^2}{3}. \tag{21}$$

The same universe equation is also derived from the recent exact solution to Einstein's field equation [28], as provided by Haug-Spavieri [29,30]. Firstly, this solution does not yield a curvature constant k, indicating a prediction of flat space. Secondly, the cosmological constant is not introduced manually into Einstein's 1916 field equation, as Einstein did in 1917 in his extended field equation [31]. Thirdly, the cosmological constant is negative rather than positive. For researchers primarily familiar with the Λ-CDM model, this may seem unfamiliar, but the possibility of a negative cosmological constant is actively discussed in the literature (see, for example, [32–38]). In this model, the cosmological constant is precisely given by $\Lambda = 3\left(\frac{H_0^2}{c^2}\right)$, simplifying the equation further to:

$$H_0^2 = \frac{4\pi G\rho}{3}. \tag{22}$$

Next, we can utilize the fact that $H_0 = \mho T_0^2$ and input this into the equation above. This gives us the thermodynamic version of the Haug-Spavieri universe:

$$T_0^4\mho^2 = \frac{4\pi G\rho}{3} \tag{23}$$

that can be rewritten as

$$T_0 = \left(\frac{4\pi G\rho}{3\mho^2}\right)^{\frac{1}{4}}. \tag{24}$$

Solved with respect to ρ this gives:

$$\rho = \frac{3T_0^4\mho^2}{4\pi G} = \frac{3T_0^4\left(\frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}}\right)^2}{4\pi G} = T_0^4\frac{768k_b^4\pi^3}{c^5\hbar^3} = T_0^4\sigma\frac{46080\pi}{c^3}. \tag{25}$$

where σ is the Stefan-Boltzman constant. The only uncertainty in the mass-energy density comes from the uncertainty in the measured CMB temperature, since k_b, $\hbar$, and c are all defined as exact constants by NIST CODATA 2018. If we again use the Dhal et al. [12] $T_0 = 2.725007 \pm 0.000024$ K, we get:

$$\rho = T_0^4\sigma\frac{46080}{c^3} = 1.679896^{+0.000592}_{-0.000592} \times 10^{-26}\ kg \cdot m^{-3}. \tag{26}$$

Exactly half of this density comes from relativistic gravitational energy effects not taken into account in other metrics, such as the Schwarzschild metric. This relativistic gravitational energy can, in some sense, be labeled as dark energy, since it likely cannot be detected by any other means than observed gravitational effects. To compare with observed energy levels, one should therefore likely compare with only half of this value, which is $8.399481^{+0.000296}_{-0.000296} \times 10^{-27}\ kg \cdot m^{-3}$; see formula (12). In both the extremal universe and the Haug-Spavieri universe, 50% of the relativistic gravitational energy, which we can refer to as dark energy, arises directly from the mathematical solution itself; in other words, there is no additional dark energy input after simply solving Einstein's [28] 1916 field equation: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$, according to the boundary conditions on which these metrics are based. On the other hand, the Friedmann solution, when containing a cosmological constant, relies on Einstein's [31] extended 1917 field equation: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$, where Einstein, thoughtfully so, inputted the cosmological constant by hand and called it his extended field equation. Tatum [39]

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has earlier provided a more heuristic perspective, discussing a universe composed of 50% matter mass-energy and 50% dark energy. However, now we know that this could be the case directly from Einstein's 1916 field equation itself. The Einstein 1916 field equation has no cosmological constant, but the cosmological constant of the form we will discuss below comes from deriving a universe equation similar to the Friedmann equation when starting out from the extremal solutions or the Haug-Spavieri solution; see again [23].

The above calculations appear to suggest that both the Haug-Spavieri universe and the extremal universe models also accurately predict the observed energy density of the universe. Additionally, they appear to suggest a potentially better explanation for the cosmological constant problem. For an in-depth discussion, refer to [23,30]. Obviously, further study is necessary.

Furthermore, the critical density in past cosmic epochs of the extremal and Haug-Spavieri universe models, re-written in thermodynamic form to include gravitational energy ("dark-energy"), must be:

$$\rho_{c,t} = T_0^4(1+z)^4\sigma\frac{46080\pi}{c^3} = T_t^4\sigma\frac{46080\pi}{c^3} \tag{27}$$

where, as usual, $T_t = T_0(1+z)$. Furthermore, when not including gravitational energy (dark-energy), it must be:

$$\rho_{c,t} = T_0^4(1+z)^4\sigma\frac{23040\pi}{c^3} = T_t^4\sigma\frac{23040\pi}{c^3} \tag{28}$$

which is the same as our earlier critical Friedmann equation re-written in thermodynamic form; see formula (17).

In the extremal universe and Haug-Spavieri cosmology models, the cosmological constant in thermodynamic form is given by:

$$\Lambda = 3\left(\frac{H_0}{c}\right)^2 = 3\left(\frac{\mho T_0^2}{c}\right)^2 = 1.567677\times 10^{-52} \pm 0.000905\times 10^{-52}\ m^{-2} \tag{29}$$

when using the Dhal et. al CMB temperature of $T_0 = 2.725007 \pm 0.000024$ K.

6. Conclusion

Making use of the new Haug-Tatum cosmology model, we have demonstrated that the Friedmann equation can be re-written in thermodynamic form in terms of the Cosmic Microwave Background (CMB) temperature, instead of the Hubble constant. In doing so, we have further derived useful thermodynamic critical density and cosmological constant formulas. Since the CMB temperature is measured much more precisely than the Hubble constant, our new formulation leads to much tighter constraints on the critical density and the cosmological constant. Furthermore, our formulation facilitates comparisons between different cosmological models. With reference to the extremal and Haug-Spavieri metric models, their re-formulation herein even suggests the possibility that what is being called dark energy could be a manifestation of unaccounted-for universal relativistic gravitational energy.

Finally, it is particularly important to realize that the basis for calculating the Friedmann critical density and the cosmological constant, using the thermodynamic equations shown herein, is our new and deeper theoretical understanding of the exact mathematical relationship between the Hubble constant and CMB temperature, which can be expressed in a very compact form using the Upsilon composite constant.

References

1. A. Friedmann. Über die krüng des raumes. *Zeitschrift für Physik*, 10:377, 1922. URL https://doi.org/10.1007/BF01332580.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 26 March 2024 doi:10.20944/preprints202403.1241.v2

2. E. T. Tatum, U. V. S. Seshavatharam, and S. Lakshminarayana. The basics of flat space cosmology. *International Journal of Astronomy and Astrophysics*, 5:116, 2015. URL http://dx.doi.org/10.4236/ijaa.2015.52015.
3. M. Planck. *Natuerliche Masseinheiten*. Der Königlich Preussischen Akademie Der Wissenschaften: Berlin, Germany, 1899. URL https://www.biodiversitylibrary.org/item/93034#page/7/mode/1up.
4. M. Planck. *Vorlesungen über die Theorie der Wärmestrahlung*. Leipzig: J.A. Barth, p. 163, see also the English translation "The Theory of Radiation" (1959) Dover, 1906.
5. E. G. Haug and S. Wojnow. How to predict the temperature of the CMB directly using the Hubble parameter and the Planck scale using the Stefan-Boltzman law. *Research Square, Pre-print, under consideration by journal*, 2023. URL https://doi.org/10.21203/rs.3.rs-3576675/v1.
6. E. G. Haug. CMB, Hawking, Planck, and Hubble scale relations consistent with recent quantization of general relativity theory. *International Journal of Theoretical Physics, Nature-Springer*, 63(57), 2024. URL https://doi.org/10.1007/s10773-024-05570-6.
7. E. T. Tatum, E. G. Haug, and S. Wojnow. High precision hubble constant determinations based upon a new theoretical relationship between cmb temperature and h_0. *Hal archive*, 2023. URL https://hal.science/hal-04268732.
8. E. T. Tatum. Upsilon constants and their usefulness in Planck scale quantum cosmology. *Journal of Modern Physics*, 15:167, 2024. URL https://doi.org/10.4236/jmp.2024.152007.
9. E. G. Haug and E. T. Tatum. Planck length from cosmological redshifts solves the Hubble tension. *ResearchGate.org*, 2024. URL 1https://doi.org/0.13140/RG.2.2.21825.98407.
10. F. Melia. The $r_h = ct$ universe without inflation. *Astronomy & Astrophysics*, 553, 2013. URL https://doi.org/10.1051/0004-6361/201220447.
11. M. V. John. $r_h = ct$ and the eternal coasting cosmological model. *Monthly Notices of the Royal Astronomical Society*, 484, 2019. URL https://doi.org/10.1093/mnrasl/sly243.
12. S. Dhal, S. Singh, K. Konar, and R. K. Paul. Calculation of cosmic microwave background radiation parameters using cobe/firas dataset. *Experimental Astronomy (2023)*, 612:86, 2023. URL https://doi.org/10.1007/s10686-023-09904-w.
13. D. J. Fixsen and et. al. The temperature of the cosmic microwave background at 10 ghz. *The Astrophysical Journal*, 612:86, 2004. URL https://doi.org/10.1086/421993.
14. D. J. Fixsen. The temperature of the cosmic microwave background. *The Astrophysical Journal*, 707:916, 2009. URL https://doi.org/10.1088/0004-637X/707/2/916.
15. P. Noterdaeme, P. Petitjean, R. Srianand, C . Ledoux, and S. López. The evolution of the cosmic microwave background temperature. *Astronomy and Astrophysics*, 526, 2011. URL https://doi.org/10.1051/0004-6361/201016140.
16. Yukei S. Murakami and A. et. al Y. S., Riess. Leveraging sn ia spectroscopic similarity to improve the measurement of h_0. *arXiv:2306.00070*.
17. A. G. Riess and et. al. A comprehensive measurement of the local value of the Hubble constant with 1 km s^{-1} Mpc^{-1} uncertainty from the Hubble space telescope and the sh0es team. *The Astrophysical Journal*, 934, 2021. URL https://doi.org/10.3847/2041-8213/ac5c5b.
18. N.; et al. Planck Collaboration; Aghanim. Planck 2018 results. vi. cosmological parameters. *Astronomy & Astrophysics*, 641, 2020. URL https://doi.org/10.1051/0004-6361/201833910e.
19. A. Sneppen, D. Watson, D. Poznanski, O. Just, A. Bauswein, and R. Wojtak. Measuring the Hubble constant with kilonovae using the expanding photosphere method. *Astronomy and Astrophysics*, 678, 2023. URL https://doi.org/10.1051/0004-6361/202346306.
20. L. Balkenhol and et. al. Measurement of the CMB temperature power spectrum and constraints on cosmology from the SPT-3g 2018 TT, TE, and EE dataset. *Physical Review D*, 108:023510, 2023. URL https://doi.org/10.1103/PhysRevD.108.023510.
21. N.; et al. Planck Collaboration; Aghanim. Planck 2018 results. vi. cosmological parameters. *Astronomy & Astrophysics*, 641, 2018. URL https://doi.org/10.1051/0004-6361/201833910e.
22. E. G. Haug and E. T. Tatum. Solving the Hubble tension by extracting current CMB temperature from the Union2 supernova database. *Hal archive*, 2024. URL https://hal.science/hal-04368837.
23. E. G. Haug. The extremal universe exact solution from Einstein's field equation gives the cosmological constant directly. *Journal of High Energy Physics, Gravitation and Cosmology*, 10:386, 2024. URL https://doi.org/10.4236/jhepgc.2024.101027.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 26 March 2024 doi:10.20944/preprints202403.1241.v2

24. H. Reissner. Über die eigengravitation des elektrischen feldes nach der einsteinschen theorie. *Annalen der Physics*, 355:106, 1916. URL https://doi.org/10.1002/andp.19163550905.
25. R. P. Kerr. Gravitational field of a spinning mass as an example of algebraically special metrics. *Physical Review Letters*, 11:237, 1963. URL https://doi.org/10.1103/PhysRevLett.11.237.
26. E. T. Newman and A. I. Janis. Note on the Kerr spinning-particle metric. *Journal of Mathematical Physics*, 6: 915, 1965. URL https://doi.org/10.1063/1.1704350.
27. W. Newman. *Newton the Alchemist: Science, Enigma, and the Quest for Nature's "Secret Fire"*. Destiny Books, Rochester, Vermont, 2018.
28. A. Einstein. Näherungsweise integration der feldgleichungen der gravitation. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin*, 1916.
29. E. G. Haug and G. Spavieri. Mass-charge metric in curved spacetime. *International Journal of Theoretical Physics*, 62:248, 2023. URL https://doi.org/10.1007/s10773-023-05503-9.
30. E. G. Haug and G. Spavieri. New exact solution to einsteins field equation gives a new cosmological model. *Researchgate.org Pre-print*, 2023. URL http://dx.doi.org/10.13140/RG.2.2.36524.44161.
31. A. Einstein. Cosmological considerations in the general theory of relativity. *Sitzungsber. Preuss. Akad. Wiss, Berlin (Math.Phys.)*, page 142, 1917.
32. G. A. Monerat, Filho F. L.G., and and Silva E.V. C.and Neves C. Oliveira-Neto, G. The Planck era with a negative cosmological constant and cosmic strings. *Physics letters A*, 4741, 2010. URL https://doi.org/10.1016/j.physleta.2010.09.067.
33. T. Prokopec. Negative energy cosmology and the cosmological constant. *arXiv:1105.0078*, 2011. URL https://doi.org/10.48550/arXiv.1105.0078.
34. Maeda. K. and N. Ohta. Cosmic acceleration with a negative cosmological constant in higher dimensions. *Journal of High Energy Physics*, 2014, 2014. URL https://doi.org/10.1007/JHEP06(2014)095.
35. L. Visinelli, S. Vagnozzi, , and U. Danielsson. Revisiting a negative cosmological constant from low-redshift data. *Symmetry*, 11, 2019. URL https://doi.org/10.3390/sym11081035.
36. R. Calderón, R. Gannouji, B. L'Huillier, and D. Polarski. Negative cosmological constant in the dark sector? *Physical Review D*, 103, 2021. URL https://link.aps.org/doi/10.1103/PhysRevD.103.023526.
37. Benizri. L. and J. Troost. More on pure gravity with a negative cosmological constant. *Journal of High Energy Physics*, 2023, 2023. URL https://doi.org/10.1007/JHEP09(2023)093.
38. A. A. Sen, S. A. Adil, and S. Sen. Do cosmological observations allow a negative Λ? *Monthly Notices of the Royal Astronomical Society*, 518, 2023. URL https://doi.org/10.1093/mnras/stac2796.
39. E. T. Tatum. A universe comprised of 50% matter mass-energy and 50% dark energy. *Journal of Modern Physics*, 10:1144, 2019. URL https://doi.org/10.4236/jmp.2019.109074.

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